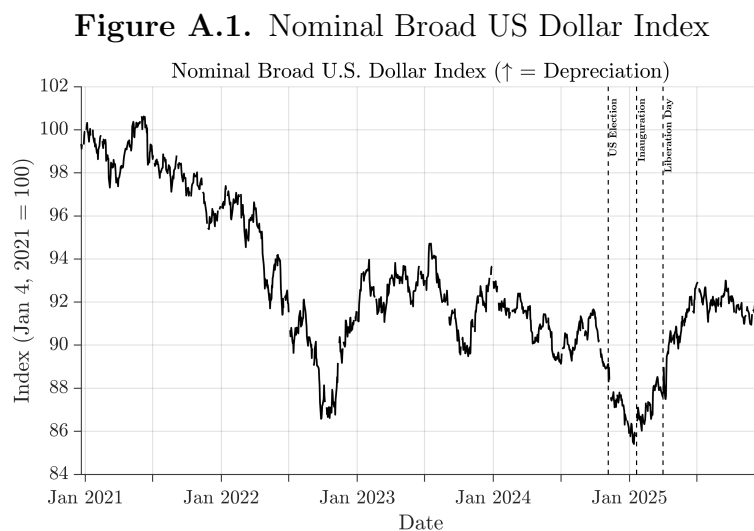


Supplemental Appendix

Global Trade, Tariff Uncertainty and the U.S. Dollar

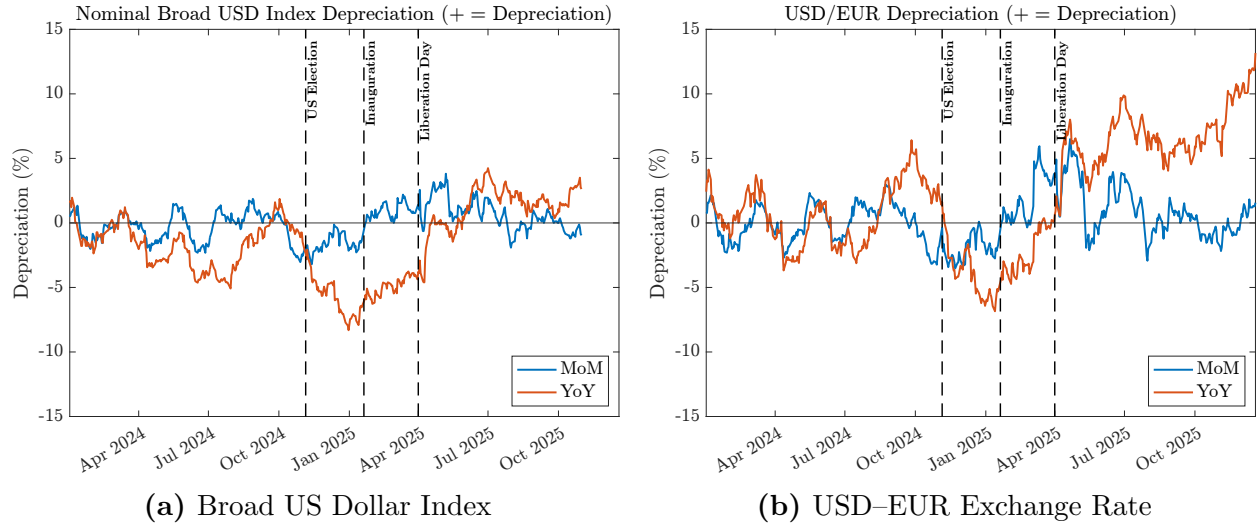
Şebnem Kalemli-Özcan, Can Soylu and Muhammed A. Yıldırım

A Additional Figures and Table



Note: The broad U.S. Dollar index since January 1, 2021. Source: FRED.

Figure A.2. US Dollar Depreciation Measures



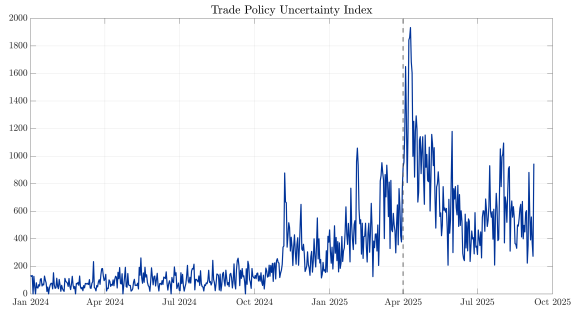
NOTE: Positive values indicate depreciation of the US dollar and negative values indicate appreciation. The left panel shows month-on-month and year-on-year changes in the broad US dollar index, highlighting the partial reversal of the strong 2024 appreciation in 2025. The right panel shows USD–EUR depreciation, where the depreciation pattern is more pronounced. SOURCE: FRED.

