Online Appendix for "Welfare Analysis of Changing Notches: Evidence from Bolsa Família"

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A Theory Appendix

A.1 Optimal Reported Incomes in Baseline Model

Type $\mu = 2$ households with $y \leq \tau$ get the following utility dependent on their choice of reported income $\hat{y}$:

$$\begin{cases} y + b - v(y - \hat{y}) & \text{if } \hat{y} < y \\ y + b & \text{if } \hat{y} \in [y, \tau] \\ y & \text{if } \hat{y} > \tau \end{cases}$$

Given that $v(y - \hat{y}) = 0$ for $\hat{y} \geq y$ and $v'(\cdot) > 0$, we know that $v(y - \hat{y}) \geq 0$. Thus, households with $y \leq \tau$ prefer to report $\hat{y} \in [y, \tau]$ (and are indifferent between reporting income levels in this range as only under-reporting is costly). We assume they break this indifference by reporting at y. Anyone with $y > \tau$ gets the following utility dependent on their choice of reported income $\hat{y}$:

$$\begin{cases} y + b - v(y - \hat{y}) & \text{if } \hat{y} \leq \tau \\ y - v(y - \hat{y}) & \text{if } \hat{y} \in (\tau, y) \\ y & \text{if } \hat{y} \geq y \end{cases}$$

Clearly, reporting $\hat{y} \in (\tau, y)$ is always dominated by reporting $\hat{y} \geq y$ as $v(y-\hat{y}) \geq 0$. Moreover, reporting $\hat{y} = \tau$ always dominates reporting $\hat{y} < \tau$ because $v' > 0$. By definition, those with $y = y^c(\mathbf{p})$ are indifferent between misreporting at the threshold τ and truthfully reporting (we break their indifference by assuming they will misreport at τ), where $y^c(\mathbf{p})$ solves $y^c(\mathbf{p}) + b - v(y^c(\mathbf{p}) - \tau) = y^c(\mathbf{p})$. The existence of a unique $y^c(\mathbf{p})$ follows from $v' > 0$. Those with $\tau < y \leq y^c(\mathbf{p})$ prefer misreporting with $\hat{y} = \tau$ over truthfully reporting and those with $y > y^c(\mathbf{p})$ prefer truthfully reporting $\hat{y} = y$ by the fact that $v' > 0$.

A.2 Proof of Lemma 1

Let us begin by calculating the WTP for the mechanical households, which are the households for which $\hat{y}^*(y, \mu; \mathbf{p}) \leq \tau$ and $\hat{y}^*(y, \mu; \mathbf{p}') \leq \tau$. For type $\mu = 1$ households, these are just households for whom $y \leq \tau$. For type $\mu = 2$ households, we appeal to the optimal choice function in Equation 3, finding that $\hat{y}^*(y, \mu = 2; \mathbf{p}') \leq \tau \iff y \leq \tau$ and $y \leq \tau \implies \hat{y}^*(y, \mu = 2; \mathbf{p}) \leq \tau$. Thus, the number of mechanical households is just the set which report an income below τ under policy $\mathbf{p}'$: $M = G(\tau, \mathbf{p}')$. Moreover, $y \leq \tau$ implies $\hat{y}^*(y, \mu; \mathbf{p}) = y$ and $\hat{y}^*(y, \mu; \mathbf{p}') = y$. Hence, WTP for mechanical households is defined by:

$$y + b = y + b' - WTP \implies WTP = b' - b = \Delta b$$

Next, let us discuss the WTP for bunching households. Bunching households are those with $\hat{y}^*(y, \mu; \mathbf{p}) = \tau$ and $\hat{y}^*(y, \mu; \mathbf{p}') > \tau$. From Equation 3, one can immediately see that only households with $y \in (\tau, y^c(\mathbf{p})]$ and $\mu = 2$ satisfy these criterion. Equation 3 also implies that $\hat{y}^*(y, \mu = 2; \mathbf{p}) \leq \tau \iff y \leq y^c(\mathbf{p})$ and $\hat{y}^*(y, \mu = 2; \mathbf{p}') \leq \tau \iff y \leq \tau$ so that the number of bunching households is given by $B = G(\tau; \mathbf{p}) - G(\tau; \mathbf{p}')$. In words, only bunching households and mechanical households report incomes $\leq \tau$ under policy $\mathbf{p}$; under policy $\mathbf{p}'$ bunching households strictly increase their reported income while mechanical households do not. Based on their optimal choices from Equation 3, bunching households have utility $y + b - v(y - \tau)$ under policy $\mathbf{p}$. Also based on Equation 3, bunching households with $y \in (\tau, \tau']$ will set $\hat{y}^*(y, \mu = 2; \mathbf{p}') = y$, getting utility $y + b'$ under policy $\mathbf{p}'$. Those with $y \in (\tau', y^c(\mathbf{p})]$ will set $\hat{y}^*(y, \mu = 2; \mathbf{p}') = \tau'$, getting utility $y + b' - v(y - \tau')$ under policy $\mathbf{p}'$.[1] Hence, we know that the WTP for bunching households satisfies:

$$y + b - v(y - \tau) = y + b' - WTP - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$$

where $\hat{y}^*(y, \mu = 2; \mathbf{p}') = y$ if $y \in (\tau, \tau']$ and $\hat{y}^*(y, \mu = 2; \mathbf{p}') = \tau'$ if $y \in (\tau', y^c(\mathbf{p})]$. Equivalently:

$$WTP = b' - b + v(y - \tau) - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$$

Given that $v(y - \tau) - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) > 0$ (as $v' > 0$ and $\hat{y}^*(y, \mu = 2; \mathbf{p}') > \tau$ for bunching households), we know $WTP \geq b' - b = \Delta b$. Moreover, because $y + b - v(y - \tau) \geq y$ for bunching households (or, equivalently, $v(y - \tau) \leq b$), we know that $WTP \leq b' - b + b - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) = b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$. And because $v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) \geq 0$, we therefore know that $WTP \leq b'$ for bunching households.

Next, we turn to threshold households: those with $\hat{y}^*(y, \mu; \mathbf{p}) \in (\tau, \tau']$. There are $T = G(\tau'; \mathbf{p}) - G(\tau; \mathbf{p})$ such households. Equation 3 implies that anyone with $\mu = 2$ and $\hat{y}^*(y, \mu; \mathbf{p}) \in (\tau, \tau']$ must have $y = \hat{y}^*(y, \mu; \mathbf{p})$ and clearly anyone with $\mu = 1$ with $\hat{y}^*(y, \mu; \mathbf{p}) \in (\tau, \tau']$ must also have $y = \hat{y}^*(y, \mu; \mathbf{p})$. Equation 3 also then implies that all threshold households report $\hat{y}^*(y, \mu; \mathbf{p}') = y \in (\tau, \tau']$ as well. All of these households therefore get utility y under policy $\mathbf{p}$ and receive utility $y + b'$ under policy $\mathbf{p}'$. Hence, their WTP is given by:

$$y = y + b' - WTP \implies WTP = b'$$

Finally, there are jumping households for whom $\hat{y}^*(y, \mu; \mathbf{p}) > \tau'$ and $\hat{y}^*(y, \mu; \mathbf{p}') \leq \tau'$. Clearly, these must be type $\mu = 2$ households. By Equation 3, $\hat{y}^*(y, \mu; \mathbf{p}) \leq \tau' \implies \hat{y}^*(y, \mu; \mathbf{p}') \leq \tau'$ so that the number of jumping households is given by the increase in households reporting at or below τ' as a result of the reform: $J = G(\tau'; \mathbf{p}') - G(\tau'; \mathbf{p})$. Next, note that by Equation 3, if someone has $\hat{y}^*(y, \mu; \mathbf{p}) > \tau'$ then they must report $\hat{y}^*(y, \mu; \mathbf{p}) = y$ yielding utility y under policy $\mathbf{p}$. Under policy

1. Note, if $y^c(\mathbf{p}) \leq \tau'$, i.e., the change in τ is large, all bunching households will set $\hat{y}^*(y, \mu = 2; \mathbf{p}') = y$.

$\mathbf{p}'$ these households have $\hat{y}^*(y, \mu = 2; \mathbf{p}') \leq \tau'$ yielding utility $y + b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}'))$. Thus, WTP for these individuals is given by:

$$y = y + b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) - WTP$$

Because $y + b' - v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) \geq y$ by revealed preference, $WTP \geq 0$. And because $v(y - \hat{y}^*(y, \mu = 2; \mathbf{p}')) \geq 0$, $WTP \leq b'$ for jumping individuals.

Putting this all together we get:

$$\Delta b(M + B) + b'T \leq \text{Total WTP} \leq \Delta bM + b'(B + T + J)$$

A.3 Proof of Proposition 3

Proof. We start with proving the lower bound for $\frac{\frac{1}{\lambda}[\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p})]}{[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})]}$. First, welfare under policy $\mathbf{p}$ is given by:

$$\mathcal{W}(\mathbf{p}) = \int_{\Theta} \phi(\theta) u \left(y \left[\mathbf{x}^*(\theta, \mathbf{p}), \theta\right] + b\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta, \mathbf{p}\right), \theta\right) \leq \tau\right], \mathbf{x}^*(\theta, \mathbf{p}); \theta\right) dF(\theta) - \lambda bG(\tau; \mathbf{p}) \tag{18}$$

Next, note that by revealed preference, we have the following for any $\mathbf{x} \in \mathbf{X}$:

$$u\left(y\left[\mathbf{x}^*\left(\theta, \mathbf{p}\right), \theta\right] + b\mathbb{1}\left[\hat{y}(\mathbf{x}^*\left(\theta, \mathbf{p}\right), \theta) \leq \tau\right], \mathbf{x}^*(\theta, \mathbf{p}); \theta\right) \geq u\left(y\left[\mathbf{x}, \theta\right] + b\mathbb{1}\left[\hat{y}(\mathbf{x}, \theta) \leq \tau\right], \mathbf{x}; \theta\right) \tag{19}$$

Put simply, optimal decisions conditional on any given θ under $\mathbf{p}$, $\mathbf{x}^*(\theta, \mathbf{p})$, yield weakly higher utility than any other set of decisions $\mathbf{x}$ that one could make. Using Equations (18) and (19), we can bound welfare under policy $\mathbf{p}' = \{b', \tau'\}$ by evaluating utility under policy $\mathbf{p}'$, but holding household decisions constant at their values under policy $\mathbf{p}$ (i.e., by revealed preference):

$$\begin{aligned}
\mathcal{W}(\mathbf{p}') &= \int_{\Theta} \phi(\theta) u \left(y\left[\mathbf{x}^*(\theta, \mathbf{p}'), \theta\right] + b'\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta, \mathbf{p}'\right), \theta\right) \leq \tau'\right], \mathbf{x}^*(\theta, \mathbf{p}'); \theta\right) dF(\theta) - \lambda b'G(\tau'; \mathbf{p}') \\
&\geq \int_{\Theta} \phi(\theta) u \left(y\left[\mathbf{x}^*(\theta, \mathbf{p}), \theta\right] + b'\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta, \mathbf{p}\right), \theta\right) \leq \tau'\right], \mathbf{x}^*(\theta, \mathbf{p}); \theta\right) dF(\theta) - \lambda b'G(\tau'; \mathbf{p}')
\end{aligned} \tag{20}$$

So as to slightly reduce some cumbersome notation, let us define:

$$y^*(\theta, \mathbf{p}) \equiv y\left[\mathbf{x}^*\left(\theta, \mathbf{p}\right), \theta\right]$$

$$\hat{y}^*(\theta, \mathbf{p}) \equiv \hat{y}[\mathbf{x}^*\left(\theta, \mathbf{p}\right), \theta]$$

Thus, for the reform from $\mathbf{p} = \{b, \tau\}$ to $\mathbf{p}' = \{b', \tau'\}$ with $\mathbf{p}' - \mathbf{p} = \{\Delta b, \Delta \tau\}$ and $\Delta \tau > 0$ we can combine Equations (18) and (20) as well as split up the domain Θ into sets of households who receive the benefit under policy $\mathbf{p}$ and those who do not receive the benefit under policy $\mathbf{p}$ to yield:

$$\begin{aligned}\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq &\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\leq\tau} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p})+b',\mathbf{x}^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p})+b,\mathbf{x}^*(\theta,\mathbf{p});\theta\right]\right\}dF(\theta)\\ &+\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p})+b',\mathbf{x}^*\left(\theta,\mathbf{p}\right);\theta\right] - u\left[y^*(\theta,\mathbf{p}),\mathbf{x}^*\left(\theta,\mathbf{p}\right);\theta\right]\right\}dF(\theta)\\ &-\lambda[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})]\end{aligned} \tag{21}$$

Note, the change in utility for those with $\hat{y}^*(\theta,\mathbf{p}) > \tau'$ is zero as we move from $\mathbf{p}$ to $\mathbf{p}'$, holding decisions fixed, as they do not receive a transfer. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}$ as the government's average welfare weight on the households who optimally report incomes $\hat{y}^* \leq \tau$ under policy $\mathbf{p}$:

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\leq\tau} \phi(\theta)\frac{1}{b'-b}\left\{u\left[y^*(\theta,\mathbf{p})+b',\mathbf{x}^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p})+b,\mathbf{x}^*(\theta,\mathbf{p});\theta\right]\right\}dF(\theta)}{G(\tau,\mathbf{p})} \tag{22}$$

Note that in Equation (22), we divide by $\frac{1}{b'-b}$, which renormalizes by the additional amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}) \leq \tau$ due to the reform; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}$ as the average welfare gain from giving these households an extra \$1. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ as the government's average welfare weight of giving a dollar to the households who optimally report incomes $\hat{y}^* \in (\tau,\tau']$ under policy $\mathbf{p}$:[2]

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\frac{1}{b'}\left\{u\left[y^*(\theta,\mathbf{p})+b',\mathbf{x}^*(\theta,\mathbf{p});\theta\right] - u\left[y^*(\theta,\mathbf{p}),\mathbf{x}^*(\theta,\mathbf{p});\theta\right]\right\}dF(\theta)}{G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})} \tag{23}$$

