

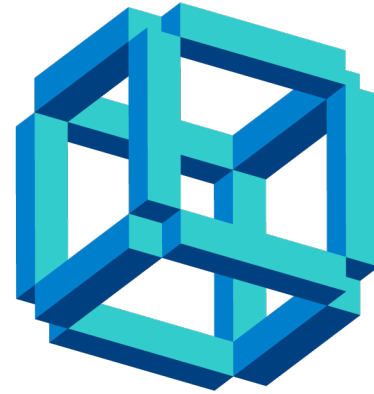
Sequential Cursed Equilibrium

Shani Cohen

Harvard → Hebrew U

Shengwu Li

Harvard



UTMD

The University of Tokyo
Market Design Center



KOJIMA
Market Design Project

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Shani Cohen

Harvard → Hebrew U

Shengwu Li

Harvard

‘Look before you leap’

- Standard game theory posits that players ‘look before they leap’
- Examples: Voting, asset markets, auctions.

Savage (1972): In view of the “look before you leap” principle, acts and decisions, like events, are timeless. The person decides “now” once for all; there is nothing for him to wait for, because his one decision provides for all contingencies.

Savage (1972): ... the task implied by making such a decision is not even remotely resembled by human possibility

People often don't look before they leap

- They make inferences from observed events
- But neglect the implications of hypothetical events
- Lab experiments:
 - Voting
 - Esponda & Vespa 2014, 2023
 - Asset markets
 - Ngangoe & Weizsacker 2021, Martinez-Marquina, Niederle, & Vespa 2019
 - Auctions
 - Levin, Kagel, & Richard 1996, Levin, Peck, & Ivanov 2016, Nagel, Niederle & Vespa 2025
- Treatment effects not explained by known solution concepts.
 - E.g. BNE, cursed equilibrium, level-k, ABEE.



Outline

1. Cursed equilibrium
 - A worked example
 - Two problems
2. Sequential cursed equilibrium
 - Players 'look after they leap'
 - Solves the two problems
3. Some limitations of sequential cursed equilibrium
4. Evidence from lab experiments

Cursed equilibrium (Eyster and Rabin 2005)

- Captures over-bidding in common-value auctions
- Bayesian game:
 - Set of players N , each with action set A_n , type set T_n .
 - Joint distribution $f \in \Delta(T)$
- A strategy for player n is $\sigma_n: T_n \rightarrow \Delta(A_n)$.
- Marginal over opponent action profiles $g_\sigma(a_{-n}|t_n)$.
- Marginal over opponent type profiles $f(t_{-n}|t_n)$.
- Strategy profile σ is a **cursed equilibrium** if for each player n and type t_n , the distribution $\sigma_n(t_n)$ maximizes

$$\sum_{a_{-n}} \sum_{t_{-n}} g_\sigma(a_{-n}|t_n) f(t_{-n}|t_n) u_n(\cdot, a_{-n}, t_n, t_{-n})$$

Example: Zero-sum trading game

- Two players, $A_1 = A_2 = \{\text{accept, decline}\}$.
- Trade happens if and only if both accept.
- Three states, each with probability $1/3$.

Sonsino, Erev, Gilat, & Shabtai (2002)
 Sørensen (2009)
 Rogers, Palfrey, & Camerer (2009)
 Brocas, Carrillo, Wang, & Camerer (2014)

State	ω_1	ω_2	ω_3
Payoffs from trade	$1, -1$	$\epsilon, -\epsilon$	$-1, 1$
Payoffs from no-trade	$0, 0$	$0, 0$	$0, 0$
1's type	t_1 accept		t'_1 decline
2's type	t_2 decline	t'_2	

- In every BNE, no trade. There is a CE with trade for $\epsilon < 1$.

Problem #1: CE players are not cursed about endogenous information

- Replace nature with a fictitious player 0, with action set $A_0 = \{\omega_1, \omega_2, \omega_3\}$.
 - Player 0 plays first. (and has constant utility.)
 - Players 1 and 2 learn about player 0's action.
 - Their payoffs depend player 0's action.
- In the Bayesian normal form:
 - Player 1 now has four actions, $A_1 = \{a_{\omega_1}, d_{\omega_1}\} \times \{a_{\omega_2, \omega_3}, d_{\omega_2, \omega_3}\}$.
 - Player 2 also has four actions.
 - All players have singleton type sets, so CE=BNE.
 - CE predicts no trade. (Even if player 0 randomizes just as nature did!)