Table A.1. U.S. Dollar Depreciation/Appreciation

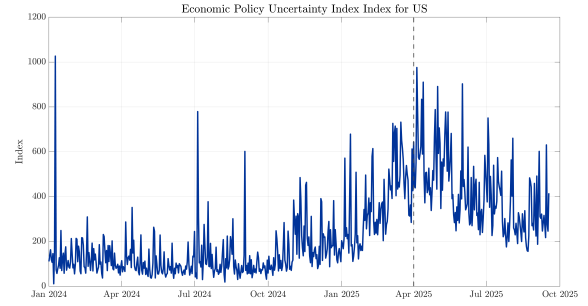
Date	Broad US Dollar Index		USD–EUR	
	MoM (30d, %)	YoY (365d, %)	MoM (30d, %)	YoY (365d, %)
2025-04-01	1.4	-3.8	3.8	0.6
2025-04-02	1.0	-3.9	3.5	0.9
2025-04-03	2.6	-2.9	4.9	2.1
2025-04-04	0.5	-3.9	2.3	1.4
2025-04-07	-0.6	-4.5	0.5	0.7
2025-04-08	-0.6	-4.6	0.5	0.5
2025-04-09	-0.3	-4.6	1.9	1.7
2025-04-10	0.8	-2.7	2.4	4.2

Note: Positive values indicate depreciation of the US dollar; negative values indicate appreciation. *Source:* FRED.

Figure A.3. Measures of Policy Uncertainty



(a) Trade Policy Uncertainty



(b) U.S. Economic Policy Uncertainty

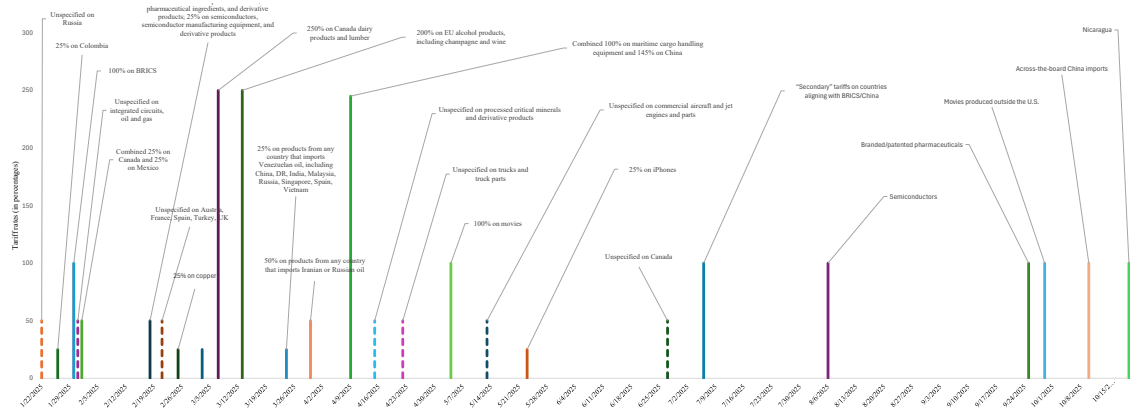
Note: Trade and economic policy uncertainty indices of Baker, Bloom and Davis (2016) and Caldara et al. (2020) from January 2024 to October 2025. Both series rise sharply around the Liberation Day tariff announcements and remain elevated afterward. Although some uncertainty dissipates, each measure stays above its level from the prior year.

Table A.2. U.S. Weighted Average Implemented Tariff Rate

Date	Tariff (%)	Date	Tariff (%)
1-Jan-25	2.5	23-Jun-25	15.6
4-Feb-25	3.9	30-Jun-25	15.6
4-Mar-25	11.7	1-Aug-25	15.6
7-Mar-25	6.3	6-Aug-25	15.8
12-Mar-25	7.4	7-Aug-25	17.0
3-Apr-25	8.5	27-Aug-25	17.4
5-Apr-25	12.2	1-Sep-25	17.4
9-Apr-25	21.4	8-Sep-25	17.3
3-May-25	23.0	16-Sep-25	17.2
14-May-25	13.8	1-Oct-25	17.2
4-Jun-25	15.5		

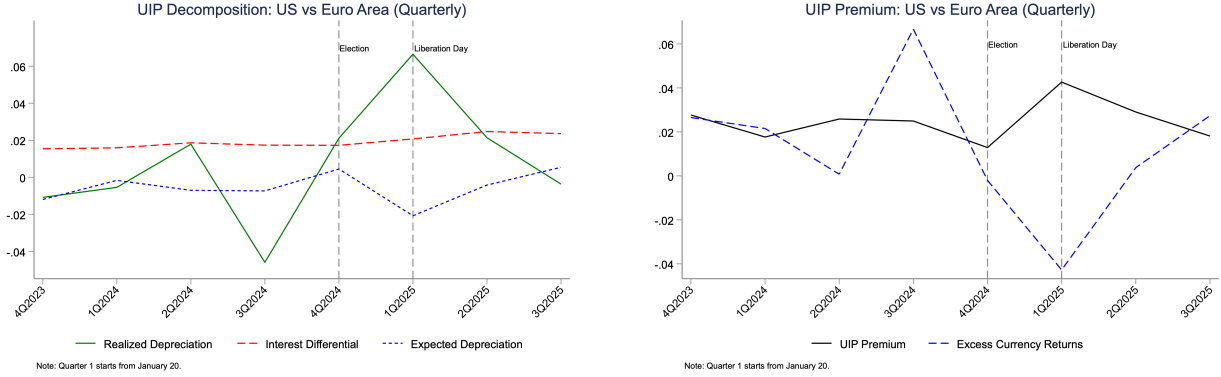
Note: Table A.2 reports the average U.S. tariff rates between January 1, 2025 and October 1, 2025. We downloaded the tariff data from *WTO and IMF (2025)* as of October 17, 2025, at the HS-6 level of classification. To compute the average U.S. tariff rate, we use 2024 import weights to aggregate tariff rates across products.

