

Supplemental Appendix

Quantifying the Role of Firms in Intergenerational Mobility

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A. Appendix Figures and Tables

Table A.1—Types of Higher Education Institution

	% Pop.	% Grads	Father Log Inc	Log Inc	% Stable Job
Type of Higher Ed					
University	8	13	10.72	11.58	100
College	10	16	10.64	11.58	100
Teaching College	7	12	10.68	11.56	100
Engineering School	9	15	10.73	11.58	100
Practical Training	7	12	10.70	11.60	100
Diploma	14	22	10.90	11.63	100
Other	6	10	10.48	11.43	100
None	38		10.49	11.46	100

Notes: This table shows descriptive statistics of each higher education institution type. The first column shows the share of our sample with a degree from each type of institution. The second column shows the same shares, but only among the ones with a degree. The third column shows the average log earnings of the graduates' fathers between 1986 and 1991. The fourth column shows the average log earnings of the graduates themselves between 2010 and 2015. The fifth column shows the share of the graduates that held a stable job—as defined in Section [B](#)—at least once between 2010 and 2015.

Table A.2— Individual and firm components of the IGE: Robustness

Panel A: Expanded birth cohorts			
$\overline{\log Y_{f(i)}}$	0.227 (0.001)	0.177 (0.001)	0.050 (0.000)
Share of IGE	1.00 .	0.78 (0.007)	0.22 (0.002)
Observations	1,351,577	1,351,577	1,351,577
Panel B: Temporary jobs			
$\overline{\log Y_{f(i)}}$	0.260 (0.002)	0.191 (0.002)	0.069 (0.001)
Share of IGE	1.00 .	0.73 (0.009)	0.27 (0.003)
Observations	737,038	737,038	737,038
Panel C: Non-employed fathers			
$\overline{\log Y_{f(i)}}$	0.190 (0.001)	0.140 (0.001)	0.050 (0.000)
Share of IGE	1.00 .	0.74 (0.006)	0.26 (0.002)
Observations	1,045,819	1,045,819	1,045,819

Notes: This table reports robustness checks for the decomposition of the intergenerational earnings elasticity (IGE) into individual and firm components, as described in Equation (6). Baseline estimates are presented in Table 3. Panel A expands the analysis to include birth cohorts from 1955 to 1990. Panel B includes all individuals who were ever employed at a firm in the AKM sample, even if only for a single month. Panel C includes individuals whose fathers reported zero earnings during the observed period. See Section D for further details on each specification.

B. Why Father Earnings?

In this project, we use parental earnings as a proxy for children’s socioeconomic background (SES). In the setting we study, fathers’ earnings is a better proxy than mothers’ or household earnings. Female labor force participation in the 1980s in Israel—when we measure parental earnings—was below 50%. In this context, having a household with two earners is often a sign of low SES. Indeed, Appendix Table B.1 shows that fathers’ earnings are more correlated with children’s earnings than mothers’ or household earnings.

Note that using fathers’ earnings as a proxy for SES is a common practice in the literature. For a review, see Black and Devereux (2011).

Table B.1—Parental earnings rank vs child earnings rank

	Family earnings Measure		
	Household	Father	Mother
Coefficient	.23 (.003)	.246 (.003)	.093 (.003)
Obs	156555	156555	156555
R^2	.049	.055	.008

Notes: This table presents the rank correlation between children’s earnings rank and their household, fathers’ and mothers’ earning ranks. Both parents’ and childrens’ earnings are the residuals from a regression of age, age-squared and year fixed effects on log earnings.

C. Validating the AKM decomposition

A. Specification test

In this appendix, we test the restrictions imposed by the AKM framework. In particular, the restriction that the log-linear structure of earnings and that the job moving probability is uncorrelated with the error term. We test this restrictions with the approach proposed by Sorkin (2018).