Again, note that in Equation (23) we have divided by $\frac{1}{b'}$, which renormalizes by the amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}) \in (\tau,\tau']$; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ as the average welfare gain from giving these households an extra \$1. We can rewrite Equation (21) using Equations (22) and (23) as follows:

$$\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq \eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}b'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right] - \lambda\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right] \tag{24}$$

Next, let us define the aggregate welfare weight, η_L, which equals the weighted average welfare weight of giving a dollar to all households, where the weights are determined by the lower bound of WTP for the reform:

$$\eta_L = \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}b'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]}{\Delta bG(\tau;\mathbf{p}) + b'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]} \tag{25}$$

Then, dividing Equation (24) through by the budgetary effect multiplied by λ, we have (recall we

2. We have used $\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} dF(\theta) = G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})$.

assume the budgetary effect is > 0):

$$\begin{aligned}\frac{\frac{1}{\lambda}\left[\mathcal{W}\left(\mathbf{p}'\right)-\mathcal{W}(\mathbf{p})\right]}{\left[b'G\left(\tau';\mathbf{p}'\right)-bG\left(\tau;\mathbf{p}\right)\right]} &\geq \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}}\Delta bG(\tau;\mathbf{p})+\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}b'\left[G(\tau';\mathbf{p})-G(\tau;\mathbf{p})\right]}{\lambda\left[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})\right]}-1 \\ &= \frac{\eta_L}{\lambda}\frac{\Delta bG(\tau;\mathbf{p})+b'\left[G(\tau';\mathbf{p})-G(\tau;\mathbf{p})\right]}{\left[b'G\left(\tau';\mathbf{p}'\right)-bG\left(\tau;\mathbf{p}\right)\right]}-1 \\ &= \omega_L MVPF_L - 1\end{aligned} \tag{26}$$

where $\omega_L = \eta_L/\lambda$ and $MVPF_L$ is given by:

$$MVPF_L = \frac{\Delta bG(\tau;\mathbf{p})+b'[G(\tau';\mathbf{p})-G(\tau,\mathbf{p})]}{b'G\left(\tau';\mathbf{p}'\right)-bG\left(\tau;\mathbf{p}\right)} = 1-\frac{b'\left[G(\tau';\mathbf{p}')-G(\tau';\mathbf{p})\right]}{b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})} \tag{27}$$

Next, we prove the upper bound for $\frac{\frac{1}{\lambda}[\mathcal{W}(\mathbf{p}')-\mathcal{W}(\mathbf{p})]}{[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})]}$. We use identical revealed preference logic to bound welfare under policy $\mathbf{p} = \{b, \tau\}$ by evaluating utility under policy $\mathbf{p}$, but holding household decisions constant at their values under policy $\mathbf{p}'$:

$$\begin{aligned}\mathcal{W}(\mathbf{p}) &= \int_{\Theta} \phi(\theta)u\left(y^*(\theta,\mathbf{p})+b\mathbb{1}\left[\hat{y}^*(\theta,\mathbf{p})\leq\tau\right],\mathbf{x}^*(\theta,\mathbf{p});\theta\right)dF(\theta)-\lambda bG(\tau;\mathbf{p}) \\ &\geq \int_{\Theta} \phi(\theta)u\left(y^*(\theta,\mathbf{p}')+b\mathbb{1}\left[\hat{y}^*(\theta,\mathbf{p}')\leq\tau\right],\mathbf{x}^*(\theta,\mathbf{p}');\theta\right)dF(\theta)-\lambda bG(\tau;\mathbf{p})\end{aligned} \tag{28}$$

Hence, for the reform from $\mathbf{p} = \{b, \tau\}$ to $\mathbf{p}' = \{b', \tau'\}$ with $\mathbf{p}' - \mathbf{p} = \{\Delta b, \Delta\tau\}$ and $\Delta\tau > 0$:

$$\begin{aligned}\mathcal{W}(\mathbf{p}')-\mathcal{W}(\mathbf{p}) \leq &\int_{\theta:\hat{y}^*(\theta,\mathbf{p}')\leq\tau} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p}')+b',\mathbf{x}^*(\theta,\mathbf{p}');\theta\right]-u\left[y^*(\theta,\mathbf{p}')+b,\mathbf{x}^*(\theta,\mathbf{p}');\theta\right]\right\}dF(\theta) \\ &+\int_{\theta:\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']} \phi(\theta)\left\{u\left[y^*(\theta,\mathbf{p}')+b',\mathbf{x}^*\left(\theta,\mathbf{p}'\right);\theta\right]-u\left[y^*(\theta,\mathbf{p}'),\mathbf{x}^*\left(\theta,\mathbf{p}'\right);\theta\right]\right\}dF(\theta) \\ &-\lambda[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})]\end{aligned} \tag{29}$$

Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}$ as the government's average welfare weight on the households who optimally report incomes $\hat{y}^* \leq \tau$ under policy $\mathbf{p}'$:

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}} = \frac{\int_{\theta:\hat{y}^*(\theta,\mathbf{p}')\leq\tau} \phi(\theta)\frac{1}{b'-b}\left\{u\left[y^*(\theta,\mathbf{p}')+b',\mathbf{x}^*(\theta,\mathbf{p}');\theta\right]-u\left[y^*(\theta,\mathbf{p}')+b,\mathbf{x}^*(\theta,\mathbf{p}');\theta\right]\right\}dF(\theta)}{G(\tau,\mathbf{p}')} \tag{30}$$

Note that in Equation (30), we divide by $\frac{1}{b'-b}$, which renormalizes by the amount of money given to households with $\hat{y}^*(\theta, \mathbf{p}') \leq \tau$; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}$ as capturing the average welfare gain from giving these households an extra \$1. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}$ as the government's average welfare weight of giving a dollar to the households who optimally report incomes $\hat{y}^* \in (\tau, \tau']$ under policy $\mathbf{p}'$:[3]

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\frac{1}{b'}\left\{u\left[y^*(\theta,\mathbf{p}')+b',\mathbf{x}^*(\theta,\mathbf{p}');\theta\right]-u\left[y^*(\theta,\mathbf{p}'),\mathbf{x}^*(\theta,\mathbf{p}');\theta\right]\right\}dF(\theta)}{G(\tau';\mathbf{p}')-G(\tau,\mathbf{p}')} \tag{31}$$

Again, note that in Equation (31) we have divided by $\frac{1}{b'}$, which renormalizes by the amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}') \in (\tau,\tau']$; hence, we can interpret $\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}$ as capturing the average welfare gain from giving these households an extra \$1. We can rewrite Equation (29) using Equations (30) and (31) as follows:

$$\mathcal{W}(\mathbf{p}')-\mathcal{W}(\mathbf{p}) \leq \eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}\Delta b G(\tau;\mathbf{p}')+\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}b'\left[G(\tau';\mathbf{p}')-G(\tau;\mathbf{p}')\right]-\lambda\left[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})\right] \tag{32}$$

Next, let us define the aggregate welfare weight, η_U, which equals the weighted average welfare weight of giving a dollar to all households, where the weights are determined by the upper bound of WTP for the reform:

$$\eta_U = \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}\Delta b G(\tau;\mathbf{p}')+\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}b'[G(\tau';\mathbf{p}')-G(\tau,\mathbf{p}')]}{\Delta b G(\tau;\mathbf{p}')+b'[G(\tau';\mathbf{p}')-G(\tau,\mathbf{p}')]}$$

Then, dividing Equation (32) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is > 0):

$$\begin{aligned}
\frac{\frac{1}{\lambda}\left[\mathcal{W}(\mathbf{p}')-\mathcal{W}(\mathbf{p})\right]}{\left[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})\right]} &\leq \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\leq\tau\}}\Delta b G(\tau;\mathbf{p}')+\eta_{\{\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']\}}b'\left[G(\tau';\mathbf{p}')-G(\tau;\mathbf{p}')\right]}{\lambda\left[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})\right]}-1 \\
&= \frac{\eta_U}{\lambda}\frac{\Delta b G(\tau;\mathbf{p}')+b'\left[G(\tau';\mathbf{p}')-G(\tau;\mathbf{p}')\right]}{\left[b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})\right]}-1 \\
&= \omega_U MVPF_U - 1
\end{aligned} \tag{33}$$

where $\omega_U = \eta_U/\lambda$ and $MVPF_U$ is given by:

$$MVPF_U = \frac{\Delta b G(\tau;\mathbf{p}')+b'[G(\tau';\mathbf{p}')-G(\tau,\mathbf{p}')]}{b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})} = 1+\frac{b\left[G(\tau;\mathbf{p})-G(\tau;\mathbf{p}')\right]}{b'G(\tau';\mathbf{p}')-bG(\tau;\mathbf{p})} \tag{34}$$

□

A.4 Welfare Impacts of Infinitesimal Reforms to Notches

In this Appendix, we show how to construct bounds for welfare impacts of infinitesimal reforms in the context of the model from Section I.A. First, let us start by explicitly writing out the expression

3. We have used $\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p}')\in(\tau,\tau']} dF(\theta) = G(\tau';\mathbf{p}')-G(\tau,\mathbf{p}')$.

for total welfare under policy $\mathbf{p}$ in Equation 4 given the optimal decision rule specified in Equation 3:

$$\begin{aligned}\mathcal{W}(\mathbf{p}) = & \int_0^{\tau} \phi(y,1)[y+b]dF(y|\mu=1)\pi(1) + \int_{\tau}^{\infty} \phi(y,1)ydF(y|\mu=1)\pi(1) \\ & + \int_0^{\tau} \phi(y,2)[y+b]dF(y|\mu=2)\pi(2) + \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)[y+b-v(y-\tau)]dF(y|\mu=2)\pi(2) \\ & + \int_{y^c(\mathbf{p})}^{\infty} \phi(y,2)ydF(y|\mu=2)\pi(2) - \lambda bG(\tau;\mathbf{p})\end{aligned} \tag{35}$$

We are interested in understanding the object:

$$\frac{d\mathcal{W}(\mathbf{p})}{d\mathbf{p}} \cdot d\mathbf{p} = \frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau$$

for some small $db, d\tau$ (this is just a first order Taylor series approximation of welfare around $\mathbf{p} = [b, \tau]$). Differentiating Equation (35), we get:

$$\begin{aligned}& \frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau \\ & = db \sum_{\mu=1,2} \int_0^{\tau} \phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau \\ & + db \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)dF(y|\mu=2)\pi(2) + d\tau \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)v'(y-\tau)dF(y|\mu=2)\pi(2) \\ & - \lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db - \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau\end{aligned} \tag{36}$$

Note, in computing Equation (36), we have used Leibniz integral rule multiple times (along with the facts that $y^c(\mathbf{p}) + b - v(y^c(\mathbf{p}) - \tau) = y^c(\mathbf{p})$ and $v(0) = 0$). For instance:

$$\begin{aligned}& \frac{\partial}{\partial \tau}\left[\int_0^{\tau} \phi(y,1)[y+b]dF(y|\mu=1)\pi(1) + \int_{\tau}^{\infty} \phi(y,1)ydF(y|\mu=1)\pi(1)\right] \\ & = [y+b]\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau - y\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau = b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau\end{aligned} \tag{37}$$

The Leibniz terms for the type $\mu = 2$ individuals all cancel out as well. For instance:

$$\phi(\tau,2)[\tau+b]f(\tau|\mu=2)\pi(2) = \phi(\tau,2)[\tau+b-v(\tau-\tau)]f(\tau|\mu=2)\pi(2) \tag{38}$$

and

$$\begin{aligned}& \left[\phi(y^c(\mathbf{p}),2)[y^c(\mathbf{p})+b-v(y^c(\mathbf{p})-\tau)]f(y^c(\mathbf{p})|\mu=2)\pi(2)\right]\frac{\partial y^c(\mathbf{p})}{\partial \tau} \\ & = \left[\phi(y^c(\mathbf{p}),2)y^c(\mathbf{p})f(y^c(\mathbf{p})|\mu=2)\pi(2)\right]\frac{\partial y^c(\mathbf{p})}{\partial \tau}\end{aligned} \tag{39}$$

Looking at Equation (36), it becomes clear that to recover the exact welfare impact, we need to understand $v'(\cdot)$, which is difficult because, to the best of our knowledge, $v'(\cdot)$ is not related to any observable behavioral effects. Nonetheless, we can bound the infinitesimal welfare impact from below by noting that $v'(\cdot) > 0$ so that:

$$\begin{aligned}
&\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau \\
&\geq db \sum_{\mu=1,2} \int_0^{\tau} \phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau \\
&+ db \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)dF(y|\mu=2)\pi(2) - \lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db - \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau
\end{aligned} \tag{40}$$

Moreover, we can bound the infinitesimal welfare impact from above by noting that $v(y^c(\mathbf{p}) - \tau) = b$ (by the indifference condition for $y^c(\mathbf{p})$) so that $\int_{\tau}^{y^c(\mathbf{p})} v'(y-\tau)dy = b$ (by the fundamental theorem of calculus and the fact that $v(0) = 0$):

$$\int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)v'(y-\tau)dF(y|\mu=2)\pi(2) \leq \int_{\tau}^{y^c(\mathbf{p})} \phi(\bar{y},2)v'(y-\tau)f(\bar{y}|\mu=2)dy\pi(2) = b\phi(\bar{y},2)f(\bar{y}|\mu=2)\pi(2)$$

where $\bar{y}$ is the value of $y \in [\tau, y^c(\mathbf{p})]$ that maximizes $\phi(y,2)f(y|\mu=2)$. Conceptually, we're bounding the welfare impact of moving the threshold for the bunching households by b times $\phi(\bar{y},2)f(\bar{y}|\mu = 2)\pi(2)$, which is a maximium density weighted welfare weight that we put on bunching households. Hence, we have that:

$$\begin{aligned}
&\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau \\
&\leq db \sum_{\mu=1,2} \int_0^{\tau} \phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau \\
&+ db \int_{\tau}^{y^c(\mathbf{p})} \phi(y,2)dF(y|\mu=2)\pi(2) + d\tau\phi(\bar{y},2)bf(\bar{y}|\mu=2)\pi(2) \\
&- \lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db - \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau
\end{aligned} \tag{41}$$

Next, let us discuss the budgetary impacts of the reform. First, note that:

$$bG(\tau;\mathbf{p}) = b\left[\int_0^{\tau} dF(y|\mu=1)\pi(1) + \int_0^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2)\right]$$

Again using Leibniz integral rule:

$$\begin{aligned}
&\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau = \left[\int_0^{\tau} dF(y|\mu=1)\pi(1) + \int_0^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2)\right]db \\
&+ b\left\{f(\tau|\mu=1)\pi(1)d\tau + \left[\frac{\partial y^c(\mathbf{p})}{\partial b}db + \frac{\partial y^c(\mathbf{p})}{\partial \tau}d\tau\right]f(y^c(\mathbf{p})|\mu=2)\pi(2)\right\}
\end{aligned} \tag{42}$$

Note, for an infinitesimal reform, jumping households have a WTP of 0 as the only households who jump are those that are indifferent between locating at the notch and reporting truthfully. However, jumping households still have a first order impact on social welfare through their effect on the government's budget as each jumping household costs the government b' dollars (despite the fact that the mass of jumpers is measure 0 for an infinitesimal reform; see Bergstrom and Dodds (2021) for further discussion).