Figure A.4. Tariff Threats - not implemented and future implementation uncertain



Note: Figure A.4 visualizes tariff threats between January 20, 2025 and December 24, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (<https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates>). It presents a list from Reed Smith’s International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs.

Figure A.5. Decomposing the UIP Condition and Excess Currency Returns



(a) Interest Differential and Depreciation

(b) UIP and Excess Currency Returns

Note: Figure A.5a plots the components, whereas Figure A.5b plots the UIP premium calculated with Consensus survey data and realized excess currency returns. All calculations are for dollar vs euro. We use the convention of Kalemlı-Özcan (2019) and Kalemlı-Özcan and Varela (2021) while constructing UIP premiums, where the latter paper also links the UIP premium to policy uncertainty. To capture the timeline of tariff policy changes, in this figure and throughout the paper, we calculate quarterly figures with the first quarter of 2025 starting on January 20, 2025 with the inauguration and move each quarter by 20 days accordingly. With this definition of quarters, we are able to capture both Liberation Day announcements and the major tariff rate increases in February and March 2025 in the same quarter.

B Model Appendix

Our model is a stylized two-country version of KSY, whereby the supply side of the economy is simplified to an endowment economy under flexible prices. It corresponds to the version of the model in Section 4. To that baseline, we add risk-sensitive households (via CARA utility) and risk-sensitive financial intermediaries (via a mean-variance optimization problem). This document is intended to serve as a standalone derivation of the stylized model.

The model leads to higher precautionary savings and UIP premium. Both of these, in turn, lead to a decline in demand for the dollar, which is consistent with the behavior observed around the Liberation Day announcement, as documented by [Jiang et al. \(2025\)](#). An alternative mechanism is proposed by [Itskhoki and Mukhin \(2025\)](#), who argue that when the tariff-imposing country has a negative net foreign asset position (as in the case of the United States) and its liabilities are denominated in domestic currency while its assets are denominated in foreign currency, an improvement in the trade balance can require a depreciation of the domestic currency. Such a depreciation reduces the real value of liabilities while increasing the value of foreign assets.

B.1 Set-up

Timing and countries. Home (subscript H) and Foreign (subscript F) have representative households. Each country is endowed with its own tradable good, $y_{H,t}$ and $y_{F,t}$. One-period nominal discount bonds in each currency are traded internationally.

Preferences and intratemporal aggregator. Each household consumes a CES bundle of Home and Foreign Goods:

$$\max_{\{C_{H,t}, c_{H,H,t}, c_{H,F,t}\}} \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(C_{H,t}) \right], \quad C_{H,t} = \left[(1-\gamma_H) c_{H,H,t}^{\frac{\theta-1}{\theta}} + \gamma_H c_{H,F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 0, \theta \neq 1.$$

(Home bias parameter $1 - \gamma_H \in (0, 1)$; Foreign reliance on Home is $1 - \gamma_F$.)

Given home-currency good prices $p_{H,H,t} = p_{H,t}, p_{H,F,t}$, the unit-expenditure (CPI) index and Hicksian demands are

$$P_{H,t} = \left[(1 - \gamma_H) p_{H,t}^{1-\theta} + \gamma_H p_{H,F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad c_{H,H,t} = (1 - \gamma_H) \left(\frac{p_{H,t}}{P_{H,t}} \right)^{-\theta} C_{H,t}, \quad c_{H,F,t} = \gamma_H \left(\frac{p_{H,F,t}}{P_{H,t}} \right)^{-\theta} C_{H,t}.$$

Foreign utility maximization, CPI $P_{F,t}$ and demands $c_{F,H,t}, c_{F,F,t}$ are defined analogously with prices in Foreign currency $p_{F,H,t}, p_{F,t}$.

Law of One Price, tariffs and exchange rates. Let $\mathcal{E}_t$ be the nominal exchange rate (Home currency per unit of Foreign currency). For each good,

$$p_{F,H,t} = \frac{p_{H,t}}{\mathcal{E}_t} \quad p_{H,F,t} = \mathcal{E}_t p_{F,t} \tau_t$$

where τ_t is a gross import tariff rate imposed by the home country on the foreign country.

The gross tariff rate is a random variable, whereby $\tau_t = e^{\hat{\tau}_t}$ and $\hat{\tau}_t$ is independently and identically distributed following a normal distribution: $\hat{\tau}_t \sim \mathcal{N}(0, \sigma_{\hat{\tau}_t}^2)$. We allow for the variance of $\hat{\tau}_t$ to exogenously vary across time. Notably, we choose the following timing convention. The next period's tariff shock depends on a variance term that is known today: $\text{Var}_t(\hat{\tau}_{t+1}) = \sigma_{\hat{\tau}_t}^2$. We do this for notational ease in capturing uncertainty. In our stylized model, to capture uncertainty, we will consider one-time changes in $\sigma_{\hat{\tau}_t}^2$, which will impact next period's state variable, $\hat{\tau}_t$. That is today uncertainty increases about tomorrow's tariffs.