From Equation (2), we have:

$$\begin{aligned}\log Y_{i,t} &= \alpha_i + \psi_{J[i,t]} + x'_{i,t} \beta^x + r_{i,t} , \\ \log Y_{i,t+1} &= \alpha_i + \psi_{J[i,t+1]} + x'_{i,t+1} \beta^x + r_{i,t+1} , \end{aligned}$$

Taking first differences:

$$\Delta \log Y_{i,t} - \Delta x'_{i,t} \beta^x = \Delta \psi_{J[i,t]} + \Delta r_{i,t}$$

We now take expectations, conditional on moving:

$$\mathbb{E}[\Delta \log Y_{i,t} - \Delta x'_{i,t} \beta^x | M_{i,t} = 1] = \Delta \mathbb{E}[\psi_{J[i,t]} | M_{i,t} = 1] + \mathbb{E}[\Delta r_{i,t} | M_{i,t} = 1]$$

where $M_{i,t}$ indicates whether worker i changed firms in year t :

$$M_{i,t} \equiv \mathbb{1}\{J(i,t) \neq J(i,t+1) \text{ \& } J(i,t) \neq Non\ Emp \text{ \& } J(i,t+1) \neq Non\ Emp\}.$$

The key assumption to estimate Equation (2) by OLS is that the probability of moving is uncorrelated with the error term, that is $\mathbb{E}[\Delta r_{i,t} | M_{i,t} = 1] = 0$. Under this assumption:

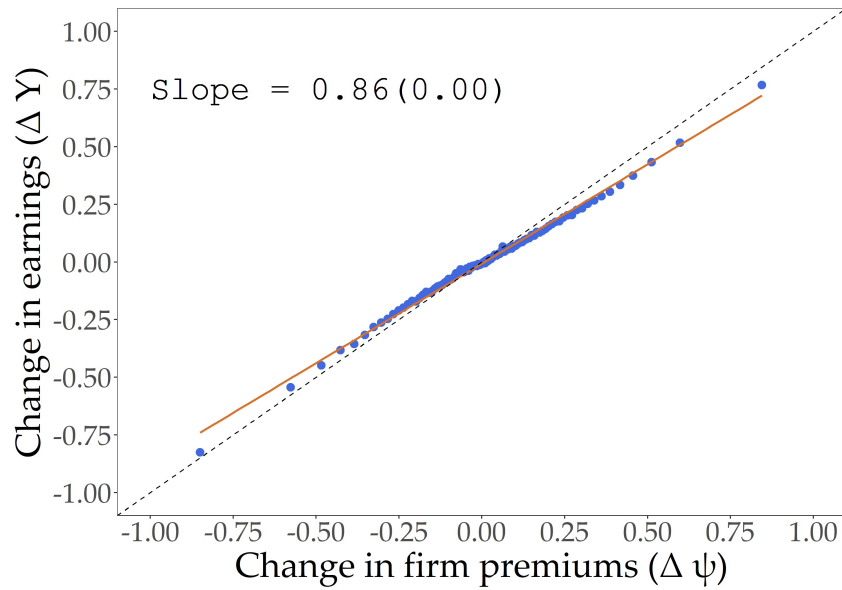
$$\mathbb{E}[\Delta \log Y_{i,t} - \Delta x'_{i,t} \beta^x | M_{i,t} = 1] = \Delta \mathbb{E}[\psi_{J[i,t]} | M_{i,t} = 1]$$

We take this restriction to the data by focusing on job switchers and comparing their residualized earnings change against their firm-effect change. The results are in Figure C.1. The solid blue line plots the best-fitting line. The dashed line plots the 45 degree line. We find that earnings changes closely follow changes in firm premiums, showing that the AKM framework fits the data well.

B. Firm premium estimates by socioeconomic background

In our main analysis, we use firm premiums estimated using all workers, not only the ones in IGM sample. A potential concern is that firm premiums estimated with the full sample are not representative for the IGM sample. In this Appendix, we show the correlation between firm

Figure C.1—Earnings Change Corresponds to Firm Fixed Effect Change



Notes: This figure shows how the magnitude of earnings changes relate to the change in firm-level pay for employer-to-employer transitions who switch annual stable jobs. The earnings are the residualized annualized earnings in the last year at the previous job and in the first year at the new job. We bin the job changers into equally sized bins on the basis of the change in the firm effects. The circles plot the bin means. The solid line plots the best-fitting line estimated based on the micro-data. The dashed line plots the 45 degree line.

premiums estimated in different sub-samples. The results are in Table C.1.

We see that the correlation between premiums estimated with the full sample and the IGM sample is 0.88. This is very similar to the correlation between premiums estimated with the full sample and with a sample with the same number of observations as the IGM sample (0.90). This indicates that the underlying premiums are the same in the full and the IGM sample, and the observed differences are due to measurement error.

