

Supplemental Appendix:
Dynamics of the Long Term Housing Yield:
Evidence from Natural Experiments

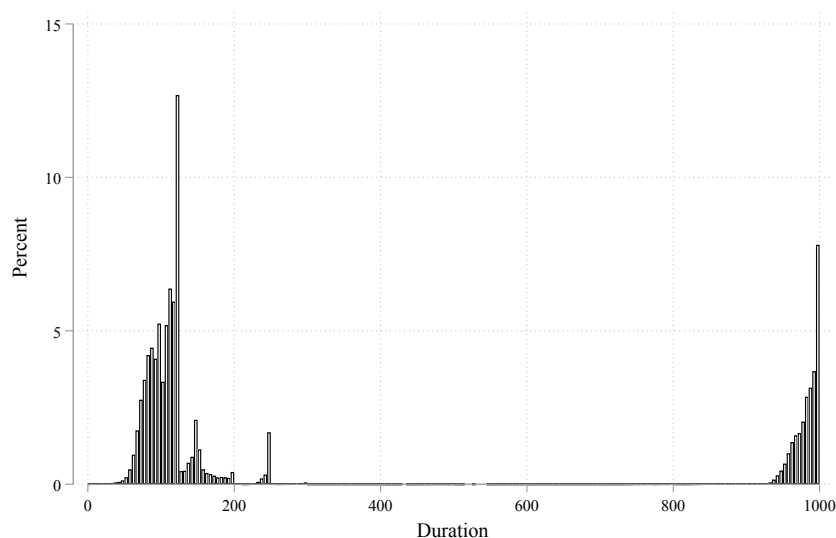
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July 19, 2025

A Appendix

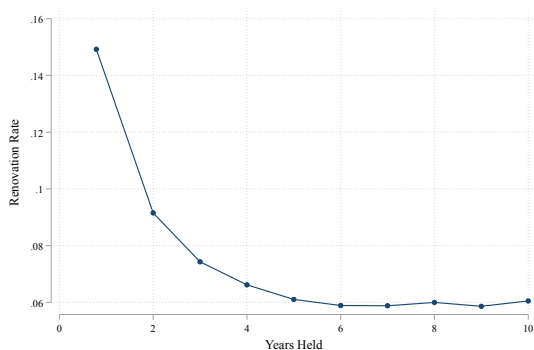
A.1 Additional Figures

Figure A.1: Distribution of Lease Term for Leasehold Flats

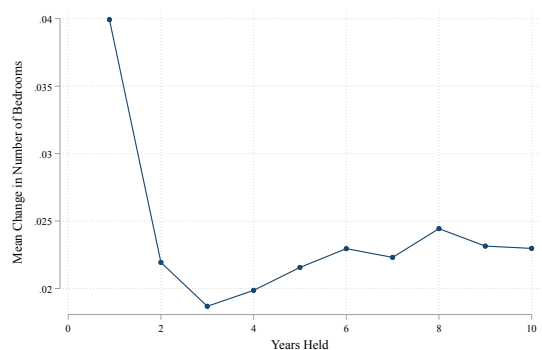


The figure is a histogram of the remaining lease term at the time of transaction. The sample is all leasehold flats that transact at least once in the Land Registry Transaction Data Set.

Figure A.2: Renovations By Holding Period



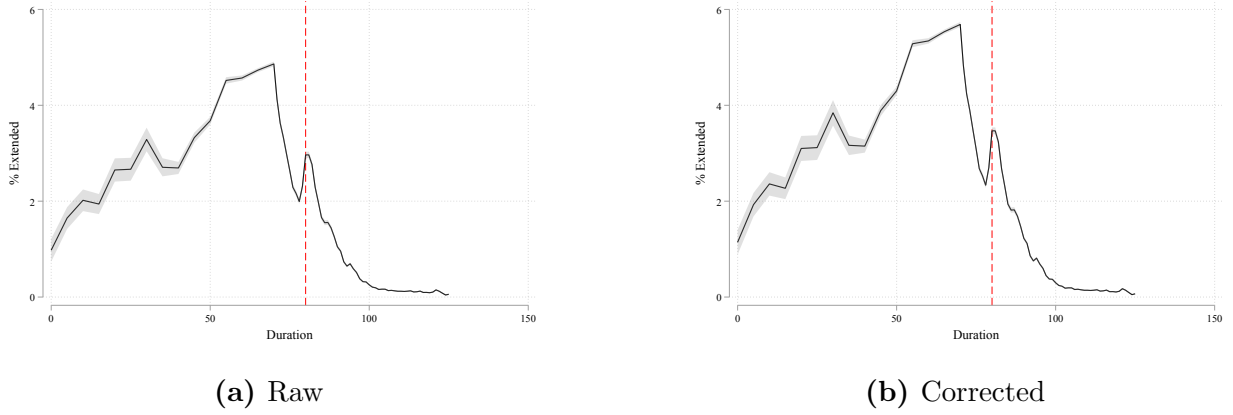
(a) Renovation Rate



(b) Change in Bedrooms

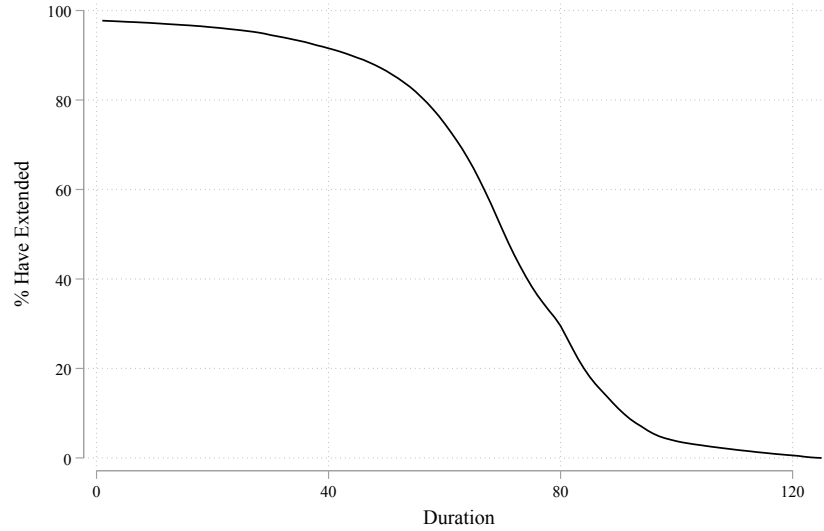
The figure shows the renovation rate and change in number of bedrooms reported relative to property holding period. The renovation rate is measured as the share of properties which have Rightmove listings that mention renovation. We see that very short holds have a disproportionately high renovation rate according to both measures. We take this as evidence that many of these are “flippers” who buy properties to re-sell them. The sample is all flats for which we observe two different Rightmove or Zoopla listings associated with two different property transactions.

Figure A.3: Hazard Rate of Lease Extension



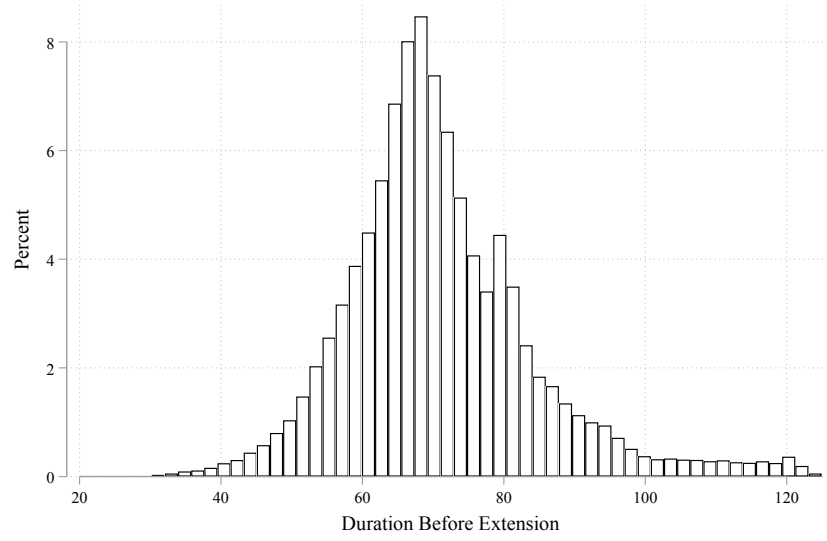
The figure shows the conditional probability of extension, $\theta(T)$ given that a property has duration T . In the first panel, the conditional probability of extension is given by $\theta_1(T) = \frac{N_T^{Ext}}{N_T}$ where N_T^{Ext} is the number of properties which extended with duration T and N_T is the number of properties that reached duration T . In the second panel, the conditional probability of extension is $\theta_2(T) = \gamma \frac{N_T^{Ext}}{N_T}$ where $\gamma = 1.16$ adjusts for the fact that our primary method does not identify properties which never transact before extension. The shaded area shows the 95% confidence interval.

Figure A.4: Cumulative Hazard Rate



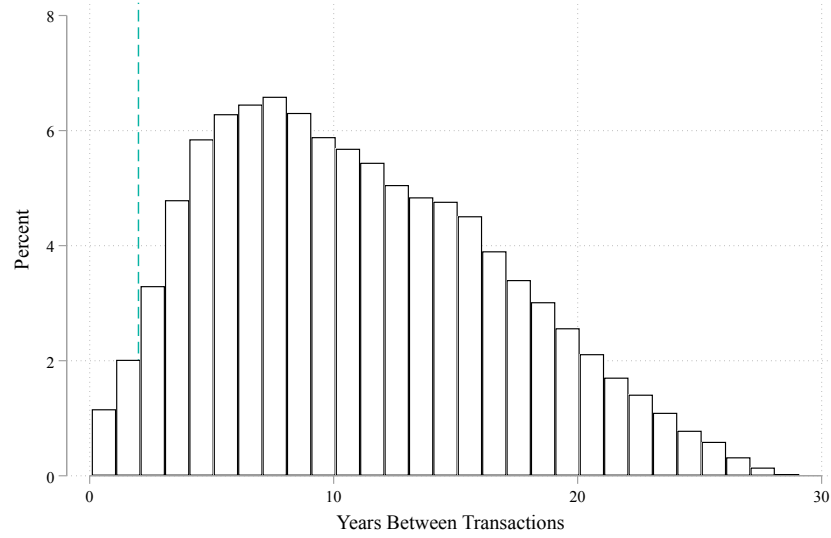
The figure shows the cumulative probability of extending over a property's lifetime. The sample includes all leases with at least one transaction and covers the 2003-2020 period. We exclude the pandemic period due to abnormally low extension rates.

Figure A.5: Histogram of Duration Before Extension



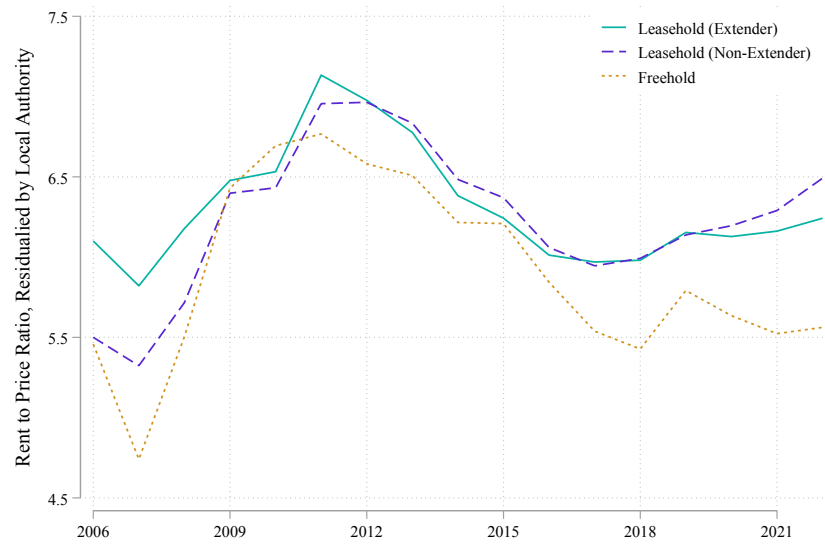
The figure presents a histogram of lease duration immediately before extension. The sample is extended flats.

Figure A.6: Histogram of Years Between Transaction



The figure shows a histogram of the holding period, h , for lease extensions which have a recorded transaction before and after extension. The dotted line shows the $h = 2$ cutoff; properties below the cutoff are not included in our primary sample. The sample is all extended flats.

Figure A.7: Rent to Price (Freeholds vs Leaseholds)



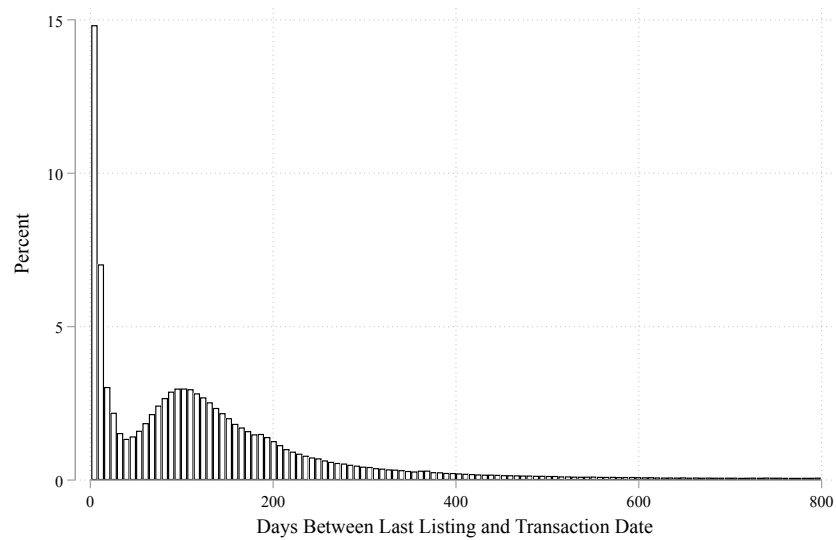
The figure shows the property-level price to rent ratio for leasehold and freehold flats. Leaseholds are subdivided into those which extend during our sample and those which do not. Property-level rental price data is collected from Rightmove and Zoopla.

Figure A.8: Heat Map of Extension Rate



The figure shows a heatmap of the number of properties extended in each Local Authority in England and Wales.

Figure A.9: Time Elapsed Between Rightmove Listing and Land Registry Transaction Date



The figure shows a histogram of the amount of time (in days) between the last property listing on Rightmove and the date in which the transaction is recorded by the Land Registry. Each bin refers to one week. The sample is transactions for properties which have Rightmove listings within two years of the Land Registry transaction date.