Home and foreign budget constraints with nominal bonds.

$$P_{H,t} C_{H,t} + V_{H,t-1} = p_{H,t} y_{H,t} + \frac{V_{H,t}}{R_{H,t}} + T_t$$

$$P_{F,t} C_{F,t} + V_{F,t-1} = p_{F,t} y_{F,t} + \frac{V_{F,t}}{R_{F,t}}$$

where $V_{H,t}$ and $V_{F,t}$ are nominal bonds denominated in local currency and T_t are lump-sum transfers to rebate households for tariff revenues.

Policy. Let us assume monetary policy in the two countries, perfectly stabilizes the aggregate price level such that $P_{H,t} = P_{F,t} = 1 \ \forall t$.

B.2 Households' Problem

The Home household maximizes expected discounted utility over an infinite horizon subject to a sequence of nominal budget constraints:

$$\begin{aligned} & \max_{\{C_{H,t}, V_{H,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_H^t u(C_{H,t}) \\ \text{s.t.} \quad & P_{H,t} C_{H,t} + V_{H,t-1} = p_{H,t} y_{H,t} + \frac{V_{H,t}}{R_{H,t}} + T_t \end{aligned}$$

for all $t \geq 0$, given initial debt $V_{H,-1}$.

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_H^t \left[u(C_{H,t}) + \lambda_{H,t} \left(p_{H,t} y_{H,t} - P_{H,t} C_{H,t} - V_{H,t-1} + \frac{V_{H,t}}{R_{H,t}} \right) \right].$$

FOCs with respect to $C_{H,t}$ and $V_{H,t}$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_{H,t}} : \quad & u'(C_{H,t}) - \lambda_{H,t} P_{H,t} = 0, \\ \frac{\partial \mathcal{L}}{\partial V_{H,t}} : \quad & \frac{\lambda_{H,t}}{R_{H,t}} - \beta_H \mathbb{E}_t[\lambda_{H,t+1}] = 0. \end{aligned}$$

Since policy perfectly stabilizes the aggregate price level we obtain the following Euler equation:

$$u'(C_{H,t}) = \beta_H R_{H,t} \mathbb{E}_t[u'(C_{H,t+1})].$$

Using the CARA form, $u'(c) = e^{-\alpha c}$,

$$e^{-\alpha C_{H,t}} = \beta_H R_{H,t} \mathbb{E}_t [e^{-\alpha C_{H,t+1}}] .$$

Since uncertainty is driven by tariff shocks, in line with our second-order approximation detailed below, $C_{H,t+1}$ is approximately conditionally normal,

$$C_{H,t+1} \mid \mathcal{F}_t \approx \mathcal{N}(\mathbb{E}_t C_{H,t+1}, \text{Var}_t(C_{H,t+1})) .$$

Then

$$\mathbb{E}_t [e^{-\alpha C_{H,t+1}}] = \exp\left(-\alpha \mathbb{E}_t C_{H,t+1} + \frac{1}{2} \alpha^2 \text{Var}_t(C_{H,t+1})\right) .$$

So the Euler equation is:

$$\begin{aligned} e^{-\alpha C_{H,t}} &= \beta_H R_{H,t} e^{\left(-\alpha \mathbb{E}_t C_{H,t+1} + \frac{1}{2} \alpha^2 \text{Var}_t(C_{H,t+1})\right)} \\ e^{\left(\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) - \frac{1}{2} \alpha^2 \text{Var}_t(C_{H,t+1})\right)} &= \beta_H R_{H,t} . \end{aligned}$$

Taking logs:

$$\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) = \ln(\beta_H R_{H,t}) + \frac{1}{2} \alpha^2 \text{Var}_t(C_{H,t+1}) .$$

We shall assume that the foreign household, has CRRA utility instead of CARA utility. The foreign household's problem then yields:

$$1 = \beta_F R_{F,t} \mathbb{E}_t \left[\left(\frac{C_{F,t+1}}{C_{F,t}} \right)^{-\alpha} \right]$$

B.3 Relative Demand Conditions

Standard CES structure yields the following expressions:

$$\frac{c_{H,H,t}}{c_{H,F,t}} = \frac{1 - \gamma_H}{\gamma_H} \left(\frac{p_{H,t}}{p_{H,F,t}} \right)^{-\theta}$$

$$\frac{c_{F,H,t}}{c_{F,F,t}} = \frac{\gamma_F}{1 - \gamma_F} \left(\frac{p_{F,H,t}}{p_{F,t}} \right)^{-\theta}$$