A related concern is that, within the IGM sample, premiums are different for high- and low-SES workers. Table C.1 reports the correlations between premiums estimated with each of these samples and the ones estimated with the full sample. As a comparison, we also show results for premiums estimated with a 50% random sample of the IGM sample. We see that these three correlations are very similar to each other. Once again, this indicates that the underlying premiums faced by this groups are the same, and the observed differences are due to measurement error.

Table C.1—Correlation between firm premiums in different samples

	Full	IGM	Random (Full)	Random (IGM)	Low-SES	High-SES
Full	1.00					
IGM	0.88	1.00				
Random (Full)	0.90	0.77	1.00			
Random (IGM)	0.83	0.92	0.75	1.00		
Low-SES	0.78	0.90	0.72	0.81	1.00	
High-SES	0.84	0.95	0.77	0.88	0.71	1.00

Notes: This table shows the correlations between firm premiums (ψ) estimated in different samples. Firm premiums are defined in Equation (2). “Full” includes all workers with a stable job in the Israeli labor market in 2010-2015. “IGM” only includes the ones that have fathers with positive earnings. “Random (Full)” is a random sub-sample of the full sample with the same size as the IGM sample. “Random (IGM)” is a 50% random sample of the IGM sample. “High-SES” and “Low-SES” are, respectively, workers above and below the median father earnings in the IGM sample.

D. Proof of Equation (6):

From the definition of $\overline{\log Y_i}$, we have:

$$\overline{\log Y_i} \equiv \frac{1}{N_i} \sum_{t \in \mathcal{T}_i} \log \tilde{Y}_{it} = \frac{1}{N_i} \sum_{t \in \mathcal{T}_i} \left\{ \hat{\alpha}_i + \psi_{J(i,t)} + r_{i,t} \right\} = \hat{\alpha}_i + \hat{\psi}_i. \quad (\text{D.1})$$

Above, we used that $\sum_{t \in \mathcal{T}_i} r_{i,t} = 0$ because $r_{i,t}$ is the residual of an OLS regression with individual fixed effects. Moreover, by definition, $\beta^{IGE} \equiv \frac{\text{Cov}\left(\overline{\log Y_i}, \overline{\log Y_{f(i)}}\right)}{\text{Var}\left(\overline{\log Y_{f(i)}}\right)}$. Replacing Equation (D.1) into this definition:

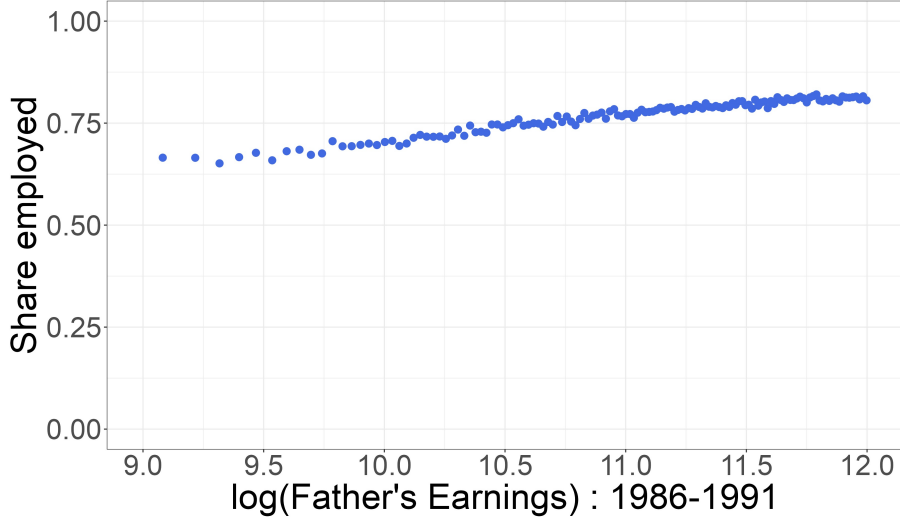
$$\beta^{IGE} = \frac{\text{Cov}\left(\hat{\alpha}_i + \hat{\psi}_i, \overline{\log Y_{f(i)}}\right)}{\text{Var}\left(\overline{\log Y_{f(i)}}\right)} = \frac{\text{Cov}\left(\hat{\alpha}_i, \overline{\log Y_{f(i)}}\right)}{\text{Var}\left(\overline{\log Y_{f(i)}}\right)} + \frac{\text{Cov}\left(\hat{\psi}_i, \overline{\log Y_{f(i)}}\right)}{\text{Var}\left(\overline{\log Y_{f(i)}}\right)} = \beta^{\alpha|Y_f} + \beta^{\psi|Y_f}$$