Figure A.10: Cumulative Density

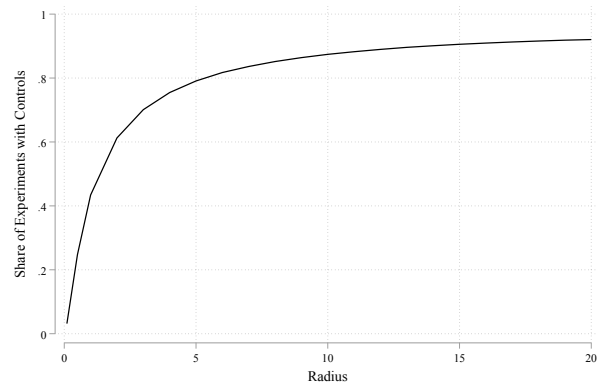


Figure A.11: Control Radius Relative to Population Density

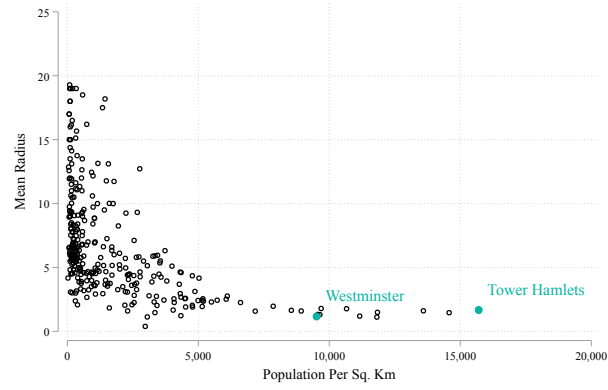
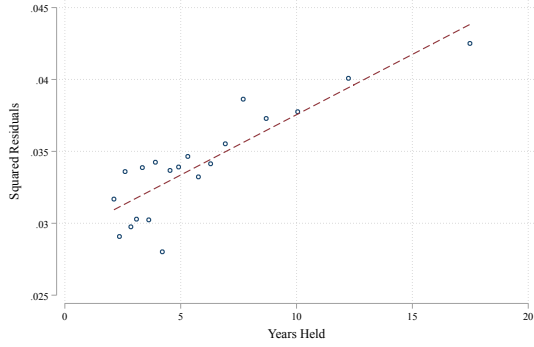
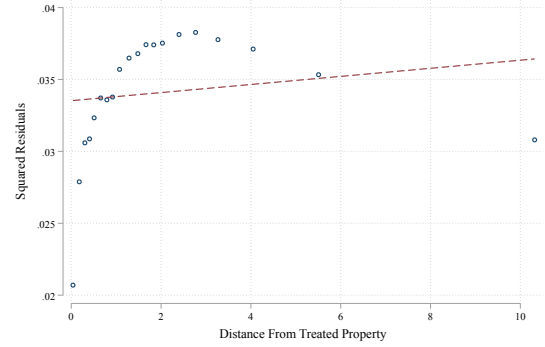


Figure A.12: Binscatter of Repeat-Sales Residuals



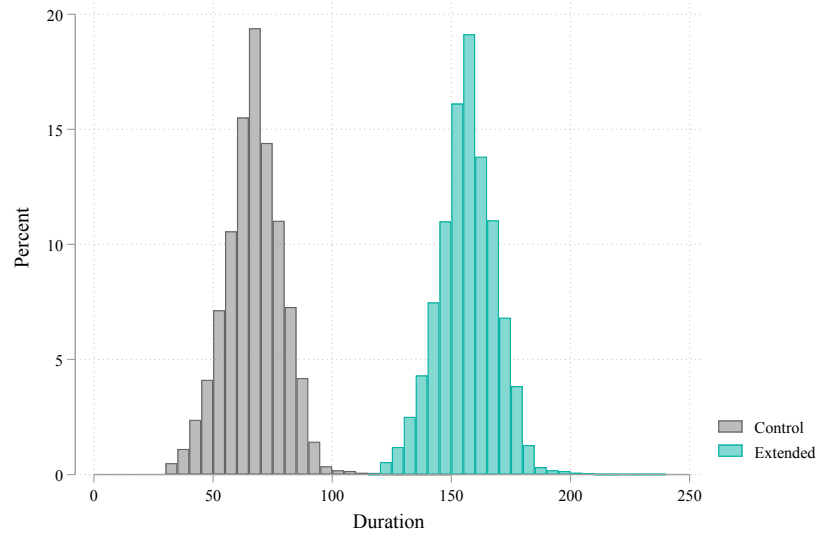
(a) Holding Period



(b) Haversine Distance

The figures are binscatters of the squared residuals from the first step of the repeat-sales methodology against two variables which may be correlated with the variance of the estimate: (1) the time between transactions and (2) the distance between the treated and control property.

Figure A.13: Histogram of Remaining Lease Term At Sale



The figure shows a histogram of remaining duration at sale time t for the approximately 40 thousand experiments which were extended for 90 years. The blue-shaded histogram includes the sale duration for the controls in the experiment and the grey-shaded histogram includes the sale duration for the extensions.

Figure A.14: Y-star Estimates for Buy-to-Let and Owner Occupied Properties

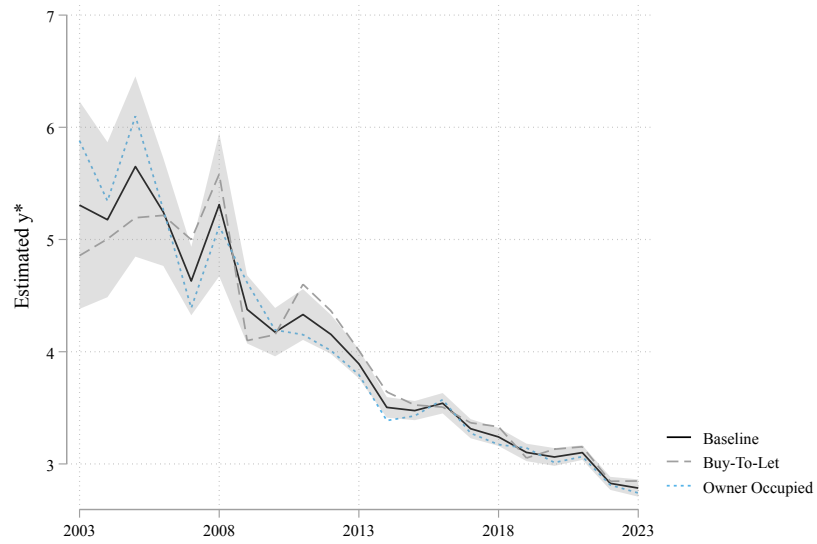


Figure A.15: y^* and Second Homes

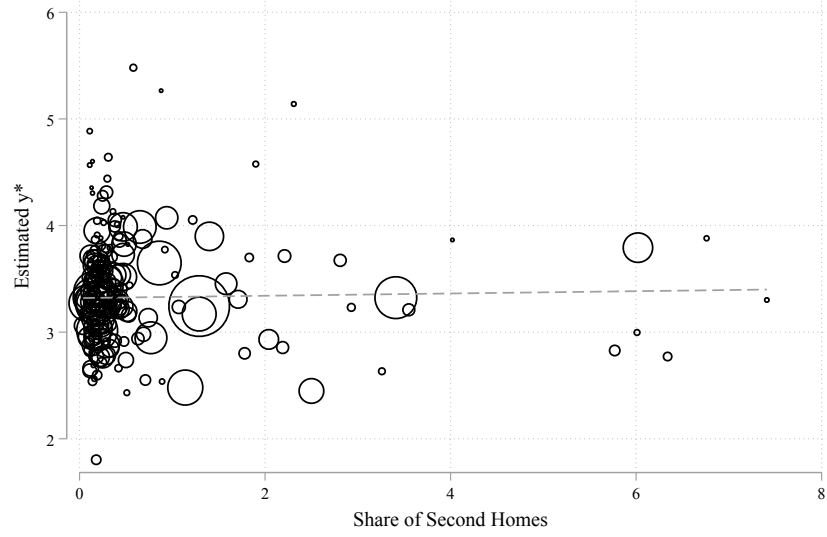
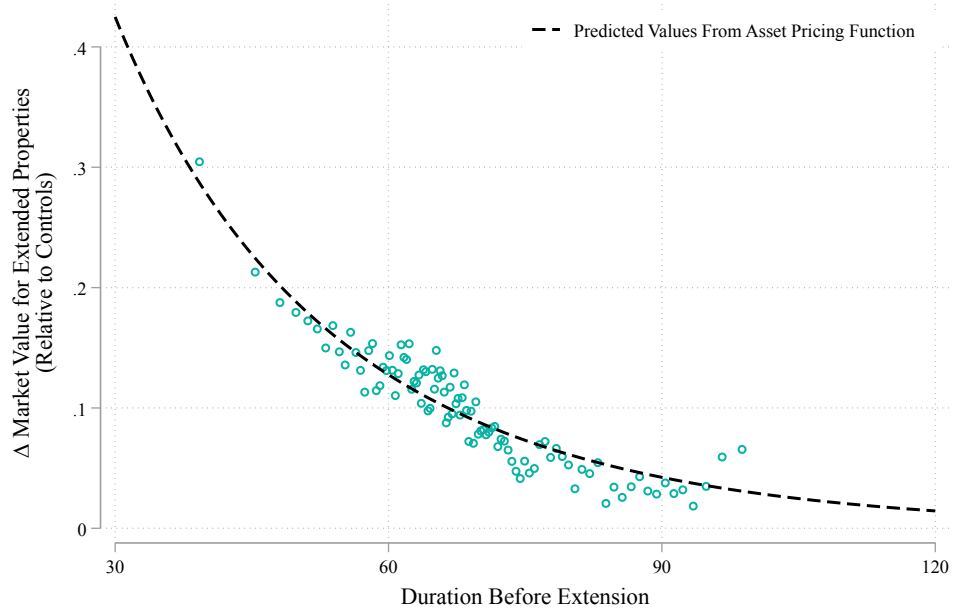
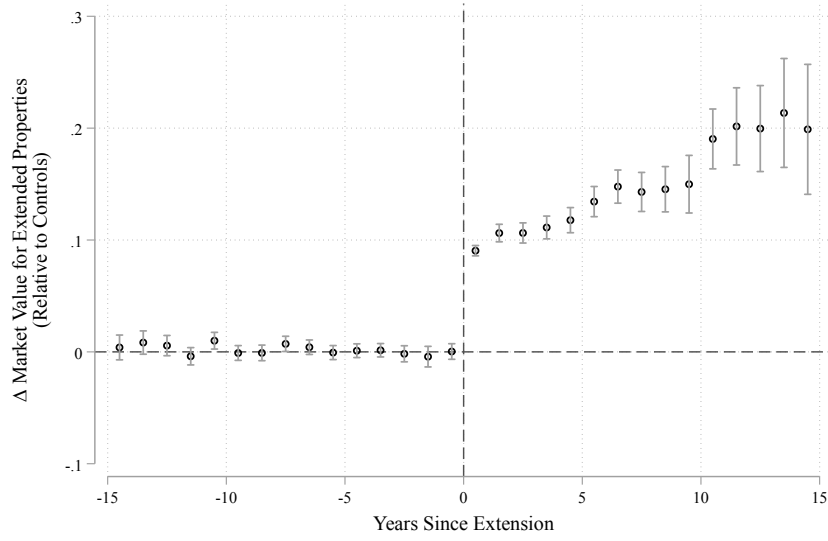


Figure A.16: Duration vs. Price Gain After Extension, Leases with 700+ Years



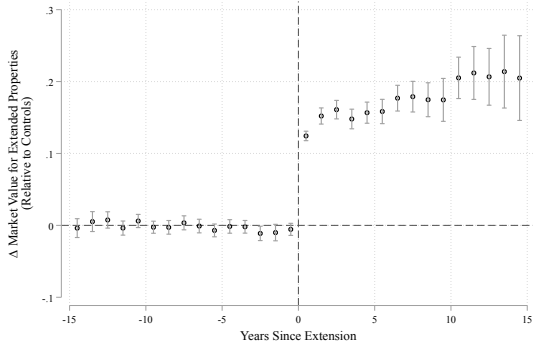
The figure is a binscatter of our difference-in-difference estimator against duration before extension, T , with 100 bins. The sample includes leases that were extended for more than 700 years. The black line shows fitted estimates of [equation \(6\)](#).

Figure A.17: Event Study Representation of Lease Extension, 90-Year Extensions

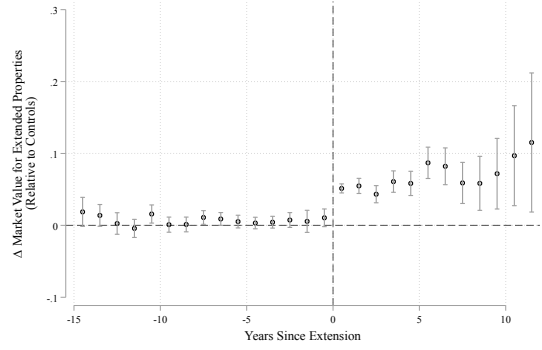


The figure replicates Figure 6 for properties extended by 90 years.

Figure A.18: Event Study By Duration



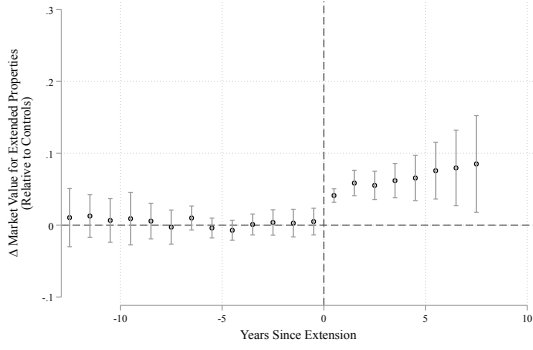
(a) $T < 70$



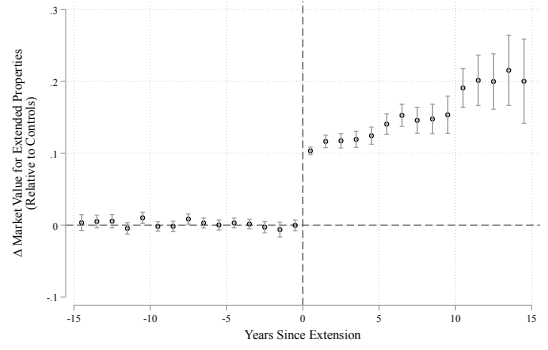
(b) $T \geq 70$

The figure replicates Figure 6 for properties that were extended with below and above median durations.

Figure A.19: Event Study By Time Period



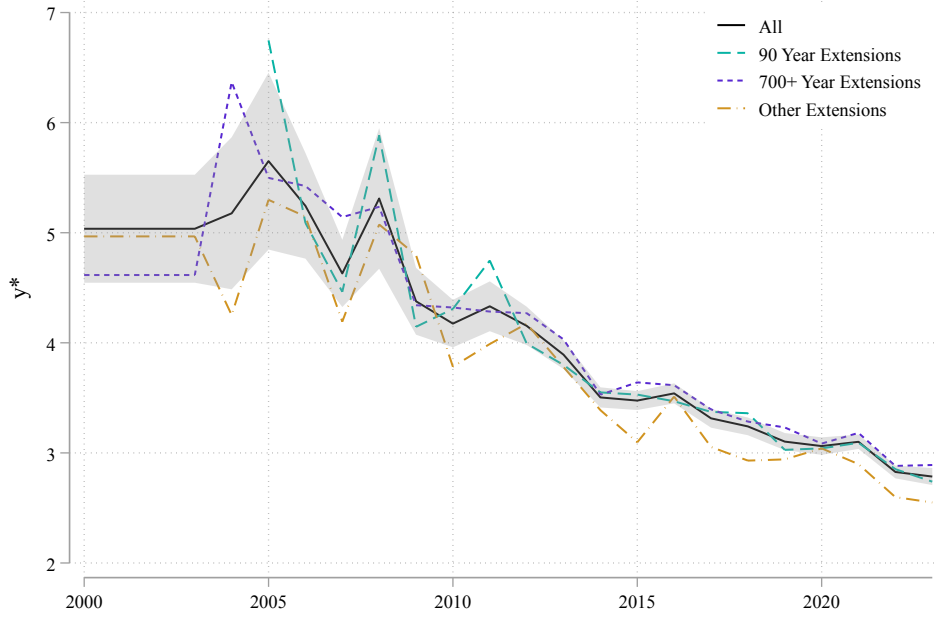
(a) Pre 2012



(b) Post 2012

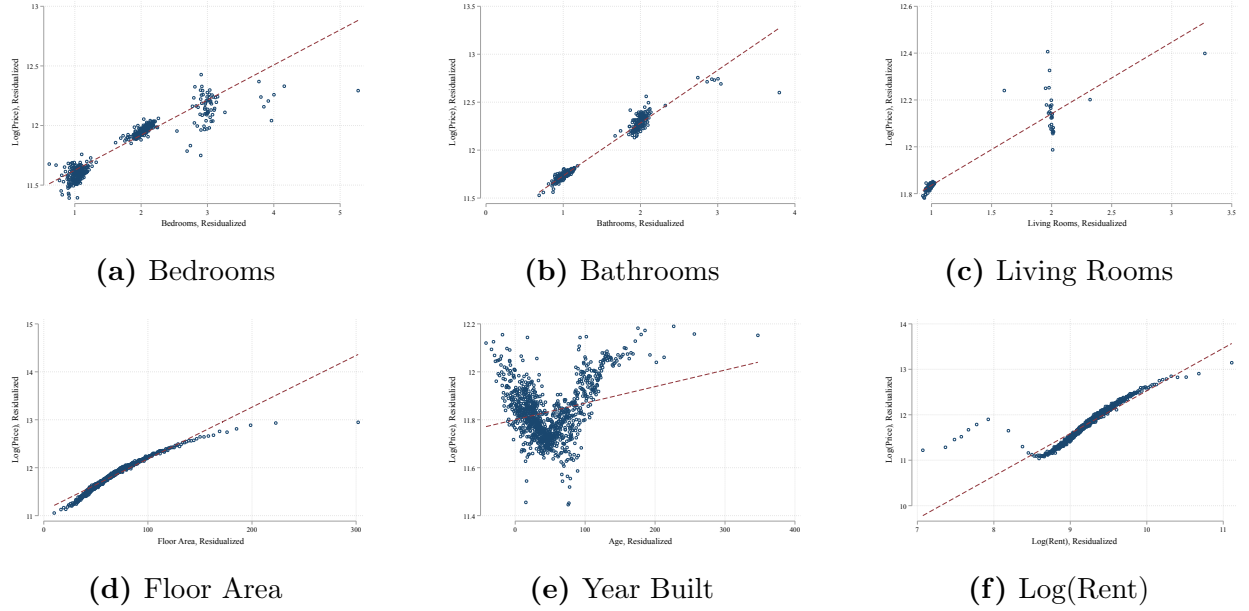
The figure replicates Figure 6 for two different sub-periods: pre 2012 (inclusive) and post 2012.