To simplify expressions, let us note that in the infinitesimal case, the number of mechanical households, M, equals $\sum_{\mu=1,2} \int_0^\tau dF(y|\mu)\pi(\mu)$, the number of threshold households, T, equals $f(\tau|\mu=1)\pi(1)d\tau$, the number of bunching households, B, is equal to $\int_\tau^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2)$, and the number of jumping households, J, is equal to $\left[\frac{\partial y^c(\mathbf{p})}{\partial b} db + \frac{\partial y^c(\mathbf{p})}{\partial \tau} d\tau\right] f(y^c(\mathbf{p})|\mu=2)\pi(2)$. Thus:

$$\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right] db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right] d\tau = dbM + bT + dbB + bJ \tag{43}$$

Assuming that money spent on the transfer goes up as a result of the reform (so that $\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right] db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right] d\tau > 0$):

$$\begin{aligned} &\frac{\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b} db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau} d\tau}{\lambda \frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right] db + \lambda \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right] d\tau} \\ &\geq \left\{ db \sum_{\mu=1,2} \int_0^\tau \phi(y,\mu) dF(y|\mu)\pi(\mu) + b\phi(\tau,1) f(\tau|\mu=1)\pi(1)d\tau \right. \\ &\left. + db \int_\tau^{y^c(\mathbf{p})} \phi(y,2) dF(y|\mu=2)\pi(2) \right\} \Big/ \left\{ dbM + bT + dbB + bJ \right\} - 1 \end{aligned} \tag{44}$$

Next, let us convert the above equation into an expression involving a lower bound for the MVPF by defining the incidence-weighted welfare weight:

$$\omega_L \equiv \frac{db \sum_{\mu=1,2} \int_0^\tau \phi(y,\mu) dF(y|\mu)\pi(\mu) + b\phi(\tau,1) f(\tau|\mu=1)\pi(1)d\tau + db \int_\tau^{y^c(\mathbf{p})} \phi(y,2) dF(y|\mu=2)\pi(2)}{db \sum_{\mu=1,2} \int_0^\tau dF(y|\mu)\pi(\mu) + b f(\tau|\mu=1)\pi(1)d\tau + db \int_\tau^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2)} \tag{45}$$

Next, note that:

$$db \sum_{\mu=1,2} \int_0^\tau dF(y|\mu)\pi(\mu) + b f(\tau|\mu=1)\pi(1)d\tau + db \int_\tau^{y^c(\mathbf{p})} dF(y|\mu=2)\pi(2) = dbM + bT + dbB \tag{46}$$

Plugging Equations (43), (45), and (46) into Equation (44) we get:

$$\frac{\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b} db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau} d\tau}{\lambda \frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right] db + \lambda \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right] d\tau} \geq \omega_L \frac{dbM + bT + dbB}{dbM + bT + dbB + bJ} - 1 \equiv \omega_L MVPF_L - 1 \tag{47}$$

where $MVPF_L = \frac{dbM+bT+dbB}{dbM+bT+dbB+bJ}$. Note that the lower bound for the MVPF in the infinitesimal case is the same as in the non-infinitesimal case discussed in Section I.A.

As far as the upper bound for the infinitesimal welfare change, let us define:

$$\omega_U \equiv \frac{db\sum_{\mu=1,2}\int_0^\tau \phi(y,\mu)dF(y|\mu)\pi(\mu) + b\phi(\tau,1)f(\tau|\mu=1)\pi(1)d\tau + db\int_\tau^{y^c(\mathbf{p})}\phi(y,2)dF(y|\mu=2)\pi(2) + d\tau\phi(\bar{y},2)bf(\bar{y})\pi(2)}{db\sum_{\mu=1,2}\int_0^\tau dF(y|\mu)\pi(\mu) + bf(\tau|\mu=1)\pi(1)d\tau + db\int_\tau^{y^c(\mathbf{p})}dF(y|\mu=2)\pi(2) + d\tau bf(\bar{y}|\mu=2)\pi(2)} \tag{48}$$

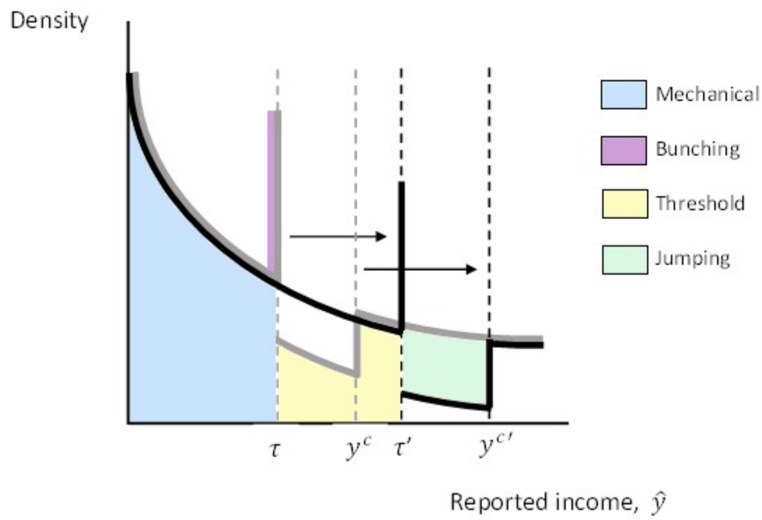
Plugging in Equation (48) into Equation (41), substituting the above expressions for M, T, B, J and dividing by $\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau > 0$, we get:

$$\frac{\frac{\partial \mathcal{W}(\mathbf{p})}{\partial b}db + \frac{\partial \mathcal{W}(\mathbf{p})}{\partial \tau}d\tau}{\lambda\frac{\partial}{\partial b}\left[bG(\tau;\mathbf{p})\right]db + \lambda\frac{\partial}{\partial \tau}\left[bG(\tau;\mathbf{p})\right]d\tau} \leq \omega_U\frac{dbM + bT + dbB + d\tau bf(\bar{y}|\mu=2)\pi(2)}{dbM + bT + dbB + bJ} - 1 \equiv \omega_U MVPF_U - 1 \tag{49}$$

where $MVPF_U = \frac{dbM+bT+dbB+d\tau bf(\bar{y}|\mu=2)\pi(2)}{dbM+dbB+bT+bJ}$. There are two key differences for the upper bound MVPF in the infinitesimal case relative to the bound given in Proposition 3. First, note that by the envelope theorem, the welfare impact for jumping households is zero, so they do not enter the numerator of the MVPF (whereas for large reforms, jumping households may have a positive WTP). As such, the upper bound for the MVPF may be below one in the infinitesimal case. Second, in the infinitesimal case, the upper bound for the MVPF contains the object $d\tau bf(\bar{y}|\mu=2)\pi(2)$, which captures the welfare impact of increasing the threshold τ for bunching households (due to decreasing their costs of misreporting). Note that this object is different than the bound we use in the non-infinitesimal case: $b \times B$. It is worthwhile to mention that $b \times B$ *also* is a bound for the welfare impact of increasing the threshold for bunching households in the infinitesimal case, but is vacuous because $b \times B \not\to 0$ as $db, d\tau \to 0$, so that using $b \times B$ in the infinitesimal case merely allows us to say the welfare impact of an infinitesimal change (which is also infinitesimal for small $db, d\tau$) is smaller than a constant. Similarly, $d\tau bf(\bar{y}|\mu=2)\pi(2)$ is also a bound for the welfare impact of increasing the threshold for bunching households in the non-infinitesimal case, but we do not use this bound for three reasons. First, in order to gauge $d\tau bf(\bar{y}|\mu=2)\pi(2)$, we need an estimate of $f(\bar{y})$ which requires one to estimate the density of true (rather than reported) incomes along with the fraction of households who are willing to misreport in the first place, $\pi(2)$. Second, because $\bar{y}$ is the value of $y \in [\tau, y^c(\mathbf{p})]$ that maximizes $\phi(y,2)f(y|\mu=2)$, we also need to know welfare weights $\phi(y,\mu)$, meaning that $f(\bar{y}|\mu=2)$ is not a purely positive object. Third, the economic interpretation of $d\tau bf(\bar{y}|\mu=2)\pi(2)$ is far less clear than the economic interpretation of $b \times B$.

A.5 Figure 1 when $\tau' > y^c(\mathbf{p})$

Figure 8 redraws Figure 1 in the main text under the scenario where $\tau' > y^c(\mathbf{p})$.



Note: This figure shows a hypothetical density of reported incomes under the initial policy $\mathbf{p} = \{b, \tau\}$ (solid grey curve) and how this density changes as a result of a reform that increases the policy to $\mathbf{p}' = \{b', \tau'\}$ (solid black curve). Note, the vertical solid grey line at τ and the vertical solid black line at τ' represent the bunching households under the initial policy and new policy, respectively. This figure also depicts which households are classified as mechanical households, bunching households, threshold households, and jumping households.

FIGURE 8: A HYPOTHETICAL DENSITY OF REPORTED INCOMES UNDER $\mathbf{p}$ AND $\mathbf{p}'$

A.6 Incidence-Weighted Welfare Weights and MVPF Bounds

It may not be immediately clear from Proposition 1 how exactly the MVPF can be combined with incidence-weighted welfare weights to make welfare statements. To illustrate this point, we work through a simple example. Suppose in the setup from Section I.A that the government places a welfare weight of 1 on the "honest" individuals with $\mu = 1$ and places a welfare weight of 0 on the "dishonest" individuals with $\mu = 2$. That is to say that the government's welfare function is given by:

$$\mathcal{W}(\mathbf{p}) = \sum_{\mu \in \{1,2\}} \int_Y \mathbb{1}[\mu = 1] U^*(y, \mu; \mathbf{p}) dF(y|\mu)\pi(\mu) - \lambda b G(\tau; \mathbf{p}) \tag{50}$$

Let us now explain the incidence-weighted welfare weights from Proposition 2 in this case. Let us begin with ω_L, the lower bound incidence-weighted welfare weight. Let us denote the fraction of mechanical households that are type $\mu = 1$ as $P_{M,1}$. Similarly, let $P_{B,1}$, $P_{T,1}$, $P_{J,1}$ denote the fraction of bunching, threshold, and jumping households that are type $\mu = 1$ (note $P_{B,1} = 0$ and $P_{J,1} = 0$ because type $\mu = 1$ households do not change their reported income in response to a policy change). The incidence-weighted welfare weight is as follows (this can be derived from the more general formulas in Appendix A.3, see Equations (22), (23), and (25) or, alternatively, see Equation 1 and related discussion from

Hendren and Sprung-Keyser (2020)):

$$\begin{aligned}\omega_L &= \frac{1}{\lambda}\frac{[\Delta bMP_{M,1} + \Delta bBP_{B,1} + b'TP_{T,1}] \times 1 + [\Delta bM(1-P_{M,1}) + \Delta bB(1-P_{B,1}) + b'T(1-P_{T,1})] \times 0}{\Delta b(M+B) + b'T} \\ &= \frac{1}{\lambda}\frac{\Delta bMP_{M,1} + b'TP_{T,1}}{\Delta b(M+B) + b'T}\end{aligned} \tag{51}$$

Hence, the lower bound welfare weight ω_L captures the fact that $P_{M,1}$ of the total WTP for mechanical households goes to $\mu = 1$ households we care about whereas $(1 - P_{M,1})$ of this WTP is "wasted" on $\mu = 2$ households that the government does not care about. Likewise, for the threshold households, $(1 - P_{T,1})$ of the total WTP for these households is "wasted" on the type $\mu = 2$ households that the government does not care about. Finally, all of the bunching and jumping households are type $\mu = 2$, so all of their WTP is "wasted" in this example. When we use ω_L to translate the lower bound $MVPF_L$ into a welfare statement, we find that:

$$\omega_L \times MVPF_L = \frac{1}{\lambda}\frac{\Delta bMP_{M,1} + b'TP_{T,1}}{\Delta b(M+B) + b'T}\frac{\Delta b(M+B) + b'T}{\Delta b(M+B) + b'(T+J)} = \frac{1}{\lambda}\frac{\Delta bMP_{M,1} + b'TP_{T,1}}{\Delta b(M+B) + b'(T+J)}$$

In words, to determine the lower bound for the welfare effect of the reform (relative to the government cost), the government takes the lower bound $MVPF_L$ and scales it by the fraction of the total WTP that goes to type $\mu = 1$ households. The same logic holds for how we use the upper bound incidence-weighted welfare weight ω_U to translate the upper bound $MVPF_U$ into an upper bound for the total welfare impact of the reform.

