

ONLINE APPENDIX

AN AMERICAN MACROECONOMIC PICTURE:

SUPPLY AND DEMAND SHOCKS IN THE FREQUENCY DOMAIN

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A. FREQUENCY DOMAIN IDENTIFICATION

In this Appendix we show how to target multiple elements of the matrix $\Psi(\underline{\theta}, \bar{\theta})$ appearing in equation (5). We can write the l, k element as $\Psi_{lk}(\underline{\theta}, \bar{\theta}) = \mathcal{E}_l \Psi(\underline{\theta}, \bar{\theta}) \mathcal{E}_k'$ where $\mathcal{E}_l$ is the l -th row of the n -dimensional identity matrix. Using equation (5), we have¹

$$\Psi_{lk}(\underline{\theta}, \bar{\theta}) = h' \left[\int_{\underline{\theta}}^{\bar{\theta}} \Re \left(D(e^{-i\theta})' \mathcal{E}_l' \mathcal{E}_k D(e^{i\theta}) \right) d\theta \right] h.$$

The contribution of the shock $h'\eta_t$ to the weighted sum $\sum_{k=1}^m \omega_k \Psi_{M_k N_k}(\underline{\theta}, \bar{\theta})$ is given by

$$h' \left[\int_{\underline{\theta}}^{\bar{\theta}} \Re \left(D(e^{-i\theta})' \sum_{k=1}^m \omega_k \mathcal{E}_{M_k}' \mathcal{E}_{N_k} D(e^{i\theta}) \right) d\theta \right] h$$

where ω_k are the weights, to be chosen by the researcher. The weighted sum $\sum_{k=1}^m \omega_k \mathcal{E}_{M_k}' \mathcal{E}_{N_k} = P_M' \Omega P_N$, where

$$P_M = (\mathcal{E}_{M_1}', \mathcal{E}_{M_2}', \dots, \mathcal{E}_{M_m}')'$$

and

$$P_N = (\mathcal{E}_{N_1}', \mathcal{E}_{N_2}', \dots, \mathcal{E}_{N_m}')'$$

are $m \times n$ matrices, and $\Omega = \text{diag}(\omega_1, \omega_2, \dots, \omega_m)$ is $m \times m$ matrix.

Hence the multi-target can be written as

$$\sum_{k=1}^m \omega_k \Psi_{M_k N_k}(\underline{\theta}, \bar{\theta}) = h' O_{MN}(\underline{\theta}, \bar{\theta}) h \quad (11)$$

where

$$O_{MN}(\underline{\theta}, \bar{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \Re \left(D(e^{i\theta})' P_M' \Omega P_N D(e^{-i\theta}) \right) d\theta. \quad (12)$$

This is the objective function of our identification problem, in the case of multiple targets. Of course, this objective function reduces to the single target objective function in the case $m = 1$.

An example of multiple-target identification is the cyclical variance of a set of real economic activity variables: one could jointly maximize the cyclical variance of GDP growth and unemployment. Assuming that GDP growth and unemployment are the first two variables in x_t , we have $m = 2$, $M_1 = N_1 = 1$ and

¹To see this, notice that $\mathcal{E}_l D(e^{-i\theta})' h$ is a scalar so that it is equal to $h' D(e^{-i\theta})' \mathcal{E}_l'$. The same reasoning applies to $h' D(e^{i\theta})' \mathcal{E}_k'$

$$M_2 = N_2 = 2,$$

$$P'_M = P'_N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix}, \quad \Omega = \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}.$$

In this case, a reasonable choice for the weights is to take the reciprocals of the cyclical variances of the variables, i.e. $\omega_1 = \frac{1}{V_{11}(\underline{\theta}, \bar{\theta})}$ and $\omega_2 = \frac{1}{V_{22}(\underline{\theta}, \bar{\theta})}$.

B. DFM ESTIMATION AND RANK REDUCTION

B.1. DFM ESTIMATION

FIRST STEP. We set a value for the number r of the static factors, using the criterion by [Bai and Ng \(2002\)](#) with the penalty modification proposed in [Alessi et al. \(2010\)](#), finding a number of static factors $\hat{r} = 11$.² The static factors $F_t = (F_{1t} \dots F_{rt})'$ are estimated by the first $\hat{r}$ principal components of the variables in our dataset. The estimated loading matrix, $\hat{\Lambda}$, is the $n \times \hat{r}$ matrix having on the columns the normalized eigenvectors corresponding to the $\hat{r}$ -largest eigenvalues of the sample covariance matrix of the data, $\hat{\Sigma}_x$. The estimated common component vector is given by $\hat{\chi}_t = \hat{\Lambda} \hat{F}_t$.

SECOND STEP. We run a VAR(p) for the estimated factors $\hat{F}_t$ to get estimates $\hat{C}(L)$ and $\hat{\epsilon}_t$ of $C(L)$ and the VAR innovations ϵ_t . The estimated Moving Average representation is $\hat{F}_t = \hat{C}(L)^{-1} \hat{\epsilon}_t$. The number of lags p is determined according to the BIC criterion ($\hat{p}_{BIC} = 1$). In the robustness section we repeat the analysis with different lags order. To orthogonalize the shocks we use the Cholesky factor $\hat{S}$ of $\hat{\Sigma}_\epsilon$. Therefore, the Cholesky IRFs of the common components are obtained according to (4) as

$$\hat{D}(L) = \hat{\Lambda} \hat{C}(L)^{-1} \hat{S}.$$

From this matrix, we estimate the spectral density of the common components at the Fourier frequencies $\theta = 2\pi s/T$, $s = 1, \dots, T$, and take the real part, so that the resulting off-diagonal terms are co-spectra rather than cross-spectra. This is useful when we take an off-diagonal term as a target, since the integral of

²In the robustness section, we take into account the uncertainty in estimating the number of static factors, and repeat the analysis with different specifications of $\hat{r}$.

the co-spectrum of two variables over a given frequency band is the covariance of the two variables over that band. Finally, we compute $V(\underline{\theta}, \bar{\theta})$ by replacing the integral with the simple average of the real part of the spectral density matrix, across the frequencies belonging to the relevant interval. $\Psi(\underline{\theta}, \bar{\theta})$ and $O_{MN}(\underline{\theta}, \bar{\theta})$ are estimated in a similar way.

B.2. RANK REDUCTION

In the standard DFM estimation procedure the identification techniques are applied to the residuals of the VAR estimated for F_t after estimating q , the number of common shocks, and the rank reduction. The estimated factors $\hat{F}_t$ are not exactly singular, as they contain a residual of the idiosyncratic components that disappears completely only asymptotically. As a consequence, the vector $\hat{\epsilon}_t$ has rank $r > q$, although the last $r - q$ eigenvalues of $\hat{\Sigma}_\epsilon$ are close to zero (Forni et al., 2020). In the standard procedure, singularity is forced on $\hat{\epsilon}_t$ by means of rank-reduction techniques. In Forni et al. (2009), the rank reduction is obtained by using the spectral decomposition of $\hat{\Sigma}_\epsilon$, so that the vector $\hat{\epsilon}_t$ is replaced by the $\hat{q}$ -dimensional vector $V^{-1}\hat{\epsilon}_t$, where V^{-1} is the matrix whose rows are the normalised eigenvectors corresponding to the q -largest eigenvalues of the variance-covariance matrix of $\hat{\epsilon}_t$. This is equivalent to assume that the static rank of the common components is r , which is the rank of its covariance matrix, while the dynamic rank is q , which is the rank of its spectral density. In empirical situation, the number q of dynamic factors or common shocks is unknown and has to be determined by existing information criteria. For instance, the criterion proposed by Hallin and Liška (2007) is based on the properties of dynamic eigenvalues of the data and looks for the value q that minimizes the contribution of the idiosyncratic component. Alternative methods are proposed by Onatski (2009), Amengual and Watson (2007) and Bai and Ng (2007). Recently, Avarucci et al. (2021) introduce a novel consistent criterion to estimate the number of common shocks that can be applied to single frequencies as well as to frequency bands. Such criteria, albeit consistent, often give different results each other.

Forni et al. (2020) shown that the rank reduction step can be ignored with no consequences on the (IRFs) estimation accuracy. Since different information criteria often give different results, the estimation of q and the rank reduction can be a potential source of error, in particular whether $\hat{q}$ underestimates the true q , leading to large estimation errors implied by a possible mis-specification of q . Therefore, we apply the identification techniques to the not exactly singular

Cholesky-transformed residuals of the estimated VAR without reducing the rank.

Moreover, by reducing the number of shocks of interest in the first stage of our identification strategy, where we select the two shocks maximizing the explained variance of targeted variables on the band $[0, 2\pi/6]$, rather than across all frequencies, we do not need to implement the rank reduction step in our estimation procedure.

C. DATA DESCRIPTION AND DATA TREATMENT

For the description of each variable see McCracken and Ng (2020). For variables not in the FRED-QD dataset, refer to the Mnemonic and note. Treatment codes: 1 = no treatment; 2 = first difference, Δx_t ; 4 = $\log(x_t)$; 5 = log of the first difference, $\Delta \log(x_t)$.

ID	FRED-QD ID	MNEMONIC	TREATMENT CODE	NOTE
1	1	GDPC1/CNP16OV	5	
2	2	PCECC96/CNP16OV	5	
3	3	PCDGx/CNP16OV	5	
4	4	PCESVx/CNP16OV	5	
5	5	PCNDx/CNP16OV	5	
6	6	GPDIC1/CNP16OV	5	
7	7	FPIx/CNP16OV	5	
8	8	Y033RC1Q027SBEAx/CNP16OV	5	
9	9	PNFIx/CNP16OV	5	
10	10	PRFIx/CNP16OV	5	
11	11	A014RE1Q156NBEA	1	
12	12	GCEC1/CNP16OV	5	
13	13	A823RL1Q225SBEA	1	
14	14	FGRECPTx/CNP16OV	5	
15	15	SLCEx/CNP16OV	5	
16	16	EXPGSC1/CNP16OV	5	
17	17	IMPGSC1/CNP16OV	5	
18	18	DPIC96/CNP16OV	5	
19	19	OUTNFB/CNP16OV	5	
20	20	OUTBS/CNP16OV	5	
21		(PCESVx+PCNDx)/CNP16OV	5	
22		(PCDGx+FPIx)/CNP16OV	5	
23	22	INDPRO/CNP16OV	5	
24	23	IPFINAL/CNP16OV	5	
25	24	IPCONGD/CNP16OV	5	
26	25	IPMAT/CNP16OV	5	
27	28	IPDCONGD/CNP16OV	5	
28	30	IPNCONGD/CNP16OV	5	
29	31	IPBUSEQ/CNP16OV	5	
30	35	PAYEMS/CNP16OV	2	
31	36	USPRIV/CNP16OV	2	
32	38	SRVPRD/CNP16OV	2	
33	39	USGOOD/CNP16OV	2	
34	51	USGOVT/CNP16OV	2	
35	57	CE16OV/CNP16OV (EMRATIO)	2	
36	58	CIVPART	2	
37	59	UNRATE	1	
38	60	UNRATESTx	1	
39	61	UNRATELTx	1	
40	62	LNS14000012	1	
41	63	LNS14000025	1	
42	64	LNS14000026	1	