Figure A.20: y_t^* Estimates For Various Extension Amounts



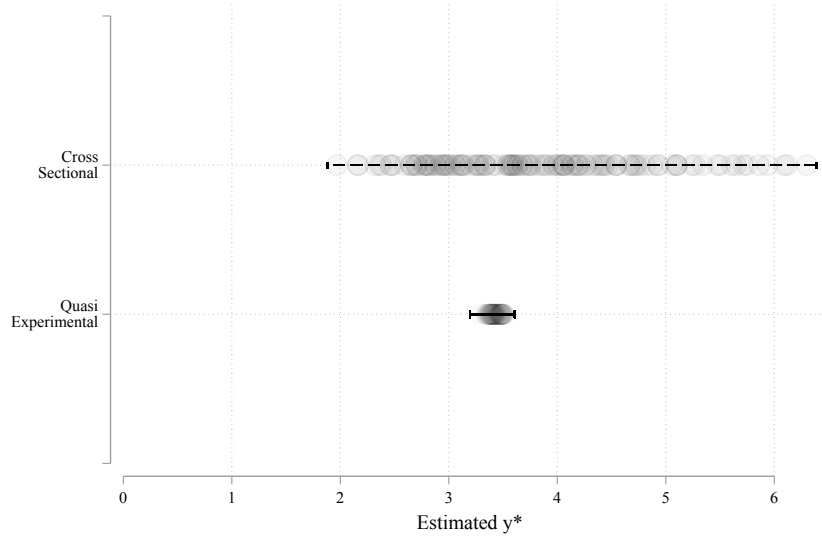
The figure presents estimates of y_t^* for every year of our sample period for lease extensions that were extended for 90 years, for more than 700 years, or for another amount, separately. The estimate of y^* for 90-year extensions in 2008 is excluded, because the mean estimated difference-in-difference is negative due to low sample size and volatility from the housing crisis, which leads to an extreme y^* estimate.

Figure A.21: Binscatter Log(Price) on Hedonics



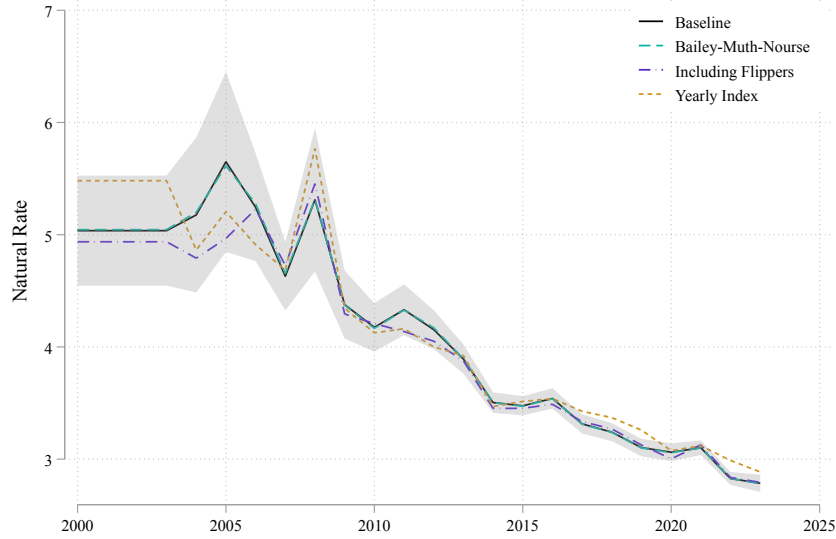
The figures are binscatters of log transaction price against the following hedonic characteristics: number of bedrooms, number of bathrooms, number of living rooms, floor area (sq. meters), year that the property was built, and log yearly rental price. Both the x and y-axis variables are residualized by Local Authority fixed effects, Γ_{it} . In particular, the y-axis variable is $\overline{\log(P_{it})} + \epsilon_{it}$ where $\overline{\log(P_{it})}$ is the mean log transaction price and ϵ_{it} is the residual from the following specification: $\log(P_{it}) = \Gamma_{it} + \epsilon_{it}$. The x-axis variable for each hedonic characteristic X_{it} is $\overline{X_{it}} + \eta_{it}$ where $\overline{X_{it}}$ is the mean level of X_{it} and η_{it} is the residual from the following specification: $X_{it} = \Gamma_{it} + \eta_{it}$. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set.

Figure A.22: Stability of y^* , Controlling for Observables Interacted with Time



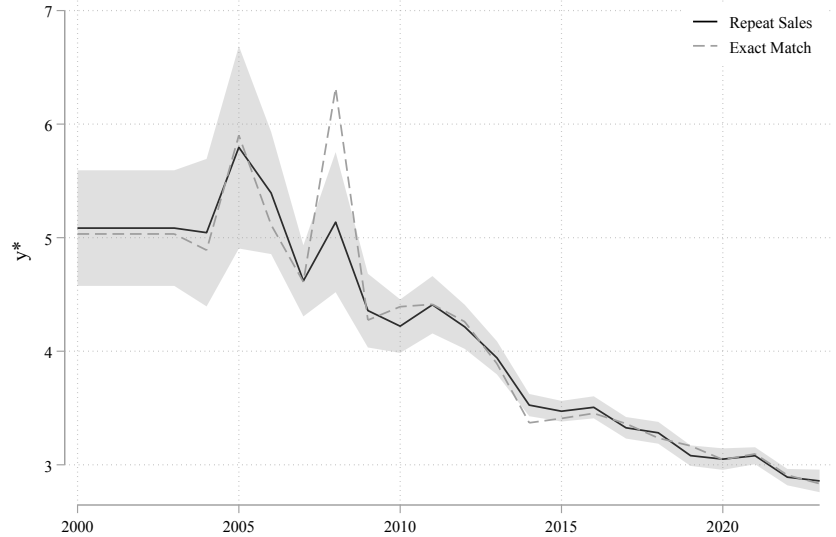
The figure replications [Figure 10](#) controlling for hedonic interacted with time fixed effects.

Figure A.23: Robustness Checks



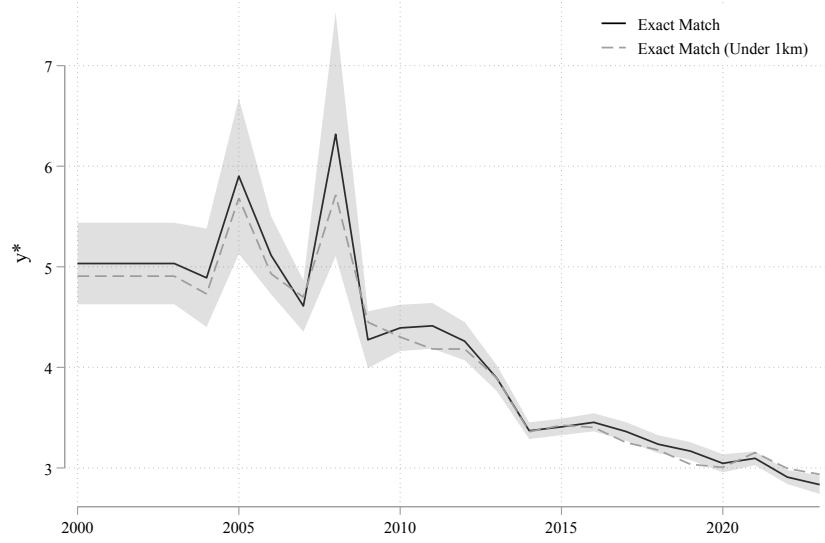
The figure presents robustness for our estimates of y_t^* . The solid black line presents our baseline estimates from Figure 1. The dashed grey line presents estimates when using the methodology from Bailey et al. (1963) to produce the repeat sales index of control properties. The dash-dot blue line presents estimates if we include “flipper” properties that were held for 2 years or less. The dotted green line presents estimates when constructing the repeat sales index at an annual rather than quarterly frequency.

Figure A.24: Exact Control Method



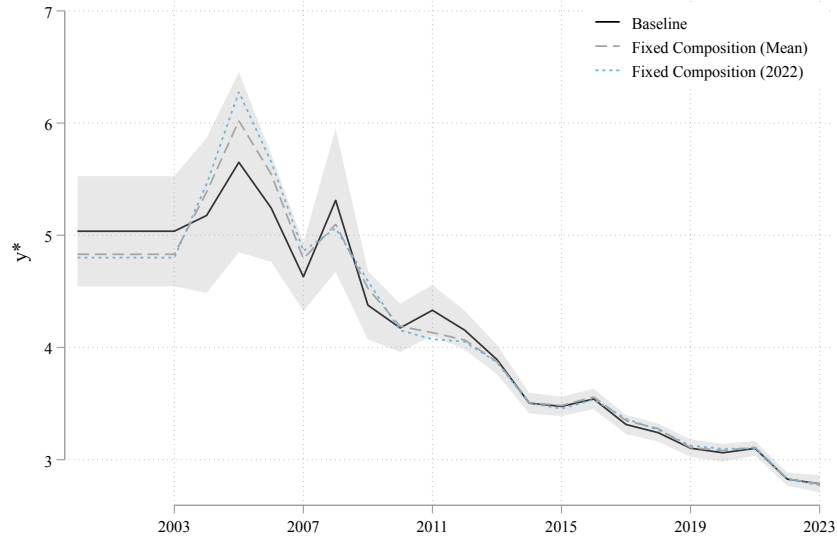
The figure presents estimates of y^* when we pick controls that transact in the same purchase and sale years as the treated property. Our difference-in-difference then becomes $\Delta_{it} = (\log P_{it}^{T+90} - \log P_{i,t-h}^{T+h}) - (\log P_{jt}^T - \log P_{j,t-h}^{T+h})$, where i is a treated property and j is a neighboring control property of similar duration. The black line presents our baseline estimates on the sample for which we are able to find exact controls. The dashed grey line presents estimates using exact controls. A 95% confidence interval is shaded. The sample is all lease extensions.

Figure A.25: Exact Control Method, Under 1km



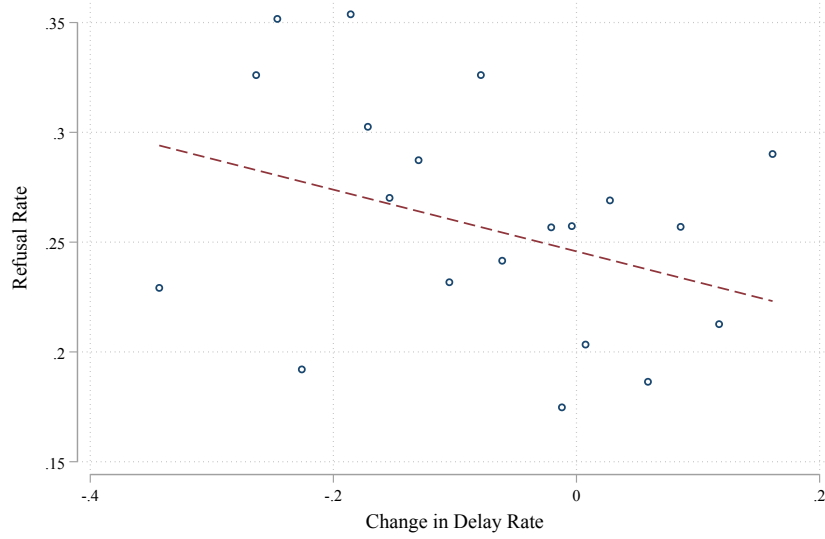
The figure presents replicates [Figure A.24](#) but presents estimates when we restrict the sample to properties for which the control is within 1km of the treated property.

Figure A.26: Fixed Regional Composition



The figure presents estimates of y^* controlling for the geographic composition of the data. The solid black line presents our baseline estimates. In the dashed grey line, we fix the composition to the mean composition of Local Authorities over the sample period. In particular, we produce weights, $w_{it} = \bar{N}_i / N_{it}$, where $\bar{N}_i$ is the average number of observations in Local Authority i over the entire period and N_{it} is the number of observations in i in year t . We then run a weighted NLLS regression using the weights w_{it} . The dotted blue line presents estimates where we fix the composition of the data to its 2022 level using the same method. The 95% confidence interval of the baseline estimates is shaded. The sample is all lease extensions.

Figure A.27: Refusal Rate Instrument, First Stage



The figure presents the first stage binscatter for the IV regression in [Table 6](#). The x-axis variable is the change in application delay rate after the 2002 policy discussed in [Section 6.3](#). The y-axis variable is the mean refusal rate of major construction applications. The slope coefficient is -0.22, with standard error of 0.05. The binscatter has 20 bins.

A.2 Additional Tables

Table A.1: Total Number of Extensions

Extension Amount	90	700+	Other	Total
2000-2005	4,077	13,313	10,280	27,670
2006-2010	14,153	25,416	13,473	53,042
2011-2015	33,446	38,977	20,244	92,667
2016-2020	46,570	49,616	23,489	119,675
2021-2023	10,376	11,945	4,098	26,419
Total	108,622	139,267	71,584	319,473

The table reports the number of extended leases that have transaction data before extension time and may or may not have transaction data after extension time. The first column includes 90 year extensions, the next column includes 700+ year extensions, and the third column includes others, which are almost all non-90 under 200 year extensions. The first column denotes the time of extension.

Table A.2: Freehold vs Leasehold Statistics (English Housing Survey)

	Freehold	Leasehold
Income	29,628.73 (52.95)	25,653.20 (138.48)
Age	53.95 (0.03)	51.49 (0.10)
% Have Mortgage	54.82 (0.10)	59.07 (0.28)
LTV	72.17 (0.14)	76.16 (0.39)
N	305,135	

This table reports the mean characteristic for freehold owners, in Column (1), and leasehold owners, in Column (2). The standard error of the mean is in parentheses.

A.3 Details on Merge Between Lease and Transaction Data

This section briefly describes our procedure to merge Land Registry data on lease lengths, and house price transaction data. In the UK, every property can be uniquely identified by three items: the first address number, the second address number and the 6-digit postcode. Therefore, we merge according to the following procedure: First, we conduct a perfect merge using address as our merge key. This method accounts for 93% of our matches. Second, we conduct a fuzzy merge on all observations not matched by step 1. The fuzzy merge matches observations in which (1) all numeric elements of the address are the same, (2) all single letters (e.g. A, B, C, etc.) are the same, (3) a select set of identifying terms (e.g. first floor, second floor, basement, etc.) are the same and (4) the postcode is the same in both addresses. For example, this allows for the property “3 Swan Court 59-61 TW13 6PE” in the transaction data set to be matched to “Flat 3 Swan Court 59-61 Swan Road Feltham TW13 6PE” in the lease term data set. This method accounts for 7% of our matches.

Additionally, we purchase the leasehold titles from the Land Registry for approximately 20 thousand transactions for which we are unable to identify a lease term based on the fuzzy merge. These include cases where the lease term address has typos, or has been accidentally omitted from the main public data set.