□

E. Non-employment

A common limitation in research estimating intergenerational elasticities—including the IGE—is the exclusion of non-employed individuals, since $\log(0)$ is undefined. This exclusion is potentially problematic because non-employment is not randomly distributed: as shown in Figure E.1, individuals from lower-SES backgrounds are more likely to be non-employed.

Figure E.1—Employment Rates by Father’s earnings



Notes: The figure plots the share of employed children by father’s log earnings. “Employed” is defined as holding a stable job for at least one year during the sample period.

To address this issue, we extend the IGE framework to incorporate non-employment. Let $\bar{Y}_i = \mathbb{E}[Y_{it} \mid i]$ denote the average earnings of individual i over the observation window, and $\bar{Y}_{f(i)}$ the analogous measure for their father. Rather than modeling $\mathbb{E}[\log \bar{Y}_i \mid \bar{Y}_{f(i)}]$, which is undefined when $\bar{Y}_i = 0$, we consider the log of expected earnings: $\log \mathbb{E}[\bar{Y}_i \mid \bar{Y}_{f(i)}]$. Since the expectation of earnings is strictly positive once we average over a given father earnings level, this formulation includes all individuals, regardless of employment status.

We define the extended IGE as:

$$\log \mathbb{E}[\bar{Y}_i \mid \bar{Y}_{f(i)}] = \beta^{\widehat{IGE}} \cdot \log \bar{Y}_{f(i)} + \text{residual}.$$

A. Decomposing Expected Earnings

We can decompose expected earnings into employment probability times expected earnings conditional on employment:

$$\mathbb{E}[\bar{Y}_i \mid \bar{Y}_{f(i)}] = P(\text{employed}_i \mid \bar{Y}_{f(i)}) \cdot \mathbb{E}[\bar{Y}_i \mid \bar{Y}_{f(i)}, \text{employed}_i = 1],$$

where “employed” indicates whether the individual holds a stable job for at least one year within our sample period.

For employed individuals, residualized earnings follow the AKM decomposition:²⁰

$$\log Y_{it} = \alpha_i + \psi_{J(i,t)} + r_{it}.$$

Therefore, using the law of iterated expectations, we write:

$$\begin{aligned} \mathbb{E}[\bar{Y}_i | \bar{Y}_{f(i)}] &= P(\text{employed}_i | \bar{Y}_{f(i)}) \cdot \mathbb{E} \left[\mathbb{E}[Y_{it} | i] | \bar{Y}_{f(i)}, \text{employed}_i = 1 \right] \\ &= P(\text{employed}_i | \bar{Y}_{f(i)}) \cdot \mathbb{E} \left[\mathbb{E}[e^{\alpha_i + \psi_{J(i,t)} + r_{it}} | i] | \bar{Y}_{f(i)}, \text{employed}_i = 1 \right] \\ &= P(\text{employed}_i | \bar{Y}_{f(i)}) \cdot \mathbb{E}[e^{\alpha_i + \psi_{J(i,t)} + r_{it}} | \bar{Y}_{f(i)}, \text{employed}_i = 1]. \end{aligned}$$

Taking logs:

$$\log \mathbb{E}[\bar{Y}_i | \bar{Y}_{f(i)}] = \log P(\text{employed}_i | \bar{Y}_{f(i)}) + \log \mathbb{E}[e^{\alpha_i + \psi_{J(i,t)} + r_{it}} | \bar{Y}_{f(i)}, \text{employed}_i = 1].$$

We approximate the second term using a log-linear expansion:

$$\log \mathbb{E}[e^{\alpha_i + \psi_{J(i,t)} + r_{it}} | \bar{Y}_{f(i)}] \approx \mathbb{E}[\alpha_i | \bar{Y}_{f(i)}] + \mathbb{E}[\psi_{J(i,t)} | \bar{Y}_{f(i)}] + \mathbb{E}[r_{it} | \bar{Y}_{f(i)}].$$

This approximation becomes more accurate as the conditional variance of earnings components decreases. Conditioning on $\bar{Y}_{f(i)}$ reduces the dispersion of α_i , $\psi_{J(i,t)}$, and r_{it} , making the linearization more reliable. Still, some approximation error remains.