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ID	FRED-QD ID	MNEMONIC	TREATMENT CODE	NOTE
43	74	HOABS/CNP16OV	4	
44	76	HOANBS/CNP16OV	4	
45	77	AWHMAN	1	
46	79	AWOTMAN	1	
47	81	HOUST/CNP16OV	5	
48	95	PCECTPI	5	
49	96	PCEPILFE	5	
50		GDPDEF	5	GDP: Implicit Price Deflator
51	97	GDPCTPI	5	
52	98	GPDICTPI	5	
53	120	CPIAUCSL	5	
54	121	CPILFESL	5	
55	122	WPSFD49207	5	
56	123	PPIACO	5	
57	124	WPSFD49502	5	
58	126	PPIIDC	5	
59	129	WPU0561	5	
60	130	OILPRICE _x	5	
61	135	COMPRNFB	5	
62	138	OPHNFB	5	
63	139	OPHPBS	5	
64	140	ULCBS	5	
65	142	ULCNFB	5	
66	143	UNLPNBS	5	
67		dtfp	1	Fernald's TFP growth
68		dtfp util	1	Fernald's TFP growth CU adjusted
69		dtfp I	1	Fernald's TFP growth - Inv
70		dtfp C	1	Fernald's TFP growth - Con
71		dtfp I util	1	Fernald's TFP growth CU - Inv
72		dtfp C util	1	Fernald's TFP growth CU - Con
73	144	FEDFUNDS	1	
74	145	TB3MS	1	
75	146	TB6MS	1	
76	147	GS1	1	
77	148	GS10	1	
78	150	AAA	1	
79	151	BAA	1	
80	152	BAA10YM	1	
81	156	GS10TB3M _x	1	
82		BAA-AAA	1	
83		GS10-FEDFUNDS	1	
84		GS1-FEDFUNDS	1	
85		BAA-FEDFUNDS	1	
86	158	BOGMBASEREAL _x /CNP16OV	5	
87	160	M1REAL/CNP16OV	5	
88	161	M2REAL/CNP16OV	5	
89	163	BUSLOANS _x /CNP16OV	5	
90	164	CONSUMER _x /CNP16OV	5	
91	166	REALLN _x /CNP16OV	5	
92	168	TOTALSL _x /CNP16OV	5	
93	188	UMCSENT _x	1	
94		Business Condition 12 Months	1	Michigan Consumer Survey
95		Business Condition 5 Years	1	Michigan Consumer Survey
96		Current Index	1	Michigan Consumer Survey
97		Expected Index	1	Michigan Consumer Survey
98		News Index: Relative	1	Michigan Consumer Survey
99	197	UEMPMEAN	1	
100	201	GS5	1	
101	210	CUSR0000SAC	5	
102	211	CUSR0000SAD	5	
103	212	CUSR0000SAS	5	
104	213	CPIULFSL	5	
105	245	S&P 500	5	
106	246	S&P: indust	5	
107		S&P 500/GDPDEF	5	
108		S&P: indust/GDPDEF	5	
109		JLN Macro Unc 1-month	1	Jurado Ludvigson and Ng Uncertainty
110		JLN Macro Unc 3-month	1	JLN Uncertainty
111		JLN Macro Unc 12-month	1	JLN Uncertainty

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ID	FRED-QD ID	MNEMONIC	TREATMENT CODE	NOTE
112		DPCCRC1Q027SBEAx/CNP16OV	5	Real PCE Excluding food and energy
113		DFXARC1M027SBEAx/CNP16OV	5	Real PCE: Food
114		DNRGRC1Q027SBEAx/CNP16OV	5	Real PCE: Energy goods

D. ACD UNDER THE MICROSCOPE: INFLATION AND THE LONG-RUN

In this Appendix we study whether our data support a representation in which a shock presents the distinctive feature of ACD’s MBC shock: the contemporaneous disconnection from both inflation and long-run real economic activity. Is there an orthogonalization of our two shocks such that both disconnections hold?

Let us start from the results of Identification II. Clearly, the temporary shock is, by construction, disconnected from the long-run real economic activity. Yet it is far from being disconnected from the inflation (see Table ??) in the main text. So, imposing the long-run disconnection yields a shock with no inflation disconnection. What about the other way around? We now search for a linear combination of our two shocks with minimum contribution to the cyclical fluctuations of the inflation. The result of this exercise is also at odds with ACD: the shock we found is not disconnected from the long-run economic activity as it explains 57% of the long-run GDP growth variance (see Table D.1, Panel A). Hence, imposing the inflation disconnection yields a shock with no long-run disconnection.

VARIABLES	A: NO-INFL SHOCK		B: GDP TARGETING SHOCK	
	CYCLICAL V.	LONG-RUN V.	CYCLICAL V.	LONG-RUN V.
GDP	56.1	57.0	59.5	34.4
Consumption	60.9	62.4	54.1	43.0
Investment	52.2	34.3	59.1	17.3
Unemployment Rate	46.4	27.0	51.0	12.7
Hours Worked	51.2	54.0	49.6	38.4
Inflation	8.0	6.0	13.6	23.6
Labor Productivity	26.9	20.8	27.0	6.9
TFP	11.6	31.2	7.9	13.9
FFR	17.0	28.0	33.3	52.3
Baa-GS10	56.8	83.2	73.0	77.0
S&P500	51.7	15.1	45.9	7.0
JLN uncertainty 3M	59.5	6.3	59.2	7.3
BC5Y	63.7	19.4	47.6	4.5

Table D.1: PERCENTAGE OF CYCLICAL AND LONG-RUN VARIANCE EXPLAINED BY THE SHOCK DISCONNECTED FROM INFLATION (PANEL A), AND BY THE GDP TARGETING SHOCK (PANEL B) FOR A FEW SELECTED VARIABLES, BY FREQUENCY BAND. BUSINESS CYCLE FREQUENCY BAND: $[2\pi/32 \leq \omega \leq 2\pi/6]$ CORRESPONDING TO CYCLES WITH PERIODICITY BETWEEN 18 MONTHS AND 8 YEARS. LONG RUN FREQUENCY BAND: $[0 \leq \omega \leq 2\pi/80]$, CORRESPONDING TO PERIODICITY GREATER THAN 20 YEARS, WITH QUARTERLY DATA.

Finally, let us see what happens if we apply to our two shocks the same identification imposed in ACD, that is, maximizing the cyclical variance of a single real economic activity variable (here we use GDP growth). This identification yields a shock which is partially disconnected from the inflation but again not disconnected from the long-run — it explains 13.6% of the cyclical variance of inflation and 34.4% of GDP growth’s long-run variance (see Table D.1, Panel B).

Figure D.1 explores all possible linear combinations of our two shocks: given all rotation angles, we show that a shock explaining less than 20% of both cyclical inflation and long-run GDP cannot be obtained.

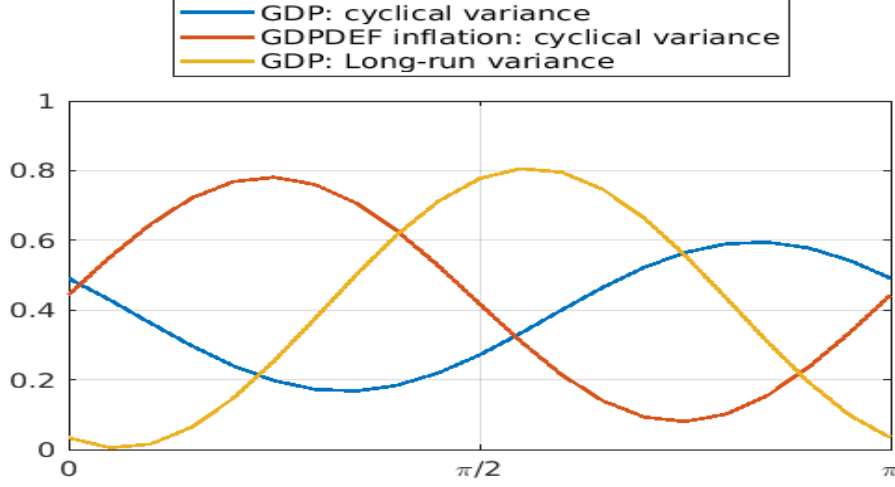


Figure D.1: PERCENTAGE OF VARIANCES EXPLAINED BY ALL LINEAR COMBINATION OF THE TWO SHOCKS $g_1'\eta_t$, $g_2'\eta_t$ DESCRIBED IN THE FIRST STEP OF SECTION ???. ANY GENERIC LINEAR COMBINATION IS OBTAINED AS $\mathcal{R}(\theta) (g_1'\eta_t \ g_2'\eta_t)'$, WHERE $\mathcal{R}(\theta) \equiv \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$ IS A TWO DIMENSIONAL ROTATION MATRIX. THE ANGLE $0 \leq \theta \leq \pi$ IS IN ABSCISSA.

ORTHOGONALITY				
PRINCIPAL COMPONENTS	1 lags	2 lags	3 lags	4 lags
r=6	0.00	0.00	0.00	0.03
r=7	0.01	0.00	0.00	0.00
r=8	0.01	0.00	0.00	0.01
r=9	0.01	0.00	0.00	0.01
r=10	0.02	0.01	0.00	0.04
r=11	0.02	0.02	0.01	0.08

Table D.2: p -VALUES OF THE ORTHOGONALITY F -TEST (Forni and Gambetti, 2014), ONE TO FOUR LAGS, FOR THE MBC SHOCK, ESTIMATED WITH ACD'S VAR SPECIFICATION. r IS THE NUMBER OF PRINCIPAL COMPONENTS USED IN THE TEST. SOURCE: GraneSE (2024).

Why our result are different from ACD? A possible explanation is the following. It is well-known that, while large dimensional factor models are generally unaffected by non-invertibility issues, VAR systems could be *informationally deficient*. GraneSE (2024) investigates whether the 10-variable VAR considered by ACD contains enough information to recover the MBC shock obtained by targeting the unemployment rate. To do so, the author uses the invertibility test of Forni and Gambetti (2014), which tests for the orthogonality of the estimated

shock with respect to the past of the principal components of a large macroeconomic dataset (the author uses the same data set used here). He finds that informational sufficiency is rejected, since the MBC shock is predicted by the lags of the principal components. The p-values are reported in Table D.2.

E. ADDITIONAL RESULTS AND ROBUSTNESS

TABLES

FREQUENCIES	DDR	DGR	DER
$0 \leq \omega \leq 2\pi/6$	2	2	1
$0 \leq \omega \leq 2\pi/8$	2	2	1
$0 \leq \omega \leq \pi$	2	1	1

Table E.1: NUMBER OF ESTIMATED DYNAMIC FACTORS BY DDR, DGR AND DER EVALUATED AT SELECTED FREQUENCIES OR FREQUENCY BANDS. THE SIZE OF THE SPECTRAL WINDOW - BANDWIDTH PARAMETER - IS $M_T = \lfloor a\sqrt{T} \rfloor$ WITH $a = 0.5$. DDR: DYNAMIC DIFFERENCE RATIO ESTIMATOR; DGR: DYNAMIC GROWTH RATIO ESTIMATOR; DER: DYNAMIC EIGENVALUE RATIO ESTIMATOR.