A.4 Calculating the Extension Hazard Rate

In this section, we explain how we calculate the extension hazard rate, shown in [Figure A.3](#) and [Figure A.4](#). We define the conditional probability that a property i extends given that it has duration T as $\theta_i(T) = P(\text{Extended At } S | \text{Duration} = S)$. To get the total cumulative probability that a T duration property i will extend over the course of its lifetime, we must convert our conditional probabilities to unconditional probabilities as follows,

$$\begin{aligned}\pi_i(S) &= P_i(\text{Extended At } S) \\ &= P_i(\text{Extended At } i | \text{Duration} = S) P_i(\text{Duration} = i) \\ &= \theta_i(S) \prod_{U=S+1}^T (1 - \theta_i(U))\end{aligned}$$

The cumulative probability that property i extends over its lifetime is then given by $\Pi_i(T) = \sum_{S=1}^T \pi_i(S)$. In [Figure A.4](#), we scale the hazard rate up by a factor of 1.16. This is because our method to identify lease extensions does not capture extensions that have no transactions before extension. We estimate that there are about 17% more extensions that have been extended but do not have pre-extension transaction data.¹

Then, the price of a T duration property at time t is given by the following recursive formula,

$$P_{i,t}^T = \frac{R_{i,t+1} + \theta_i(T)(P_{i,t+1}^{T+90-1} - \kappa_{t+1}^{T-1}) + (1 - \theta_i(T))P_{t+1}^{T-1}}{1 + r^* + \zeta^*} \quad (1)$$

Intuitively, the price of a T duration asset is the discounted dividends it yields next period, plus with probability $1 - \theta_i(T)$, the price of a $P_{i,t+1}^{T-1}$ asset, and with probability $\theta_i(T)$, the price of a $P_{i,t+1}^{T+90-1}$ duration asset minus the cost of extending, all appropriately discounted.

A.5 Measuring the Option Value of Lease Extension

So far, we have assumed that there is no option value from lease extension, because leaseholders pay the market value of extension to freeholders when they extend. This section studies option value and its consequences for estimates of y^* , in three steps. First, we present a simple framework that encompasses a key feature of the UK law on extensions, namely discontinuities in the cost of lease extensions when leaseholds have 80 years remaining. Second,

¹To obtain this estimate, we utilize the fact that the most common lease extension amount is for 90 years and the most common initial lease terms are for 99 and 125 years. Therefore, almost all leases with recorded terms of 189(=99+90) and 215(=125+90) have almost certainly been extended. As such, we infer that the share of extensions missing pre-transaction data is equal to the share of 189/215 leases missing pre-transaction data.

we use this framework to derive discontinuity based tests about whether there is option value from lease extension. These tests indicate that the baseline estimate of the fall in y^* , which ignores option value, is a lower bound for the true fall in y^* . Third, we generalize our the difference-in-differences estimator to point identify the size of option value. We find that our baseline estimate of y^* , which ignores option value, is an excellent approximation.

A.5.1 A Framework for Lease Extension Costs

We now summarize the institutional framework of lease extensions and develop a simple model of this framework.

Tribunals. Leaseholders are legally entitled to a 90-year extension by the 1993 Leasehold Reform, Housing and Urban Development Act. According to the act, lease extensions ought to be priced at their market value, the present value of service flows from the lease. However the leaseholder and freeholder negotiate the size of the market value, by independently hiring surveyors. If there is no agreement, a Residential Property Tribunal determines the value of the extension after a costly legal process. Tribunals require that the extension is 90 years long, and price extensions using a two part formula, which requires an estimate of *reversion value* and *marriage value*.

Reversion value. The reversion value is the value of the lease extension according to a yield assumed by the tribunal. Formally, the reversion value of property i at time t satisfies the formula

$$RV_t^T = \frac{R_{it}}{r_{RV}} \left(e^{-r_{RV}T} - e^{-r_{RV}(T+90)} \right). \quad (2)$$

This equation is the value of the lease extension according to a Gordon Growth model, with a discount rate of r_{RV} and a current service flow R_{it} . The first term in the brackets on the right hand side is the value of an infinite duration property, and the second term is the value of an infinite duration property 90 years hence—meaning the difference is the value of the lease extension. In practice, the service flow R_{it} is imputed from the price of an observably similar freehold property. The discount rate is fixed by the tribunal at $r_{RV} = 5\%$.

Marriage value. We will define the market value of the extension as $V_{it}^T \equiv P_{it}^{T+90} - P_{it}^T$, which is the difference in price between property to be extended, with duration T , and the price of the same property with duration $T + 90$. The marriage value is the tribunal's estimate of the gap between the market value and the reversion value of the lease. That is, the marriage value is $MV_{it}^T \equiv V_{it}^T - RV_{it}^T$. To calculate the marriage value, the tribunal imputes market values from the transacted prices of observably similar properties. Provided

that the tribunal imputes correctly, the market value satisfies

$$V_{it}^T = \frac{R_{it}}{y_t^*} (e^{-y_t^* T} - e^{-y_t^* (T+90)}) . \quad (3)$$

Here, we have written the marriage value as the market value of a lease extension according to a Gordon Growth model, where the natural rate expected by the market, y_t^* , enters the equation for market value. The first term in brackets on the right hand side is the market value of an infinite duration property, and the second term is the market value of an infinite duration property 90 years later—the difference is the market value of extension.

Reversion value vs. marriage value. Our approach makes extensive use of the following observation. Compare [equation \(2\)](#) and [equation \(3\)](#) for reversion and market value. Reversion value calculated by the tribunal is greater than market value, if and only if y^* is greater than the 5% yield assumed by the tribunal.

Discontinuity at 80 years duration. There is a discontinuity in the tribunal assessed cost of the lease extension when 80 years remain on the lease. For leases with more than 80 years remaining, the tribunal dictates that only the reversion value must be paid, whereas for leases with less than 80 years remaining, the cost is the average of the reversion value and the market value; or equivalently, the reversion value plus half of the marriage value.

Tribunal costs to leaseholder. There are significant costs to the leaseholder of appealing to the tribunal. These costs include time and information costs, uncertainty from the outcome of the tribunal, costs of hiring a survey to value the lease, and legal costs associated with the tribunal. Moreover the law dictates that leaseholders must cover all of the freeholder’s legal and surveyor fees associated with extension. We will denote these costs for property i by γR_{it} , which we assume for simplicity scales with the current service flow R_{it} .

Leaseholder’s participation constraint. The leaseholder also has a participation constraint—if the tribunal assessed costs of extending are greater than the value of extending, the leaseholder can opt not to extend. Therefore the freeholder should reduce costs down to the market value of extension, so that a transaction can occur. The converse is not true: if the tribunal associated costs are less than the market value of extending, the leaseholder will choose the tribunal costs instead of paying the freeholder the market value of extension.

Total extension costs. We can summarize the cost of extension κ_{it}^T , of a property i with T years remaining at time t , as

$$\kappa_{it}^T = \begin{cases} \min \left\{ RV_{it}^T + \gamma R_{it}, V_{it}^T \right\} & T \geq 80 \\ \min \left\{ \frac{RV_{it}^T + V_{it}^T}{2} + \gamma R_{it}, V_{it}^T \right\} & T < 80 \end{cases} \quad (4)$$

Equation (4) recognizes that the cost of extension κ_{it}^T is the minimum of the market value of extension from the present value of rents; and the tribunal recommended value of extension plus the costs of a tribunal settlement. The tribunal recommended settlement varies discontinuously at 80 years, provided that the market value and the reversion value are not equal.²

Price of a T duration property. We assume property prices satisfy a simple no arbitrage assumption. Therefore the price of a property P_{it}^T , that has not extended, must equal the price of an otherwise similar property that has extended, after deducting lease extension costs. That is, prices satisfy

$$P_{it}^T = P_{it}^{T+90} - \kappa_{it}^T. \quad (5)$$

Intuitively, suppose that the cost of buying a property and extending is less than the price of an extended property—then demand would fall for the extended property until prices equalize. We denote $\alpha_t^T = \kappa_{it}^T/V_{it}^T \leq 1$ as the share of the extension value that is paid by the leaseholder to the freeholder, noting that under our assumptions α does not depend on the service flow i .

Remark on option value. When $\alpha_t^T = 1$ we say there is *zero option value* and when $\alpha_t^T < 1$ there is *positive option value*. This terminology acknowledges that when $\alpha_t^T = 1$ for all T , then all of the gains of lease extension are paid to the freeholder. As a result, the option to extend the lease has no value to the leaseholder ex ante.

A.5.2 A Discontinuity Based Test for Option Value

We now use our framework for lease extension costs to derive a discontinuity based test of whether there is positive option value—that is, we provide a simple test of whether the baseline assumption in the main text, of no option value, is correct. Using this test, we show that there was no option value in the early part of the data, before 2010, and positive option value afterwards. Given the behavior of option value, our estimates of the fall in y^* will be smaller than the true fall in y^* .

We summarize our predictions in the following proposition.

Proposition A.1. *There exists some value $\bar{r}_K \leq r_{RV}$ such that:*

1. *If y^* satisfies $y_t^* \geq \bar{r}_K$ then: (i) there is zero option value at all years of duration remaining, that is, $\alpha_t^T = 1$ for all T ; (ii) the price of a leasehold is continuous in duration as the property's duration falls below 80 years, so $\lim_{T \rightarrow 80^-} P_{it}^T = \lim_{T \rightarrow 80^+} P_{it}^T$.*

²For exposition, we assume that the tribunal, the leaseholder and the freeholder all agree on the market value of the extension. Our qualitative and quantitative results are not affected by this assumption.

2. If y^* satisfies $y_t^* < \bar{r}_K$ then: (i) there is positive option value above 80 years in duration, that is, $\alpha_t^T < 1$ for all $T > 80$ and option value discontinuously falls at 80 years, so that α_t^T discontinuously increases at $T = 80$; (ii) the price of a leasehold discontinuously falls as the property's duration falls below 80 years, so $\lim_{T \rightarrow 80^-} P_{it}^T < \lim_{T \rightarrow 80^+} P_{it}^T$.

This proposition, which we prove in [Appendix A.17](#), has two implications. First, we should expect zero option value at all durations, including above 80 years remaining, only if y^* is relatively high. Second, we can test for the presence of zero option value by searching for discontinuities in the price of leaseholds at 80 years.

Part (1) of the proposition shows that when y^* is high, there is zero option value at all durations. Intuitively, suppose that y^* is greater than the yield assumed by the tribunal to calculate the reversion value of the extension. Then, the reversion value of the lease extension calculated by the tribunal is greater than the market value. The freeholder will only require the leaseholder to pay the market value—given participation constraints the freeholder cannot force the leaseholder to pay more than the market value of extension. Beneath 80 years, the tribunal assessed value remains above the market value of the lease extension—again, by the participation constraint, the freeholder can only force the leaseholder to pay the market value. Part (1) of the proposition also shows how to detect whether the economy has zero option value everywhere—in that case prices are continuous around 80 years of duration. Note that we can extrapolate from the 80 year threshold to conclude that there is no option value at any durations, because we know the functional form of lease extension costs.

Part (2) of the proposition shows that when y^* is low, there will be positive option value when lease durations are greater than 80 years. Suppose that y^* is significantly lower than the yield used by the tribunal to calculate the reversion value of the lease extension. Then the cost paid by leaseholders to extend via the tribunal, if more than 80 years remain, is less than the market value of extension plus time and legal costs—meaning positive option value. Beneath 80 years, the tribunal assessed cost of lease extension discontinuously increases, since the tribunal assessed extension cost is now a weighted average of the reversion value and market value plus additional costs, and market value is greater than reversion value. As a result, the price of leases must discontinuously fall. Importantly, Part (2) of the proposition does not rule out zero option value for leases with less than 80 years remaining.

We use [Proposition A.1](#) to test for whether there is option value. The long-term housing yield seems to have been declining over time. As a result, our proposition suggests that there should be zero option value at all durations, only in the early part of the sample. Later on, there should be positive option value, at least for long duration leases. Our discontinuity based test confirms these predictions.

We test for holdup by estimating whether there is a discontinuity in prices at 80 years,

before and after 2010. To determine whether there is a discontinuity in property price at $T = 80$, we can estimate,

$$\frac{\Delta_h \log P_{it}^T}{h} = \alpha + \beta \cdot \mathbb{1}\text{Crossed } 80 + \Gamma_{i,t,t-h} \quad (6)$$

where the left hand side variable is the annualized log change in price of a property i between time $t - h$ and t and the right hand side variable is a dummy which checks whether i fell below 80 between time $t - h$ and t . More precisely, we say that a property crossed 80 if at time t , $T < 80$ and at time $t - h$, $T > 82$. We choose a cutoff of 82 because leaseholders cannot extend through the tribunals during the first two years of ownership, so any property purchased with less than 82 years remaining must pay discontinuously larger costs upon extension. $\Gamma_{i,t,t-h}$ are Purchase Year x Sale Year x Local Authority fixed effects. We restrict the sample to properties with duration between 70 and 90, to get the local effect around 80.

Table A.3 reports the estimated coefficient from equation (6). The first column presents estimates before 2010. There is no statistically significant discontinuity in price at 80. In fact, Appendix Figure A.28 shows that in the pre-2010 period, the estimated coefficient is approximately in the middle of a distribution of “placebo” experiments using 30 placebo cutoffs between 70-100. Column (2) of Table A.3 presents estimates after 2010. There is a significant discontinuity in the price of properties that fall below 80 years duration remaining. This fall is much greater in magnitude than any placebo test with cutoffs ranging from 70-100, shown in Appendix Figure A.28. Taken together, the price discontinuity results show that there is zero option value at all durations before 2010, and positive option value for long duration leases after 2010. However the results do not pin down whether there is zero option value for leases with less than 80 years remaining.

Table A.3: Test for Discontinuity at 80

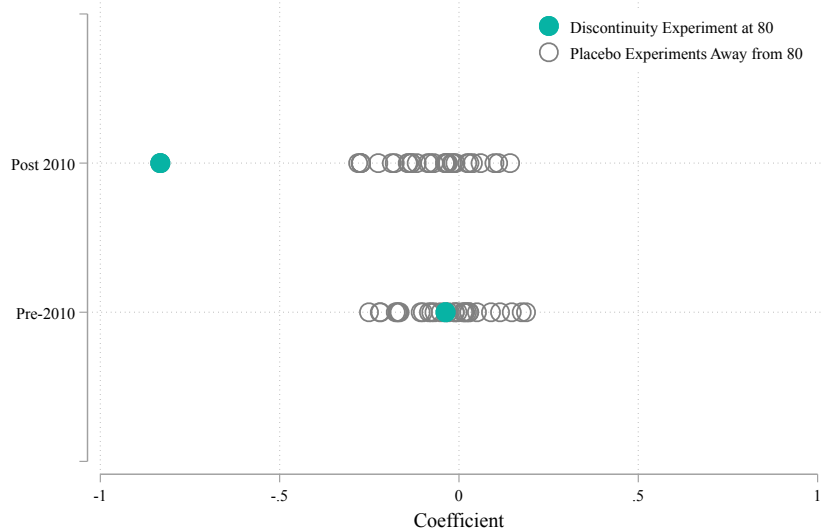
	(1)	(2)
Crossed Cutoff	-0.04	-0.83***
	(0.09)	(0.14)
Sale Year x Purchase Year x LA FE	✓	✓
Period	Pre 2010	Post 2010
N	55,325	10,153

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table provides an estimate of the discontinuity in log price at $T = 80$. The sample includes properties with duration between 70 and 90. The first column is run on the pre-2010 data, and the second column is run on post-2010 data. Standard errors are clustered at the purchase year, sale year and Local Authority (LA) level.

Proposition A.1 also suggests there should be time varying bunching of lease extensions. When y^* is relatively high, there is no gain to extending slightly before 80 years remaining; whereas when the yield is low there can be large gains to extending before 80 years. Therefore lease extensions should bunch in a time varying fashion—leases should be more likely to extend shortly before 80 years remain, only if y^* is relatively low.