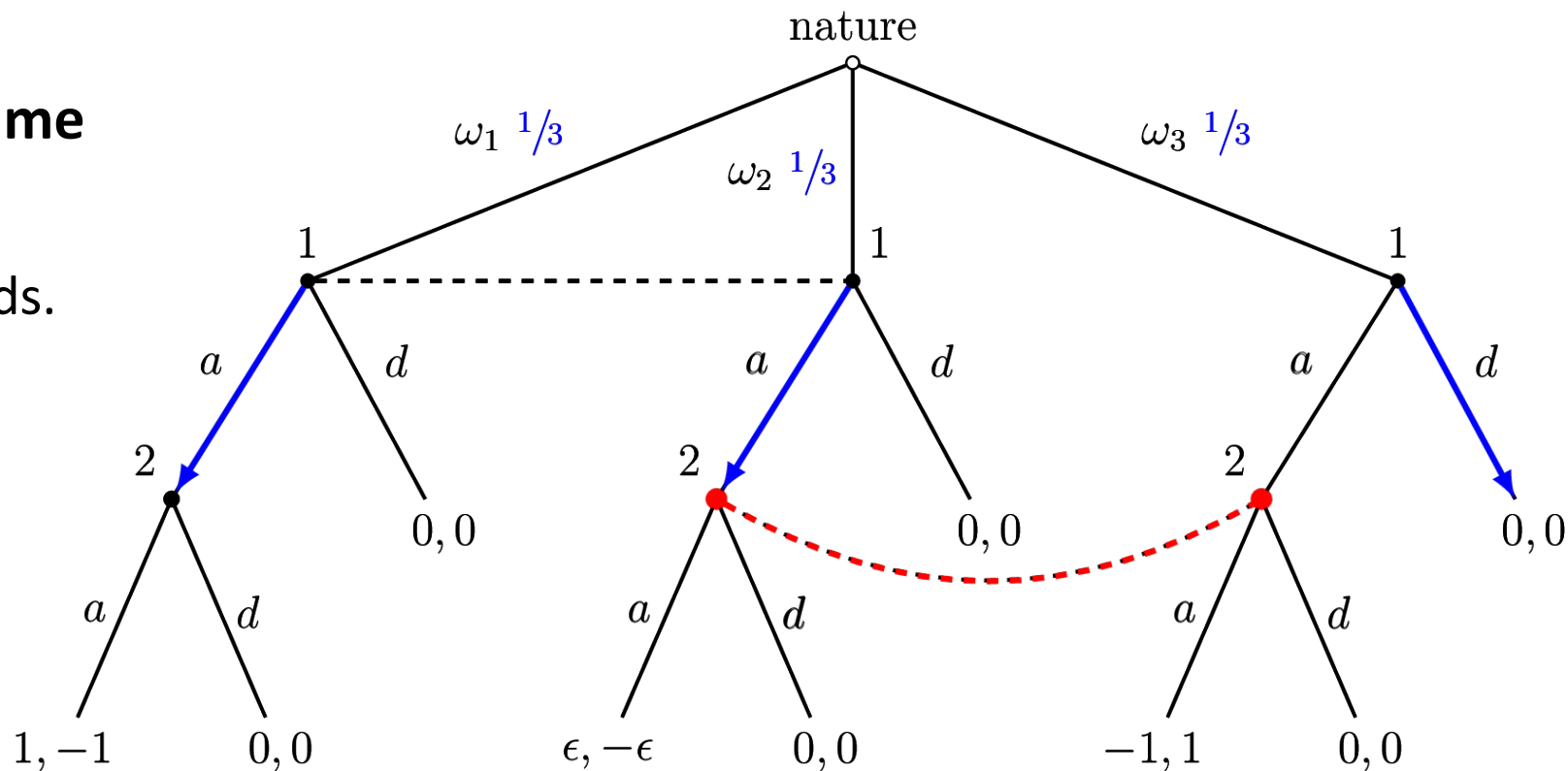
Problem #1: CE players are not cursed about endogenous information

Eyster & Rabin: Treating “exogenous” and “endogenous” private information differently not only seems to us intuitively and psychologically wrong, but also creates some highly artificial differences in predictions based on the way a game is formally written down.

Eyster & Rabin: [...] observing actions seems likely to induce more strategic sophistication. Hence, players in certain sequential games may be less cursed than they would be in corresponding simultaneous-move games.

The sequential trading game

- Player 1 moves first.
- If 1 declines, the game ends.
- If 1 accepts, then player 2 moves.