VARIABLES	P=1		P=2		P=3		P=4	
	PERCENTAGE OF EXPLAINED CYCLICAL VARIANCE							
	PERM	TRANS	PERM	TRANS	PERM	TRANS	PERM	TRANS
GDP	29.6	46.6	29.6	46.6	28.8	47.7	27.1	54.5
Consumption	43.9	25.9	50.4	16.5	52.7	18.1	51.3	21.0
Investment	25.5	53.4	27.1	51.4	30.0	50.3	30.2	53.2
Unemployment	30.1	48.4	31.8	51.4	36.4	45.1	36.4	45.5
Hours Worked	29.2	29.0	27.7	35.6	29.6	31.0	31.7	29.6
Inflation	37.2	48.8	44.0	43.4	52.1	28.9	43.6	35.7
Labor Productivity	23.0	23.9	26.1	29.5	22.0	38.5	17.4	39.2
TFP	20.7	6.7	19.1	10.0	17.3	13.3	16.3	7.3
FFR	10.9	64.5	15.0	62.2	23.1	44.9	17.9	51.2
Baa-GS10	12.9	74.8	16.2	70.7	21.6	60.6	20.0	59.3
S&P500	36.2	20.9	37.2	16.2	29.5	30.1	35.4	24.9
JLN Uncertainty 3M	39.8	43.1	43.7	40.2	44.0	41.2	45.0	40.9
BC5Y	71.2	17.9	72.5	16.8	70.6	18.1	71.0	19.5
	PERCENTAGE OF EXPLAINED LONG RUN VARIANCE							
	PERM	TRANS	PERM	TRANS	PERM	TRANS	PERM	TRANS
	PERM	TRANS	PERM	TRANS	PERM	TRANS	PERM	TRANS
GDP	79.3	1.7	72.9	2.0	73.0	0.7	76.5	1.4
Consumption	69.7	5.9	57.6	6.1	59.2	1.5	59.4	2.4
Investment	67.2	5.1	73.3	2.2	76.9	1.5	79.6	1.3
Unemployment	68.7	13.2	80.1	7.1	83.9	5.3	83.9	6.7
Hours Worked	57.3	6.3	58.8	12.9	68.0	8.4	71.3	6.7
Inflation	15.5	69.9	24.2	45.8	22.0	25.0	22.3	27.2
Labor Productivity	58.3	5.1	64.1	1.5	76.4	0.2	78.9	0.1
TFP	64.5	1.6	60.2	0.7	67.4	0.9	68.3	1.1
FFR	0.9	85.0	5.6	73.3	5.9	45.8	5.6	52.2
Baa-GS10	49.2	36.9	31.6	27.3	31.2	11.3	31.5	7.7
S&P500	30.2	0.7	36.9	0.8	43.6	0.2	46.2	1.6
JLN Uncertainty 3M	49.2	42.5	59.8	32.4	62.5	23.8	61.0	29.0
BC5Y	71.5	11.9	81.6	4.9	85.7	3.8	86.8	5.4

Table E.2: IDENTIFICATION II: PERCENTAGE OF VARIANCE EXPLAINED BY THE PERMANENT SHOCK AND THE TRANSITORY SHOCK FOR A FEW SELECTED VARIABLES, BY FREQUENCY BAND, ACCORDING TO DIFFERENT LAGS ORDER: $p = [1 \ 2 \ 3 \ 4]$. BASELINE SPECIFICATION: $p = 1$. BUSINESS CYCLE FREQUENCY BAND: $[2\pi/32 \leq \omega \leq 2\pi/6]$ CORRESPONDING TO CYCLES WITH PERIODICITY BETWEEN 18 MONTHS AND 8 YEARS. LONG RUN FREQUENCY BAND: $[0 \leq \omega \leq 2\pi/80]$, CORRESPONDING TO PERIODICITY GREATER THAN 20 YEARS, WITH QUARTERLY DATA.

VARIABLES	R=11		R=13		R=15		R=17		R=20	
	PERCENTAGE OF EXPLAINED CYCLICAL VARIANCE									
	PERM	TRANS	PERM	TRANS	PERM	TRANS	PERM	TRANS	PERM	TRANS
GDP	29.6	46.6	24.0	50.1	25.7	47.1	18.7	50.4	19.0	49.9
Consumption	43.9	25.9	33.3	25.4	33.3	23.1	27.1	27.6	28.3	25.8
Investment	25.5	53.4	25.5	53.0	26.8	51.6	22.4	54.9	21.9	53.4
Unemployment	30.1	48.4	30.7	46.5	31.3	44.1	26.9	52.3	27.3	50.8
Hours Worked	29.2	29.0	23.3	28.8	26.2	25.6	20.5	27.5	21.9	28.8
Inflation	37.2	48.8	36.9	38.6	34.4	35.5	36.7	37.5	34.5	38.8
Labor	23.0	23.9	18.3	28.8	20.4	27.1	17.3	31.9	18.8	32.8
TFP	20.7	6.7	13.6	6.6	15.4	6.8	19.9	4.5	17.3	4.4
FFR	10.9	64.5	18.3	58.9	18.1	58.6	11.6	51.8	10.7	52.0
Baa-GS10	12.9	74.8	14.4	70.7	14.2	66.3	13.7	48.2	13.7	48.2
S&P500	36.2	20.9	31.1	26.8	31.9	26.7	23.3	34.3	21.8	31.9
JLN Uncertainty	39.8	43.1	46.7	33.8	45.8	34.6	44.3	32.5	44.2	32.4
BC5Y	71.2	17.9	56.1	18.1	50.1	13.1	47.7	12.1	46.8	10.2
	PERCENTAGE OF EXPLAINED LONG RUN VARIANCE									
	PERM	TRANS	PERM	TRANS	PERM	TRANS	PERM	TRANS	PERM	TRANS
GDP	79.3	1.7	78.7	2.1	78.8	2.1	70.3	2.5	73.2	2.4
Consumption	69.7	5.9	65.6	4.8	64.9	4.4	59.9	5.6	62.0	5.2
Investment	67.2	5.1	68.3	2.5	68.0	3.0	64.6	1.7	64.8	1.4
Unemployment	68.7	13.2	74.8	11.0	70.2	11.4	71.4	12.6	70.9	13.2
Hours Worked	57.3	6.3	58.4	7.0	55.5	6.4	60.1	6.0	59.9	4.8
Inflation	15.5	69.9	15.1	55.3	14.0	54.9	13.4	53.1	12.3	54.4
Labor	58.3	5.1	59.4	4.0	59.8	4.2	68.1	2.3	68.0	2.5
TFP	64.5	1.6	64.4	1.2	64.3	1.2	70.1	0.6	68.7	0.8
FFR	0.9	85.0	2.1	74.5	1.8	74.8	1.3	72.3	1.0	71.8
Baa-GS10	49.2	36.9	46.5	25.4	44.4	26.9	33.9	18.5	35.3	17.9
S&P500	30.2	0.7	31.0	0.6	30.2	0.7	23.4	0.4	23.1	0.3
JLN Uncertainty	49.2	42.5	57.9	32.8	53.8	33.1	47.1	36.5	47.1	37.2
BC5Y	71.5	11.9	77.5	9.4	75.4	9.2	76.8	8.5	74.2	9.1

Table E.3: IDENTIFICATION II: PERCENTAGE OF VARIANCE EXPLAINED BY THE TRANSITORY SHOCK AND THE PERMANENT SHOCK FOR A FEW SELECTED VARIABLES, BY FREQUENCY BAND, ACCORDING TO THE NUMBER OF STATIC FACTORS: $r = [11 \ 13 \ 15 \ 17 \ 20]$. BASELINE SPECIFICATION: $r = 11$ STATIC FACTORS. BUSINESS CYCLE FREQUENCY BAND: $[2\pi/32 \leq \omega \leq 2\pi/6]$ CORRESPONDING TO CYCLES WITH PERIODICITY BETWEEN 18 MONTHS AND 8 YEARS. LONG RUN FREQUENCY BAND: $[0 \leq \omega \leq 2\pi/80]$, CORRESPONDING TO PERIODICITY GREATER THAN 20 YEARS, WITH QUARTERLY DATA.

(a) IDENTIFICATION II: MAXIMUM AND MINIMUM PERCENTAGE VALUE OF EXPLAINED VARIANCE ACCORDING TO DIFFERENT LAGS ORDER: $p = [1\ 2\ 3\ 4]$. BASELINE SPECIFICATION: $p = 1$ LAG.

VARIABLES	CYCLICAL VARIANCE				LONG RUN VARIANCE			
	PERM		TRANS		PERM		TRANS	
	MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX
GDP	27.1	29.6	46.6	54.4	72.9	79.3	0.7	2.0
Consumption	43.9	52.7	16.5	25.9	57.6	69.7	1.5	6.1
Investment	25.5	30.2	50.3	53.4	67.2	79.6	1.3	5.1
Unemployment	30.1	36.4	45.1	51.4	68.7	83.9	5.3	13.2
Hours Worked	27.7	31.7	29.0	35.6	57.3	71.3	6.3	12.9
Inflation	37.2	52.1	28.9	48.8	15.5	24.2	25.0	69.9
Labor Productivity	17.4	26.1	23.9	39.2	58.3	78.9	0.1	5.1
TFP	16.3	20.7	6.7	13.3	60.2	68.3	0.7	1.6
FFR	10.9	23.1	44.9	64.5	0.9	5.9	45.8	85.0
Baa-GS10	12.9	21.6	59.3	74.8	31.2	49.2	7.7	36.9
S&P500	29.5	37.2	16.2	30.1	30.2	46.2	0.2	1.6
JLN Uncertainty 3M	39.8	45.0	40.2	43.1	49.2	62.5	23.8	42.5
BC5Y	70.6	72.5	16.8	19.5	71.5	86.8	3.8	11.9

(b) IDENTIFICATION II: MAXIMUM AND MINIMUM VALUE OF EXPLAINED VARIANCE ACCORDING TO THE NUMBER OF STATIC FACTORS: $r = [11\ 13\ 15\ 17\ 20]$. BASELINE SPECIFICATION: $r = 11$ STATIC FACTORS.

VARIABLES	CYCLICAL VARIANCE				LONG RUN VARIANCE			
	PERM		TRANS		PERM		TRANS	
	MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX
GDP	18.7	29.6	46.6	50.4	70.3	79.3	1.7	2.5
Consumption	27.1	43.9	23.1	27.6	59.9	69.7	4.4	5.9
Investment	21.9	26.8	51.6	54.9	64.6	68.3	1.4	5.1
Unemployment	26.9	31.3	44.1	52.3	68.7	74.8	11.0	13.2
Hours Worked	20.5	29.2	25.6	29.0	55.5	60.1	4.8	7.0
Inflation	34.4	36.9	35.5	48.8	12.3	15.5	53.1	69.9
Labor Productivity	17.3	23.0	23.9	32.8	58.3	68.1	2.3	5.1
TFP	13.6	20.7	4.4	6.8	64.3	70.1	0.6	1.6
FFR	10.7	18.3	51.8	64.5	0.9	2.1	71.8	85.0
Baa-GS10	12.9	14.4	48.2	74.8	33.9	49.2	17.9	36.9
S&P500	21.8	36.2	20.9	34.3	23.1	31.0	0.3	0.7
JLN Uncertainty 3M	39.8	46.7	32.4	43.1	47.1	57.9	42.5	32.8
BC5Y	46.8	71.2	10.2	18.1	71.5	77.5	8.5	11.9

Table E.4: PERCENTAGE OF VARIANCE EXPLAINED BY THE PERMANENT SHOCK AND THE TRANSITORY SHOCK (IDENTIFICATION II) FOR A FEW SELECTED VARIABLES, BY FREQUENCY BAND. BUSINESS CYCLE FREQUENCY BAND: $[2\pi/32 \leq \omega \leq 2\pi/6]$ CORRESPONDING TO CYCLES WITH PERIODICITY BETWEEN 18 MONTHS AND 8 YEARS. LONG RUN FREQUENCY BAND: $[0 \leq \omega \leq 2\pi/80]$, CORRESPONDING TO PERIODICITY GREATER THAN 20 YEARS, WITH QUARTERLY DATA.