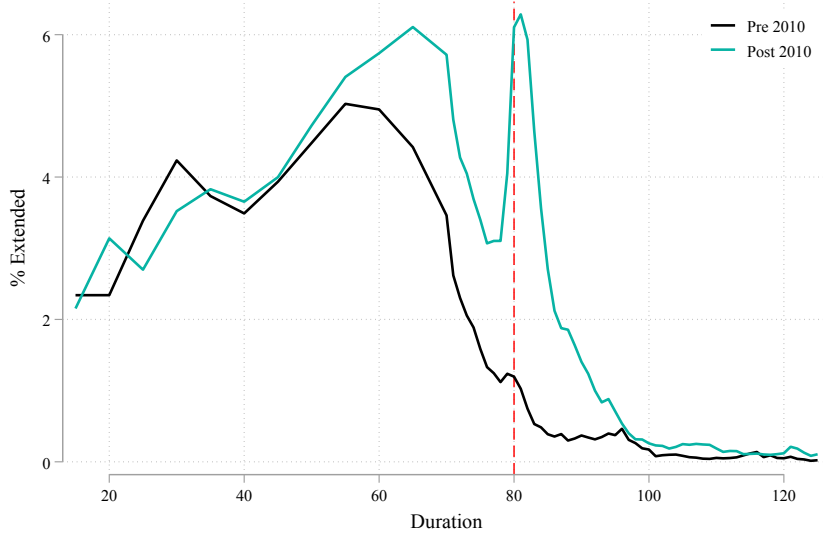
Figure A.28: Test for Discontinuity at 80 Years (Placebos)

The figure presents the estimates from **Table A.3** in red. In grey, we run 30 placebo tests using alternative cutoffs between 70-100. In each case, the sample includes properties within 10 years above and below each cutoff.

Figure A.29 shows precisely this pattern of bunching. In the figure, we plot the likelihood

that a lease extends when it has T years remaining, separately before and after 2010. Before 2010, the likelihood that a lease extends smoothly increases as leases cross the 80 year threshold. After 2010, the likelihood jumps just before 80 years, and there is a missing mass after 80 years. This time varying bunching strongly suggests that there is a difference in option value above versus below 80 years, only when y^* is low—consistent with [Proposition A.1](#).

Figure A.29: Hazard Rate of Extension, Before and After 2010



The figure shows the conditional probability of extension for each T . The black line shows the probability before 2010 and the blue line shows the probability after 2010. We exclude the post-2020 pandemic period due to disruptions to the lease registration process, which resulted in lower extension rates than usual.

Our results on option value so far imply an informative bound—the estimates of the fall in y^* from the main analysis of [Section 5](#), which assumes zero option value at all durations and times, are a lower bound on the magnitude of the true fall. Intuitively, if there is positive option value then the *true* duration of non-extended leases is higher than their *notional* duration. As such the price gain from lease extension is associated with a smaller increase in duration after lease extension. Therefore incorrectly assuming zero option value biases estimates of y^* upward. Given that option value emerges later in the sample, this bias increases over time, meaning the estimates that ignore this source of bias will under-estimate the fall in y^* . We now introduce more structure to show that this bias is small.

A.5.3 A Difference-in-Differences Estimator of Option Value

This subsection directly estimates the degree of option value before and after 2010, both above and below the 80 year threshold, and explores the implications for y^* . To do so, we introduce more structure by embedding the framework for lease extension costs into our

difference-in-differences estimator of y^* . Doing so lets us point identify option value, using a different source of variation from the discontinuity based tests of the previous subsection. With this estimator, we find that the effects of option value are small.

In order to incorporate option value and the threshold in a simple fashion, we take as given the probability that a lease of duration T extends at time t . We also assume that the share of extension value lost by leaseholders is piecewise constant in duration and discontinuous at 80 years remaining, which captures the discontinuities imposed by the tribunal. Therefore we have $\alpha_t^T = \alpha_t^H$ if $T > 80$ and $\alpha_t^T = \alpha_t^L$ otherwise. In this case, [Appendix A.16](#) shows that the difference-in-differences estimator of the price gain upon lease extension becomes

$$\begin{aligned} \underline{\Delta}_{it}^T = & \log(1 - e^{-y_t^*(T_{it}+90)}) - \log\left((1 - e^{-y_t^*T}) \right. \\ & \left. + [\Pi_{Tt}^H(1 - \alpha_t^H) + \Pi_{Tt}^L(1 - \alpha_t^L)] e^{-y_t^*T_{it}} (1 - e^{-y_t^*90})\right) \end{aligned} \quad (7)$$

For simplicity, this derivation makes the additional assumption that leases extend only once.³ In this equation, Π_{Tt}^H is the probability that a lease with $T > 80$ years remaining extends with more than 80 years remaining. Π_{Tt}^L is the probability that a lease extends with less than 80 years remaining. The cumulative probability of extension is derived from the observed extension hazard rate, as shown in [Figure A.29](#). [Equation \(7\)](#) is the same as the baseline estimator [equation \(5\)](#) either when $\alpha_H = \alpha_L = 1$, or $\Pi_H^T = \Pi_L = 0$. In either case, there is no option value from lease extension and the final term in square brackets vanishes.

We jointly estimate y_t^* , α_t^H and α_t^L , by using the difference-in-differences estimator with option value, [equation \(7\)](#), as an input into a non-linear least squares estimation similar to the baseline procedure. The two additional parameters, α_t^H and α_t^L are estimated from the covariance between T_{it} and $\underline{\Delta}_{it}^T$, i.e. between the length of time remaining before extension and the difference-in-difference estimate of the price increase from extension. Relative to the baseline estimation, we also add information on the probabilities of extension Π_H^T and Π_L , which helps to identify α_t^H and α_t^L .

The estimated y^* , α_t^H and α_t^L parameters are presented in [Table A.4](#). In the estimation, we constrain the α parameters to lie between zero and one. The results suggest, once again, that $\alpha = 1$ for all T in the pre-2010 period. In the post-2010 period, there is positive option value when leases have more than 80 years remaining, but there is zero option value below 80 years remaining.

Our difference-in-difference estimates of option value in this subsection are consistent with the discontinuity-based results from the previous subsections—even though the two

³The value of subsequent extensions, in the very far future, is quantitatively small but complicates the algebra.

Table A.4: Estimating Alpha

	(1)	(2)
y^*	4.77*** (0.08)	3.25*** (0.01)
α_t^H	1.00*** (0.01)	0.58*** (0.05)
α_t^L	1.00*** (0.01)	1.00*** (0.00)
Period	Pre 2010	Post 2010
N	18,871	110,542

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table presents estimates of α_t^H and α_t^L , estimated jointly with y^* from a nonlinear least squares estimate of [equation \(7\)](#). Estimates of α_t^L and α_t^H are constrained to lie within $[0,1]$. Standard errors are bootstrapped.

subsections use different sources of variation. In both subsections, there is zero option value for leases with less than 80 years remaining, at all times; and positive option value after 2010 for leases that extend with more than 80 years remaining. Our estimates of the change in option value before versus after 2010 for leases above 80 years, is also similar across the two subsections. In this section, we estimate a change in option value, before versus after 2010 for leases above 80 years, of 0.5. In the previous subsection we estimate a change of 0.47.⁴

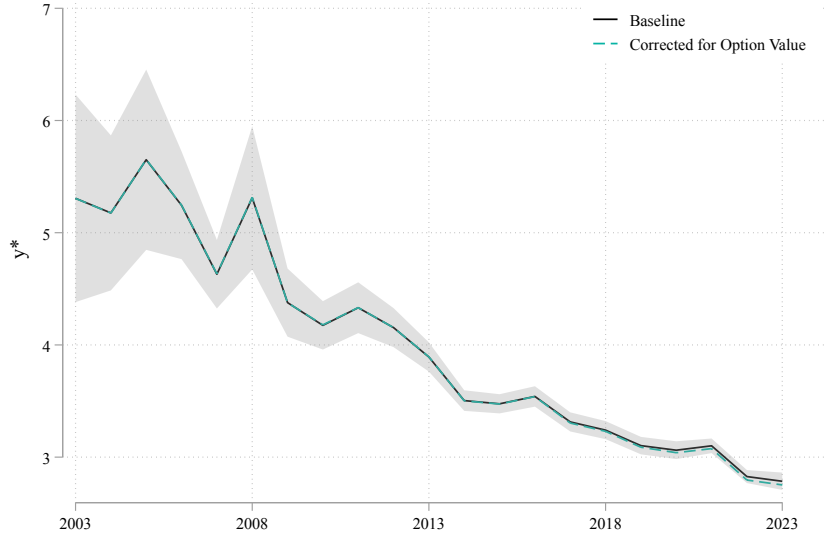
A.5.4 Estimates of y^* Corrected for Option Value

Finally, we present estimates of y^* that correct for option value using the estimates of option value. [Figure A.30](#) presents a version of our y^* timeseries which corrects for $\alpha_t^H = 50\%$ in the post-2010 period. The solid line is the estimate of y^* from [Section 5](#), using our baseline assumption of zero option value at all times and durations. The dashed line is the estimate of y^* using our estimates of the degree of option value.

The estimate of y^* that corrects for option value is very similar to the baseline estimates that assume no option value. The reason is that most leases do not extend with more than 80 years remaining, as [Figure A.29](#) shows. Therefore the possibility of extending with more than 80 years remaining has a small effect on equilibrium prices, meaning departures from zero option value are quantitatively small.

⁴The details of this calculation are presented in [Appendix A.5.5](#).

Figure A.30: Estimate of y^* Correcting for Option Value



The figure presents a corrected y^* timeseries, adjusting for the fact that $\alpha_t^H = 0.69$ for the post-2010 period. The black line presents the unadjusted estimates from [Section 5.3](#).

A.5.5 Estimating Change in Option Value Using Discontinuities

This subsection briefly presents our estimates of how option value changes, after 2010 and for leases with more than 80 years remaining, using the discontinuity based estimator from [Section A.5.2](#).

As before, assume that $\alpha_t^H = \alpha_t^T$ for $T \geq 80$ and $\alpha_t^L = \alpha_t^T$ for $T < 80$, such that the option value in a given time period t is fixed above and below 80, separately. In this section, we aim to estimate $\alpha_t^L - \alpha_t^H$ for the post-2010 period, using the discontinuity in prices at $T = 80$ observed in [Table A.3](#). From the preceding subsection, the price of a property i with duration T is

$$P_{it}^T = \frac{R_{it}}{y^*} (1 - e^{-y^*T} + [\Pi_{Tt}^H (1 - \alpha_t^H) + \Pi^L (1 - \alpha_t^L)] e^{-y_t^* T_{it}} (1 - e^{-y_t^* 90})) \quad (8)$$

To condense notation, denote the option value term

$$\Omega(T) \equiv [\Pi_{Tt}^H (1 - \alpha_t^H) + \Pi^L (1 - \alpha_t^L)] e^{-y_t^* T_{it}} (1 - e^{-y_t^* 90}).$$

Then, the difference in change in price between time $t - h$ and t of properties i and j is,

$$\begin{aligned} \Delta \log P_{it}^T - \Delta \log P_{jt}^T &= \log(1 - e^{-y^*T_i} + \Omega(T_i)) - \log(1 - e^{-y^*(T_i+h)} + \Omega(T_i + h)) \\ &\quad - (\log(1 - e^{-y^*T_j} + \Omega(T_j)) - \log(1 - e^{-y^*(T_j+h)} + \Omega(T_j + h))). \end{aligned}$$

This equation acknowledges that option value will discontinuously change around the 80 year threshold, via changes in the Ω terms. We can then estimate α_t^H by nonlinear least squares on the same sample as for regression [equation \(6\)](#), by setting $\alpha_t^L = 1$, which is what we estimate in [Table A.4](#), and y^* to its mean for that period. We obtain an estimate of $\alpha_t^H = 0.53$ in the post-2010 period. Overall, using discontinuities for variation, we identify a similar change in option value to the difference-in-differences estimator of [Section A.5.3](#).

A.5.6 The 2006 Sportelli Case

In [Appendix A.5](#), we have presented the asset pricing formula used by British property tribunal courts to price lease extensions, which utilizes a court-determined discount rate that we denote r_{RV} . Following the September 2006 *Cadogan v. Sportelli* case, and the subsequent October 2007 appeals case in the Court of Appeals, British property courts decided to set a fixed discount rate of $r_{RV} = 5\%$. Therefore, the decline in our estimator in the subsequent period cannot be attributed to arbitrary changes in the tribunal discount rate, and instead reflects changes in the market's expectations of long-run interest rates. Before the Sportelli decision, freeholders and leaseholders would negotiate the discount rate r_{RV} in court, which was not fixed at any particular value, but on average was approximately 6% in years immediately before the Sportelli case. Given our estimates of $\alpha_t^T = 1$ from [Appendix A.5](#), which imply that there was no option-value from extending for leaseholders during this period, the Sportelli case should not have had an effect on leasehold market prices, and should therefore have no effect on our estimates.

We can verify this empirically by examining the price and extension rate of leaseholds before and after the 2006 Sportelli case. In particular, we will compare the price and extension rate around the cutoff of $T = 80$, which as we saw in [Appendix A.5](#), is informative as to the difference between the market value of lease extension and the court dictated outside option. First, in [Figure A.31](#), we plot the extension hazard rate before and after 2006. There is no bunching around 80 in either of these two periods, suggesting that the Sportelli case had no effect on the option-value from extension for leaseholders. Moreover, [Table A.5](#) indicates that there is no significant discontinuity in price around $T = 80$ both before and after 2006, again implying that the Sportelli case did not significantly affect the actual cost of extension. In both [Figure A.31](#) and [Table A.5](#), we include the post-2010 period for reference, which

does experience significant bunching and a discontinuity in price at $T = 80$.

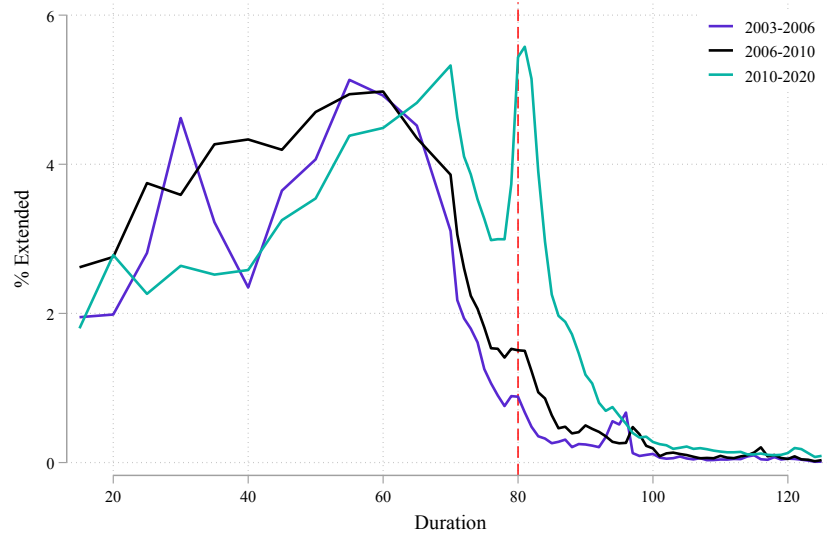


Figure A.31: Hazard Rate of Extension, Three Periods

The figure shows the conditional probability of extension for each T . The purple line shows the probability from 2003-2006, the black line shows the probability from 2006-2010 and the blue line shows the probability after 2010. We exclude the post-2020 pandemic period due to disruptions to the lease registration process, which resulted in lower extension rates than usual.

Table A.5: Test for Discontinuity at 80, Three Periods

	(1)	(2)	(3)
Crossed Cutoff	-0.09 (0.08)	0.18 (0.12)	-0.83*** (0.14)
Sale Year x Purchase Year x LA FE	✓	✓	✓
Period	2003-2006	2006-2010	2010-2023
N	41,854	13,471	10,153

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table provides an estimate of the discontinuity in log price at $T = 80$. The sample includes properties with duration between 70 and 90. The first column is run on 2003-2006 data, the second column is run on 2006-2010 data, and the third column is run on 2010-2023 data. Standard errors are clustered at the purchase year, sale year and Local Authority (LA) level.

The small effect of the Sportelli case on leasehold prices and extension rates around the $T = 80$ cutoff provides additional evidence that in the pre-2010 period, the full value of extension was held by the freeholder. The tribunal discount rate fell from an average of about 6% before the Sportelli decision to 5% after. Therefore, by [equation \(2\)](#) and [equation \(4\)](#),

we would expect for the Sportelli case to increase α_t^T , resulting in a shift in bunching and prices, if and only if $\alpha_t^T < 1$. The only scenario in which bunching and prices would not change as a result of a decline in r_{RV} is if leaseholds were already priced at their market value in the first place. The lack of a significant change in either bunching or prices in the aftermath of *Cadogan v. Sportelli* therefore suggests that $\alpha_t^T = 1$ for this entire period.