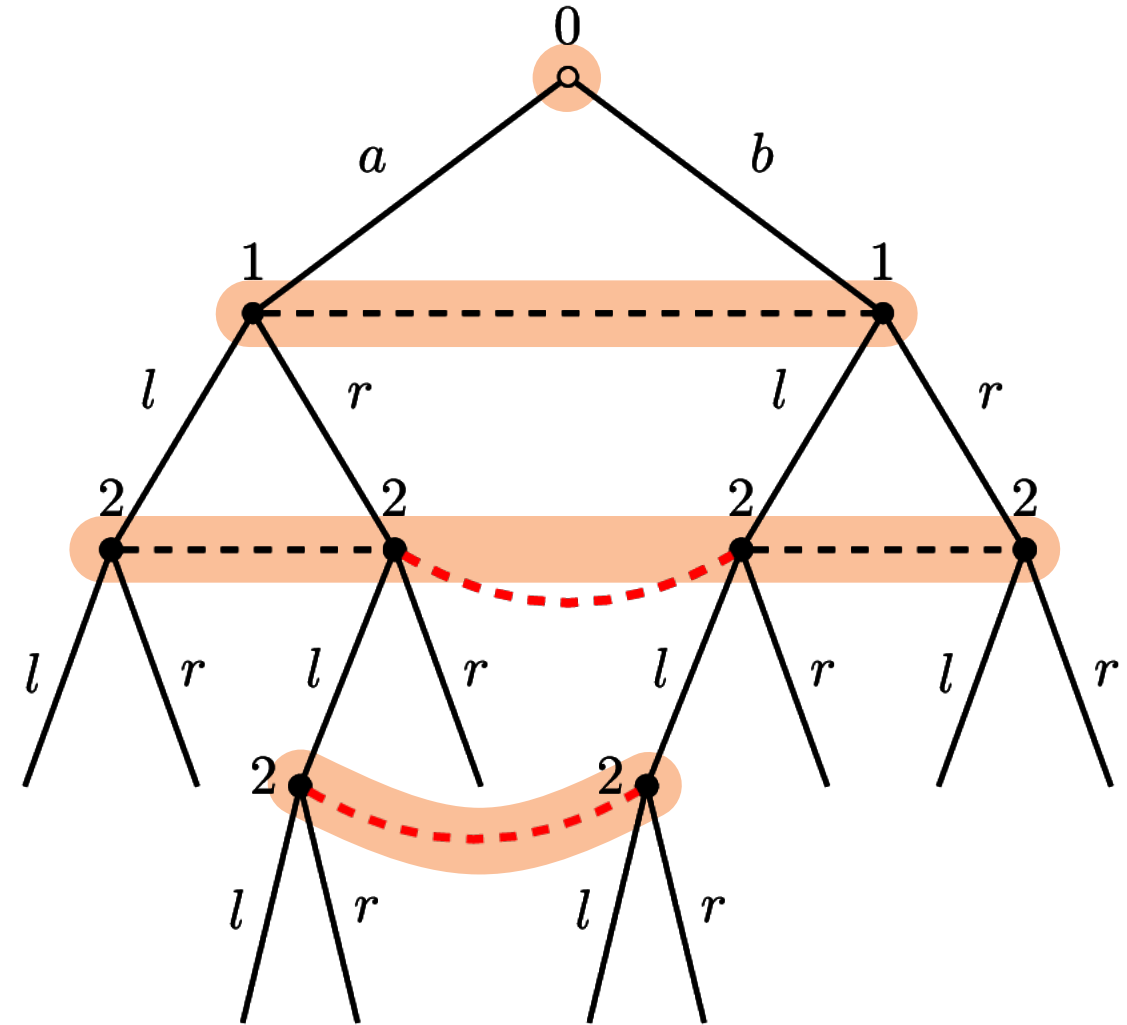
A solution: Sequential Cursed Equilibrium (SCE)

- Recap:
 1. CE players are not cursed about endogenous information.
 2. CE players do not learn from observed events.
- We can solve these issues by generalizing CE to extensive games.