FIGURES

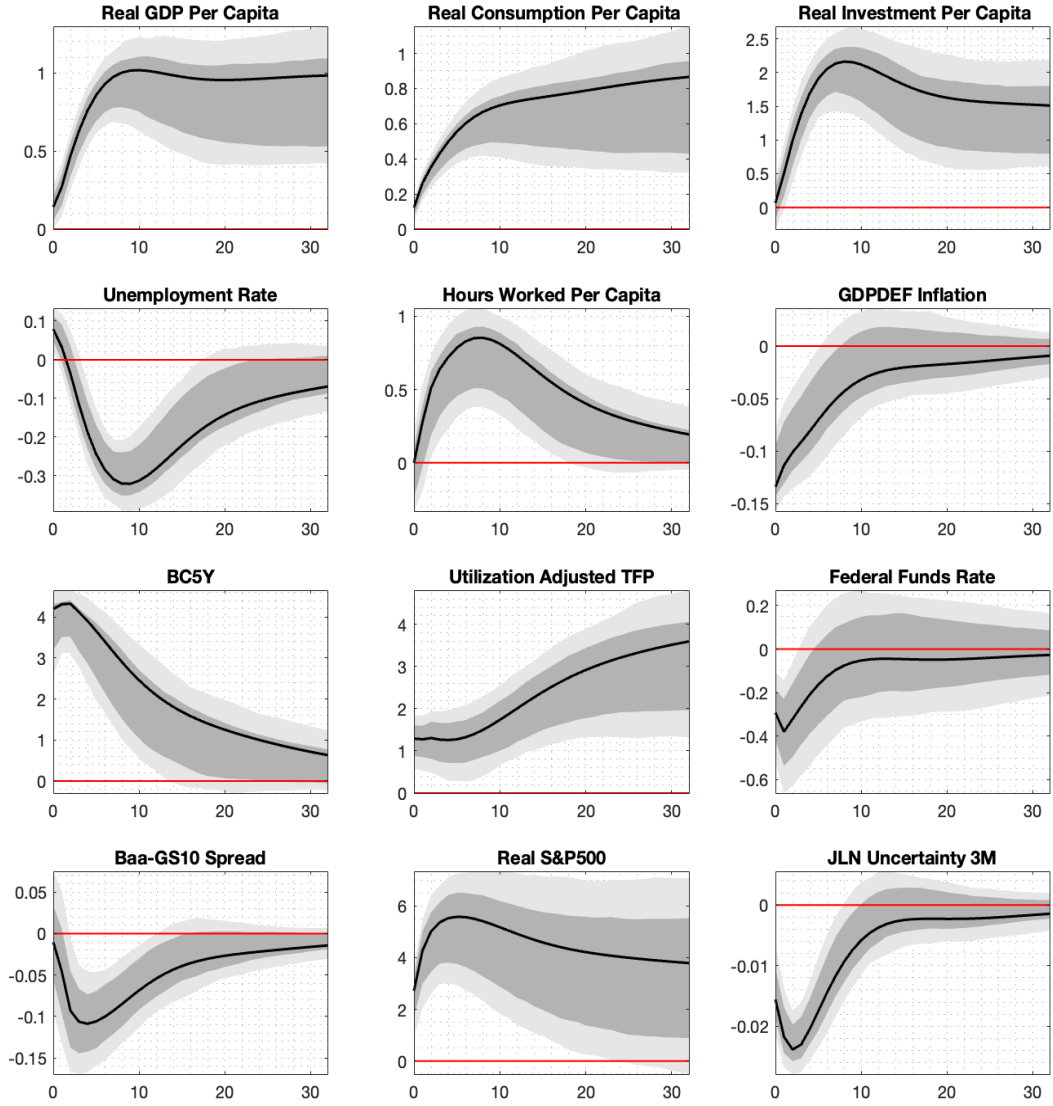


Figure E.1: IDENTIFICATION I: POINT ESTIMATES OF THE IMPULSE RESPONSE FUNCTIONS OF THE SUPPLY SHOCK. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY.

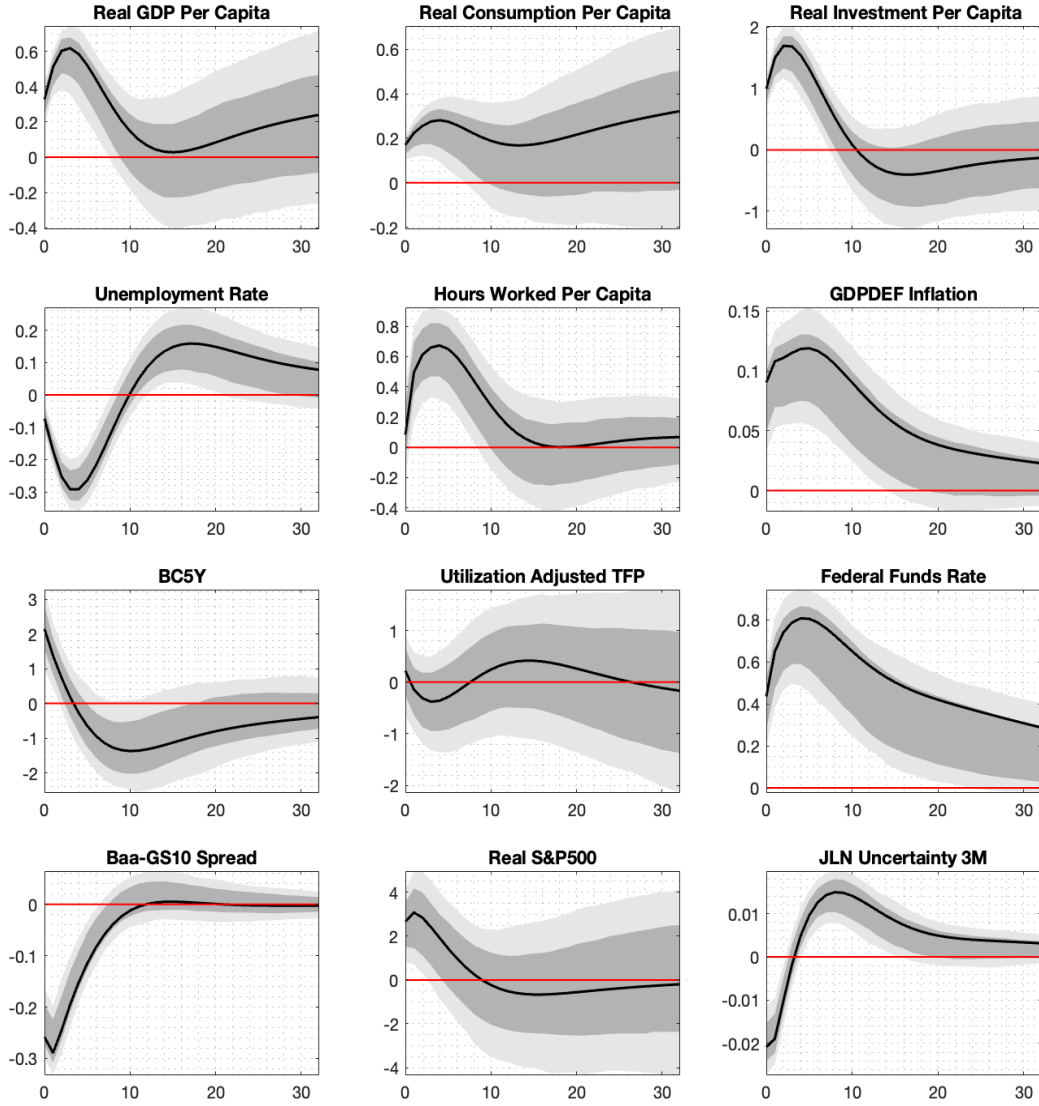


Figure E.2: IDENTIFICATION I: POINT ESTIMATES OF THE IMPULSE RESPONSE FUNCTIONS OF THE DEMAND SHOCK. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY.

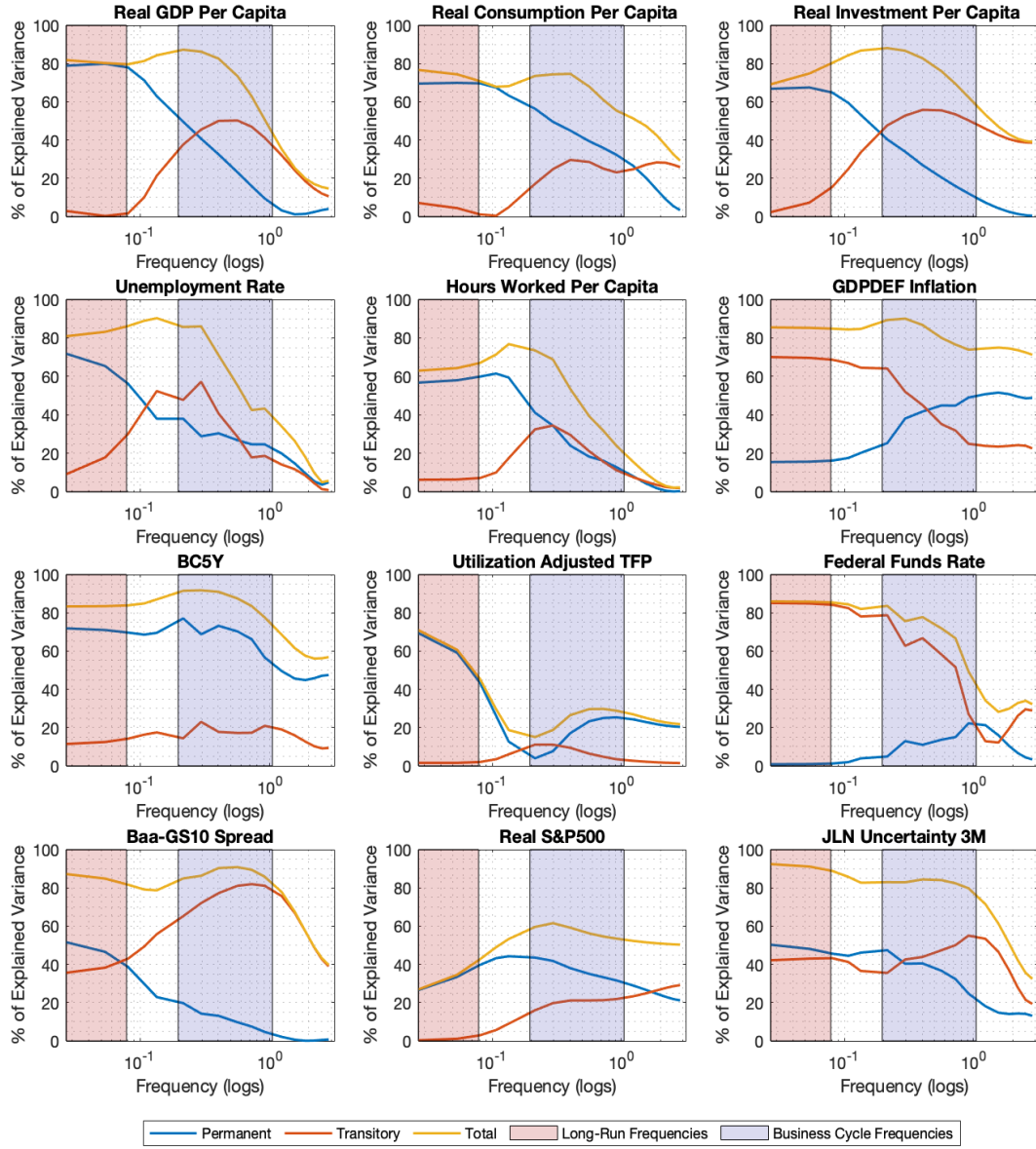


Figure E.3: IDENTIFICATION II: SPECTRAL DECOMPOSITION FOR A FEW SELECTED VARIABLES, FREQUENCY BY FREQUENCY. THE FIGURE REPORTS THE PERCENTAGE OF EXPLAINED VARIANCE. BLUE LINE: CONTRIBUTION OF THE PERMANENT SHOCK; RED LINE: CONTRIBUTION OF THE TRANSITORY SHOCK; YELLOW LINE: SUM. PINK SHADOWED AREA: LONG RUN FREQUENCIES (>80 QUARTERS); LILAC SHADOWED AREA: BUSINESS CYCLE FREQUENCIES (6-32 QUARTERS).