A.5.7 Leasehold Reform

In recent years, there have been some efforts in England and Wales to reform the process of purchasing and extending leaseholds. In 2017, the government published a [white paper](#) promising to “promote fairness and transparency for the growing number of leaseholders.” Here the focus was primarily on ground rent fees that leaseholders must pay to freeholders, which as we have discussed are for the most part negligible, but in some rare instances may be onerous. More recently, there has also been a movement to reduce the legally recommended cost of extension. In 2024, the [Leasehold and Freehold Reform Act](#) expressed a commitment to reduce the cost of lease extension. However, the most significant provisions of the act have not yet been implemented “will need to be commenced through an extensive programme of secondary legislation.” Due to legislative inertia and freeholder backlash, it is unclear when and to what degree reform will be implemented.

We now show that expectations of leasehold reform bias our estimate of y^* upwards. Recall that we assume a no arbitrage condition, that

$$P_{it}^{T+90} = P_{it}^T + \kappa_{it}^T.$$

That is, the price of an extended property P_{it}^{T+90} must equal the price of a property that has not extended, P_{it}^T , plus extension costs that leaseholders expect to pay, κ_{it}^T . The no arbitrage condition is natural: if it were cheaper to buy a property and extend, than to own an extended property, then demand would be higher for properties that have not been extended. Rearranging the no arbitrage condition, we have

$$P_{it}^{T+90} - P_{it}^T = \kappa_{it}^T.$$

We can now investigate leasehold reform. Leasehold reform proposes to lower extension costs κ_{it}^T , which by the no arbitrage condition leads the price increase from extension $P_{it}^{T+90} - P_{it}^T$ to fall as well. Our estimate of y^* increases when $P_{it}^{T+90} - P_{it}^T$ falls. If the price increase from extension has fallen, our procedure infers that the value of duration has fallen, meaning that the estimate of y^* goes up. Altogether, leasehold reform implies lower extension costs,

meaning a smaller price gain from extension and as a result a higher estimate of y^* .

A.6 DiD Estimator with Stamp Duty Land Tax

This section shows that multiplicative taxes on value, such as the Stamp Duty Land Tax, do not affect the difference in difference estimate. In the data, we observe prices paid for properties, which do not contain stamp duty. However, the relevant price for agents is the price including tax. We will now show that because the tax is multiplicative, the estimating equation (3) identifies price growth including taxes—i.e. the relevant economic concept.

Suppose that prices including taxes are $\tilde{P}_{it}^{T+h} = (1 + T_t)P_{it}^{T+h}$, where T_t is the stamp duty tax and P_{it}^{T+h} is the price excluding tax. We would like to estimate the growth in prices including taxes for the extending property versus its control, i.e.

$$\underline{\Delta}_{it} \equiv \left[\log \tilde{P}_{it}^{T+90} - \log \tilde{P}_{i,t-h}^{T+h} \right] - \left[\log \tilde{P}_{jt}^T - \log \tilde{P}_{j,t-h}^{T+h} \right].$$

Substituting the definition of prices including taxes into the formula implies

$$\begin{aligned} \underline{\Delta}_{it} &\equiv \left[\log \tilde{P}_{it}^{T+90} - \log \tilde{P}_{i,t-h}^{T+h} \right] - \left[\log \tilde{P}_{jt}^T - \log \tilde{P}_{j,t-h}^{T+h} \right] \\ &= \left[\log (1 + T_t) P_{it}^{T+90} - \log (1 + T_{t-h}) P_{i,t-h}^{T+h} \right] - \left[\log (1 + T_t) P_{jt}^T - \log (1 + T_{t-h}) P_{j,t-h}^{T+h} \right] \\ &= \left[\log P_{it}^{T+90} - \log P_{i,t-h}^{T+h} \right] - \left[\log P_{jt}^T - \log P_{j,t-h}^{T+h} \right] \\ &\quad + (\log (1 + T_t) - \log (1 + T_{t-h})) - (\log (1 + T_t) - \log (1 + T_{t-h})) \\ &= \left[\log P_{it}^{T+90} - \log P_{i,t-h}^{T+h} \right] - \left[\log P_{jt}^T - \log P_{j,t-h}^{T+h} \right]. \end{aligned}$$

The final line is the difference-in-differences estimate from the data, i.e. price growth excluding taxes. As such, price growth excluding taxes identifies the object of interest, that is, price growth including taxes. The reason is that the stamp duty tax is multiplicative. Therefore in log terms, the price change for the treatment versus control “differences out” the effect of the tax.

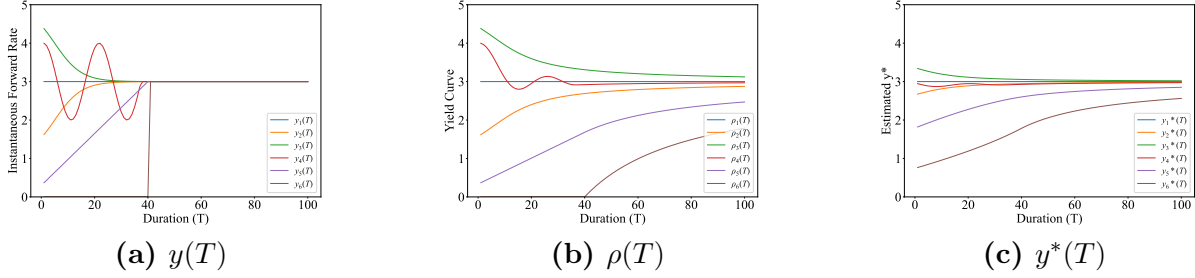
A.7 Simulation Results For Flexible Forward Curve

In [Section 4.2](#) we presented one possible parameterization of $y(T)$. In this section, we explore other parameterizations and present several insights from our simulation. We assume that for $T \geq 40$, $y(T) = y^*$ is constant. Hence, for $T < 40$ we assume $y(\cdot)$ can have any shape as long as it asymptotes to y^* as $T \rightarrow 40$.

[Figure A.32a](#) shows several possible choices of $y(T)$, all of which asymptote to the same value. [Figure A.32b](#) presents the yield curves associated with each of these forward curves,

which we denote by $\rho(T)$; and Figure A.32c shows the corresponding $\hat{y}^*(T)$ curves that we estimate by equation (6) using simulation data. We also plot the point estimate of $\hat{y}^*$ we obtain at the median of our true distribution.

Figure A.32: Long Run Discount Rates Using Simulation Data



Panel (a) presents multiple possible parameterizations of $y(T)$, all of which asymptote to 3% by $T = 40$. Panel (b) presents the corresponding yield curve, $\rho(T)$, for each choice of $y(T)$. Panel (c) presents estimates of $y^*(T')$ obtained by NLLS corresponding to each choice of $y(T)$, where T' is the average between the control and extension sale duration. The point presents the estimate of y^* we would obtain at the median of our true distribution.

These simulation results yield several key insights. First, when the yield curve is flat, $y(T) = \rho(T) = \hat{y}^*(T) = y^*$, as exemplified with the by the dark blue line in Figure A.32. When the yield curve is not flat, however, our estimate will differ from the true asymptotic value of $y(T)$ by some amount $\eta \equiv y^* - \hat{y}^*$. When the yield curve is upwards sloping, $\eta > 0$ and when the yield curve is downwards sloping, $\eta < 0$.

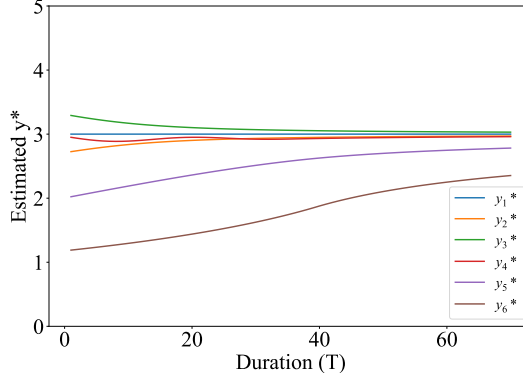
Notice that for $T \geq 40$, $\hat{y}^*(T)$ converges to y^* much more quickly than $\rho(T)$. The reason for this is that our difference-in-difference estimate differences out a large portion of the short-end of the yield curve. To see this, consider a property with duration T that extends by k years to a total of 160 years ($T + k = 160$). The shorter T is, the less of the short-end that will be differenced out by our estimate. We present simulation evidence for this in Figure A.33. For this reason, it is important that our experiments take the difference between two long-duration properties, as opposed to one long duration property and one short duration property.

A.8 Repeat Sales Index Methodology

This subsection explains the details of how we estimate the repeat sales index for the control group associated with a given extending property i .

First, we estimate a Bailey, Muth and Nourse (1963) repeat sales index for the control

Figure A.33: Estimated y^* When Extending From T to 160



The figure indicates the point estimate of $\hat{y}^*$ we obtain by NLLS for an extension from duration T to duration 160. As the duration before extension increases, the estimate of $\hat{y}^*$ approaches the limit of the forward curve. We repeat this for each example forward curve, $y(T)$, from Figure A.32a.

group of each treated property i . We fit a regression

$$\log P_{jt'} - \log P_{jt} = \alpha + \sum_{s=t_0+1}^{t_1} \gamma_s D_s + \varepsilon_{j,t,t'} \quad (9)$$

for each property j in i 's control group, where D_s is a dummy variable that is equal to 1 if $s = t'$, -1 if $s = t$, and 0 otherwise. t_0 is the first date for which a control property is available and t_1 is the last date. The coefficients γ_s capture the estimated change in log price since time t_0 . We include a constant, α , in the estimation equation as in ?.

As in Case and Shiller (1989), we correct for the effects of heteroskedasticity by allowing the error term to be related to the interval of time between sales, $t' - t$, and also to the (Haversine) distance, d_j , between the treated property i and the control j . We estimate,

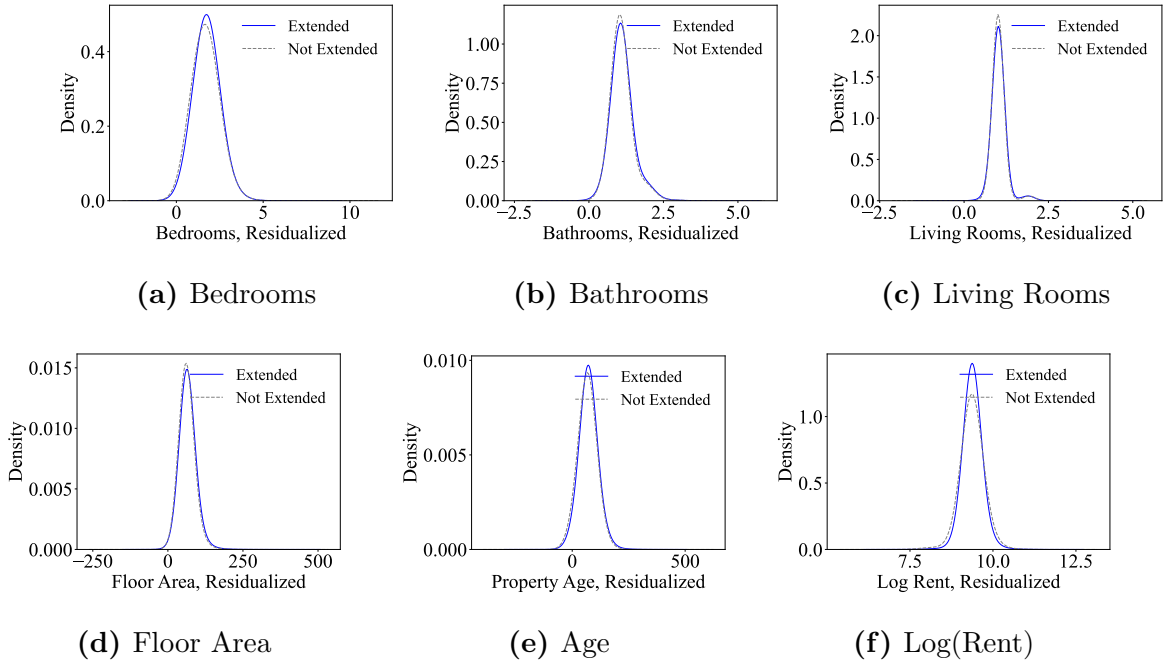
$$\hat{\varepsilon}_{j,t,t'}^2 = \beta_0 + \beta_1(t' - t) + \beta_2 d_j + \mu_{j,t,t'} \quad (10)$$

where $\hat{\varepsilon}_{j,t,t'}$ are the predicted residuals from the regression in equation (9). Figure A.12 presents binscatters of $\hat{\varepsilon}_{j,t,t'}$ against $t' - t$ and d_j , showing that both are positively correlated. We estimate equation (10) for all properties together to reduce noise. Then, we calculate weights, $w_{j,t,t'} = 1/(\hat{\beta}_0 + \hat{\beta}_1(t' - t) + \hat{\beta}_2 d_j)$, equal to the inverse predicted values from equation (10). The last step is to rerun equation (9) using these weights.

A.9 Balance Test

In this subsection, we show that extended and control properties are balanced on observable characteristics. First, we regress our main hedonic variables—bedroom number, bathroom number, living room number, floor area, age, and rental price—on an experiment $\times$ year of listing fixed effect. We plot the density distribution of the residuals for extended and control properties separately in [Figure A.34](#). The distributions are visually very similar.

Figure A.34: Hedonics Residuals Density Curve for Extended and Non-Extended Properties



The figures show the distribution of residuals for extended and non-extended flats after regressing hedonic characteristics on a experiment $\times$ year of listing fixed effect. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set.

The average differences between treatment and control are also very small. For instance, the mean difference in number of bedrooms between extended and control properties is 0.08 bedrooms, a tiny difference to the average bedroom count of 1.8. The mean difference in floor area is 3.5 sq. meters, compared to an average floor area of 67 sq. meters. As a whole, rental prices, which ought to measure the quality of the property, are about 2% higher in extended properties, or about £20 per month. These differences are even smaller if we consider properties extended for 90 years. Importantly, what matters for our identification assumption is whether extended properties *change* their hedonic characteristics more than their controls, which we test for in [Section 4.4](#). Therefore, even if there were economically

significant in differences in the level of these hedonic variables across extended and control properties, our identification assumption still holds.

A.10 Liquidity Premium

One institutional factor which could raise concerns about our estimator is the difficulty for owners of short leases to obtain a mortgage. If short leases have more limited access to financing than longer leases, we might worry that part or all of the observed price gain upon extension is a result of increased access to financing opportunities; in other words, we may wonder if the effect is driven by a “liquidity premium.” Indeed, important lenders, such as Barclays, Halifax and The Co-Operative Bank, refuse to lend to leaseholds with less than 70 years remaining. Others have different thresholds, such as 55 years, and some have a preference for longer leases but allow for case-by-case exceptions.⁵

Reassuringly, using detailed micro-data from the English Housing Survey 1993-2019, we find that mortgage access and conditions are not vastly different for shorter and longer leaseholds, especially for those with more than 30 years remaining, as indicated in [Table A.6](#). Approximately 60% of short (under 80) duration leaseholds were purchased with a mortgage, relative to 58% of longer (over 80) duration leaseholds. The typical mortgage length and Loan-To-Value (LTV) ratios are similar across the duration spectrum, at around 23 years and 75-80%, respectively. Additionally, short leaseholds have similar interest rate types as long leaseholds, with around 30% choosing adjustable rate mortgages (as opposed to mortgages with a fixed interest rate for a number of years, or tracker mortgages which are indexed to the Bank of England bank rate). These results suggest that financing constraints are unlikely to drive the very large extension price changes we observed in [Section 5](#).

Another common way to test for the existence of a liquidity premium resulting from financing frictions is to use the amount of time a property was listed on the market (i.e. sale time minus the time of the first listing) as a proxy for its liquidity (???). The intuition is that properties for which the buyer cannot obtain a mortgage have a smaller pool of potential buyers, which ought to increase the amount of time that the property is on the market. We find limited evidence of this in the data, as illustrated in [Figure A.35](#). After controlling for quarter $\times$ 3-digit postcode fixed effects and hedonics, the typical listing time for a 50-70 year lease is only 3-4 days longer than for a long lease of more than 100 years. This is negligible given that the average listing time is 5 months. For shorter leases, the listing period actually decreases further; all else equal, a leasehold with less than 50 years remaining will sell about a week faster than a leasehold with more than 50 years.