Additionally, using the law of iterated expectations, we have that:

$$\mathbb{E}[\psi_{J(i,t)} | \bar{Y}_{f(i)}] = \mathbb{E}[\mathbb{E}[\psi_{J(i,t)} | i] | \bar{Y}_{f(i)}] = \mathbb{E}[\bar{\psi}_i | \bar{Y}_{f(i)}],$$

and that:

$$\mathbb{E}[r_{it} | \bar{Y}_{f(i)}] = \mathbb{E}[\mathbb{E}[r_{it} | i] | \bar{Y}_{f(i)}] = \mathbb{E}[0 | \bar{Y}_{f(i)}] = 0.$$

Combining terms yields the decomposition:

$$\log \mathbb{E}[\bar{Y}_i | \bar{Y}_{f(i)}] \approx \log P(\text{employed}_i | \bar{Y}_{f(i)}) + \mathbb{E}[\alpha_i | \bar{Y}_{f(i)}] + \mathbb{E}[\bar{\psi}_i | \bar{Y}_{f(i)}]. \quad (\text{E.1})$$

²⁰The full AKM model includes covariates, $x'_{it}\beta^x$, which we omit here because we are interested in residualized earnings, after removing age effects.

B. Decomposing the Extended IGE

We estimate the following regressions:

$$\begin{aligned}\log P(\text{employed}_i \mid \bar{Y}_{f(i)}) &= \beta^W \cdot \overline{\log Y}_{f(i)} + \text{residual}, \\ \mathbb{E}[\alpha_i \mid \bar{Y}_{f(i)}] &= \beta^\alpha \cdot \overline{\log Y}_{f(i)} + \text{residual}, \\ \mathbb{E}[\bar{\psi}_i \mid \bar{Y}_{f(i)}] &= \beta^\psi \cdot \overline{\log Y}_{f(i)} + \text{residual}.\end{aligned}$$

Using Equation E.1 and following the same steps as in Appendix D, we can show that the extended intergenerational elasticity can be expressed as:

$$\beta^{IGE} \approx \beta^W + \beta^\alpha + \beta^\psi. \quad (\text{E.2})$$

Using Equation (E.2), we find that employment (β^W) accounts for 25% of the IGE, while the firm (β^ψ) and individual (β^α) components account for 17% and 58%, respectively. The interpretation of the participation margin depends on its underlying drivers. If low-SES individuals are less likely to work due to limited access to certain firms, then firm effects may be larger than our baseline estimates suggest. Conversely, if non-employment reflects skill deficits or preferences, the role of firms may be overstated. In this paper, we focus on employed individuals and leave the extensive-margin interpretation for future research.

F. Sector Persistence

A potential mechanism behind the firm-IGE is sector persistence. Indeed, individuals are 27% more likely than chance to work in the same sector as their fathers. To further investigate this pattern, we perform an AKM decomposition of earnings using sector fixed effect, instead of firm fixed effects:

$$\log Y_{i,t} = \alpha_i^S + \psi_{J(i,t)}^S + x_{it}'\beta^x + r_{i,t}^S,$$

where ψ_i^S are sector-level earnings premium. We estimate this regression separately for fathers and children. Since there are much fewer sector than firms, the connected set in this specification includes virtually all workers in the IGM sample (defined in Table 1) in both generations.

We then calculate the average earnings premium of each worker ($\hat{\psi}^S$) regress it against their father log earnings. We find an elasticity of 0.028, which we name sector-IGE. Note that the sector-IGE is half of the firm-IGE, indicating that the firm-IGE is, to a large extent, driven high-SES children work in high-paying sectors.