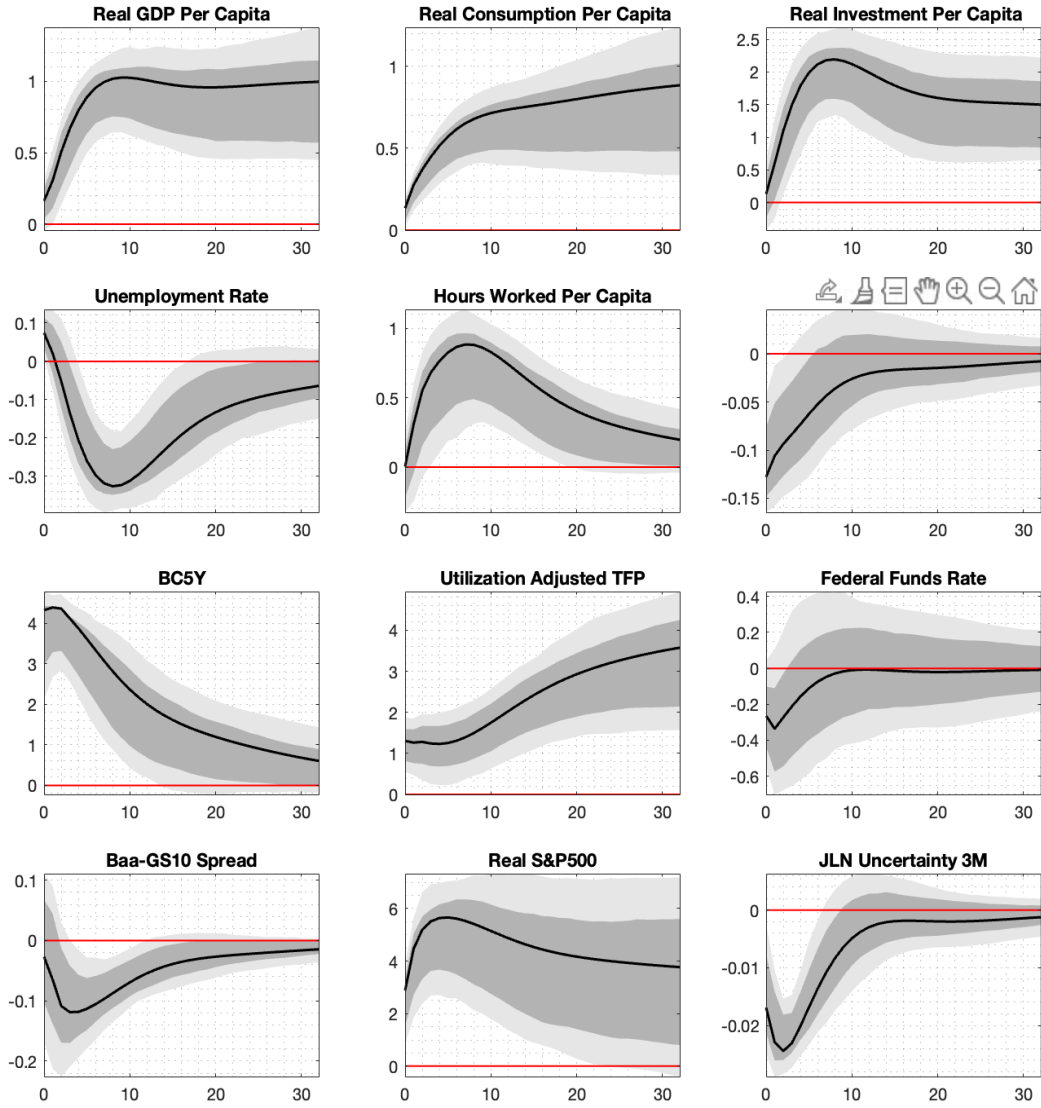


Figure E.4: IDENTIFICATION II: POINT ESTIMATES OF THE IMPULSE RESPONSE FUNCTIONS OF THE PERMANENT SHOCK. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY.

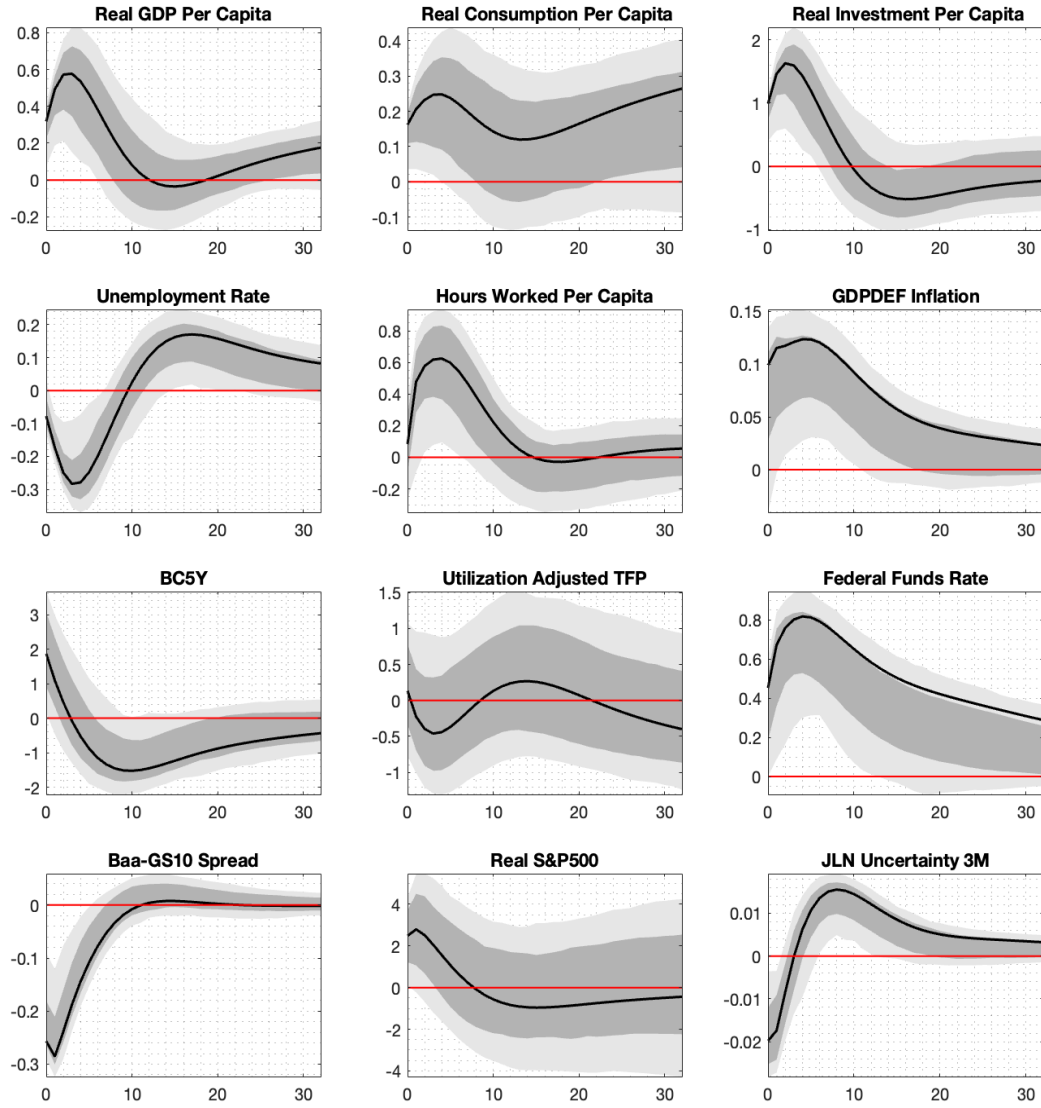


Figure E.5: IDENTIFICATION II: POINT ESTIMATES OF THE IMPULSE RESPONSE FUNCTIONS OF THE TRANSITORY SHOCK. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY.

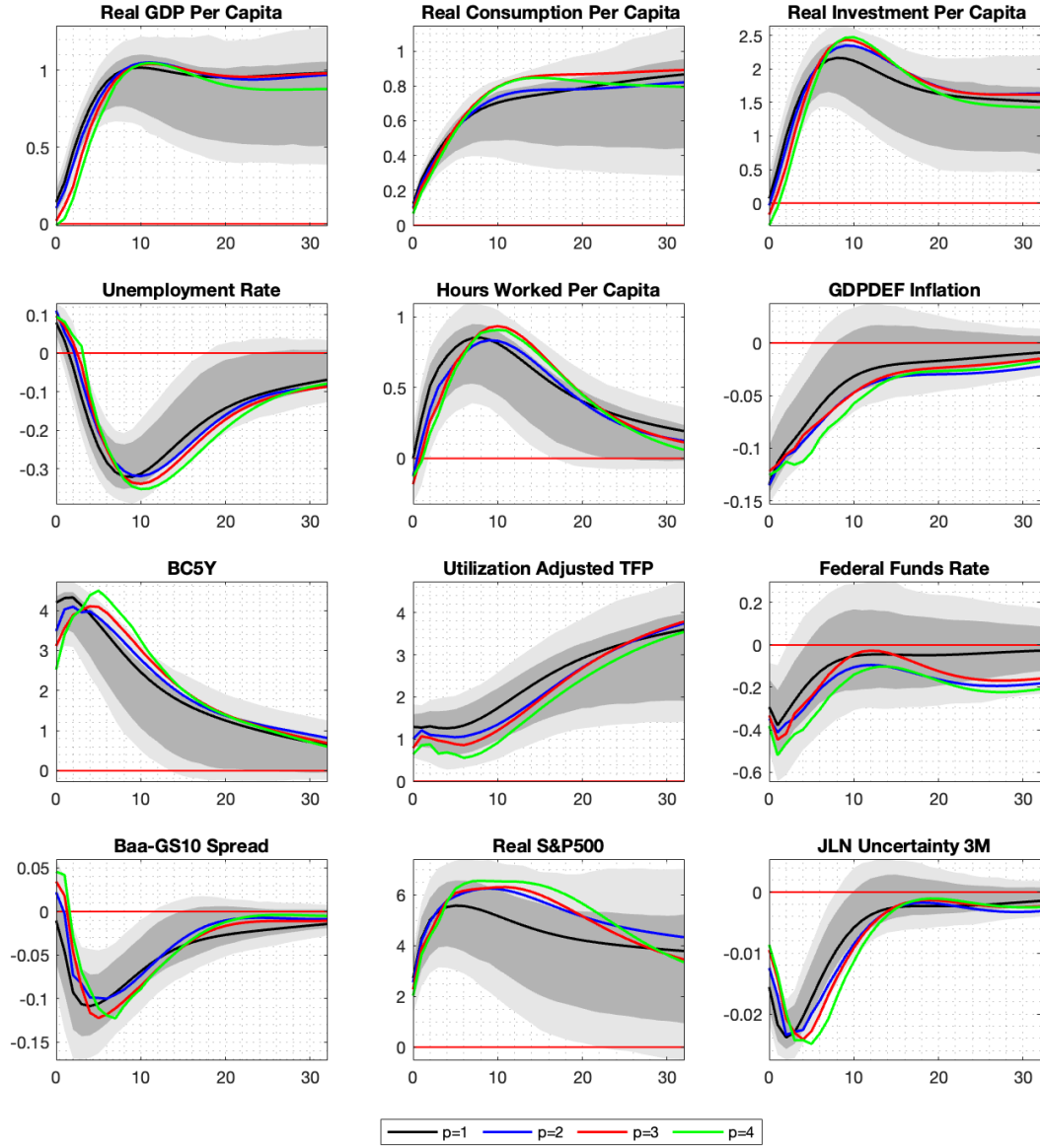


Figure E.6: IDENTIFICATION I: IMPULSE RESPONSE FUNCTIONS OF THE SUPPLY SHOCK, ACCORDING TO DIFFERENT LAG ORDERS: $p = [1 \ 2 \ 3 \ 4]$. BASELINE SPECIFICATION: $p = 1$. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