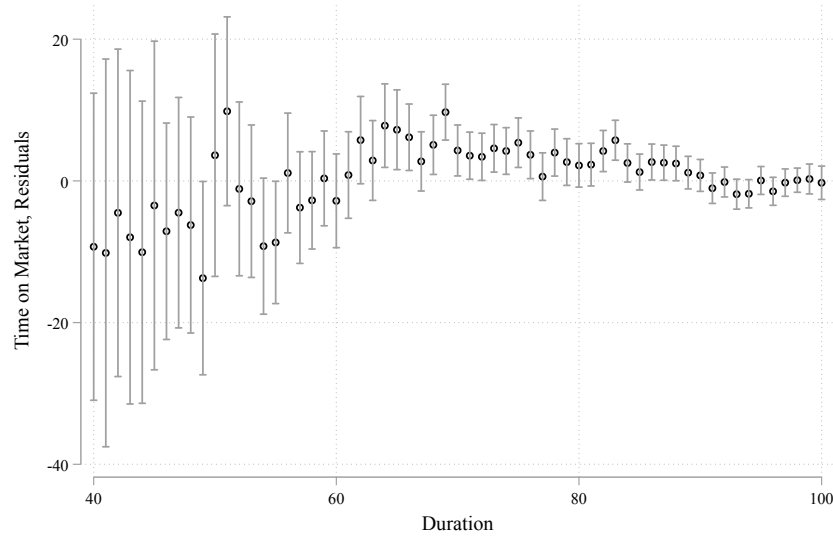
⁵A comprehensive list of lease length policies for banks in England can be found in the [UK Finance Lenders’ Handbook For Conveyancers](#).

Table A.6: Mortgage Statistics For Short Leaseholds

	Less Than 50 Years	50-60 Years	60-70 Years	70-80 Years	80-99 Years	100+ Years	Total
Mortgage Length	22.1 (0.6)	22.1 (0.5)	23.9 (0.5)	23.0 (0.3)	23.9 (0.2)	23.1 (0.1)	23.3 (0.1)
LTV	76.3 (3.3)	80.8 (2.6)	81.4 (1.8)	77.7 (1.7)	73.3 (1.0)	76.5 (0.6)	76.2 (0.5)
% Have Mortgage	59.9 (2.4)	60.4 (2.4)	62.1 (1.6)	58.1 (1.4)	63.9 (0.8)	55.6 (0.5)	58.2 (0.4)
% Adjustable Rate	24.2 (5.3)	40.0 (5.2)	38.3 (4.1)	32.8 (3.4)	25.3 (1.5)	31.0 (1.0)	30.2 (0.8)
N	18,292						

mean reported; standard error of mean in parentheses

The table reports several summary statistics for leases of various durations. The first row presents the average mortgage length; the second presents the average Loan-To-Value (LTV) ratio for the mortgage, calculated as the initial mortgage value divided by the market price of the property; the third row presents the percent of properties of that duration which have a mortgage; the fourth row presents the percent of properties with a mortgage that have a fully adjustable rate mortgage.

Figure A.35: Time on Market by Lease Duration

The figure shows the mean time on market for every duration under 125, de-measured by quarter $\times$ Local Authority fixed effects and controls for bedroom count, floor area and property age.

To further test for the existence of a liquidity premium, we are able to model the case of discontinuous financing frictions and reject the existence of a liquidity premium using the data. Consider that under the threshold of 70 years—which is the most prominent bank mortgage cutoff duration—it becomes significantly more difficult to finance a leasehold property.

Then the price of a $T > 70$ duration leasehold is given by,

$$P_t^T = R_t \left[\int_t^{t+(T-70)} e^{-(y^*)(s-t)} ds + e^{-(y^*)(T-70)} \int_{t+(T-70)}^{t+T} e^{-(y^*+\sigma)(s-(t+(T-70)))} ds \right]$$

$$= R_t \left[\frac{1 - e^{-(y^*)(T-70)}}{y^*} + \frac{(1 - e^{-(y^*+\sigma) \cdot 70})}{y^* + \sigma} (e^{-(y^*)(T-70)}) \right]$$

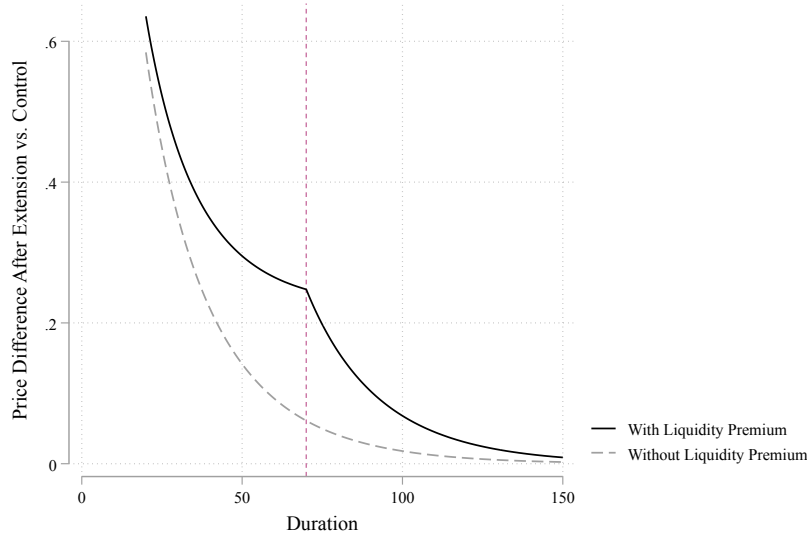
where rents below the cutoff of 70 are discounted at an additional rate σ . One way to think about σ is as the difference between the return on housing and outside investment options. Therefore, our difference in difference will yield the following equation:

$$\log P_t^{T+90} - \log P_t^T = \log \left[\frac{1 - e^{-(y^*)(T+90-70)}}{y^*} + \frac{1 - e^{-(y^*+\sigma)70}}{y^* + \sigma} (e^{-(y^*)(T+90-70)}) \right]$$

$$- \log \left[\frac{1 - e^{-(y^*)(\max\{0, T-70\})}}{y^*} + \frac{1 - e^{-(y^*+\sigma)(\min\{70, T\})}}{y^* + \sigma} (e^{-(y^*)(\max\{0, T-70\})}) \right] \quad (11)$$

When $\sigma > 0$, the price change for extending a lease will have a kink at 70 as shown in [Figure A.36](#). The reason for this is that as $T \rightarrow 70$, there are two incentives to extend: first is the value of 90 additional years of discounted rents and the second is the value of postponing the liquidity discount, σ . Once $T < 70$, the first incentive continues to grow, as we have discussed in earlier sections, but the second incentive becomes increasingly weaker, since there are less periods at which rents will be discounted at rate σ . Moreover, as $T \rightarrow 0$, $T + 90$ grows closer to 70, so the value of the extended lease also starts to decrease as it approaches the liquidity premium cutoff. This kink is not present in the data, which can be verified visually in [Figure 7](#). The existence of a discontinuous liquidity premium at $T = 70$ can be formally rejected by estimating [equation \(11\)](#) by NLLS, which uses variation in $\underline{\Delta}_{it}^T$ for different values of T to test for the existence of a liquidity premium.

Figure A.36: Liquidity Premium Example



The figure shows the effect of a liquidity premium on the value of extending for 90 years ($P_t^{T+90} - P_t^T$). The dark line plots equation (11) when $\sigma = 1\%$ and the dashed light line plots the same equation absent a liquidity premium, i.e. $\sigma = 0\%$. The liquidity premium is assumed to start at $T = 70$. We can see that when there is a liquidity premium, the value of extension will exhibit a kinked shape, with a kink at $T = 70$.

A.11 House Price Seasonality

An important feature of the UK housing market, as well as in other countries including the United States, is that it is highly seasonal, with systematically higher sale prices in the second and third quarter (the “hot months”) and lower prices in the first and fourth quarters (?). This may be associated with a number of factors, including mortgage conditions and market tightness, which we would not expect to affect the long-run and therefore ought to be differenced out by our estimator.

To determine whether our estimates are seasonal, we must select control properties which transact in the same quarter as extended properties. Then, we can estimate y_t^* at a quarterly frequency, as described in the main text. We then test for seasonality by regressing our time series of y_t^* on quarter dummy variables, controlling for year fixed effects. The regression results are presented in Table A.7, which indicate that there is no statistically significant difference in the level of y_t^* across quarters.

A.12 Bootstrap Standard Errors

Our results report heteroskedasticity consistent standard errors. One concern is that our difference-in-difference estimator uses a repeat sales index. The repeat sales index is in

Table A.7: Test for Estimate Seasonality

	(1)
2nd Quarter	-0.13 (0.17)
3rd Quarter	0.03 (0.21)
4th Quarter	-0.08 (0.15)
Year FE	✓
N	84
Standard errors in parentheses	
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$	

The table indicates the regression results for a regression of quarter dummies on y_t^* . We control for year fixed effects. Standard errors are heteroskedasticity robust.

effect a “generated regressor”, and our standard errors do not account for the extra variance. To incorporate the variance of the repeat sales index, this section implements a Bayesian bootstrap procedure. We find the increase in the standard errors is minimal.

In our setting, there are some complications to implementing a traditional bootstrap. As we resample for the bootstrap, the composition of the repeat sales index changes, and we cannot always build a repeat sales index for all extending properties. To elaborate on the challenge, let i be an extended property and let C_i be the set of controls for this property according to the baseline methodology. The controls are within radius d_i of the property. Let $C = \cup_i C_i$ be the union of all the controls and let C^b be a bootstrap sample of C , where we sample with replacement. Then the bootstrap set of controls for i , C_i^b , will necessarily have weakly fewer distinct properties than C_i . To generate a repeat sales index for i , we need C_i to contain properties that are sold at connected dates, as we described in the main text. For example, if i is transacted in 2000 and 2010, a set of connected dates would be controls that sell at $a = (2000, 2003)$, $b = (2003, 2007)$ and $c = (2007, 2010)$, where $C_i = \{a, b, c\}$. However, if when we resample $b \notin C_i^b$, then the experiment dates are no longer connected and so we cannot build a repeat sales index for i using the same control properties.

To address this, we utilize a Bayesian bootstrap procedure (?). Instead of resampling with replacement, which is effectively equal to assigning integer weights to an operation, the Bayesian bootstrap assigns non-integer weights sampled from a Dirichlet distribution. The Bayesian bootstrap has the same asymptotic behavior as the traditional bootstrap but has the added advantage that no observations receive zero weight. This allows us to circumvent the problem described above.

In particular, we conduct a two-stage Bayesian bootstrap, bootstrapping independently over the treated and control properties in our sample. We conduct 100 bootstrap iterations, in each of which we resample the weights on the OLS regressions used to estimate the repeat sales index as well as the weights on the NLLS regression used to estimate y^* . Thus, we can capture variation across experiments and within the experiment, across components of the repeat sales index. The bootstrap standard error is 0.016pp, compared to 0.013pp under our baseline methodology. Therefore, our estimates of y^* continue to be highly precise, even adjusting for the variability in the repeat sales methodology.

A.13 Components of y^* for UK Housing—A VAR Analysis

This section uses a VAR to estimate long run risk premia and expected dividend growth for UK housing, in order to show that neither of these can account for the trend fall in the long-term yield for UK housing that we have documented. We stress that this exercise is tentative, given the inherent uncertainty of any VAR based procedure.

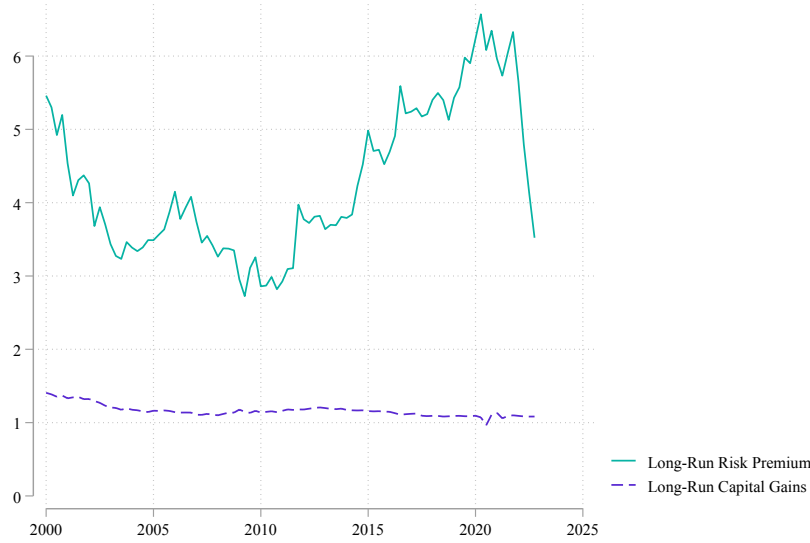
To estimate risk premia and dividend growth, we follow the standard VAR approach, with growth rate of rents, the rent to price ratio, and the GDP growth rate as inputs. To estimate long run expected dividend growth, in every quarter, we estimate forecasts of the rental growth for the subsequent 30 years and take the mean, recalling that long run expected rent growth is equal to long run expected dividend growth. To estimate long run housing risk premia, we calculate the 30-year ahead predicted Rent-to-Price ratio (again using forecasts from the VAR). Then $\zeta = [\text{Predicted RTP}] + g - r$, where r is the 10 year 15 year real forward rate.

Figure A.37 displays the results. The estimates for long run expected dividend growth, g_P^* , are stable. Estimates of the long run housing risk premium, ζ^* , have been rising in the post-Great Recession era. Therefore the estimates suggest that neither dividend growth nor risk premia for UK housing can account for the trend fall in the long-term housing yield, $y^* \equiv r^* + \zeta^* - g_P^*$, that we have documented.

A.14 Housing Supply Elasticity Instrument

In this section, we provide more details on the housing supply elasticity instrument developed by Hilber and Vermeulen (2016). As described in the main text, Hilber and Vermeulen (2016)’s primary measure of supply inelasticity is the average refusal rate of major construction applications for each Local Authority, average over a nearly thirty year period. The problem with this measure is that the housing application rate is endogenous. More restrictive Local Authorities will on average receive less applications if developers expect that their

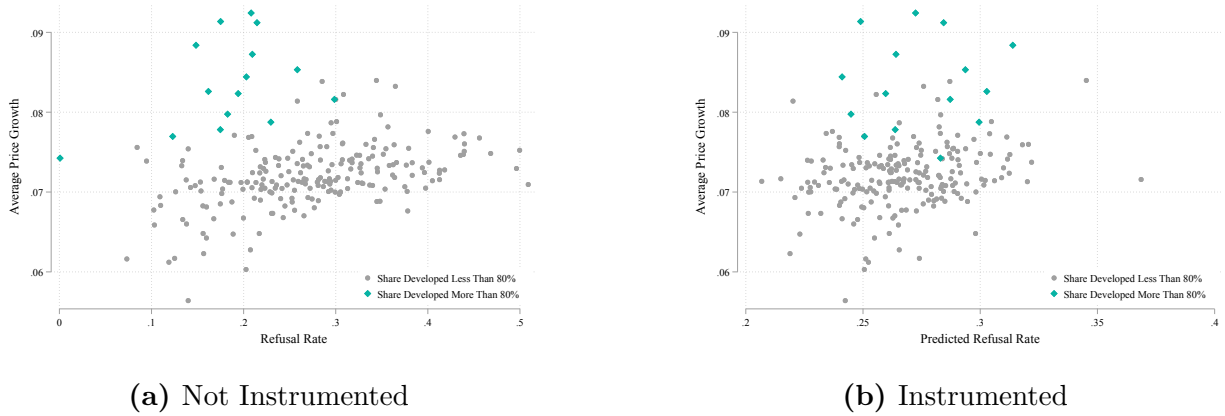
Figure A.37: Risk Premia and Dividend Growth for UK Housing in the Long Run



The figure presents estimates of long-run housing risk premia and dividend growth estimated using a VAR. Estimates are produced using data on the UK rent-to-price ratio from the OECD Housing Price Index, as well as the mean level of house prices from Land Registry data and the mean level of rents from the Valuation Office Agency. UK GDP data is obtained from the St. Louis Fed, Federal Reserve Economic Data and long-run UK interest rates are obtained from the Bank of England.

applications will be refused. This in turn lowers the measured refusal rate.