A.7 Incorporating Misperceptions of the Schedule

We now assume that households do not necessarily understand how the policy impacts their consumption. Households solve the following problem:

$$\begin{aligned}&\max_{x \in X} \; u(c, \mathbf{x}; \theta) \\ &\text{s.t. } \; c = f(y(\mathbf{x}, \theta), \hat{y}(\mathbf{x}, \theta), \mathbf{p}, \theta)\end{aligned} \tag{52}$$

In words, households make decisions under the assumption that their consumption is some function of their true income y, their reported income $\hat{y}$, the $\mathbf{p}$, and state variables θ. For instance, this framework allows for households to misperceive the threshold τ or the benefit level b (e.g., $f(y(\mathbf{x}, \theta), \hat{y}(\mathbf{x}, \theta), \mathbf{p}, \theta) = y(\mathbf{x}, \theta) + (b + \theta_1)\mathbb{1}[\hat{y}(\mathbf{x}, \theta) \leq (\tau + \theta_2)]$, for some values of θ_1, θ_2). Total welfare is still given by:

$$\mathcal{W}(\mathbf{p}) = \int_{\Theta} \phi(\theta) u\left[y\left(\mathbf{x}^*(\theta, \mathbf{p}), \theta\right) + b\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta, \mathbf{p}\right), \theta\right) \leq \tau\right], \mathbf{x}^*(\theta, \mathbf{p}); \theta\right] dF(\theta) - \lambda b G(\tau; \mathbf{p}) \tag{53}$$

In order for Proposition 3 to hold under the more general model with misperceptions (Problem (52)), we need to make two additional assumptions. We need to assume that when the policy changes from $\mathbf{p}$ to $\mathbf{p}'$, household behavioral re-optimization improves welfare, *on average*. In other words, misperceptions of the policy reform cannot be so severe that households make themselves worse off (on average) by responding to the reform. Mathematically, we require that:

$$\begin{aligned}\mathcal{W}(\mathbf{p}') &= \int_{\Theta} \phi(\theta)u\left[y\left(\mathbf{x}^*(\theta,\mathbf{p}'),\theta\right) + b'\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau'\right], \mathbf{x}^*(\theta,\mathbf{p}');\theta\right] dF(\theta) - \lambda b' G(\tau';\mathbf{p}') \\ &\geq \int_{\Theta} \phi(\theta)u\left[y\left(\mathbf{x}^*(\theta,\mathbf{p}),\theta\right) + b'\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau'\right], \mathbf{x}^*(\theta,\mathbf{p});\theta\right] dF(\theta) - \lambda b' G(\tau';\mathbf{p}')\end{aligned} \tag{54}$$

Note, that the previous inequality holds by revealed preference if households correctly perceive the schedule (i.e., behavioral re-optimization can only improve utility). If households misperceive the schedule, we need to assume that behavioral responses improve welfare on average. Correspondingly, our second assumption is that if, hypothetically, the policy were to change from $\mathbf{p}'$ to $\mathbf{p}$, household behavioral re-optimization would also improve welfare, *on average*. Mathematically, this amounts to assuming:

$$\begin{aligned}\mathcal{W}(\mathbf{p}) &= \int_{\Theta} \phi(\theta)u\left[y\left(\mathbf{x}^*(\theta,\mathbf{p}),\theta\right) + b\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau\right], \mathbf{x}^*(\theta,\mathbf{p});\theta\right] dF(\theta) - \lambda b G(\tau;\mathbf{p}) \\ &\geq \int_{\Theta} \phi(\theta)u\left[y\left(\mathbf{x}^*(\theta,\mathbf{p}'),\theta\right) + b\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau\right], \mathbf{x}^*(\theta,\mathbf{p}');\theta\right] dF(\theta) - \lambda b G(\tau;\mathbf{p})\end{aligned} \tag{55}$$

If we are willing to make these two assumptions, the rest of the proof to Proposition 3 goes through, so that we can bound the welfare impact of changing notches if individuals misperceive the schedule. Hence, we can state:

Proposition 4. *Suppose households solve Problem (52), welfare is given by Equation (53) and $\tau' > \tau$. If we assume:*

$$\mathcal{W}(\mathbf{p}') \geq \int_{\Theta} \phi(\theta)u\left[y\left(\mathbf{x}^*(\theta,\mathbf{p}),\theta\right) + b'\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau'\right], \mathbf{x}^*(\theta,\mathbf{p});\theta\right] dF(\theta) - \lambda b' G(\tau';\mathbf{p}')$$

and

$$\mathcal{W}(\mathbf{p}) \geq \int_{\Theta} \phi(\theta)u\left[y\left(\mathbf{x}^*(\theta,\mathbf{p}'),\theta\right) + b\mathbb{1}\left[\hat{y}\left(\mathbf{x}^*\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau\right], \mathbf{x}^*(\theta,\mathbf{p}');\theta\right] dF(\theta) - \lambda b G(\tau;\mathbf{p})$$

Then as long as $b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p}) > 0$ have:

$$\omega_L MVPF_L - 1 \leq \frac{\frac{1}{\lambda}\left[\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p})\right]}{b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})} \leq \omega_U MVPF_U - 1$$

where $MVPF_L$ is given by Equation 8, $MVPF_U$ is given by Equation 9, and ω_L (ω_U) denotes the weighted average money-metric welfare gain from giving a dollar to households impacted by the reform *where the weights are determined by the relative size of each households's lower bound (upper bound) WTP for the reform.*

A.8 Discounted Welfare Impact of Reform

Suppose households have several decisions variables at time t denoted by the vector $\mathbf{x}_t$ (within a potentially limited choice set $\mathbf{X}_t$). Household decisions are made conditional on state variables denoted by the vector $\theta_t \in \Theta_t$ and the policy $\mathbf{p}$. Households get the benefit b if their reported income $\hat{y}_t$, which is a function of decision variables $\mathbf{x}_t$, is below τ. Household income, denoted y_t, is also potentially a function of decisions $\mathbf{x}_t$.[4] Households in period t solve the following problem:

$$\begin{aligned} V(\theta_t) &= \max_{\mathbf{x}_t \in \mathbf{X}_t} \; u\left(c_t, \mathbf{x}_t; \theta_t\right) + \beta \mathbb{E}_{\theta_{t+1}}[V(\theta_{t+1}(\theta_t, \mathbf{x}_t))] \\ \text{s.t. } \; c_t &= y_t(\mathbf{x}_t, \theta_t) + b\mathbb{1}(\hat{y}_t(\mathbf{x}_t, \theta_t) \leq \tau) \end{aligned} \tag{56}$$

where c_t denotes consumption in period t, β is a discount factor, and $\mathbb{E}_{\theta_{t+1}}[V(\theta_{t+1}(\theta_t, \mathbf{x}_t))]$ represents the expected value of starting period $t+1$ with state variables θ_{t+1}, noting that the state variables tomorrow, θ_{t+1}, may be impacted by current state variables θ_t and current decisions $\mathbf{x}_t$ along with random variation. Equivalently, we can write out cumulative individual utility over T time periods from the perspective of time period 0 as:

$$\sum_{t=0}^{T} \beta^t \mathbb{E}_{\theta_t}[u\left(c_t, \mathbf{x}_t; \theta_t(\theta_0, \{\mathbf{x}_t\}_{t-1})\right]$$

where $\mathbb{E}_{\theta_t}$ represents the expectation over θ_t from the perspective of time period 0 (taking into account the impact of all conditional decisions $\{\mathbf{x}_t\}_{t-1}$ between time 0 and time $t-1$ on the underlying expectations).

So as to slightly reduce some cumbersome notation, let us define:

$$y_t^*(\theta_t, \mathbf{p}) \equiv y_t\left(\mathbf{x}_t^*\left(\theta_t, \mathbf{p}\right), \theta_t\right)$$

$$\hat{y}_t^*(\theta_t, \mathbf{p}) \equiv \hat{y}_t(\mathbf{x}_t^*\left(\theta_t, \mathbf{p}\right), \theta_t)$$

Using this notation, we again assume total discounted welfare is given by a weighted discounted sum of utilities, with welfare weights given by $\phi(\theta_0)$, less the total discounted budgetary cost of the policy

4. For example, in a dynamic misreporting model, $\mathbf{x}_t = \hat{y}_t$ and θ_t could include current income y_t, aversion to misreporting μ, prior reported income $\hat{y}_{t-1}$, and a parameter governing expected future income growth. Households may also make savings decisions if assets are a state variable in θ_t, current savings is included in $\mathbf{x}_t$, and c_t represents post-transfer income.

multiplied by a shadow value of public funds λ:

$$\sum_{t=0}^{T} \beta^t \mathcal{W}_t(\mathbf{p}) = \int_{\Theta_0} \phi(\theta_0) \left[\sum_{t=0}^{T} \beta^t \mathbb{E}_{\theta_t} [u\left(y_t^*(\theta_t, \mathbf{p}) + b\mathbb{1}\left[\hat{y}_t^*(\theta_t, \mathbf{p}) \leq \tau\right], \mathbf{x}_t^*(\theta_t, \mathbf{p}); \theta_t\right)] dF(\theta_0) \right] - \lambda \sum_{t=0}^{T} \beta^t b G_t(\tau; \mathbf{p}) \tag{57}$$

where we have dropped the arguments of $\theta_t(\theta_0, \{\mathbf{x}_t^*(\theta_t, \mathbf{p})\}_{t-1})$ for brevity, β represents the governments discount rate, λ represents the shadow value of public funds at time $t = 0$ (so that the shadow value of public funds in future periods equals $\beta^t \lambda$), and:

$$G_t(\tau; \mathbf{p}) = \int_{\Theta_0} \mathbb{E}_{\theta_t} [\mathbb{1}(\hat{y}_t^*(\theta_t, \mathbf{p}) \leq \tau)] dF(\theta_0)$$

represents the *expected* number of households receiving the benefit under policy $\mathbf{p}$ in period t. More generally, we define:

$$G_t(z; \mathbf{p}) = \int_{\Theta_0} \mathbb{E}_{\theta_t} [\mathbb{1}(\hat{y}_t^*(\theta_t, \mathbf{p}) \leq z)] dF(\theta_0)$$

as the expected number of households with a reported income below z under policy $\mathbf{p}$ at time t.

This setup allows us to bound the cumulative welfare impacts of a policy reform over T time periods:

Proposition 5. *Suppose households solve Problem (56), total welfare is given by Equation (57), and $\tau' > \tau$. Defining:*

$$MVPF_{L,T} \equiv 1 - \frac{\sum_{t=0}^{T} \beta^t b' \left[G_t(\tau'; \mathbf{p}') - G_t(\tau'; \mathbf{p})\right]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 - \frac{b' \sum_{t=0}^{T} \beta^t J_t}{\sum_{t=0}^{T} \beta^t \, Total\ Cost_t}$$

$$MVPF_{U,T} \equiv 1 + \frac{b \sum_{t=0}^{T} \beta^t [G_t(\tau; \mathbf{p}) - G_t(\tau; \mathbf{p}')]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 + \frac{b \sum_{t=0}^{T} \beta^t B_t}{\sum_{t=0}^{T} \beta^t \, Total\ Cost_t}$$

Then as long as $\sum_t \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})] > 0$ have:

$$\omega_{L,T} MVPF_{L,T} - 1 \leq \frac{\frac{1}{\lambda} \sum_{t=0}^{T} \beta^t [\mathcal{W}_t(\mathbf{p}') - \mathcal{W}_t(\mathbf{p})]}{\sum_{t=0}^{T} \beta^t \left[b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})\right]} \leq \omega_{U,T} MVPF_{U,T} - 1$$

where $\omega_{L,T}$ ($\omega_{U,T}$) denotes the discounted weighted average money-metric welfare gain from giving a dollar to households impacted by the reform where the weights are determined by the relative size of each households's lower bound (upper bound) discounted WTP for the reform.

Proof. We start with proving the lower bound for $\frac{\frac{1}{\lambda}\sum_{t=0}^{T}\beta^t[\mathcal{W}_t(\mathbf{p}')-\mathcal{W}_t(\mathbf{p})]}{\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]}$. First, note that by revealed preference, we have the following:

$$\begin{aligned}
&\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\right],\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right)]\\
&\equiv\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t\left(\mathbf{x}_t^*\left(\theta_t,\mathbf{p}\right),\theta_t\right)+b\mathbb{1}\left[\hat{y}_t(\mathbf{x}_t^*\left(\theta_t,\mathbf{p}),\theta_t\right)\leq\tau\right],\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right)]\\
&\geq\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t\left(\mathbf{x}_t,\theta_t\right)+b\mathbb{1}\left[\hat{y}_t(\mathbf{x}_t,\theta_t]\leq\tau\right),\mathbf{x}_t;\theta_t\right]
\end{aligned}$$

Put simply, optimal decisions conditional on any given θ_t under $\mathbf{p}$, $\mathbf{x}_t^*(\theta_t,\mathbf{p})$, yields weakly higher utility than any other set of decisions $\mathbf{x}_t$ that one could make. This yields the following bound on welfare under policy $\mathbf{p}'=\{b',\tau'\}$, which ensues by evaluating utility under policy $\mathbf{p}'$, but holding household decisions constant at their values under policy $\mathbf{p}$ (i.e., by revealed preference):

$$\begin{aligned}
&\sum_{t=0}^{T}\beta^t\mathcal{W}_t(\mathbf{p}')=\\
&\int\limits_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b'\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau'\right],\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau';\mathbf{p}')\\
&\geq\int\limits_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b'\mathbb{1}\left[\hat{y}_t(\mathbf{x}_t^*\left(\theta_t,\mathbf{p}),\theta_t\right)\leq\tau'\right],\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau';\mathbf{p}')
\end{aligned}$$

Hence, for the reform from $\mathbf{p}=\{b,\tau\}$ to $\mathbf{p}'=\{b',\tau'\}$ with $\mathbf{p}'-\mathbf{p}=\{\Delta b,\Delta\tau\}$ and $\Delta\tau>0$ (noting that we have written $\mathbb{E}_{\theta_t}$ as an integral over θ_t conditional on θ_0):

$$\begin{aligned}
&\sum_{t=0}^{T}\beta^t\left[\mathcal{W}_t\left(\mathbf{p}'\right)-\mathcal{W}_t(\mathbf{p})\right]\geq\\
&\sum_{t=0}^{T}\beta^t\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p})+b,\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\
&+\sum_{t=0}^{T}\beta^t\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}),\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\
&-\lambda\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]
\end{aligned}\tag{58}$$