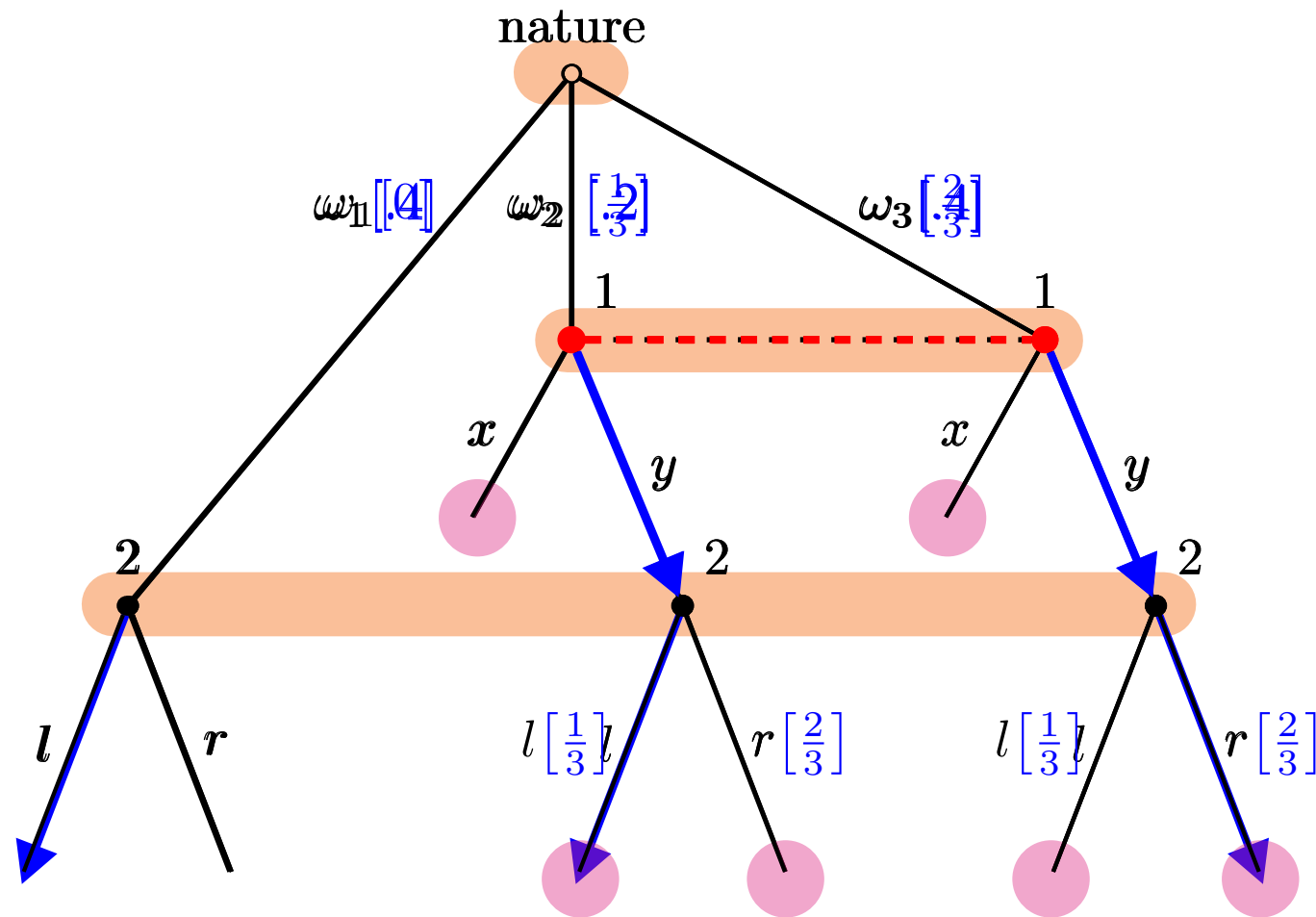
At each type, the player understands the marginal distributions of the other players' actions and of the state, conditional on that type, but responds as if the other players' actions are independent of their types.

As if ‘actions are independent of infosets’.

- Given a game tree, only some partitions of non-terminal histories can be infosets for a game of perfect recall.
- There is a coarsest such partition, $\mathcal{C}$.
 - $C(I)$ denotes cell of $\mathcal{C}$ containing infoset I . (a **coarse set**)
 - Nature’s coarse sets are singleton.
- Extend notation: σ_n denotes a behavioral strategy.
- A strategy is **coarse** if it is measurable with respect to the coarse sets.
 - $C(I) = C(I') \implies \sigma_n(I) = \sigma_n(I')$