Figure A.38: Endogeneity of Application Refusal Rate



This problem is evident in [Figure A.38a](#), where we plot average price growth over the period of 1974-2022, a measure which we expected to be very correlated with housing supply elasticity, against the average refusal rate measures. In blue, we mark the Local Authorities for which more than 80% of land is developed, all of which are in the three most central Tube zones of London. More than 60% of these LAs are in “Central Activities Zone,” London’s very dense touristic and cultural center. The application refusal rates in these

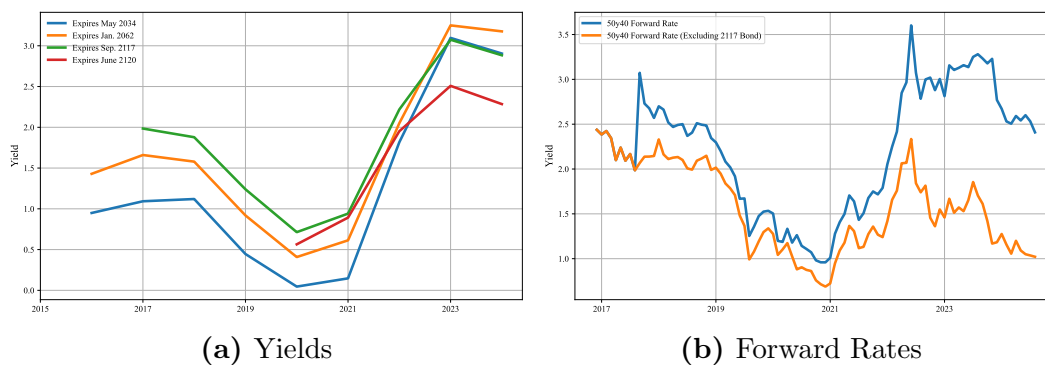
LAs are somewhat below average, probably because many developers in these areas do not take the time to file applications which almost certainly will be refused. On aggregate, the implied relationship between the refusal rate and house price growth is weakly positive and statistically insignificant. However, if we instrument the refusal rate using the delay rate instrument introduced by Hilber and Vermeulen (2016), the slope coefficient is nearly an order of magnitude larger and statistically significant.

A.15 Dynamics of Long Term Bond Yields

This subsection shows that very long term safe interest rates measured from other sources seem to have fallen. However there is uncertainty due to liquidity issues with the bonds. We study two data sources.

First, we study long-term Austrian bonds. These data begin in 2017, meaning they are not informative about the long-run decline in yields. The data do provide information about the recent rise in rates. In particular, the Austrian government has issued two 100 year bonds, the first expiring in 2117 and the other in 2120. The Austrian government has also issued several other long-term bonds with terms of 20-70 years, in addition to traditional shorter-term bonds. The yields for some of these bonds are plotted in Figure A.39a. A discrepancy emerges — the yield for the bond expiring in 2120 has risen much less than that expiring in 2117. This gap is surprising. Taken literally, there must have been enormous movements in the 3 year forward rate starting in 2117, which seems unlikely.

Figure A.39: Yield on Long-Run Austrian Bonds



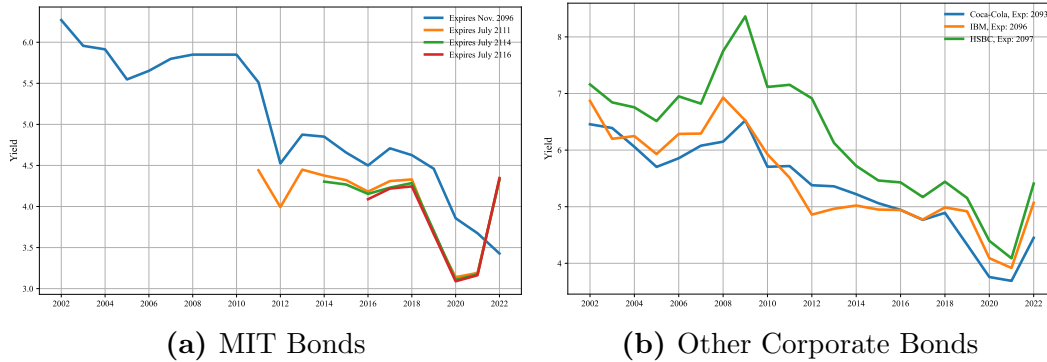
We attempt to calculate r^* for Austrian bonds, that is, the long run expected forward rate. We use observed prices for bonds of different maturities and assume the spot curve is piecewise linear between each maturity, while taking into account differences in coupon rates across bonds. The implied rise in 50 to 90 year forward rate depends on whether we include or exclude the 2117 bond.

We plot r^* for Austrian bonds in Figure A.39b. If we use data for the 2120 bond, long-run forwards have not changed much relative to 2020. Using data from the 2117 bond suggests that yields are close to their 2017 level. Even using the 2117 data, the increase in the yield is much less than at shorter horizons. For example, at the end of 2024 the US 10 year bond rate was over 2pp higher than its 2017 level.

Overall, we believe the behavior of Austrian bonds are broadly consistent with our evidence that r^* , at the very long end of the yield curve, has risen by less than at shorter horizons.

Second, we investigate data on over 70 corporate bonds with 100-year terms. These bonds include the MIT bond that you identify, but also long-run bonds for Coca Cola, IBM, HBSC, and a variety of other firms across several industries. Nearly all of the bonds we surveyed experienced consistent declines in yields between 2010-2021 of magnitudes similar to what we observe for y^* . For example, Figure A.40a presents yields for the four 100-year bonds MIT has issued in the last two decades, which expire in 2096 through 2116. The Nov. 2096 bond was first issued in 1996, and we have data on yields starting in 2002. We can see that since 2010, yields have declined by 2-3pp, similar to what we see for y^* . We see similar patterns for other bonds, as shown in Figure A.40b.

Figure A.40: Yield on Long-Run Bonds



One important shortcoming of this approach relative to our y^* estimates is that there is typically not sufficient variation in bond terms within firms to calculate spot or forward curves. Therefore, we cannot make claims about long-run forward rates without making additional assumptions.

A.16 Difference-in-Differences Estimator with Option Value

This section derives our differences-in-differences estimator of y^* in the presence of option value from lease extensions. Let Π_{Tt}^H be the likelihood that a lease, with $T > 80$ years of

duration remaining, extends before its duration reaches 80 years. Let Π_{Tt}^L be the likelihood that a lease with $T \leq 80$ years of duration remaining is extended at some point before expiration. Assume a constant discount rate y^* , and also assume that the event of extending is uncorrelated with the stochastic discount factor of the extender. Then the price of a leasehold is the present value of its cashflows, i.e.

$$P_t^T = \int_0^T e^{-y^*s} R_{t+s} ds + \Pi_{Tt}^H (1 - \alpha_t^H) \int_T^{T+90} e^{-y^*s} R_{t+s} ds + \Pi_{Tt}^L (1 - \alpha_t^L) \int_T^{T+90} e^{-y^*s} R_{t+s} ds.$$

In this equation, the first term is the present value of the first T years of service flow. The second term is the next 90 years of service flow, scaled by the share going to the leasholder, $(1 - \alpha^H)$; and the likelihood that the lease extends at any time before it falls below 80 years remaining, Π_H^T . The third term is the analogous option value if the lease extends with less than 80 years remaining. Rearranging this expression implies

$$\begin{aligned} P_t^T &= \int_0^T e^{-y_t^*s} R_{t+s} ds + \Pi_{Tt}^H (1 - \alpha_t^H) \int_T^{T+90} e^{-y_t^*s} R_{t+s} ds + \Pi_{Tt}^L (1 - \alpha_t^L) \int_T^{T+90} e^{-y_t^*s} R_{t+s} ds \\ &= \int_0^T e^{-y_t^*s} R_t ds + \Pi_{Tt}^H (1 - \alpha_t^H) e^{-y_t^*T} R_t \int_0^{90} e^{y_t^*s} ds + \Pi_{Tt}^L (1 - \alpha_t^L) e^{-y_t^*T} R_t \int_0^{90} e^{-y_t^*s} ds \\ &= \int_0^T e^{-y_t^*s} R_t ds + e^{-y_t^*T} R_t \int_0^{90} e^{y_t^*s} ds [\Pi_{Tt}^H (1 - \alpha_t^H) + \Pi_{Tt}^L (1 - \alpha_t^L)] \\ &= \int_0^T e^{-y_t^*s} R_t ds + e^{-y_t^*T} R_t \int_T^{T+90} e^{y_t^*s} ds [\Pi_{Tt}^H (1 - \alpha_t^H) + \Pi_{Tt}^L (1 - \alpha_t^L)] \\ &= \frac{1 - e^{-y_t^*T}}{y_t^*} R_t + e^{-y_t^*T} R_t \frac{1 - e^{-y_t^*90}}{y_t^*} [\Pi_{Tt}^H (1 - \alpha_t^H) + \Pi_{Tt}^L (1 - \alpha_t^L)]. \end{aligned}$$

Then for two properties i and j with identical service flow growth, where property i extends and j does not, we have

$$\begin{aligned} \underline{\Delta}_{it}^T &= \log \left(\frac{1 - e^{-y_t^*(T+90)}}{y_t^*} \right) - \log \left(\frac{1 - e^{-y_t^*T}}{y_t^*} + e^{-y_t^*T} \frac{1 - e^{-y_t^*90}}{y_t^*} [\Pi_{Tt}^H (1 - \alpha_t^H) + \Pi_{Tt}^L (1 - \alpha_t^L)] \right) \\ &= \log (1 - e^{-y_t^*(T+90)}) - \log ((1 - e^{-y_t^*T}) + e^{-y_t^*T} (1 - e^{-y_t^*90}) [\Pi_{Tt}^H (1 - \alpha_t^H) + \Pi_{Tt}^L (1 - \alpha_t^L)]), \end{aligned}$$

which is the expression in the main text.

A.17 Proofs

Proposition A.2. *There exists some value $\bar{r}_K < r_{RV}$ such that:*

1. *If y^* satisfies $y_t^* \geq \bar{r}_K$ then*

(a) There is zero option value at all years of duration remaining, that is, $\alpha_t^T = 1$ for all T .

(b) The price of a leasehold is continuous in duration as the property's duration falls below 80 years, so

$$\lim_{T \rightarrow 80^-} P_{it}^T = \lim_{T \rightarrow 80^+} P_{it}^T.$$

2. If y^* satisfies $y_t^* < \bar{r}_K$ then

(a) There is positive option value above 80 years in duration, that is, $\alpha_t^T < 1$ for all $T > 80$ and option value discontinuously falls at 80 years, so that α_t^T discontinuously increases at $T = 80$.

(b) The price of a leasehold discontinuously falls as the property's duration falls below 80 years, so

$$\lim_{T \rightarrow 80^-} P_{it}^T < \lim_{T \rightarrow 80^+} P_{it}^T.$$

Proof. From [equation \(4\)](#) and [equation \(5\)](#), the price of a property is

$$P_{it}^T = \begin{cases} P_{it}^{T+90} - \min [RV_{it}^T + \gamma R_{it}, V_{it}^T] & T \geq 80 \\ P_{it}^{T+90} - \min \left[\frac{RV_{it}^T + V_{it}^T}{2} + \gamma R_{it}, V_{it}^T \right] & T < 80. \end{cases} \quad (12)$$

Recall the definitions of reversion value and market value

$$V_{it}^T = \frac{R_{it}}{y_t^*} (e^{-y_t^* T} - e^{-y_t^* (T+90)}) \quad (13)$$

$$RV_{it}^T = \frac{R_{it}}{r_{RV}} (e^{-r_{RV} T} - e^{-r_{RV} (T+90)}) \quad (14)$$

We will define $\bar{r}_K$ as the value of y_t^* such that $RV_{it}^T + \gamma R_{it} = V_{it}^T$, that is, the tribunal costs for a lease above 80 years are exactly the market value. The value of $\bar{r}_K$ satisfies

$$\begin{aligned} \frac{R_{it}}{r_{RV}} (e^{-r_{RV} T} - e^{-r_{RV} (T+90)}) + \gamma R_{it} &= \frac{R_{it}}{\bar{r}_K} (e^{-\bar{r}_K T} - e^{-\bar{r}_K (T+90)}) \\ \implies \frac{e^{-r_{RV} T} - e^{-r_{RV} (T+90)}}{r_{RV}} + \gamma &= \frac{e^{-\bar{r}_K T} - e^{-\bar{r}_K (T+90)}}{\bar{r}_K} \end{aligned} \quad (15)$$

where in the first line we have substituted in the definitions of marriage value ([equation \(13\)](#)) and reversion value ([equation \(14\)](#)). The right hand side of [equation \(15\)](#) is strictly decreasing in $\bar{r}_K$. Therefore there is a unique value of $\bar{r}_K$ satisfying the equation.

Now we will prove part (1) of the proposition, in which $y_t^* \geq \bar{r}_K$. [equation \(15\)](#) implies

that for all $y_t^* \geq \bar{r}_K$ we must have

$$RV_{it}^T + \gamma R_{it} \geq V_{it}^T. \quad (16)$$

equation (16) implies

$$\begin{aligned} & RV_{it}^T + \gamma R_{it} \geq V_{it}^T \\ \implies & RV_{it}^T + \gamma R_{it} + V_{it}^T \geq 2V_{it}^T \\ \implies & \frac{RV_{it}^T + \gamma R_{it} + V_{it}^T}{2} \geq V_{it}^T \\ \implies & \frac{RV_{it}^T + V_{it}^T}{2} + \gamma R_{it} \geq V_{it}^T \end{aligned} \quad (17)$$

Therefore for $y_t^* \geq \bar{r}_K$, prices satisfy

$$P_{it}^T = \begin{cases} P_{it}^{T+90} - V_{it}^T & T \geq 80 \\ P_{it}^{T+90} - V_{it}^T & T < 80, \end{cases} \quad (18)$$

where we have substituted equation (16) and equation (17) into equation (12) for $y_t^* \geq \bar{r}_K$. Recall the definition of α_t^T as the ratio of the lease extension cost to V_{it}^T . equation (18) shows that $\alpha_t^T = 1$ for all t , which proves part (1a) of the proposition. Since the top and bottom of equation (18) are equal at $T = 80$, prices are continuous at $T = 80$, which proves part (1b) of the proposition.

Now we will prove part (2) of the proposition, in which $y_t^* < \bar{r}_K$. equation (15) implies that for all $y_t^* < \bar{r}_K$ we must have

$$RV_{it}^T + \gamma R_{it} < V_{it}^T. \quad (19)$$

Then by equation (12), the price of a property with more than 80 years duration remaining is

$$P_{it}^T = P_{it}^{T+90} - (RV_{it}^T + \gamma R_{it}).$$

The price of a property with less than 80 years remaining is

$$P_{it}^T = P_{it}^{T+90} - \min \left[\frac{RV_{it}^T + V_{it}^T}{2} + \gamma R_{it}, V_{it}^T \right]. \quad (20)$$

Also, note that

$$RV_{it}^T + \gamma R_{it} < \min \left[\frac{RV_{it}^T + V_{it}^T}{2} + \gamma R_{it}, V_{it}^T \right], \quad (21)$$

since $RV_{it}^T + \gamma R_{it} < V_{it}^T$ by inequality [equation \(19\)](#) and also by inequality [equation \(19\)](#) we have

$$\begin{aligned}
RV_{it}^T + \gamma R_{it} &< V_{it}^T \\
\implies RV_{it}^T &< V_{it}^T \\
\implies 2RV_{it}^T &< RV_{it}^T + V_{it}^T \\
\implies RV_{it}^T &< \frac{RV_{it}^T + V_{it}^T}{2} \\
\implies RV_{it}^T + \gamma R_{it} &< \frac{RV_{it}^T + V_{it}^T}{2} + \gamma R_{it}.
\end{aligned}$$

[equation \(20\)](#) and [equation \(21\)](#) imply that for $T < 80$

$$P_{it}^T < P_{it}^{T+90} - (RV_{it}^T + \gamma R_{it})$$

Therefore prices discontinuously fall when T falls below 80 which proves part (2b) of the proposition. Since lease extension costs rise when T falls below 80, α_t^T also discontinuously rises when T falls below 80, which is part (2a) of the proposition.

□