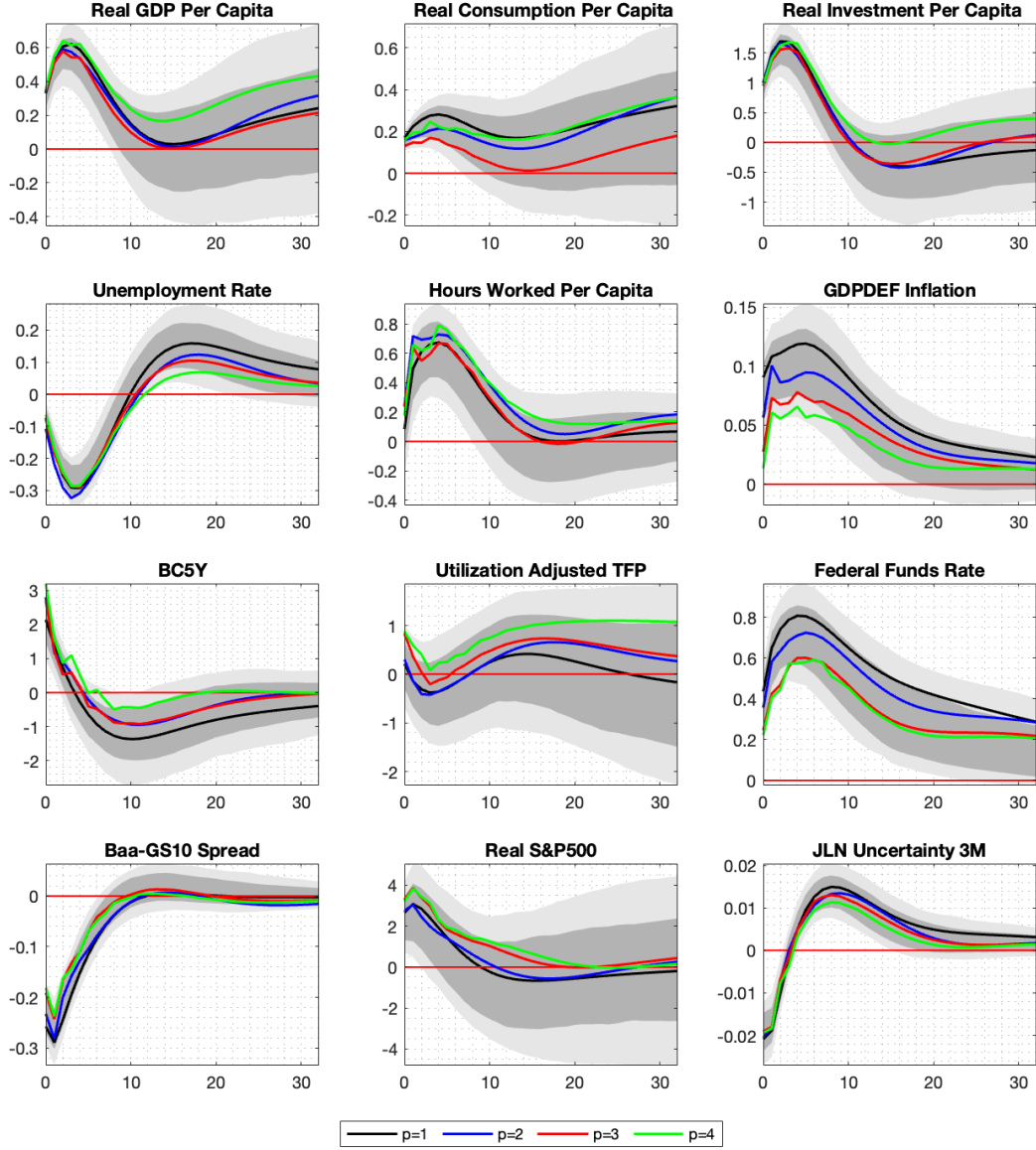


Figure E.7: IDENTIFICATION I: IMPULSE RESPONSE FUNCTIONS OF THE DEMAND SHOCK, ACCORDING TO DIFFERENT LAG ORDERS: $p = [1 \ 2 \ 3 \ 4]$. BASELINE SPECIFICATION: $p = 1$. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

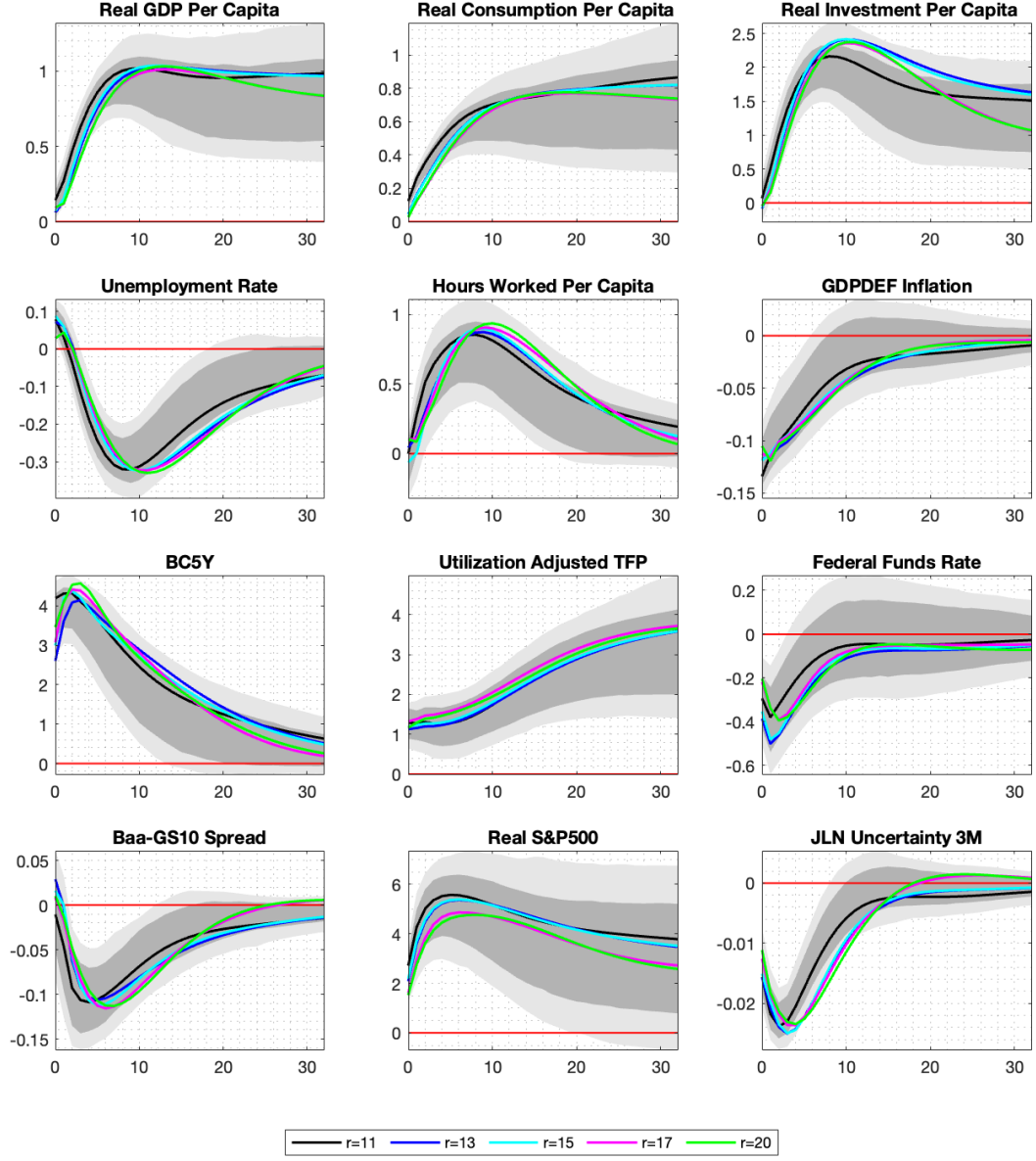


Figure E.8: IDENTIFICATION I: IMPULSE RESPONSE FUNCTIONS OF THE SUPPLY SHOCK, ACCORDING TO DIFFERENT NUMBER OF STATIC FACTORS: $r = [11 \ 6 \ 9 \ 13 \ 15]$. BASELINE SPECIFICATION: $r = 11$. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

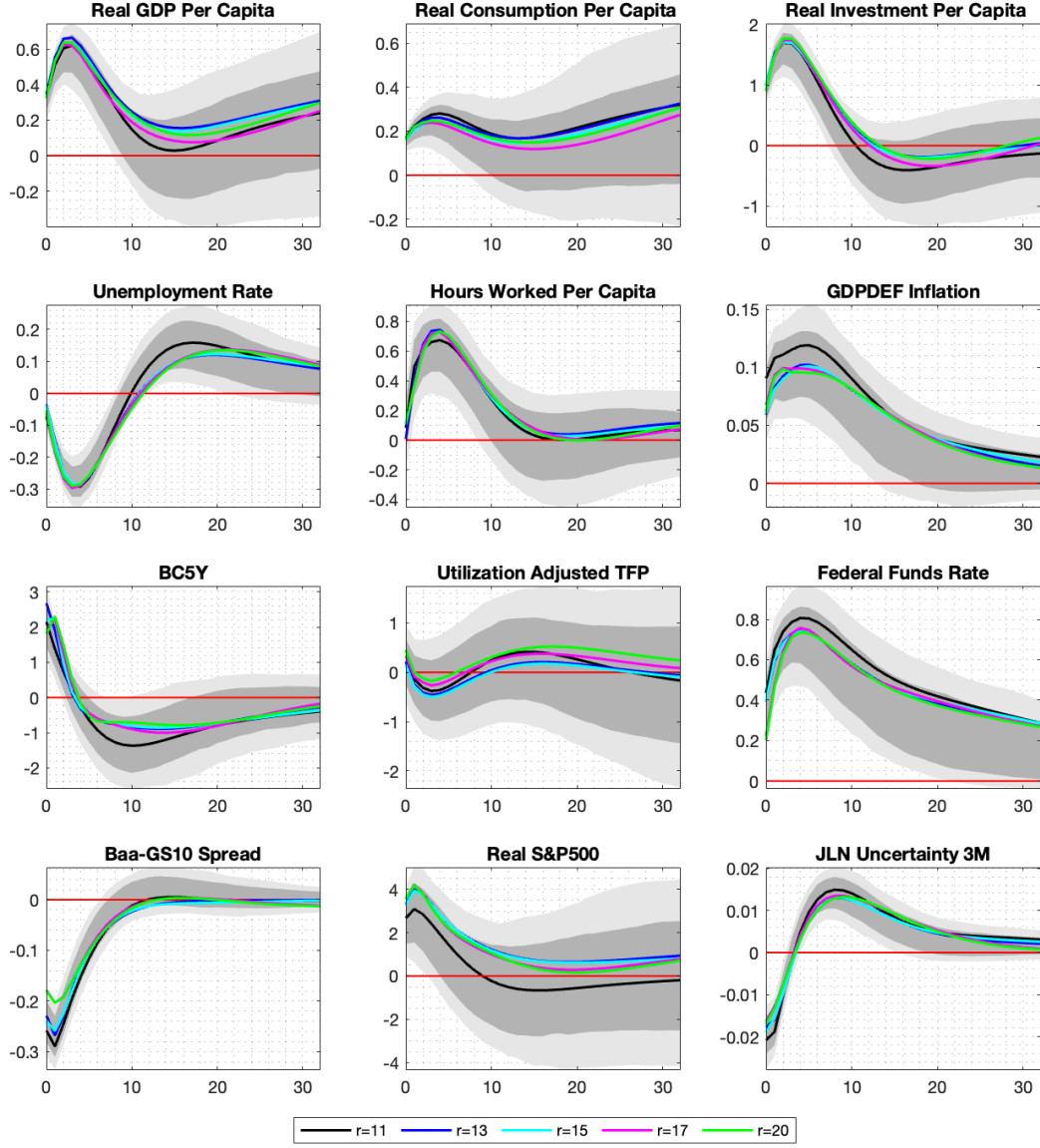


Figure E.9: IDENTIFICATION I: IMPULSE RESPONSE FUNCTIONS OF THE DEMAND SHOCK, ACCORDING TO DIFFERENT NUMBER OF STATIC FACTORS: $r = [11 \ 6 \ 9 \ 13 \ 15]$. BASELINE SPECIFICATION: $r = 11$. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