Conditioning on an information set

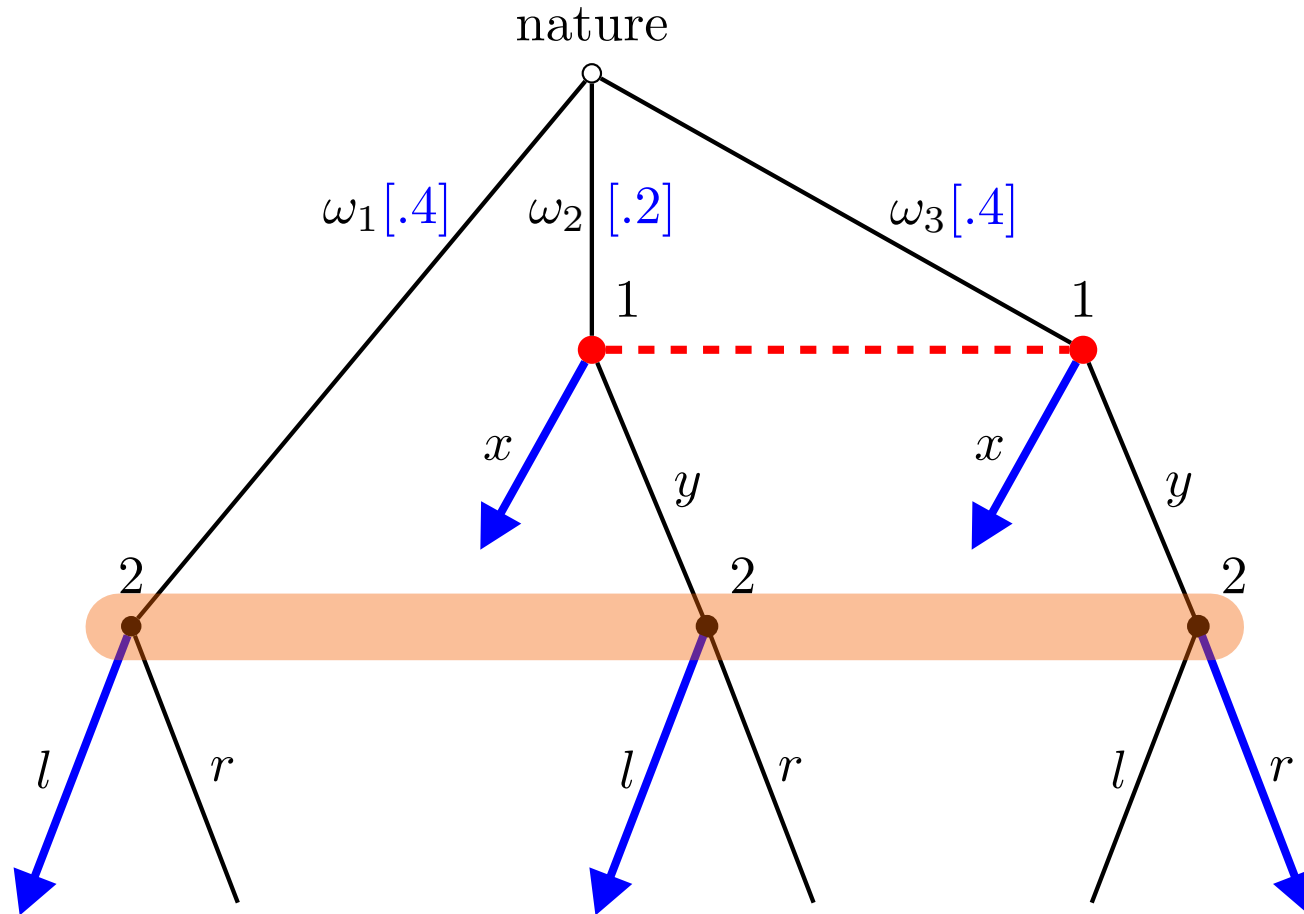


- At the red info set, cursed player 1 believes:
 - nature plays $\omega_2 \ 1/3, \omega_3 \ 2/3$.
 - 2 plays the coarse strategy $l \ 1/3, r \ 2/3$.
- Note:
 - Not the unweighted average $l \ 1/2, r \ 1/2$
 - Not the *ex ante* average $l \ 3/5, r \ 2/5$. (Unlike ABEE)

Cursed-plausible conjectures

- Player strategy profile σ_N , nature strategy σ_λ .
- A **conjecture** at info set I is a *partial* strategy profile $\hat{\sigma}^I$. It specifies what players and nature do at info sets not ruled out by I .
- Given some $(\sigma_N, \sigma_\lambda)$, the conjecture $\hat{\sigma}^I$ is **cursed-plausible** if:
 1. The active player n 's strategy $\hat{\sigma}_n^I$ plays the unique sequence of actions that can reach info set I .
 2. Each opponent's strategy $\hat{\sigma}_m^I$ is a **coarse strategy** that matches the probabilities of m 's actions at each coarse set $C \in \mathcal{C}$, conditional on I and C , 'if possible'.
- Note that Clause 2 also 'updates about' nature's strategy. (Nature has $|C| = 1$.)

Cursed-plausibility doesn't always pin down the conjectures.



Cursed-plausibility does not tie 1's conjecture to 2's actual behavior.

BUT: At any **fully mixed** strategy profile, the cursed plausible conjectures are pinned down by Bayes' rule.

Sequential Cursed Equilibrium

- An **assessment** consists of:
 - A player strategy profile σ_N .
 - A system of conjectures $(\hat{\sigma}^I)_{I \in \mathcal{I}}$, one for each info set.
- 1. An assessment is **cursed-consistent** if $\exists$ fully mixed $\{\sigma_N^k\} \rightarrow \sigma_N$ such that the cursed-plausible conjectures induced by $(\sigma_N^k, \sigma_\lambda)$ converge to $(\hat{\sigma}^I)_{I \in \mathcal{I}}$.
- 2. An assessment is **locally rational** if at any info set I , (i) the active player's self-conjecture $\hat{\sigma}_n^I$ agrees with σ_n at I and (ii) $\hat{\sigma}_n^I$ best-responds to $\hat{\sigma}^I$.
- A **sequential cursed equilibrium (SCE)** is an assessment that satisfies (1) and (2).

Existence.

Theorem: Every finite extensive game of perfect recall has a SCE.

Proof sketch:

1. Consider the ε -perturbed game; each action played w.p. $\geq \varepsilon$.
2. Each (fully mixed) σ_N induces a cursed-plausible system of conjectures.
3. By Kakutani's theorem, there is a fixed point for the ε -perturbed game.
4. Take $\varepsilon \rightarrow 0$, pass to a convergent subsequence.