B.4 Intermediaries' Problem

Redefine bond variable inclusive of interest payments. Intermediaries solve a mean-variance optimization problem:

$$\max_{V_{H,t}, V_{F,t}} \quad \mathbb{E}_t \pi_{H,t+1} - \frac{\chi_t}{2} \text{Var}_t(\pi_{H,t+1})$$

subject to the resource constraint at t ,

$$0 = \mathcal{E}_t \frac{V_{F,t}}{R_{F,t}} + \frac{V_{H,t}}{R_{H,t}},$$

and with profits (in Home currency) realized at $t + 1$,

$$\pi_{H,t+1} = V_{H,t} + \mathcal{E}_{t+1} V_{F,t}.$$

Using the constraint to substitute out $V_{H,t} = -R_{H,t} \mathcal{E}_t \frac{V_{F,t}}{R_{F,t}}$, we obtain

$$\pi_{H,t+1} = V_{F,t} \left(\mathcal{E}_{t+1} - \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}} \right) \tag{B.1}$$

Therefore, the Lagrangian can be written as

$$\mathcal{L}_t = \mathbb{E}_t \left[V_{F,t} \left(\mathcal{E}_{t+1} - \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}} \right) \right] - \frac{\chi_t}{2} (V_{F,t})^2 \text{Var}_t(\mathcal{E}_{t+1}),$$

where $\chi_t > 0$ is an aversion to risk. First order condition (interior) yields:

$$0 = \mathbb{E}_t \left[\left(\mathcal{E}_{t+1} - \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}} \right) \right] - \chi_t (V_{F,t}) \text{Var}_t(\mathcal{E}_{t+1}), \quad (\text{B.2})$$

$$\iff \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}} = \mathbb{E}_t[\mathcal{E}_{t+1}] - \chi_t (V_{F,t}) \text{Var}_t(\mathcal{E}_{t+1}) \quad (\text{B.3})$$

Dividing both sides by $\mathcal{E}_t$:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} - \chi_t \frac{V_{F,t}}{\mathcal{E}_t} \text{Var}_t(\mathcal{E}_{t+1})$$

Rearranging:

$$\begin{aligned} \frac{R_{H,t}}{R_{F,t}} &= \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} - \chi_t \underbrace{\frac{V_{F,t} \mathcal{E}_t}{}}_{\text{Quantity of Risk}} \underbrace{\text{Var}_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right)}_{\text{Price of Risk}} \\ \frac{R_{H,t}}{R_{F,t}} &= \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \chi_t V_{H,t} \frac{R_{F,t}}{R_{H,t}} \text{Var}_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \end{aligned}$$

This yields a UIP condition with a deviation based on the quantity of risk (based on the open position of intermediaries) multiplied by the price of risk (variance of depreciation). In the usual interpretation, with $R_{H,t} > R_{F,t}$, $V_{F,t} < 0$ as intermediaries borrow in foreign currency (short foreign bonds) and lend in home currency (long home bonds), so as variance increases the UIP wedge increases.

Let us suppose risk aversion is such that $\chi_t = \chi \frac{R_{H,t}}{R_{F,t}} \frac{1}{V_{H,t}}$. For the stylized model, this allows us to simplify the last equation above into an expression with a convenient risk premium expression, ρ_t , which is not sensitive to the quantity of risk and instead is only

sensitive to the price of risk:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \underbrace{\chi \text{Var}_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right)}_{\rho_t}$$

We assume that the intermediation profits are transferred to the foreign household.¹⁴

B.5 Equilibrium Definition

An equilibrium comprises 8 sequences $\{p_{H,t}, p_{F,t}, \mathcal{E}_t, R_{H,t}, R_{F,t}, V_{H,t}, C_{H,t}, C_{F,t}\}_{t=0}^{\infty}$ such that for a given sequence of exogenous variables $\{\tau_t, y_{H,t}, y_{F,t}\}_{t=0}^{\infty}$, equations (B.4)-(B.11) hold:

- Euler equations:

$$\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) = \ln(\beta_H R_{H,t}) + \frac{1}{2} \alpha^2 \text{Var}_t(C_{H,t+1}) \quad (\text{B.4})$$

$$1 = \beta_F R_{F,t} \mathbb{E}_t \left[\left(\frac{C_{F,t+1}}{C_{F,t}} \right)^{-\alpha} \right] \quad (\text{B.5})$$

- CPI equations with price level substituted out:

$$1 = \left[(1 - \gamma_H) p_{H,t}^{1-\theta} + \gamma_H (\mathcal{E}_t p_{F,t} \tau_t)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (\text{B.6})$$

$$1 = \left[(1 - \gamma_F) p_{F,t}^{1-\theta} + \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (\text{B.7})$$

- UIP condition:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \chi \text{Var}_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \quad (\text{B.8})$$

¹⁴We make this simplification for analytical simplicity and quantitatively this does not make a significant difference as noted in [Kekre and Lenel \(2024\)](#).