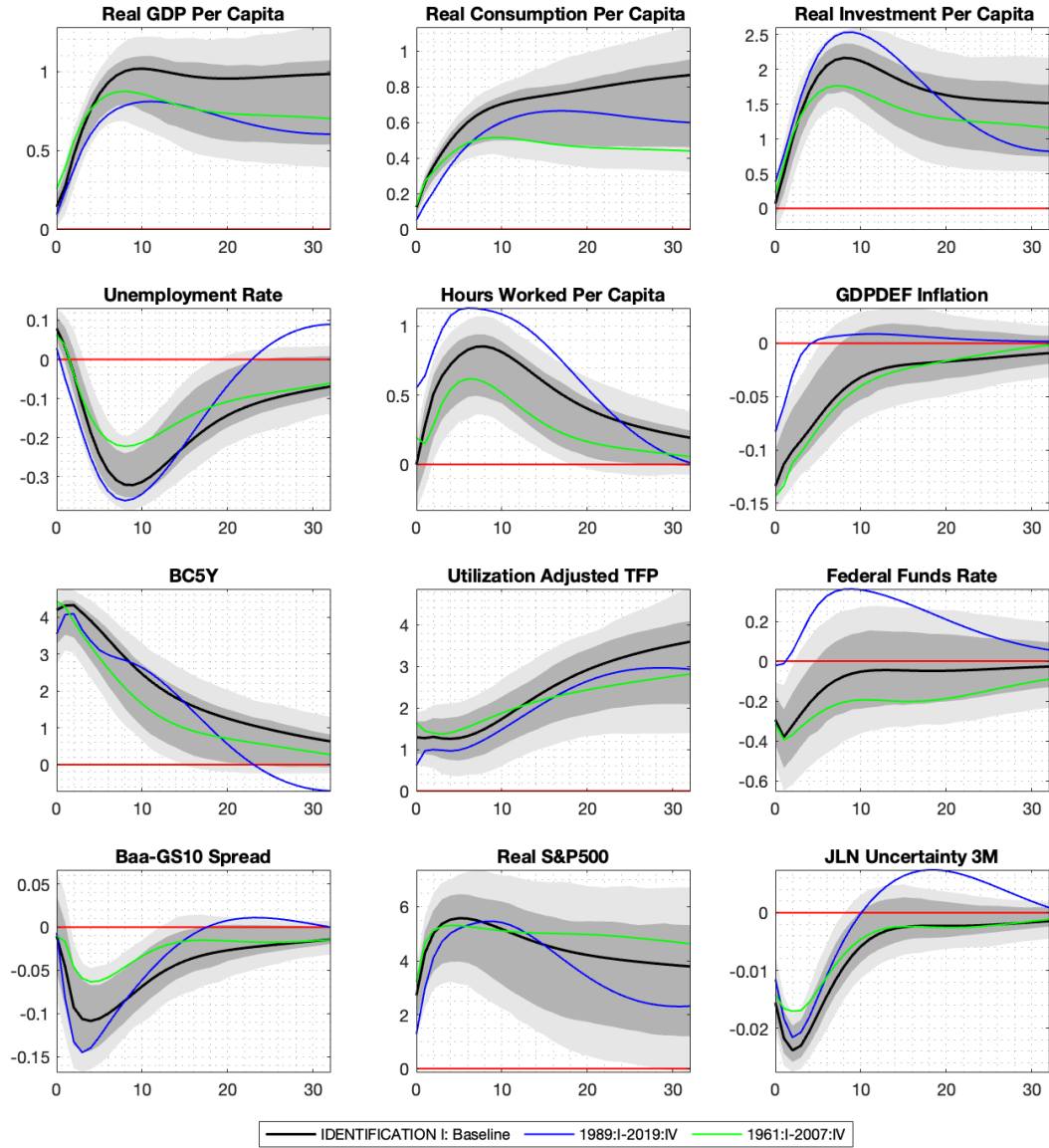


Figure E.10: IDENTIFICATION I: IMPULSE RESPONSE FUNCTIONS OF THE SUPPLY SHOCK, ACCORDING TO DIFFERENT SUBSAMPLES. BASELINE SPECIFICATION: 1961:I - 2019:IV. THE DARK GRAY AND LIGHT GRAY ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION. BLUE LINE: 1989:I - 2019:IV. GREEN LINE: 1961:I - 2007:IV.

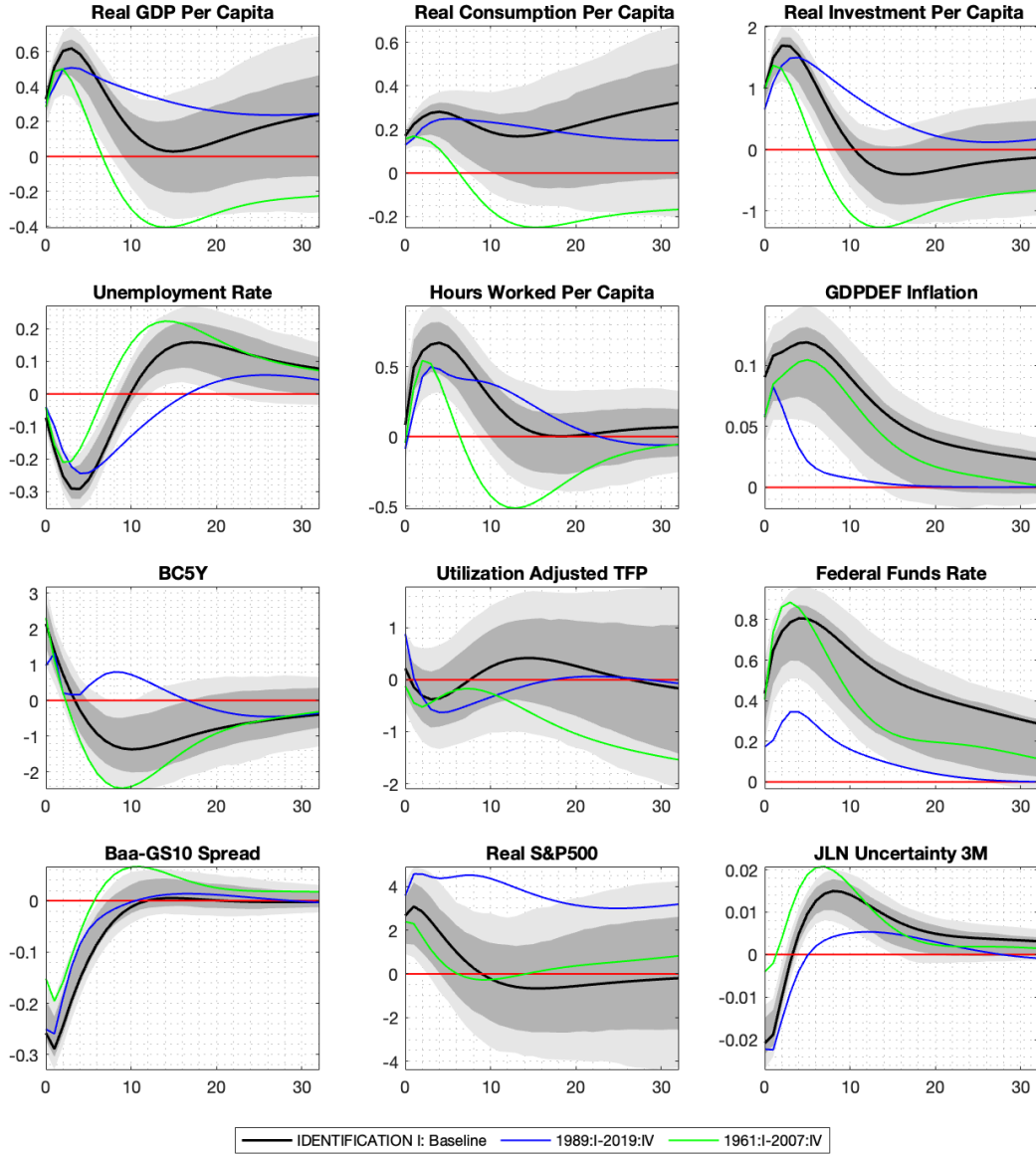


Figure E.11: IDENTIFICATION I: IMPULSE RESPONSE FUNCTIONS OF THE DEMAND SHOCK, ACCORDING TO DIFFERENT SUBSAMPLES. BASELINE SPECIFICATION: 1961:I - 2019:IV. THE DARK GRAY AND LIGHT GRAY ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION. BLUE LINE: 1989:I - 2019:IV. GREEN LINE: 1961:I - 2007:IV.

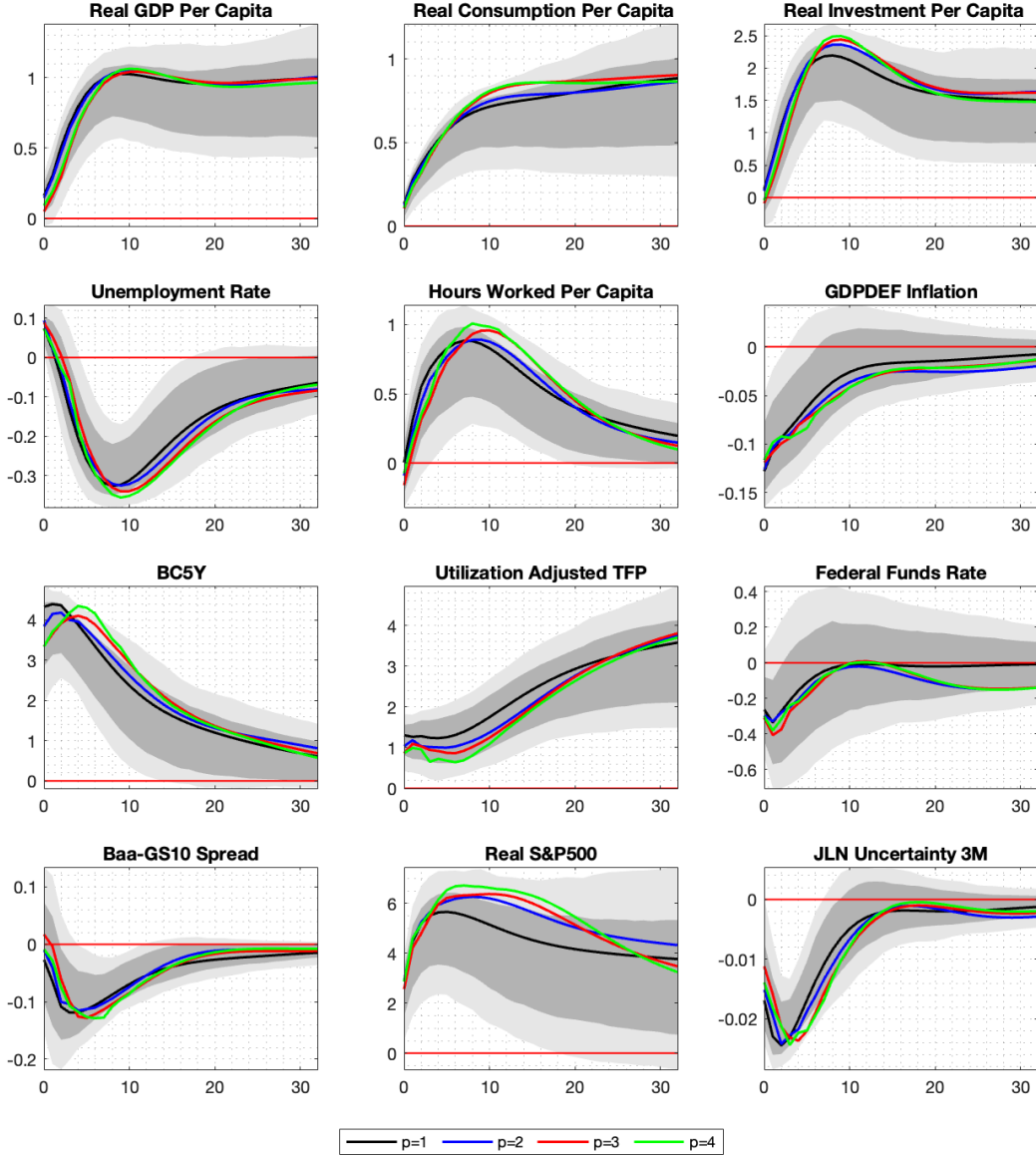


Figure E.12: IDENTIFICATION II: IMPULSE RESPONSE FUNCTIONS OF THE PERMANENT SHOCK, ACCORDING TO DIFFERENT LAGS ORDER: $p = [1 \ 2 \ 3 \ 4]$. BASELINE SPECIFICATION: $p = 1$. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