Comparing predictions of BNE, CE, and SCE

 indicates there exists an equilibrium with trade.

Trading game	BNE	CE	SCE
Simultaneous			

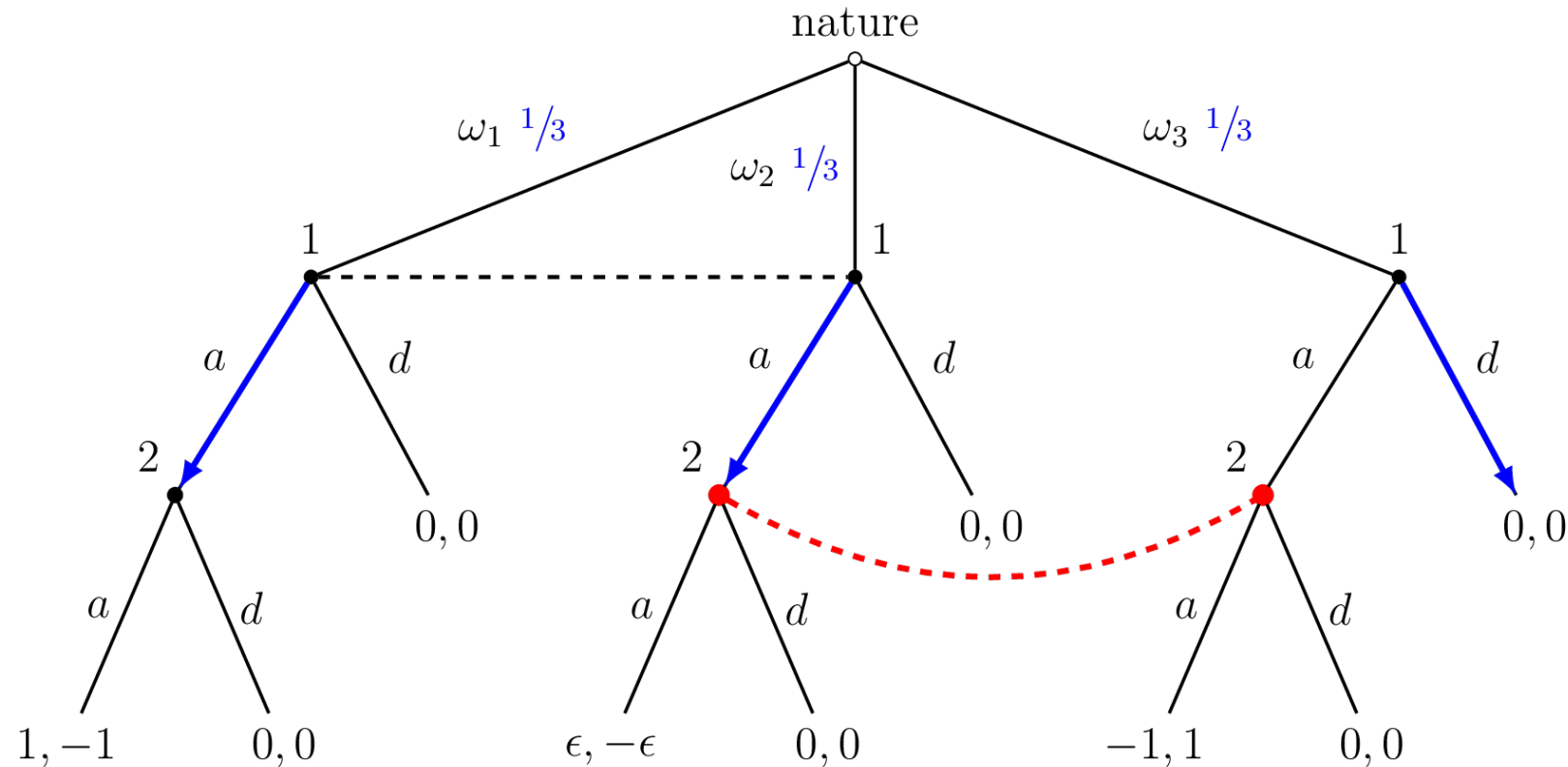
Def: Independently cursed equilibrium (ICE). As in CE, but players respond as if opponents' actions are independent of each other.

Theorem: For any finite simultaneous Bayesian game, ICE and SCE predict the same behavior.

Corollary: For any finite **two-player** simultaneous Bayesian game, **CE** and SCE predict the same behavior.