Next, define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\}}$ as the government's average expected welfare weight on the households who optimally report incomes $\leq\tau$ under policy $\mathbf{p}$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\leq\tau\}}$ captures the average expected welfare gain from giving these households an extra \$1:

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\le\tau\}} = \frac{\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\le\tau}\phi(\theta_0)\frac{1}{b'-b}\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p})+b,\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau,\mathbf{p})}$$

And define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}$ as the government's average expected welfare weight of giving a dollar to the households who optimally report incomes $\in(\tau,\tau']$ under policy $\mathbf{p}$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}$ captures the average expected welfare gain from giving these households an extra \$1:[5]

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}} = \frac{\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']}\phi(\theta_0)\frac{1}{b'}\left\{u\left[y_t^*(\theta_t,\mathbf{p})+b',\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}),\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})}$$

Next, let us define an aggregate discounted welfare weight, $\eta_{L,t}$, which equals the weighted average discounted expected welfare weight of giving a dollar to all households, where the weights are determined by the (discounted) lower bound of expected WTP for the reform:

$$\eta_{L,T} = \frac{\sum_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\le\tau\}}\Delta bG_t(\tau;\mathbf{p})+\sum_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}b'[G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})]}{\sum_{t=0}^{T}\beta^t[\Delta bG_t(\tau;\mathbf{p})+b'[G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})]]}$$

Then, dividing Equation (58) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is >0):

$$\begin{aligned}\frac{\sum_{t=0}^{T}\beta^t\frac{1}{\lambda}\left[\mathcal{W}_t(\mathbf{p}')-\mathcal{W}_t(\mathbf{p})\right]}{\sum_{t=0}^{T}\beta^t\left[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})\right]} &\ge \frac{\sum_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\le\tau\}}\Delta bG_t(\tau;\mathbf{p})+\sum_{t=0}^{T}\beta^t\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']\}}b'\left[G_t(\tau';\mathbf{p})-G_t(\tau;\mathbf{p})\right]}{\sum_{t=0}^{T}\beta^t\lambda\left[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})\right]}-1\\ &= \frac{\eta_{L,T}}{\lambda}\frac{\sum_{t=0}^{T}\beta^t[\Delta bG_t(\tau;\mathbf{p})+b'\left[G_t(\tau';\mathbf{p})-G_t(\tau;\mathbf{p})\right]]}{\sum_{t=0}^{T}\beta^t\left[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})\right]}-1\\ &= \omega_{L,T}MVPF_{L,T}-1\end{aligned}$$

where $\omega_{L,T}=\eta_{L,T}/\lambda$ and $MVPF_{L,T}$ is given by:

$$MVPF_{L,T} = \frac{\sum_{t=0}^{T}\beta^t[\Delta bG_t(\tau;\mathbf{p})+b'[G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})]]}{\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]} = 1-\frac{\sum_{t=0}^{T}\beta^t b'\left[G_t(\tau';\mathbf{p}')-G_t(\tau';\mathbf{p})\right]}{\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]}$$

5. We have used $\int\limits_{\Theta_0}\int\limits_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p})\in(\tau,\tau']}dF(\theta_t|\theta_0)dF(\theta_0)=G_t(\tau';\mathbf{p})-G_t(\tau,\mathbf{p})$.

Next, we prove the upper bound for $\frac{\frac{1}{\lambda}\sum_{t=0}^{T}\beta^t[\mathcal{W}_t(\mathbf{p}')-\mathcal{W}_t(\mathbf{p})]}{\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]}$. We use identical revealed preference logic to bound welfare under policy $\mathbf{p} = \{b, \tau\}$ by evaluating utility under policy $\mathbf{p}$, but holding household decisions constant at their values under policy $\mathbf{p}'$:

$$\begin{aligned}\sum_{t=0}^{T}\beta^t\mathcal{W}_t(\mathbf{p}) &= \int_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p})+b\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p})\le\tau\right],\mathbf{x}_t^*(\theta_t,\mathbf{p});\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau;\mathbf{p})\\ &\ge \int_{\Theta_0}\phi(\theta_0)\left[\sum_{t=0}^{T}\beta^t\mathbb{E}_{\theta_t}[u\left(y_t^*(\theta_t,\mathbf{p}')+b\mathbb{1}\left[\hat{y}_t^*(\theta_t,\mathbf{p}')\le\tau\right],\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right)]dF(\theta_0)\right]-\lambda\sum_{t=0}^{T}\beta^t bG_t(\tau;\mathbf{p})\end{aligned}$$

Hence, for the reform from $\mathbf{p} = \{b, \tau\}$ to $\mathbf{p}' = \{b', \tau'\}$ with $\mathbf{p}' - \mathbf{p} = \{\Delta b, \Delta\tau\}$ and $\Delta\tau > 0$ (again noting that we have written $\mathbb{E}_{\theta_t}$ as an integral over θ_t conditional on θ_0):

$$\begin{aligned}&\sum_{t=0}^{T}\beta^t\left[\mathcal{W}_t\left(\mathbf{p}'\right)-\mathcal{W}_t(\mathbf{p})\right]\le\\ &\sum_{t=0}^{T}\beta^t\int_{\Theta_0}\int_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\le\tau}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}')+b,\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\ &+\sum_{t=0}^{T}\beta^t\int_{\Theta_0}\int_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']}\phi(\theta_0)\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}'),\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)\\ &-\lambda\sum_{t=0}^{T}\beta^t[b'G_t(\tau';\mathbf{p}')-bG_t(\tau;\mathbf{p})]\end{aligned} \tag{59}$$

Next, define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\le\tau\}}$ as the government's average expected welfare weight on the households who optimally report incomes $\le\tau$ under policy $\mathbf{p}'$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\le\tau\}}$ captures the average expected welfare gain from giving these households an extra \$1:

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\le\tau\}} = \frac{\int_{\Theta_0}\int_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\le\tau}\phi(\theta_0)\frac{1}{b'-b}\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}')+b,\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau,\mathbf{p}')}$$

And define $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}}$ as the government's average expected welfare weight of giving a dollar to the households who optimally report incomes $\in(\tau,\tau']$ under policy $\mathbf{p}'$ at time t. $\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}}$ captures the average expected welfare gain from giving these households an extra \$1:[6]

$$\eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}} = \frac{\int_{\Theta_0}\int_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']}\phi(\theta_0)\frac{1}{b'}\left\{u\left[y_t^*(\theta_t,\mathbf{p}')+b',\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]-u\left[y_t^*(\theta_t,\mathbf{p}'),\mathbf{x}_t^*(\theta_t,\mathbf{p}');\theta_t\right]\right\}dF(\theta_t|\theta_0)dF(\theta_0)}{G_t(\tau';\mathbf{p}')-G_t(\tau,\mathbf{p}')}$$

6. We have used $\int_{\Theta_0}\int_{\theta_t:\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']}dF(\theta_t|\theta_0)dF(\theta_0) = G_t(\tau';\mathbf{p}')-G_t(\tau,\mathbf{p}')$.

Next, let us define an aggregate discounted welfare weight, $\eta_{U,t}$, which equals the weighted average discounted expected welfare weight of giving a dollar to all households, where the weights are determined by the (discounted) upper bound of expected WTP for the reform:

$$\eta_{U,T} = \frac{\sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\}} \Delta b G_t(\tau;\mathbf{p}') + \sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}} b'[G_t(\tau';\mathbf{p}') - G_t(\tau,\mathbf{p}')]}{\sum_{t=0}^{T} \beta^t [\Delta b G_t(\tau;\mathbf{p}') + b'[G_t(\tau';\mathbf{p}') - G_t(\tau,\mathbf{p}')]]}$$

Then, dividing Equation (59) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is > 0):

$$\begin{aligned}
\frac{\frac{1}{\lambda}\sum_{t=0}^{T} \beta^t [\mathcal{W}_t(\mathbf{p}') - \mathcal{W}_t(\mathbf{p})]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} &\leq \frac{\sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\leq\tau\}} \Delta b G_t(\tau;\mathbf{p}') + \sum_{t=0}^{T} \beta^t \eta_{\{\hat{y}_t^*(\theta_t,\mathbf{p}')\in(\tau,\tau']\}} b' [G_t(\tau';\mathbf{p}') - G_t(\tau;\mathbf{p}')]}{\lambda \sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} - 1 \\
&= \frac{\eta_{U,T}}{\lambda} \frac{\sum_{t=0}^{T} \beta^t [\Delta b G_t(\tau;\mathbf{p}') + b' [G_t(\tau';\mathbf{p}') - G_t(\tau;\mathbf{p}')]]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} - 1 \\
&= \omega_{U,T} MVPF_{U,T} - 1
\end{aligned}$$

where $\omega_{U,T} = \eta_{U,T}/\lambda$ and $MVPF_{U,T}$ is given by:

$$MVPF_{U,T} = \frac{\sum_{t=0}^{T} \beta^t [\Delta b G_t(\tau;\mathbf{p}') + b'[G_t(\tau';\mathbf{p}') - G_t(\tau,\mathbf{p}')]]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]} = 1 + \frac{\sum_{t=0}^{T} \beta^t b [G_t(\tau;\mathbf{p}) - G_t(\tau;\mathbf{p}')]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau';\mathbf{p}') - b G_t(\tau;\mathbf{p})]}$$

□

A.9 Proof of Proposition 3 when Eligible Households Do Not Receive Benefit With Certainty

We now prove that our bounds still hold in a model where households reporting below the threshold receive the benefit with some probability q. Those reporting above the threshold do not receive the benefit. Under this more complex policy environment, the household problem is as follows:

$$\begin{aligned}
U^*(\theta,\mathbf{p}) = \max_{\mathbf{x}\in\mathbf{X}} \mathbb{E}_\alpha [u(c,\mathbf{x};\theta)] \\
s.t.\ c = y(\mathbf{x},\theta) + \alpha b \mathbb{1}(\hat{y}(\mathbf{x},\theta) \leq \tau)
\end{aligned}$$

where α denotes a Bernoulli random variable that takes value 1 with probability q and takes value 0 with probability $1-q$. We again consider the impact on welfare from moving from policy $\mathbf{p} = \{b,\tau\}$ to $\mathbf{p}' = \{b',\tau'\}$ with $\{\Delta b, \Delta\tau\} = \{b'-b, \tau'-\tau\}$ and $\Delta\tau > 0$. We start with proving the lower bound on the welfare impact is unchanged. This proof just requires some minor adjustments to the steps used to prove the lower bound in Appendix A.3. First, welfare under policy $\mathbf{p}$ is given by:

$$\mathcal{W}(\mathbf{p}) = \int_{\Theta} \phi(\theta)\mathbb{E}_{\alpha}\left\{u\left[y\left(\mathbf{x}^{*}(\theta,\mathbf{p}),\theta\right) + \alpha b \mathbb{1}\left[\hat{y}\left(\mathbf{x}^{*}\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau\right], \mathbf{x}^{*}(\theta,\mathbf{p});\theta\right]\right\} dF(\theta) - \lambda b q G(\tau;\mathbf{p}) \quad (60)$$

Next, note that by revealed preference, we have the following for any $\mathbf{x} \in \mathbf{X}$:

$$\mathbb{E}_{\alpha}\left\{u\left[y\left(\mathbf{x}^{*}(\theta,\mathbf{p}),\theta\right) + \alpha b \mathbb{1}\left[\hat{y}\left(\mathbf{x}^{*}\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau\right], \mathbf{x}^{*}(\theta,\mathbf{p});\theta\right]\right\} \geq \mathbb{E}_{\alpha}\left\{u[y\left(\mathbf{x},\theta\right) + \alpha b \mathbb{1}\left[\hat{y}(\mathbf{x},\theta) \leq \tau\right], \mathbf{x};\theta]\right\} \quad (61)$$

This yields the following bound on welfare under policy $\mathbf{p}' = \{b', \tau'\}$, which ensues by evaluating utility under policy $\mathbf{p}'$, but holding household decisions constant at their values under policy $\mathbf{p}$ (i.e., by revealed preference):

$$\begin{aligned}
\mathcal{W}(\mathbf{p}') &= \int_{\Theta} \phi(\theta)\mathbb{E}_{\alpha}\left\{u\left[y\left(\mathbf{x}^{*}(\theta,\mathbf{p}'),\theta\right) + \alpha b' \mathbb{1}\left[\hat{y}\left(\mathbf{x}^{*}\left(\theta,\mathbf{p}'\right),\theta\right) \leq \tau'\right], \mathbf{x}^{*}(\theta,\mathbf{p}');\theta\right]\right\} dF(\theta) - \lambda b' q G(\tau';\mathbf{p}') \\
&\geq \int_{\Theta} \phi(\theta)\mathbb{E}_{\alpha}\left\{u\left[y\left(\mathbf{x}^{*}(\theta,\mathbf{p}),\theta\right) + \alpha b' \mathbb{1}\left[\hat{y}\left(\mathbf{x}^{*}\left(\theta,\mathbf{p}\right),\theta\right) \leq \tau'\right], \mathbf{x}^{*}(\theta,\mathbf{p});\theta\right]\right\} dF(\theta) - \lambda b' q G(\tau';\mathbf{p}')
\end{aligned} \quad (62)$$

So as to slightly reduce some cumbersome notation, let us define:

$$y^{*}(\theta,\mathbf{p}) \equiv y\left(\mathbf{x}^{*}\left(\theta,\mathbf{p}\right),\theta\right)$$

$$\hat{y}^{*}(\theta,\mathbf{p}) \equiv \hat{y}(\mathbf{x}^{*}\left(\theta,\mathbf{p}\right),\theta)$$