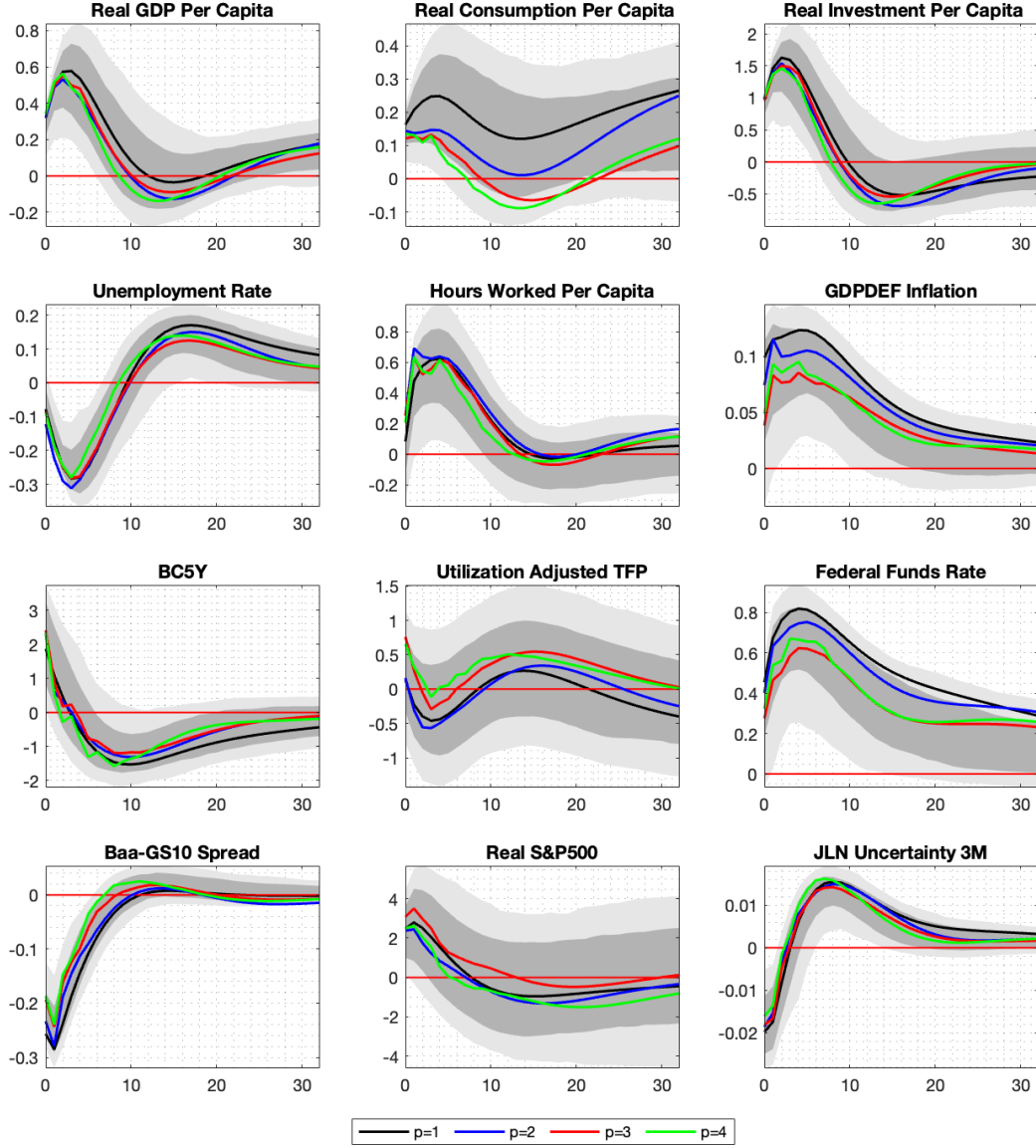


Figure E.13: IDENTIFICATION II: IMPULSE RESPONSE FUNCTIONS OF THE TRANSITORY SHOCK, ACCORDING TO DIFFERENT LAGS ORDER: $p = [1 \ 2 \ 3 \ 4]$. BASELINE SPECIFICATION: $p = 1$. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

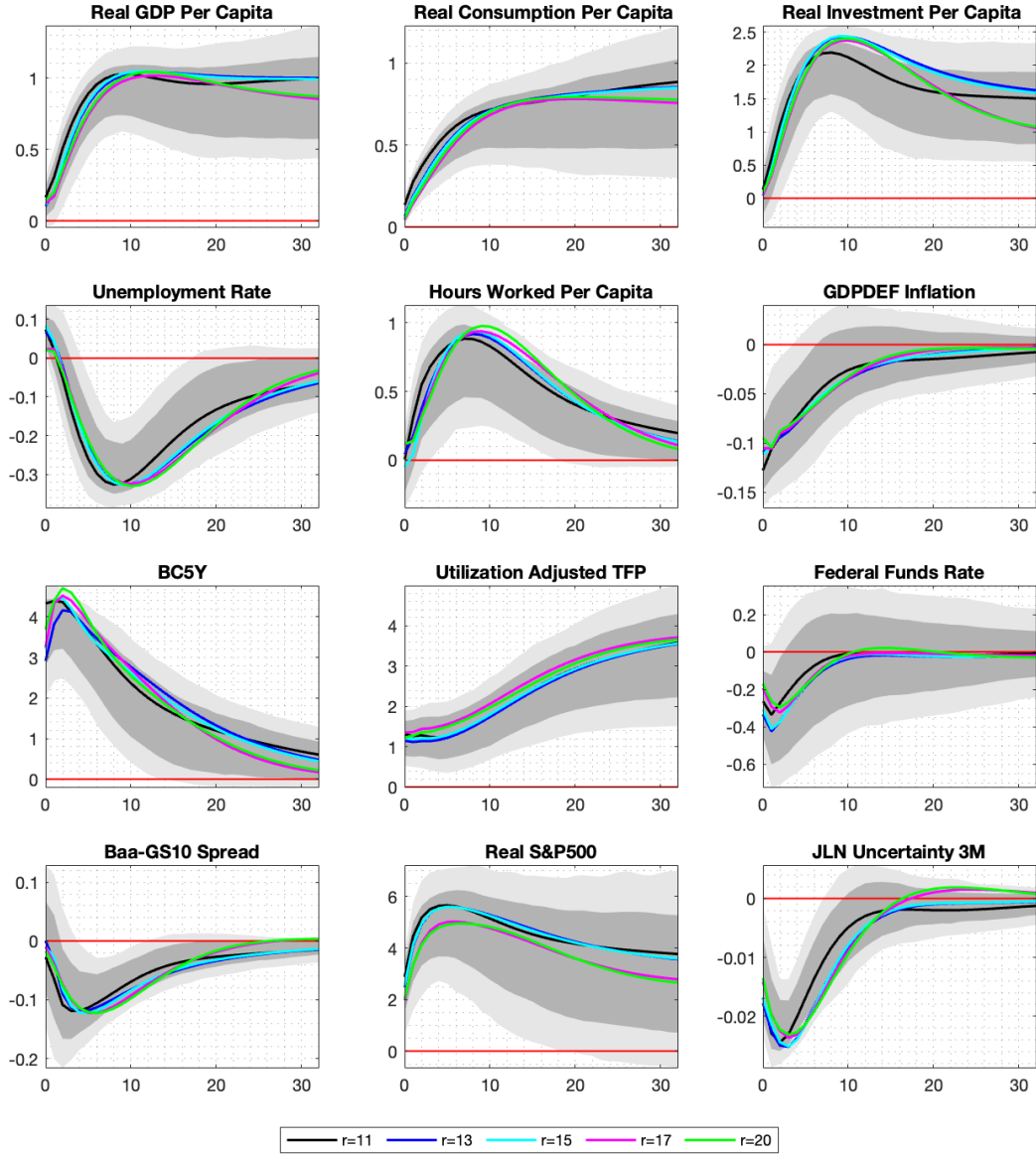


Figure E.14: IDENTIFICATION II: IMPULSE RESPONSE FUNCTIONS OF THE PERMANENT SHOCK, ACCORDING TO DIFFERENT NUMBER OF STATIC FACTORS: $r = [11 \ 6 \ 9 \ 13 \ 15]$. BASELINE SPECIFICATION: $r = 11$. THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

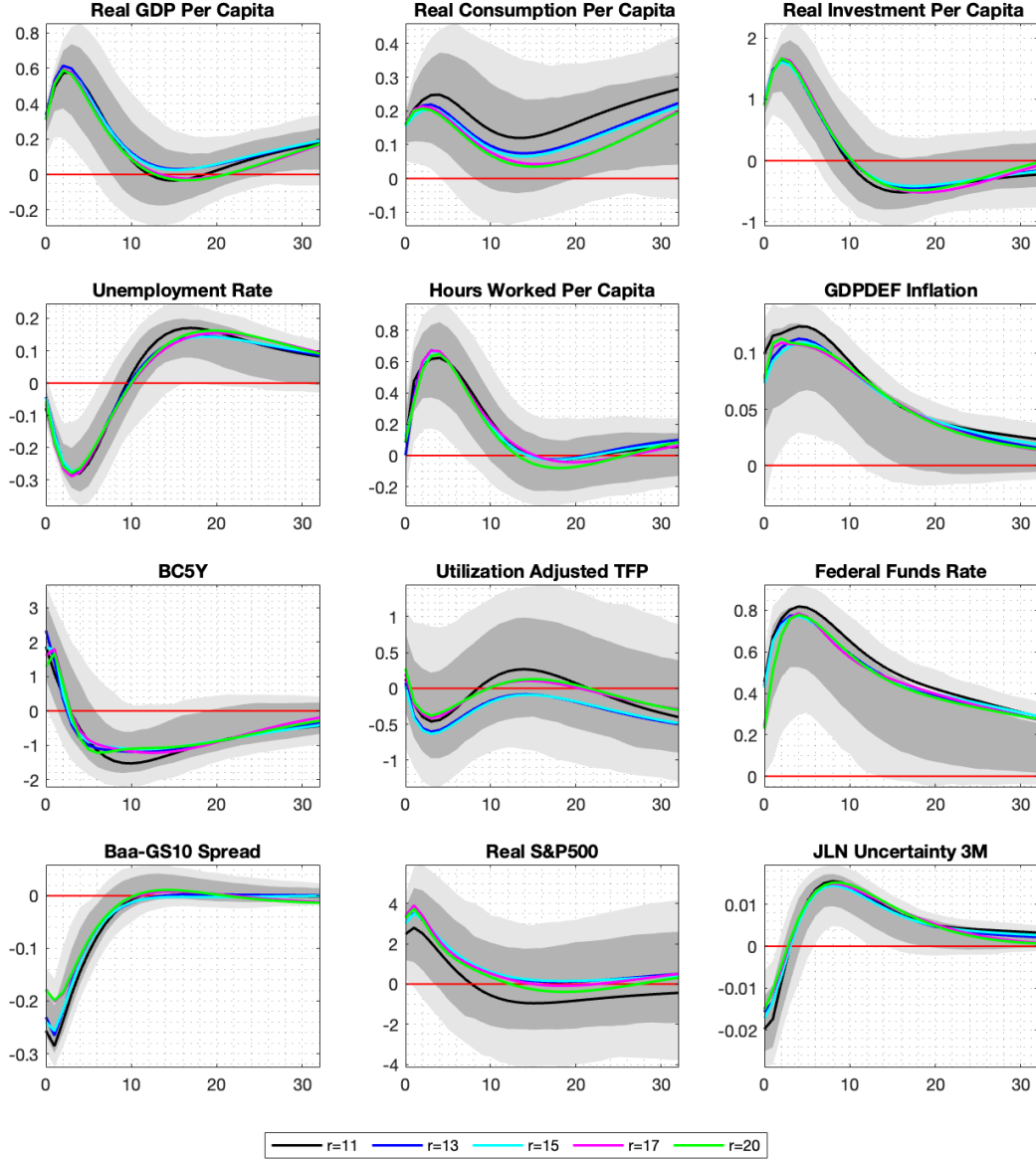


Figure E.15: IDENTIFICATION II: IMPULSE RESPONSE FUNCTIONS OF THE TRANSITORY SHOCK, ACCORDING TO DIFFERENT NUMBER OF STATIC FACTORS: $r = [11 \ 6 \ 9 \ 13 \ 15]$. BASELINE SPECIFICATION: $r = 11$. THE DARK GRAY AND LIGHT GRAY ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

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