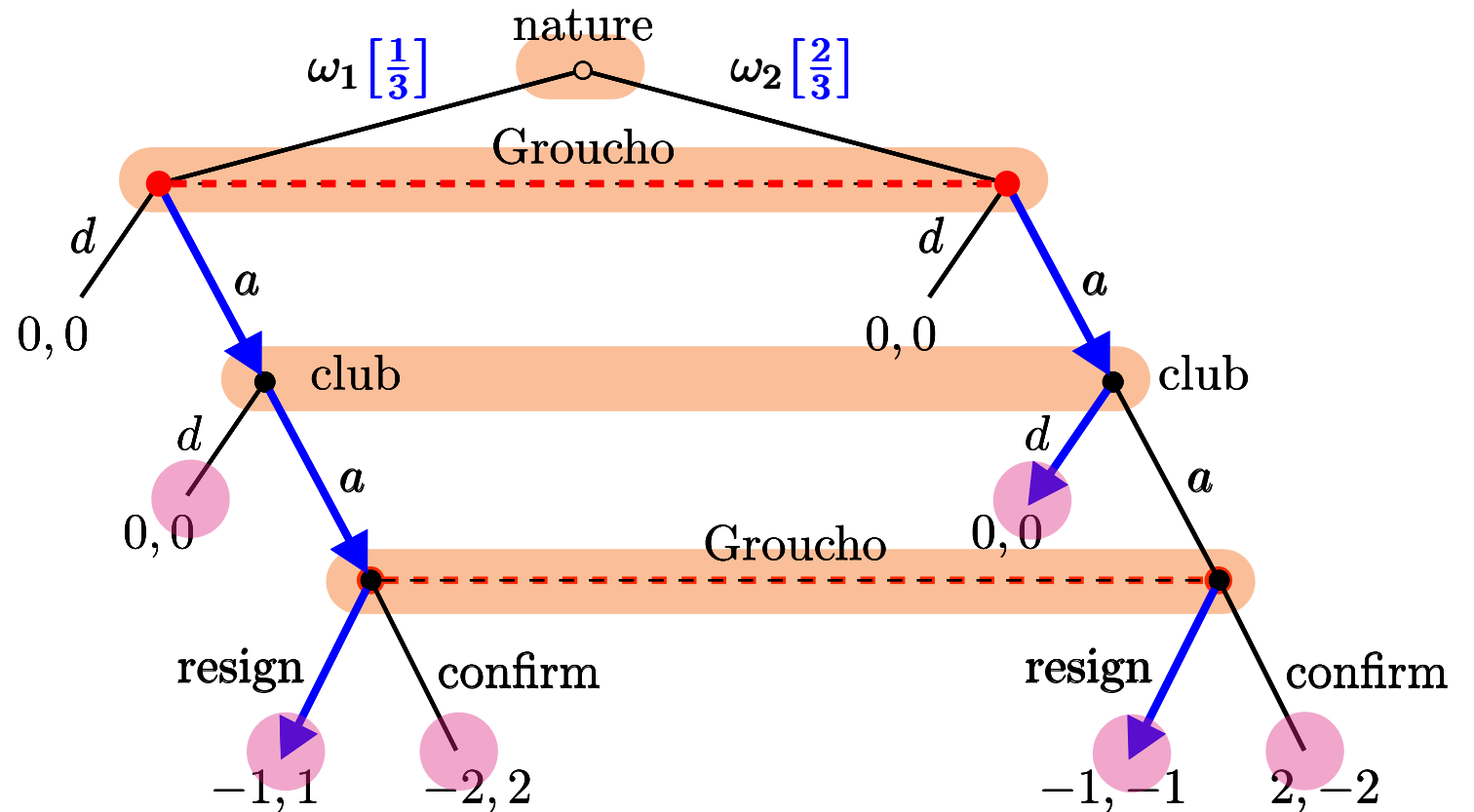
SCE players learn from observed events.



- Conditional on red info set, every cursed-consistent conjecture specifies that nature plays ω_2 and player 1 plays a .

SCE players can make mistakes *ex ante* and realize *ex post*.

- Groucho's payoff listed first.
- The club wants to accept Groucho in state ω_1 .
- Groucho wants to be a member in state ω_2 .
- Resigning is costly.



Limitations of SCE

- SCE predictions depend on action labels.
 - Labels affect the coarse partition $\mathcal{C}$.
 - Permuting labels can change predictions.
 - Similar critique applies to cursed equilibrium.
- SCE players are arguably ‘too cursed’.
 - Signaling game: Sender ignores the effect of the signal on receiver’s actions.
 - Can fix this with ‘causal SCE’.
- SCE does not capture framing.
 - Esponda & Vespa 2023.



More solution concepts compared

 indicates there exists an equilibrium with trade.