Thus, for the reform from $\mathbf{p}$ to $\mathbf{p}'$ we get:

$$\begin{aligned}
\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq & \int_{\theta:\hat{y}^{*}(\theta,\mathbf{p})\leq\tau} \phi(\theta)\mathbb{E}_{\alpha}\left\{u\left[y^{*}(\theta,\mathbf{p}) + \alpha b', \mathbf{x}^{*}(\theta,\mathbf{p});\theta\right] - u\left[y^{*}(\theta,\mathbf{p}) + \alpha b, \mathbf{x}^{*}(\theta,\mathbf{p});\theta\right]\right\} dF(\theta) \\
&+ \int_{\theta:\hat{y}^{*}(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\mathbb{E}_{\alpha}\left\{u\left[y^{*}(\theta,\mathbf{p}) + \alpha b', \mathbf{x}^{*}\left(\theta,\mathbf{p}\right);\theta\right] - u\left[y^{*}(\theta,\mathbf{p}), \mathbf{x}^{*}\left(\theta,\mathbf{p}\right);\theta\right]\right\} dF(\theta) \\
&- \lambda q[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})]
\end{aligned} \quad (63)$$

Next, define $\eta_{\{\hat{y}^{*}(\theta,\mathbf{p})\leq\tau\}}$ as the government's average welfare weight on the households who optimally report incomes $\hat{y}^{*} \leq \tau$ under policy $\mathbf{p}$:

$$\eta_{\{\hat{y}^{*}(\theta,\mathbf{p})\leq\tau\}} = \frac{\int_{\theta:\hat{y}^{*}(\theta,\mathbf{p})\leq\tau} \phi(\theta)\frac{1}{q(b'-b)}\mathbb{E}_{\alpha}\left\{u\left[y^{*}(\theta,\mathbf{p}) + \alpha b', \mathbf{x}^{*}(\theta,\mathbf{p});\theta\right] - u\left[y^{*}(\theta,\mathbf{p}) + \alpha b, \mathbf{x}^{*}(\theta,\mathbf{p});\theta\right]\right\} dF(\theta)}{G(\tau,\mathbf{p})} \quad (64)$$

Note that in Equation (64), we divide by $\frac{1}{q(b'-b)}$, which renormalizes by the expected additional amount of money given to households with $\hat{y}^{*}(\theta,\mathbf{p}') \leq \tau$ as a result of the reform; hence, we can interpret $\eta_{\{\hat{y}^{*}(\theta,\mathbf{p}')\leq\tau\}}$ as capturing the average expected welfare gain from giving these households an extra

\$1. Next, define $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ as the government's average welfare weight of giving a dollar to the households who optimally report incomes $\hat{y}^* \in (\tau, \tau']$ under policy $\mathbf{p}$:[7]

$$\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} = \frac{\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} \phi(\theta)\frac{1}{qb'}\mathbb{E}_\alpha \left\{u\left[y^*(\theta,\mathbf{p})+\alpha b', \mathbf{x}^*\left(\theta,\mathbf{p}\right);\theta\right] - u\left[y^*(\theta,\mathbf{p}), \mathbf{x}^*\left(\theta,\mathbf{p}\right);\theta\right]\right\} dF(\theta)}{G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})} \tag{65}$$

Again, note that in Equation (65), we divide by $\frac{1}{qb'}$, which renormalizes by the expected amount of money given to households with $\hat{y}^*(\theta,\mathbf{p}) \in (\tau,\tau']$; hence, $\eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}}$ captures the average expected welfare gain from giving these households an extra \$1 with probability. We can rewrite Equation (63) using Equations (64) and (65) as follows:

$$\mathcal{W}(\mathbf{p}') - \mathcal{W}(\mathbf{p}) \geq \eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}} q\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} qb'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right] - \lambda q\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right] \tag{66}$$

Next, let us define the aggregate welfare weight, η_L, which equals the weighted average welfare weight of giving a dollar to all households, where the weights are determined by the lower bound of WTP for the reform:

$$\eta_L = \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}} q\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} qb'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]}{q\Delta bG(\tau;\mathbf{p}) + qb'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]} \tag{67}$$

Then, dividing Equation (66) through by the budgetary effect multiplied by λ, we have (recall we assume the budgetary effect is > 0):

$$\begin{aligned}
\frac{\frac{1}{\lambda}\left[\mathcal{W}\left(\mathbf{p}'\right) - \mathcal{W}(\mathbf{p})\right]}{q\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} &\geq \frac{\eta_{\{\hat{y}^*(\theta,\mathbf{p})\leq\tau\}} q\Delta bG(\tau;\mathbf{p}) + \eta_{\{\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']\}} qb'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{\lambda q\left[b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})\right]} - 1 \\
&= \frac{\eta_L}{\lambda}\frac{q\Delta bG(\tau;\mathbf{p}) + qb'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{q\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} - 1 \\
&= \frac{\eta_L}{\lambda}\frac{\Delta bG(\tau;\mathbf{p}) + b'\left[G(\tau';\mathbf{p}) - G(\tau;\mathbf{p})\right]}{\left[b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)\right]} - 1 \\
&= \omega_L MVPF_L - 1
\end{aligned} \tag{68}$$

where $\omega_L = \eta_L/\lambda$ and $MVPF_L$ is given by:

$$MVPF_L = \frac{\Delta bG(\tau;\mathbf{p}) + b'[G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})]}{b'G\left(\tau';\mathbf{p}'\right) - bG\left(\tau;\mathbf{p}\right)} = 1 - \frac{b'\left[G(\tau';\mathbf{p}') - G(\tau';\mathbf{p})\right]}{b'G(\tau';\mathbf{p}') - bG(\tau;\mathbf{p})} \tag{69}$$

The upper bound can be proved in a analogous manner by adjusting the upper bound portion of the proof in Appendix A.3 for the fact that the benefit is only received with some probability q.

7. We have used $\int\limits_{\theta:\hat{y}^*(\theta,\mathbf{p})\in(\tau,\tau']} dF(\theta) = G(\tau';\mathbf{p}) - G(\tau,\mathbf{p})$.

A.10 Bounds in Proposition 3 Are Sharp

Proposition 6. *Without further assumptions on primitives, the bounds in Proposition 3 cannot be improved.*

Proof. To show that one cannot construct tighter bounds than Proposition 3 without additional structure on the household's problem, we provide examples for which these bounds are attained. In particular, we create examples for which (1) bunching households have a WTP of Δb and jumping households have a WTP of 0 as well as (2) bunching and jumping households both have a WTP of b'.[8]

Example 1: Consider our baseline, misreporting model in Section I.A with $v(0) = 0$, $v' > 0$. Suppose that $\Delta b, \Delta\tau > 0$. Consider a distribution for the misreporting types ($\mu = 2$) with a mass point at $y = \tau$, no density on $(\tau, y^{c'})$, and another mass point at $y = y^{c'}$, where $y^{c'}$ solves $b' = v(y^{c'} - \tau')$. Recall that bunching households bunch at τ under policy $\mathbf{p}$ and then move towards the new notch with the reform. All households with $y \leq \tau$ do not change behavior as a result of the reform and households with $y \geq y^{c'}$ do not bunch at τ under $\mathbf{p}$; hence, there are no bunching households in this example. Thus, total WTP of bunching households is zero. Next, all households with $y = y^{c'}$ prefer to report truthfully (and not get the benefit) under the original policy $\mathbf{p}$ as $v(y^{c'} - \tau) > b' > b$ by $v' > 0$. Under policy $\mathbf{p'}$ these households are indifferent between reporting truthfully and misreporting to get the benefit: we break their indifference by assuming that they jump to misreport at the threshold τ' under the new policy $\mathbf{p'}$, rendering these households as jumping households. The change in utility for jumping households is therefore given by:

$$y^{c'} + b' - v(y^{c'} - \tau') - y^{c'} = b' - b' = 0$$

Thus, each jumping household's WTP is equal to 0. Hence, the total WTP for the reform equals $(M + B)\Delta b + Tb' = MVPF_L$.

Example 2: Consider our baseline, misreporting model in Section I.A with $v(0) = 0$, $v' > 0$. Suppose that $\Delta b, \Delta\tau > 0$. Let y^c solve $b = v(y^c - \tau)$ and $y^{c'}$ solve $b' = v(y^{c'} - \tau')$. Finally, suppose that $\tau' - \tau$ is large enough so that $\tau' > y^c$. Consider a distribution for the misreporting types ($\mu = 2$) with no density on $[\tau, y^c)$, a mass point at $y = y^c$, and no density on $(y^c, y^{c'}]$. For the $\mu = 2$ individuals with $y > y^{c'}$, it is always optimal to report truthfully. For the $\mu = 2$ individuals with $y = y^c$, they are indifferent between bunching at τ and reporting truthfully under policy $\mathbf{p}$. In the former case, they are bunching individuals (and there are no jumping individuals) and in the latter case they are jumping individuals (and there are no bunching individuals). Regardless, the utility gain for these households is equal to:

$$y^c + b' - [y^c + b - v(y^c - \tau)] = b'$$

8. Note that our examples require mass points of the income distribution. But one can approximate our example scenarios arbitrarily well with smooth income distributions; hence, we can get arbitrarily close to the cases when either (1) all bunching households have a WTP of Δb and all jumping households have a WTP of 0 or (2) all bunching and jumping households both have a WTP of b'.

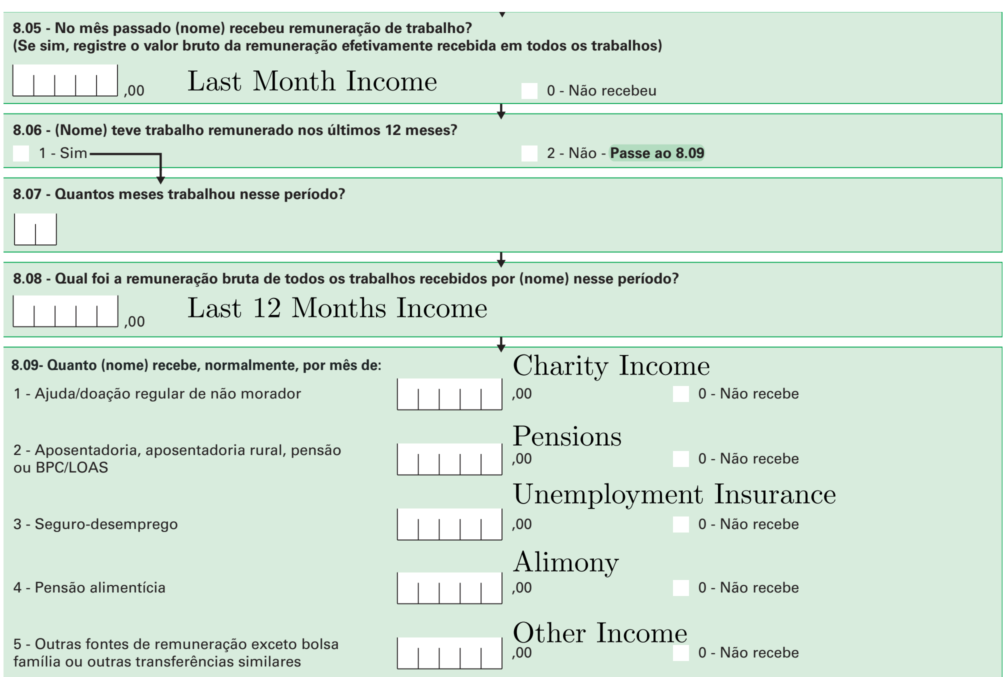
Thus the total WTP for the reform will equal $M\Delta b + (B + T + J)b' = MVPF_U$.

$\square$

B Bolsa Família Program Appendix

B.1 Bolsa Família Questionnaire

Figure 9 shows the entries on the questionnaire used to calculate each individual's total monthly income (household per-capita income will be calculated via summing total individual monthly incomes across all members of a household divided by the number of members in the household). Questions 8.05-8.08 relate to determining last month's labor income and the labor income over the last 12 months. The computer will then calculate an individual's minimum monthly labor income via taking the minimum between the individual's labor income last month and the individual's average monthly labor income over the last 12 months. Question 8.09 relates to determining the average monthly income from five other income sources: charity, pensions, unemployment insurance, alimony, and other. An individual's total monthly income is then equal to their monthly income from these five sources plus their minimum monthly labor income.


8.05 - No mês passado (nome) recebeu remuneração de trabalho?
(Se sim, registre o valor bruto da remuneração efetivamente recebida em todos os trabalhos)
,00 Last Month Income
0 - Não recebeu

8.06 - (Nome) teve trabalho remunerado nos últimos 12 meses?
1 - Sim
2 - Não - Passe ao 8.09

8.07 - Quantos meses trabalhou nesse período?

8.08 - Qual foi a remuneração bruta de todos os trabalhos recebidos por (nome) nesse período?
,00 Last 12 Months Income

8.09- Quanto (nome) recebe, normalmente, por mês de:
1 - Ajuda/doação regular de não morador — Charity Income ,00 — 0 - Não recebe
2 - Aposentadoria, aposentadoria rural, pensão ou BPC/LOAS — Pensions ,00 — 0 - Não recebe
3 - Seguro-desemprego — Unemployment Insurance ,00 — 0 - Não recebe
4 - Pensão alimentícia — Alimony ,00 — 0 - Não recebe
5 - Outras fontes de remuneração exceto bolsa família ou outras transferências similares — Other Income ,00 — 0 - Não recebe

Note: The figure depicts the income categories reported by applicants for each member of the household. Each category has been translated into English in the figure. This is a print out of the screen seen by interviewers on their computers when filling in applicants' information.

FIGURE 9: INCOME QUESTIONNAIRE

B.2 *Bolsa Família Schedule for Households with Children*

At the beginning of our dataset (December 2011), the BF program has two eligibility thresholds in the per-capita monthly income distribution for households with children: the extreme-poverty line (R$70) and the poverty line (R$140). Households with per-capita income below the extreme-poverty line are eligible for the constant basic benefit (R$70 per-month), a variable benefit proportional to the number of family members between 0 and 15 years old (R$32 per-child, per-month), and a teenager benefit proportional to the number of members aged 16 or 17 years old (R$38 per-teenager, per-month). Households with per-capita income between the extreme-poverty and poverty thresholds are only eligible for the variable and teenager benefits. Households with per-capita income above the second threshold are not eligible for any BF cash transfers. Moreover, the total variable benefit was capped at R$160 (5 children per household) and the total teenager benefit was capped at R$76 (two teenagers per household).[9]

The reform this paper studies, which occurred in June 2014, increased the extreme-poverty threshold from R$70 to R$77 and the poverty threshold from R$140 to R$154. The basic benefit was raised from R$70 to R$77, the benefit per child from R$32 to R$35, and the benefit per teenager from R$38 to R$42. Note that the thresholds are based on per-capita income but the benefits are denominated in raw amounts. This reform was announced on national television by the president in April 2014.