- Goods market clears for each country in both periods

$$y_{H,t} = (1 - \gamma_H) \left(\frac{p_{H,t}}{1} \right)^{-\theta} C_{H,t} + \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t} \right)^{-\theta} C_{F,t} \quad (\text{B.9})$$

$$y_{F,t} = \gamma_H \left(\frac{\mathcal{E}_t p_{F,t} \tau_t}{1} \right)^{-\theta} C_{H,t} + (1 - \gamma_F) \left(\frac{p_{F,t}}{1} \right)^{-\theta} C_{F,t} \quad (\text{B.10})$$

- Balance of payments equation

$$\begin{aligned} \frac{V_{H,t}}{R_{H,t}} &= V_{H,t-1} - \text{NX}_t \\ &= V_{H,t-1} - p_{H,t} C_{F,H,t} + p_{F,t} \mathcal{E}_t C_{H,F,t} \\ &= V_{H,t-1} - p_{H,t} \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t} \right)^{-\theta} C_{F,t} + p_{F,t} \mathcal{E}_t \gamma_H \left(\frac{\mathcal{E}_t p_{F,t} \tau_t}{1} \right)^{-\theta} C_{H,t} \end{aligned} \quad (\text{B.11})$$

B.6 2nd Order Approximation

Let us assume that the endowment is fixed and let us use the [Cole and Obstfeld \(1991\)](#) parametrization $\alpha = \theta = 1$.¹⁵ When the two countries are different in size, so even if home bias parameters are the same trade can be unbalanced at the steady state. To achieve this, in the general derivation, we normalize home country's consumption $C_H = 1$ and let C_F be some constant (i.e., the rest of the world can be larger). Later we will assume symmetry for the stylized model.

We conduct a second-order approximation around a steady state with unconditional moments evaluated at the ergodic distribution. Hat variables denote deviation from this steady state:

¹⁵When $\theta = 1$, the consumption price basket becomes a Cobb-Douglas function that is exactly log-linear.

B.6.1 Steady State

- Euler equations:

$$\ln(\beta R_H) = -\frac{1}{2}\sigma_c^2 \rightarrow R_H = \beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2}$$
$$R_F = \beta_F^{-1}$$

We'll assume in our work that volatility is small enough that the steady-state gross interest rate is greater than 1: $R_H = \beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2} > 1$.

- All prices and exchange rate will be 1 at the steady state:

$$P_H = P_F = \mathcal{E} = 1$$

- UIP condition:

$$\frac{R_H}{R_F} = 1 + \chi\sigma_{\mathcal{E}}^2$$

Plugging in what we know from the Euler equation we can show how the endogenous volatility of the exchange rate is related to the endogenous volatility of consumption:

$$\frac{R_H}{R_F} = 1 + \chi\sigma_{\mathcal{E}}^2 = \frac{\beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2}}{\beta_F^{-1}}$$

Importantly, σ_c^2 and $\sigma_{\mathcal{E}}^2$ are a function of the volatility of tariffs, σ^2 , at the steady state. For the existence of a steady state, with a constant exchange rate, we assume that β_H and β_F are such that the equality above holds.

- Goods market clears for each country in both periods

$$y_H = 1 + \underbrace{\gamma_H(C_F - 1)}_{\bar{N}X} = 1 + \bar{N}X$$

$$y_F = 1 - \gamma_H(C_F - 1) = 1 - \bar{N}X$$

- Balance of payments equation

$$\frac{V_H}{R_H} = V_H - \gamma_H C_{F,t} + \gamma_H$$

$$= V_H - \underbrace{\gamma_H(C_{F,t} - 1)}_{\bar{N}X}$$

$$\frac{V_H}{R_H} = V_H - \bar{N}X$$

$$V_H = \frac{R_H}{R_H - 1} \bar{N}X$$

These expressions demonstrate that setting the relative size of the countries with C_F is equivalent to setting net exports, which helps determine size of steady-state endowment and debt. If two countries both allocate 10% of their consumption to foreign goods, but one of them is twice the size of the other one, the larger country will be a net importer.

B.6.2 Approximation

Given the parametrization, with $\alpha = \theta = 1$, the model is already highly linear except for the variance terms, so we shall focus on those.¹⁶

There are two key variances in the model $\text{Var}_t(C_{t+1})$ and $\text{Var}_t\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)$. Any variable can be expressed as the steady-state value multiplied by percent deviation: $X_t = X(1 + \hat{X}_t)$.

¹⁶We assume near equivalence of log deviation and percent deviation from the steady state. Additionally with consumption and prices normalized to 1, many terms can be converted into hat variables by taking logs.

Then we can write:

$$\begin{aligned}\text{Var}(C_{t+1}) &= \text{Var}(C(1 + \hat{C}_{t+1})) \\ &= \text{Var}(\hat{C}_{t+1})\end{aligned}$$

Similarly, since $\log\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right) \approx \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t$ and $e^x \approx 1 + x$, so we have:

$$\begin{aligned}\text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) &= \text{Var}_t\left(\frac{1 + \hat{\mathcal{E}}_{t+1}}{1 + \hat{\mathcal{E}}_t}\right) \\ &= \text{Var}_t\left(e^{\log\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right)}\right) \\ &\approx \text{Var}_t\left(1 + \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t\right) \\ &= \text{Var}_t\left(\hat{\mathcal{E}}_{t+1}\right)\end{aligned}$$

With these the Euler equation and the UIP condition read as follows:

$$\begin{aligned}(\mathbb{E}_t \hat{C}_{H,t+1} - \hat{C}_{H,t}) &= \hat{i}_{H,t} + \frac{1}{2}\text{Var}(\hat{C}_{t+1}) \\ \hat{i}_{H,t} - \hat{i}_{F,t} &= \mathbb{E}_t[\hat{\mathcal{E}}_{t+1}] - \hat{\mathcal{E}}_t + \chi \text{Var}_t\left(\hat{\mathcal{E}}_{t+1}\right)\end{aligned}$$