Trading game	SCE	χ -CSE, $\chi < \frac{2\varepsilon}{1+\varepsilon}$	χ -CSE, $\chi \geq \frac{2\varepsilon}{1+\varepsilon}$	ABEE	level-k
Simultaneous					
Fictitious player		N/A	N/A		
Sequential					

Experiment #1: Learning from being pivotal

- Esponda and Vespa 2014
- State is red w.p. p , blue w.p. $1 - p$.
- Two computer players.
 - If state is red, vote red.
 - If state is blue, vote blue w.p. q . (independently)
- Subject *does not know state*, can vote red or blue.
- Subject payoff is 1 if majority vote matches state, 0 otherwise.
- Notice that if subject's vote is pivotal, then the state is blue for sure.

Experiment #1: Learning from being pivotal

- **Simultaneous** treatment: All vote at same time.
- **Sequential** treatment: Subject sees computer votes before voting.
- **Naïve** voting: Vote red if $p > .5$, blue if $p < .5$.
- **Strategic** voting: Ignore p , vote blue.

Treatment	BNE	CE	SCE
Simultaneous	strategic	naïve	naïve
Sequential	strategic	naïve	strategic

Experiment #2: Realized vs unrealized prices

- Ngangoué & Weizsäcker 2021
 - Traders face an ‘artificial market’, can buy or sell a common-value asset.
 - Each receives private signal of value.
1. Price p_1 is drawn $U[0,1]$.
 2. Trader 1 buys or sells.
 - If 1 buys, price rises. $p_2 = \frac{1+p_1}{2}$.
 - If 1 sells, price falls. $p_2 = \frac{p_1}{2}$.
- High p_2 is good news about the common value.

Experiment #2: Realized vs unrealized prices

- Trader 2 observes p_1 , and then either:
 - **Simultaneous** treatment: Trader 2 submits a limit order.
 - **Sequential** treatment: Trader 2 observes p_2 , chooses to buy or sell.
- **Naïve** trading: Ignore information content of p_2 .
- **Strategic** trading: Best-respond to trader 1's strategy.

Treatment	BNE	CE	SCE
Simultaneous	strategic	naïve	naïve
Sequential	strategic	naïve	strategic

Recap: Experiments that test the theory

Experiment	Setting	Treatment	BNE	CE	SCE	Result
Esponda & Vespa 2014	pivotal voting	simultaneous	✓	✗	✗	22% ✓
		sequential	✓	✗	✓	76% ✓
Ngangoué & Weizsäcker 2021	asset trading	limit order	✓	✗	✗	63% ✓
		observed prices	✓	✗	✓	81% ✓

- As far as we know, SCE is the only solution concept that makes these predictions.

Conclusion

- SCE treats info sets the same way that CE treats types.
- If we generalize CE in this way, the result is that:
 1. Players are cursed about endogenous information.
 2. Players learn from observed events.
- Consistent with some experiments on hypothetical thinking.
 - Esponda & Vespa 2014
 - Ngangoué & Weizsäcker 2021
 - Moser 2019
 - Nagel, Niederle, & Vespa 2025

Partial cursedness

- χ -SCE consists of a strategy profile σ_N and a sequence of fully mixed $\sigma_N^k \rightarrow \sigma_N$.
- At each info set I , the player responds as if:
 - With probability χ , he faces the cursed-consistent conjecture for I .
 - With probability $1 - \chi$, he faces the rational conjecture for I .
- SCE is $\chi = 1$.
- Sequential equilibrium is $\chi = 0$.

The ideas adapt to continuous-time auctions.

- Common-value auctions, affiliated signals (Milgrom and Weber 1982)
- Players understand the value of the object:
 - conditional on own signal (cursed equilibrium).
 - conditional on all observed information (clock cursed equilibrium).

Result (informal): Under clock cursed equilibrium

1. Bids are lower in Dutch auctions compared to 1st-price.
 - as in Levin, Peck, & Ivanov (2016)
2. Bids are higher in silent English auctions compared to 2nd-price.
3. The winner's curse is small in English auctions with many bidders.
 - as in Levin, Kagel, & Richard (1996)

Contemporaneous work

- Fong, Lin, & Palfrey (2023) propose a different generalization of CE.
 - Scope: Multi-stage games with types and observed actions.
 - χ -CSE: At each stage, each player believes that opponent actions are drawn independently of their types (with probability χ).
 - Thus, slow to update beliefs about opponent types.
- χ -CSE players:
 1. Learn fully from their own types (exogenous info)
 2. Partially neglect the public history (endogenous info)
- χ -CSE players neglect events even after observing them.

Cursed equilibrium predictions depend on labels.

- Relabel actions in the simultaneous trading game.
- Player 1's action set is now $\{ad, da\}$.
- ad accepts at type t_1 and declines at type t'_1 . da does the opposite.
- Both types of player 1 play ad . (dominant strategy)
- CE predicts no-trade.

State	ω_1	ω_2	ω_3
Payoffs from trade	1, -1	$\epsilon, -\epsilon$	-1, 1
Payoffs from no-trade	0, 0	0, 0	0, 0
1's type	t_1		t'_1
2's type	t_2	t'_2	