Table 4 summarizes these aspects of the schedule before (first column) and after (second column) the reform for households with children.

Table 4: Summary of Bolsa Família Schedule for Families with Children Before and After June 2014 Reform

	Before	After
Extreme-Poverty Threshold	70	77
Poverty Threshold	140	154
Basic Benefit (for those in extreme-poverty)	70	77
Variable Benefit Per Child 15 or Younger (max 5) (for those in poverty)	32	35
Teenager Benefit Per Teen 16-18 (max 2) (for those in poverty)	38	42

Note: The first two rows correspond to the extreme-poverty and poverty thresholds, respectively. These are measured in monthly, per-capita income. I.e., before the reform a household is below the extreme-poverty threshold if their monthly, per-capita income is below R$70. The third, fourth, and fifth rows display the benefits given to households; these are denoted in monthly amounts. I.e., before the reform a household below the extreme-poverty threshold receives R$70 per-month in the basic benefit.

Between December 2011 and February 2013 there were three other reforms to the BF program, which successively instituted a guaranteed minimum income of R$70 per-capita (along with an as-

9. Households with children need to fulfill three additional conditions to receive the variable benefit and/or teenager benefit: (1) children must maintain a minimum of 85% school attendance between ages 6 and 15 and 75% school attendance between 16 and 17; (2) households must keep track of their children's vaccines; and (3) parents must maintain at least 85% attendance in a social-education program if the household has violated child labor laws in the past. All conditionalities were held constant during the analysis period.

sociated negative income tax) for several groups of households. This guaranteed minimum income was instituted in June 2012 for households with children below 6 years of age, in November 2012 for households with children below 15 years of age, and in February 2013 for all remaining households. These reforms thus created a kink (which varies with household composition) in the benefit schedule as a function of reported, per-capita household income for households with children as well as for two adults households with no children. For example, if a two adult household without kids prior to June 2014 has a reported per-capita income of R\$20, they get R\$70 from the basic benefit and then get an additional R\$30 to bring them up to the guaranteed minimum per-capita income of R\$70. Mathematically, prior to June 2014 two adult households without kids face a benefit schedule as a function of reported per-capita income, $\hat{y}$, equal to:

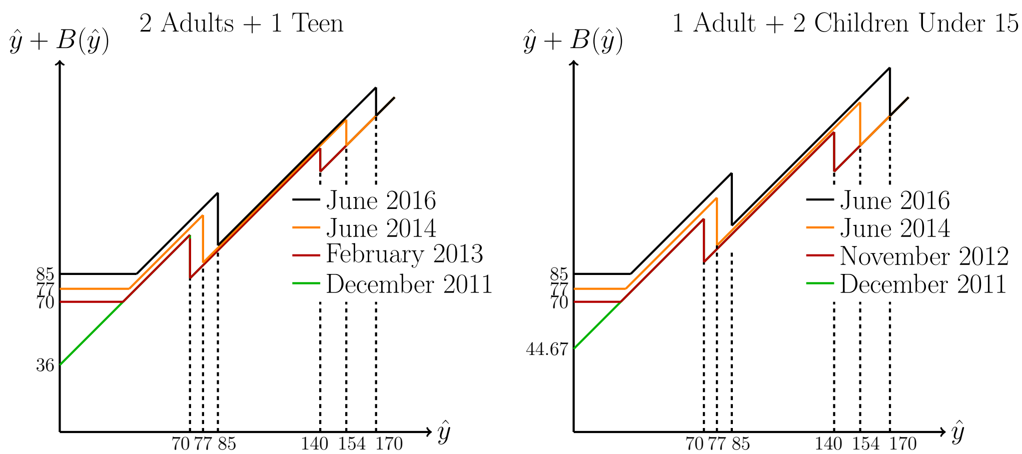
$$B(\hat{y}) = 70\mathbb{1}(\hat{y} \leq 70) + \max\{0, 70 - 2 \times \hat{y}\}$$

This benefit schedule therefore has a kink at R\$35. The kink then changed slightly with the June 2014 reform as the guaranteed minimum income was raised to R\$77 per-capita. For example, after June 2014 two adult households without kids face a benefit schedule with a kink at R\$38.5:

$$B(\hat{y}) = 77\mathbb{1}(\hat{y} \leq 77) + \max\{0, 77 - 2 \times \hat{y}\}$$

Thus, the June 2014 reform potentially impacted the reported income distribution around the kink. Because the kink is located below the first BF notch of R\$70, Identification Assumption 1 is less likely to hold for these households as households with reported incomes under R\$63 may respond to this changing kink. Finally, a reform in June 2016 further increased the extreme-poverty threshold to R\$85 (per-capita, per-month), the poverty threshold to R\$170 (per-capita, per-month), the basic benefit to R\$85 (per-month), the variable benefit R\$39 (per-child, per-month), and the teenager benefit to R\$46 (per-teenager, per-month).

For purposes of illustration, Figure 10 plots how the benefit schedules for two particular household compositions varied over time.



Note: $\hat{y}$ denotes the reported, per-capita, monthly household income. $B(\hat{y})$ denotes the monthly, per-capita benefits a household receives if they report $\hat{y}$. These benefits will also depend on household composition. For example, a household with 2 adults and 1 teenager reporting $\hat{y} = 0$ in December 2011 will receive R$70 in the basic benefit and R$38 in the teenager benefit. Thus $\hat{y} + B(\hat{y}) = (70 + 38)/3 = 36$.

Figure 10: Bolsa Família Schedule Reforms For Two Example Household Compositions

B.3 Other Social Security Programs Based on the Cadastro Único

This appendix describes other programs that set their eligibility based on information from the Cadastro Único database.

Benefício de Pretação Continuada (BPC): This benefit targets the elderly (above 65 years of age) and disabled. It gives a minimum wage to all households with per-capita income up to a quarter of the minimum wage. Table 5 reports the minimum wage and BPC threshold across all years of the analysis. The Brazilian Social Security System administers its own exam to define eligibility for this program.

Table 5: Minimum Wage and BPC Eligibility Thresholds

Year	Minimum Wage	BPC Threshold
2011	545	136.25
2012	622	155.50
2013	678	169.50
2014	724	181
2015	788	197
2016	880	220

Note: Data from the Brazilian Economy Ministry, see `http://www.ipeadata.gov.br/exibeserie.aspx?stub=1&serid1739471028=1739471028`

Carteira do Idoso: This "Elderly Card" guarantees to all individuals 60 years of age or older and with income up to two times the minimum wage at least a 50% discount on any interstate trip by road, rail, or waterway.

Créditos Instalação do Programa Nacional de Reforma Agrária: Households with per-capita income up to three times the minimum wage and that are living in camping grounds get points in a system that selects beneficiaries to be settled through the Brazilian land reform.

Facultativo de Baixa Renda: This is an option to contribute to social security at a lower rate (5% of the minimum wage). The individual cannot have any income and household income must be below two times the minimum wage.

Identidade Jovem (ID Jovem): Discounts for cultural events and trips by road, rail, or waterway for individuals between 15 and 29 years of age living in a household with up to two times the minimum wage.

Isenção de taxas de inscrição em concursos públicos: Since 2008, households with per-capita income up to half of the minimum wage or total income of up to three times the minimum wage are exempt from public tender registration payment.

Política Nacional Assistência Técnica Rural — PNATER Brasil Sem Miséria: Technical assistance for households working on activities for their own consumption in rural areas.

Programa Água para Todos — Programa Nacional de Universalização do Acesso e Uso da Água: Since July 2011, the government has installed cisterns to ensure access to clean water for all Brazilians, with priority going to those who satisfy the criteria for BF program.

Bolsa Estiagem: This is a benefit of at least R$80 per month to households with total income up to two times the minimum wage that live in areas hit by natural disasters.

Programa Bolsa Verde — Programa de Apoio à Conservação Ambiental: Since October 2011, this program transfers R$300 every 3 months to households in extreme poverty (first threshold of BF) and that follow the requirements for using natural resources.

Programa Cisternas: This program aims to provide cisterns to low-income families registered in the Cadastro Único.

Programa de Erradicação do Trabalho Infantil: This program transfers benefits similar to the BF (R$25 and R$40 per child per month in municipalities with less and more than 250,000 inhabitants, respectively) to households whose incomes are above the BF threshold with working children (up to 16 years of age) conditional on these children attending school 85% of the time instead of working.

Programa de Fomento às Atividades Produtivas Rurais: Since 2012, the government has made a one-time transfer of around R$2,400 to families that are eligible for the BF program and work on agricultural activities or belong to native or traditional communities.

Programa Minha Casa Minha Vida: Households with total monthly income up to R$1,416.67 have access to a subsidized credit line to purchase a house.

Programa Nacional de Crédito Fundiário: Households with total monthly income up to R$2,500 have access to a subsidized credit line to purchase land for production.

Serviços Socioassistenciais: MDS offers social services to poor individuals who have suffered any type of violence or neglect.

Sistema de Seleção Unificada — Sisu/Lei de Cotas: Since 2016, all federal universities in the country reserve some seats for students coming from families with per-capita income up to 1.5 times the minimum wage.

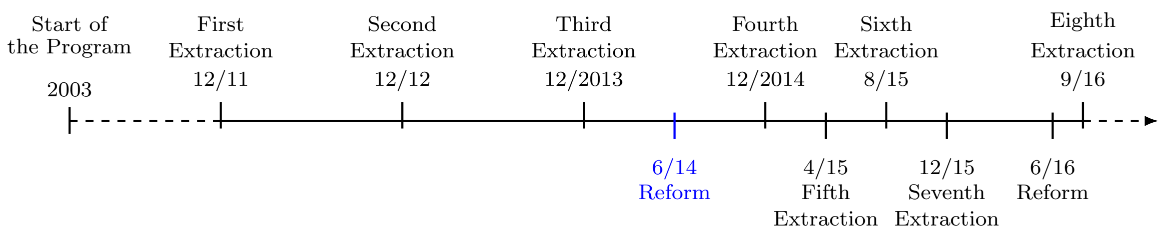
Tarifa Social de Energia Elétrica: Households with monthly per-capita income of up to a half the minimum wage have access to a discounted electricity price.

Tefone Popular — Acesso Individual Classe Especial: The government offers a landline with lower prices for individuals registered in the Cadastro Único database.

Distribuição de Conversores de TV Digital: Since the September of 2015, MDS has offered digital converters to beneficiaries of the program, which helps them transition from open TV to the new system.

B.4 Bolsa Família Program and Data Extraction Timeline

Figure 11 presents the data extraction timeline relative to when the BF program started and the June 2014 reform. As can be seen, the program started in 2003, the extractions we have span the months between December 2011 to September 2016, and the reform we study occurred in June 2014. Note, there was also another reform in June 2016.



Note: The figure describes the timeline of the program and the data extractions. BF started in 2003, and the reform we study occurred in June 2014. The final dataset is constructed from 8 extractions from December 2011 until September of 2016. Each extraction contains the most recent information on each household as of the extraction date. Note, there was another reform in June 2016.

Figure 11: Timeline

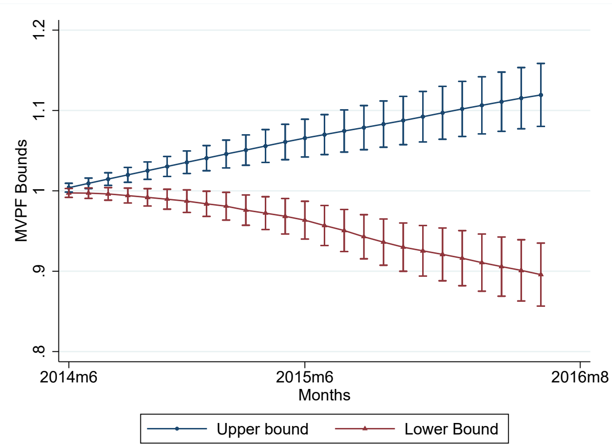
C Empirical Appendix

C.1 Calculating the MVPF Each Month in the Post-Reform Period

In this appendix, we calculate upper and lower bounds for the MVPF for each month from June, 2014 to June, 2016. To do so, we use Regression 16 with a polynomial of degree $K = 3$ to infer the jumping effect and bunching effect in each month. We then plug these estimates into Equations 11 and 12. Results are presented in Figure 12 below.

As can be seen, the MVPF bounds are initially tightly centered around 1 just after the reform. This is because the jumping effect and bunching effect are initially small (as seen in Figure 5). As the jumping effect and bunching effect grow overtime, our lower bound decreases and our upper bound increases. One might be worried that these bounds will continue to grow as we go past June 2016.

While we cannot test this with our data (as mentioned in Section II, there is another reform to BF in June 2016 and our data only goes out to September 2016), we suspect that because households are required to update their information every two years to remain eligible for BF, most behavioral responses to the reform should be observed within the first two years post-reform.



Note: This figure calculates the upper and lower bounds for the MVPF for each month from June 2014 to June 2016. We calculate the jumping effect and bunching effect for each month in the post-reform period from Regression 16 (with polynomial of degree $K = 3$) and plug these numbers into Equations 11 and 12. Confidence intervals are constructed via the delta method from the clustered standard errors estimated in Regression 16.