We have a system with three state variables: $\hat{\tau}_t, \hat{V}_{t-1}, \sigma_t^2$. With a second order approximation, every endogenous variable, $\hat{x}_t$ (e.g., including $\hat{\mathcal{E}}_{t+1}$ and $\hat{C}_{t+1}$) will be some linear and quadratic function of the state variables:

$$\begin{aligned}\hat{x}_t &= a_1 \hat{\tau}_t + a_2 \hat{V}_{H,t-1} + a_3 \sigma_t^2 + a_4 \hat{\tau}_t^2 + a_5 \hat{V}_{H,t-1}^2 + a_6 (\sigma_t^2)^2 \\ &\quad + a_7 \hat{\tau}_t \hat{V}_{H,t-1} + a_8 \hat{\tau}_t \sigma_t^2 + a_9 \hat{V}_{H,t-1} \sigma_t^2.\end{aligned}$$

Iterating one period forward:

$$\begin{aligned}\hat{x}_{t+1} = & a_1\hat{\tau}_{t+1} + a_2\hat{V}_{H,t} + a_3\sigma_{t+1}^2 + a_4\hat{\tau}_{t+1}^2 + a_5\hat{V}_{H,t}^2 + a_6(\sigma_{t+1}^2)^2 \\ & + a_7\hat{\tau}_{t+1}\hat{V}_{H,t} + a_8\hat{\tau}_{t+1}\sigma_{t+1}^2 + a_9\hat{V}_{H,t}\sigma_{t+1}^2.\end{aligned}$$

Let us assume that we are interested in one-time increases in uncertainty, so $\sigma_{t+j} = 0 \forall j > 0$ and this is known by agents. If the variance of $\hat{\tau}_{t+1}$ is σ_t^2 , then

$$\begin{aligned}\text{Var}_t(\hat{x}_{t+1}) &= \text{Var}_t\left(a_1\hat{\tau}_{t+1} + a_4\hat{\tau}_{t+1}^2 + a_7\hat{\tau}_{t+1}\hat{V}_{H,t} + a_8\hat{\tau}_{t+1}\sigma_{t+1}^2\right) \\ &= \text{Var}_t\left((a_1 + a_7\hat{V}_{H,t})\hat{\tau}_{t+1} + a_4\hat{\tau}_{t+1}^2\right) \\ &= (a_1 + a_7\hat{V}_{H,t})^2\sigma_t^2 + 2a_4^2\sigma_t^4\end{aligned}$$

We shall assume $\sigma_t < 1$. Given this, for small shocks, higher order terms will be near zero (i.e. $(a_7\hat{V}_{H,t})^2\sigma_t^2 + 2a_4^2\sigma_t^4 \approx 0$). That is we choose to focus on a variance that is only a function of the exogenous variance of $\hat{\tau}_t$, and simplify away the endogenous time-varying component.¹⁷ With that, the variance term will be directly a function of the dependence of the endogenous variable on $\hat{\tau}_t$: that is we have $\text{Var}_t(\hat{x}_{t+1}) \approx a_1^2\sigma_t^2$. With that we can define the approximated equilibrium as follows:

An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^\infty$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2\}_{t=0}^\infty$, the following equations hold:

- Euler equations:

$$(\mathbb{E}_t\hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \eta\sigma_t^2 \quad (\text{B.12})$$

$$(\mathbb{E}_t\hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t} \quad (\text{B.13})$$

¹⁷That is via the $a_7\hat{V}_{H,t}$ term, there could otherwise be endogenous time variation in how the variance of $\hat{C}_{t+1}$ and $\hat{\mathcal{E}}_{t+1}$ depends on the underlying variance of tariffs. We assume that in our model and context of shocks, this is small enough to simplify away.

- CPI equations with price level substituted out:

$$0 = (1 - \gamma_H) \hat{p}_{H,t} + \gamma_H (\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t) \quad (\text{B.14})$$

$$0 = (1 - \gamma_F) \hat{p}_{F,t} + \gamma_F (\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \quad (\text{B.15})$$

- UIP condition with a wedge that endogenously widens as outstanding debt increases and as volatility increases:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t + \kappa \sigma_t^2 \quad (\text{B.16})$$

- Goods market clears for each country in both periods

$$0 = (1 - \gamma_H) (\hat{C}_{H,t} - \hat{p}_{H,t}) + \gamma_H C_F (\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}) \quad (\text{B.17})$$

$$0 = \gamma_F (\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t) + (1 - \gamma_F) C_F (\hat{C}_{F,t} - \hat{p}_{F,t}) \quad (\text{B.18})$$

- Balance of payments equation with $\hat{V}_{H,t}$ as net debt of the home country:¹⁸

$$\hat{V}_{H,t} = R_H \hat{V}_{H,t-1} - \Xi \left[C_F (\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t) \right] + \hat{i}_{H,t} \quad (\text{B.23})$$

For the analytical simplicity of the stylized model, let us additionally assume symmetry (i.e.