Finally, we investigate the role of fathers' sectors in the sector-IGE. We perform the decomposition:

$$\overbrace{\frac{\text{Cov}(\log Y_{f(i)}, \hat{\psi}_i^S)}{\text{Var}(\log Y_{f(i)})}}^{\text{Sector-IGE}} = \overbrace{\frac{\text{Cov}(\hat{\alpha}_{f(i)}^S, \hat{\psi}_i^S)}{\text{Var}(\log Y_{f(i)})}}^{\text{individual-to-sector}} + \overbrace{\frac{\text{Cov}(\hat{\psi}_{f(i)}^S, \hat{\psi}_i^S)}{\text{Var}(\log Y_{f(i)})}}^{\text{sector-to-sector}} + \overbrace{\frac{\text{Cov}(\hat{r}_{f(i)}^S, \hat{\psi}_i^S)}{\text{Var}(\log Y_{f(i)})}}^{\text{residual-to-sector}}$$

We find that the sector-to-sector component accounts for only 9% of the sector-IGE. That is, children of high-earnings parents tend to work in high-earnings- premium sectors, even if the father himself does not.

G. Labor market and residential segregation

In this appendix, we examine the relationship between parental income and the ethnicity of coworkers and neighbors. We focus on the share of Secular Jews as they represent the group with the highest earnings.

Figure G.1 presents the results of this analysis. We find a strong positive correlation between parental income and a higher share of Secular Jews among coworkers (Panel (a)). However, this correlation disappears within demographic groups. In other words, Secular Jews are more likely than Arabs to work with other Secular Jews, while high-SES Arabs are not more likely than low-SES Arabs to work with Secular Jews. Similar patterns are observed for residential segregation at neighborhood level (Panel (c)). Moreover, we see much less segregation at commuting zone level (Panel (d)) than at neighborhood level, consistent with the results discussed above.

Figure G.1—Labor-Market and Residential Segregation



Notes: Panel (a) shows the share of Secular Jewish coworkers in one's firm as a function of her father's earnings. Panel (b) shows the share of Secular Jewish workers in one's sector as a function of her father's earnings. "Sector" is a 3-digit sector and there are 420 unique sectors. Panel (c) shows the share of Secular Jewish neighbors as a function of the individual's father's earnings. Panel (d) shows the share of individuals in one's commuting zone who are Secular Jewish as a function of their father's earnings. In all panels, father's earnings are the average yearly earnings between 1986 and 1991 and are residuals from a regression of log earnings on age, age-squared, and year fixed effects.

One potential explanation for this phenomenon is assortative matching, whereby Secular Jews tend to work together and reside nearby because they possess higher skills, and therefore are richer, on average. However, the data do not support this explanation. If segregation were driven

solely by assortative matching, we would expect the demographic composition of neighbors and coworkers to be correlated with parental income even after accounting for the worker's own demographic group, which is not what we see in Figure G.1. Moreover, differences in education do not explain segregation. Israeli-Arabs and Ultra-Orthodox Jews have, respectively, 35% and 37% fewer Secular Jew coworkers than Secular Jews do. Controlling for education²¹ reduces these differences only modestly, to 33% and 34%. These results indicate that demographic groups are not segregated based on vertically differentiated skills, but horizontal differentiation could still play an important role. If this is the case, we should see these horizontally differentiate workers self-selecting into different sectors (Roy, 1951). However, this is not consistent with the modest segregation we find at sector level (see Panel (b)).

In sum, these patterns suggest that factors other than skill are pivotal in determining the allocation of workers to firms.

²¹We use here the most granular measure of education available, which includes a dummy for each type of higher education institution, as described in Appendix Table A.1.

H. Assortative matching model: Proofs

A. Proof of Equation (12)

We start from the model in Equation (11):

$$\psi_i = \theta^\psi \cdot H_i + \xi_i^\psi.$$

Therefore:

$$\text{Cov}(\psi_i, \overline{\log Y_{f(i)}}) = \theta^\psi \cdot \text{Cov}(H_i, \overline{\log Y_{f(i)}}) + \text{Cov}(\xi_i^\psi, \overline{\log Y_{f(i)}}).$$

Dividing both sides by $\text{Var}(\overline{\log Y_{f(i)}})$ yields Equation (12):

$$\beta^{\psi|Y_f} = \theta^\psi \cdot \beta^{H|Y_f} + \beta^{\xi^\psi|Y_f}.$$

B. Proof of Proposition 1

We prove that, under assumptions (i)–(iii), the AM-share can be identified as:

$$\overline{AM} = 1 - \frac{\beta_{Y_f}^{\psi|\alpha, Y_f}}{\beta^{\psi|Y_f}},$$

where $\beta^{\psi|Y_f}$ is the firm-IGE from a regression of ψ_i on $\overline{\log Y_{f(i)}}$, and $\beta_{Y_f}^{\psi|\alpha, Y_f}$ is the coefficient on $\overline{\log Y_{f(i)}}$ from a regression of ψ_i on both $\widehat{\alpha}_i$ and $\overline{\log Y_{f(i)}}$.