¹⁸The derivation is as follows:

$$\frac{V_{H,t}}{R_{H,t}} = V_{t-1} - p_{H,t} \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t} \right)^{-\theta} C_{F,t} + p_{F,t} \mathcal{E}_t \gamma_H \left(\frac{\mathcal{E}_t p_{F,t} \tau_t}{1} \right)^{-\theta} C_{H,t} \quad (\text{B.19})$$

$$\frac{V_H}{R_H} (\hat{V}_{H,t} - \hat{i}_{H,t}) = V_H \hat{V}_{H,t-1} - \left(\gamma_H C_F (\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - \gamma_H (\hat{C}_{H,t} - \hat{\tau}_t) \right) \quad (\text{B.20})$$

$$(\hat{V}_{H,t} - \hat{i}_{H,t}) = R_H \hat{V}_{H,t-1} - \frac{R_H}{V_H} \left(\gamma_H C_F (\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - \gamma_H (\hat{C}_{H,t} - \hat{\tau}_t) \right) \quad (\text{B.21})$$

$$\hat{V}_{H,t} = R_H \hat{V}_{H,t-1} - \underbrace{\frac{R_H - 1}{\bar{N}X}}_{\Xi} \left(C_F (\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t) \right) + \hat{i}_{H,t} \quad (\text{B.22})$$

where the last two lines follow from $V_H = \frac{R_H}{R_H - 1} \bar{N}X$.

setting $\gamma_H = \gamma_F = \gamma$) and consequently an initial position of zero debt. Then equilibrium conditions read as follows:

An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2\}_{t=0}^{\infty}$, the following equations hold:

- Euler equations:

$$(\mathbb{E}_t \hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \eta \sigma_t^2 \quad (\text{B.24})$$

$$(\mathbb{E}_t \hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t} \quad (\text{B.25})$$

- CPI equations with price level substituted out:

$$0 = (1 - \gamma) \hat{p}_{H,t} + \gamma (\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t) \quad (\text{B.26})$$

$$0 = (1 - \gamma) \hat{p}_{F,t} + \gamma (\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \quad (\text{B.27})$$

- UIP condition with a wedge that endogenously widens as volatility increases:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t + \kappa \sigma_t^2 \quad (\text{B.28})$$

- Goods market clears for each country in both periods

$$0 = (1 - \gamma) (\hat{C}_{H,t} - \hat{p}_{H,t}) + \gamma (\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}) \quad (\text{B.29})$$

$$0 = \gamma (\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t) + (1 - \gamma) (\hat{C}_{F,t} - \hat{p}_{F,t}) \quad (\text{B.30})$$

- Balance of payments equation with $\hat{V}_{H,t}$ as net debt of the home country:

$$R_H^{-1} \hat{V}_{H,t} = \hat{V}_{H,t-1} - \gamma \left[(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t) \right] \quad (\text{B.31})$$

C Data and Code Availability

All data and code are available upon request.

References

- Akıncı, Özge, Şebnem Kalemli-Özcan, and Albert Queralto. 2022. “Uncertainty shocks, capital flows, and international risk spillovers.” NBER Working Paper 30026.
- Baker, Scott R, Nicholas Bloom, and Steven J Davis. 2016. “Measuring economic policy uncertainty.” *Quarterly Journal of Economics*, 131(4): 1593–1636.
- Caldara, Dario, Matteo Iacoviello, Patrick Molligo, Andrea Prestipino, and Andrea Raffer. 2020. “The economic effects of trade policy uncertainty.” *Journal of Monetary Economics*, 109: 38–59.
- Cole, Harold L, and Maurice Obstfeld. 1991. “Commodity trade and international risk sharing: How much do financial markets matter?” *Journal of Monetary Economics*, 28(1): 3–24.
- Itskhoki, Oleg, and Dmitry Mukhin. 2025. “The optimal macro tariff.” NBER Working Paper 33839.
- Jiang, Zhengyang, Arvind Krishnamurthy, Hanno N Lustig, Robert Richmond, and Chenzi Xu. 2025. “Dollar upheaval: This time is different.” Stanford Institute for Economic Policy Research (SIEPR) Working Paper 25-18.
- Kalemli-Özcan, Şebnem. 2019. “US monetary policy and international risk spillovers.” 95–191, Federal Reserve Bank of Kansas City, Jackson Hole Economic Policy Symposium Proceedings.

- Kalemli-Özcan, Şebnem, and Liliana Varela.** 2021. “Five Facts about the UIP Premium.” NBER Working Paper 28923.
- Kalemli-Özcan, Şebnem, Can Soylu, and Muhammed A Yildirim.** 2025. “Global Networks, Monetary Policy and Trade.” NBER Working Paper 33686.
- Kekre, Rohan, and Moritz Lenel.** 2024. “Exchange rates, natural rates, and the price of risk.” University of Chicago, Becker Friedman Institute for Economics Working Paper 2024-114.
- Rogoff, Kenneth.** 2025. *Our dollar, your problem: an insider’s view of seven turbulent decades of global finance, and the road ahead.* Yale University Press.
- WTO and IMF.** 2025. “WTO – IMF Tariff Tracker.” <https://ttd.wto.org/en/analysis/tariff-actions>, last accessed on October 17, 2025.