Step 1: Express ψ_i as a function of α_i .

Under assumption (ii), $\xi_i^\alpha = 0$, so equation (11) implies $\alpha_i = \theta^\alpha H_i$. Substituting this into the equation for ψ_i yields:

$$\psi_i = \frac{\theta^\psi}{\theta^\alpha} \alpha_i + \xi_i^\psi.$$

Step 2: Substitute the residual decomposition of ξ_i^ψ .

By definition of $\beta^{\xi^\psi|Y_f}$, we can write:

$$\xi_i^\psi = \beta^{\xi^\psi|Y_f} \cdot \overline{\log Y_{f(i)}} + \epsilon_i^{\xi^\psi|Y_f},$$

where $\epsilon_i^{\xi^\psi|Y_f}$ is the residual from regressing ξ_i^ψ on $\overline{\log Y_{f(i)}}$. Substituting into the expression for ψ_i gives:

$$\psi_i = \frac{\theta^\psi}{\theta^\alpha} \alpha_i + \beta^{\xi^\psi|Y_f} \cdot \overline{\log Y_{f(i)}} + \epsilon_i^{\xi^\psi|Y_f}. \quad (\text{H.1})$$

Step 3: Identify the coefficient on $\overline{\log Y_{f(i)}}$.

Consider the regression of ψ_i on α_i and $\overline{\log Y_{f(i)}}$. From equation (H.1), the coefficient on

$\overline{\log Y_{f(i)}}$ will be equal to $\beta^{\xi^\psi|Y_f}$ if the following two conditions hold:

- (a) $\text{Cov}(\overline{\log Y_{f(i)}}, \epsilon_i^{\xi^\psi|Y_f}) = 0$, which holds by construction of the regression residual;
- (b) α_i is independent of $\epsilon_i^{\xi^\psi|Y_f}$ conditional on $\overline{\log Y_{f(i)}}$.

To verify (b), note that under assumption (ii), $\alpha_i = \theta^\alpha H_i$. From equation (13), $H_i = \beta^{H|Y_f} \cdot \overline{\log Y_{f(i)}} + \epsilon_i^{H|Y_f}$, so:

$$\alpha_i = \theta^\alpha \beta^{H|Y_f} \cdot \overline{\log Y_{f(i)}} + \theta^\alpha \epsilon_i^{H|Y_f}.$$

Assumption (iii) states that $\epsilon_i^{H|Y_f}$ and $\epsilon_i^{\xi^\psi|Y_f}$ are independent, implying that α_i is conditionally independent of $\epsilon_i^{\xi^\psi|Y_f}$ given $\overline{\log Y_{f(i)}}$. Therefore, condition (b) holds, and we conclude that:

$$\beta_{Y_f}^{\psi|\alpha, Y_f} = \beta^{\xi^\psi|Y_f}.$$

Step 4: Complete the proof.

From equation (12):

$$\beta^{\psi|Y_f} = \theta^\psi \cdot \beta^{H|Y_f} + \beta^{\xi^\psi|Y_f}.$$

Solving for the assortative matching component:

$$\theta^\psi \cdot \beta^{H|Y_f} = \beta^{\psi|Y_f} - \beta^{\xi^\psi|Y_f} = \beta^{\psi|Y_f} - \beta_{Y_f}^{\psi|\alpha, Y_f}.$$

Substituting into equation (14) gives:

$$\overline{AM} = \frac{\theta^\psi \cdot \beta^{H|Y_f}}{\beta^{\psi|Y_f}} = 1 - \frac{\beta_{Y_f}^{\psi|\alpha, Y_f}}{\beta^{\psi|Y_f}}.$$