FIGURE 12: MVPF BOUNDS FOR EACH MONTH IN THE POST-REFORM PERIOD

Related to the above analysis, we can also calculate bounds for the cumulative MVPF of the reform. As shown in Appendix A.8, to do this, we need to observe the jumping effect and bunching effect for every period post-reform (i.e., for all months beyond June 2014). We show in Appendix A.8 that the lower and upper bounds for the cumulative MVPF over T time periods are given by:

$$MVPF_{L,T} \equiv 1 - \frac{\sum_{t=0}^{T} \beta^t b' \left[G_t(\tau'; \mathbf{p}') - G_t(\tau'; \mathbf{p})\right]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 - \frac{b' \sum_{t=0}^{T} \beta^t J_t}{\sum_{t=0}^{T} \beta^t \text{Total Cost}_t}$$

$$MVPF_{U,T} \equiv 1 + \frac{b \sum_{t=0}^{T} \beta^t [G_t(\tau; \mathbf{p}) - G_t(\tau; \mathbf{p}')]}{\sum_{t=0}^{T} \beta^t [b' G_t(\tau'; \mathbf{p}') - b G_t(\tau; \mathbf{p})]} = 1 + \frac{b \sum_{t=0}^{T} \beta^t B_t}{\sum_{t=0}^{T} \beta^t \text{Total Cost}_t}$$

where β denotes the discount rate. The limitation here is that we can't estimate the jumping effect and bunching effect beyond June 2016. Thus, we make the assumption that the jumping effect and bunching effect beyond June 2016 are equal to the jumping effect and bunching effect in June 2016 (we also assume the total cost for periods beyond June 2016 is equal to the total cost for June 2016). Under this assumption, we evaluate the MVPF bounds setting $T = \infty$ and using an annual discount rate $\beta = 0.98$. We estimate the jumping effect and bunching effect using Regression 16 for polynomial degrees $K = \{2,3,4,5\}$. Results are presented in Table 6. The cumulative MVPF bounds in Table 6 are close to the monthly MVPF bounds for June 2016 presented in Table 3. This is not surprising given our assumption that the jumping effect, bunching effect, and total cost post-June 2016 are equal to the number of jumping effect, bunching effect, and total cost in June 2016. Thus, our lower bound of the cumulative MVPF is given by:

$$MVPF_{L,\infty} = 1 - \frac{b' \sum_{t=0}^{24} \beta^{t/12} J_t + b' \sum_{t=25}^{\infty} \beta^{t/12} J_{24}}{\sum_{t=0}^{24} \beta^{t/12} \text{Total Cost}_t + \sum_{t=25}^{\infty} \beta^{t/12} \text{Total Cost}_{24}} \approx 1 - \frac{b' J_{24}}{\text{Total Cost}_{24}}$$

where $t = 0$ denotes June 2014, and $t = 24$ denotes June 2016 (we divide t by 12 in $\beta^{t/12}$ because β is an annual discount rate). Similarly, $MVPF_{U,\infty} \approx 1 + \frac{bB_{24}}{\text{Total Cost}_{24}}$.

Table 6: MVPF Bounds for All Months Post-Reform

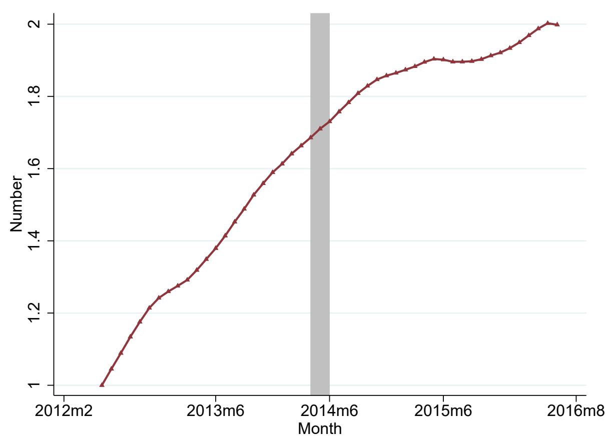
Polynomial Degree, K	$MVPF_{L,\infty}$	$MVPF_{U,\infty}$
Quadratic, $K = 2$	0.90	1.10
	(0.02)	(0.02)
Cubic, $K = 3$	0.91	1.10
	(0.02)	(0.02)
Quartic, $K = 4$	0.91	1.11
	(0.02)	(0.02)
Quintic, $K = 5$	0.91	1.11
	(0.03)	(0.03)

Note: This table presents the cumulative MVPF bounds of the reform (i.e., across all months after the June 2014 reform). We calculate the jumping effect and bunching effect for each month in the post-reform period from Regression 16 for various polynomial degress and we assume the jumping effect and bunching effect post-June 2016 are equal to the jumping effect and bunching effect in June 2016. Standard errors are presented in parentheses and are constructed via the delta method from the clustered standard errors estimated in Regression 16.

C.2 Discussion on the Presence of Income Effects

Identification Assumption 1 relies on households not changing their reported incomes in response to a higher benefit, i.e., households do not experience income effects. If households did experience income effects, then households reporting below R\$63 under policy $\mathbf{p}$ would change their reported income under policy $\mathbf{p}'$ (all else equal) as their benefit rises from b to b'. While their are no income effects in the baseline model of Section I.A, this may not be the case in the more general model of Section I.C as households may reduce their labor supply and, in turn, their reported income in response to a higher benefit. However, we believe that our placebo tests in Section IV.F provide suggestive evidence that income effects are not present in our empirical application. The placebo tests shown in Figure 7 suggest that the trends in the number of individuals reporting in income bins below R\$63 all evolve very similarly over time. This could result from either (1) income effects being negligible or (2) income effects impacting each reported income bin in the same way so that their trends remain similar post-reform. However, (2) is inconsistent with the fact that the income distribution is highly non-smooth (see Figure 19). The reasoning is that if income effects were affecting the distribution, we would expect to see some relatively constant *proportion* of individuals in each income bin move to a neighboring bin; however, this would lead to substantially different trends in the post-reform period due to differing numbers of people in each bin to start with. As an example, note that there are relatively few individuals in the income bin R$(42,49] (see Figure 19) but there are a comparatively large number of individuals in the income bin R$(49,56]. If income effects lead some households to reduce their income, then we might expect some proportion of households initially in R$(49,56] to move down to R$(42,49]. But because the number of households in R$(42,49] is so small relative to the number in R$(49,56], even a small proportion of households in R$(49,56] moving down to R$(42,49] would have a large impact on the post-reform trend for the R$(42,49] bin. Hence, because the trends in the number of individuals reporting in income bins below R$63 all evolve very similarly over time, we believe that income effects are unlikely to be impacting the reported income distribution in our setting (consistent with our hypothesis that most of the observed responses are reporting, rather than real, responses). While we cannot rule out some sort of non-standard, non-monotonic income effects affecting the distribution, this seems unlikely.

C.3 Number of Households Reporting an Income in R$(63,70]



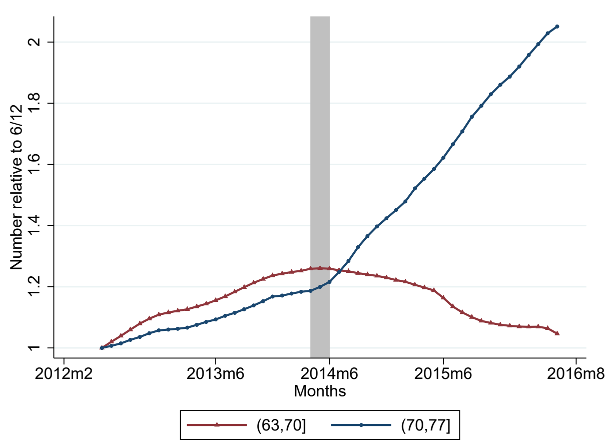
Note: This figure shows the number of single individual households with reported incomes in R$(63,70]. The number in each bin is normalized to 1 in June 2012. The timing of the reform (from the announcement in April 2014 to the enactment in June 2014) is indicated by the gray, shaded region.

FIGURE 13: NUMBER OF HOUSEHOLDS REPORTING AN INCOME IN R$(63,70]

C.4 Results for Two Adult Households with No Children

In this appendix, we present results for households with two adults and no children. As discussed in Section II.B and Appendix B.2, the guaranteed minimum income, which was instituted in February 2013 for two adult households without kids, generates a kink in the benefit schedule that occurs at a per-capita household income of R$35. Hence, after February 2013 and prior to June 2014, two adult families in extreme poverty (i.e., with household incomes below R$70) get the basic benefit of R$70 plus additional transfers to bring their per-capita income up to R$70. For example, a two adult family with a combined household income of R$40 gets R$70 *plus* an additional R$30 (giving them a total income of R$140) to reach the guaranteed minimum per-capita income of R$70. This generates a kink in the benefit schedule for two adult families at the per-capita household income of R$35 after February 2013 and prior to June 2014.

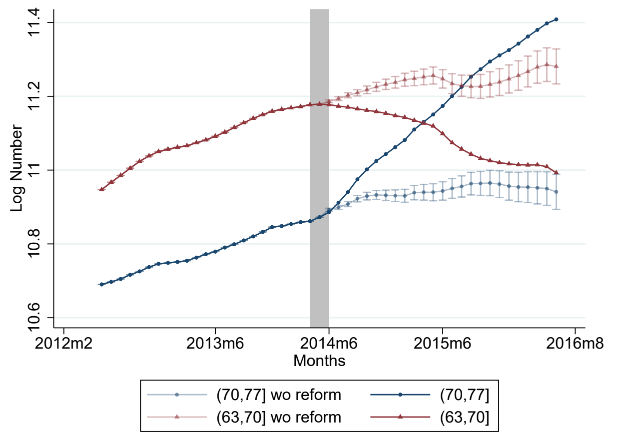
The June 2014 reform not only changed the extreme-poverty threshold and the basic benefit both from R$70 to R$77 but it also changed the guaranteed minimum income from R$70 per-capita to R$77 per-capita. Hence, the June 2014 reform changed the location and level of both the notch and the kink in the benefit schedule (the location of the notch moved from R$70 to R$77 per-capita while the location of the kink moved from R$35 to R$38.5 per-capita). Identification Assumption 1 is now harder to justify given that the June 2014 reform changed incentives for households to locate around R$35 by changing the kink in the benefit schedule from R$35 to R$38.5. However, many studies have shown that behavioral responses to kinks are typically very small; kinks generally induce substantially less bunching than do notches (Kleven, 2016). Hence, we just ignore the presence of the kink and use all seven-increment income bins below R$63 as control groups just as in our main analysis. Figure 14 shows the raw data for how the number of two adult households reporting incomes in bins R$(63, 70] and R$(70, 77] evolved in a four year window around the reform from June 2012 to June 2016. As in the corresponding figure for one adult households (Figure 3b), Figure 14 depicts a clear trend departure for bin R$(70, 77] commensurate with the reform, providing highly suggestive evidence that the BF reform induced a substantial behavioral response that increased the number of households bunching at and just below the new threshold. Figure 14 also depicts a clear decrease in the number of two adult households reporting incomes in R$(63, 70], providing highly suggestive evidence that the number of households reporting at and just below the old threshold decreased as a result of the reform.



Note: This figure shows the number of two adult households with no children that report incomes in the intervals R$(63, 70] and R$(70, 77] for each month between June 2012 to June 2016. The number in each bin is normalized to 1 in June 2012. The timing of the reform (from the announcement in April 2014 to the enactment in June 2014) is indicated by the gray, shaded region.

Figure 14: Number of Two Adult Households Reporting an Income in R$(63,70] or R$(70,77]

Figure 15 shows the analogue of Figure 5 for two adult households with no kids. There is a clear increase in the number of households locating in R$(70, 77] and a clear decrease in the fraction locating between R$(63, 70] just as for single individual households.



Note: This figure shows the log number of two adult households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The sample is restricted to two adult households without children. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation 16 where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 3$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

Figure 15: Actual and Counterfactual Paths for Treatment Bins, Two Adult Households with No Children

Results from estimating Equation 16 for various polynomial degrees K can be found in Table 7. The MVPF bounds are roughly similar in magnitude as for the single individual households discussed in Section IV but are a bit more sensitive to the degree of polynomial used.[10]

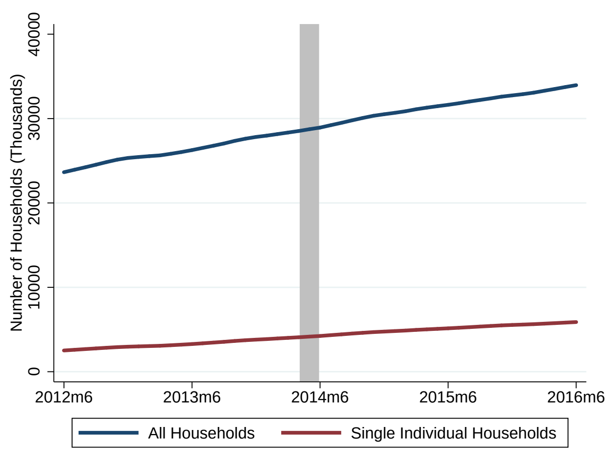
10. Note these bounds are for the MVPF associated with changing the location and level of the notch only, i.e., we ignore any welfare impacts of changing the level and location of the kink.

Table 7: Impacts of Reform and Efficiency Loss as of June, 2016 Estimated from Equation 16, Two Adult Households with No Children

	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	-19,897	37,055	19,897	17,157	0.88	1.12
	(2,586)	(1,729)	(2,586)	(4,315)	(0.03)	(0.02)
Cubic, $K = 3$	-19,862	33,645	19,862	13,783	0.91	1.12
	(1,919)	(1,366)	(1,919)	(3,284)	(0.02)	(0.01)
Quartic, $K = 4$	-14,347	21,043	14,347	6,696	0.96	1.08
	(1,058)	(990)	(1,058)	(2,048)	(0.01)	(0.01)
Quintic, $K = 5$	-14,157	25,714	14,157	11,557	0.93	1.08
	(1,773)	(1,551)	(1,773)	(3,324)	(0.02)	(0.01)

Note: Columns (1) and (2) show the estimated impacts of the reform on the number of two adult households reporting incomes in bins R$(63,70] and R$(70,77] for June 2016: $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated from Equation 16 with various polynomial degrees $K \in \{2, 3, 4, 5\}$, restricting the sample to two adult households without children. Columns (3) and (4) show the estimated bunching effect and jumping effect for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations 13 and 14. Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations 11 and 12. Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation 16.

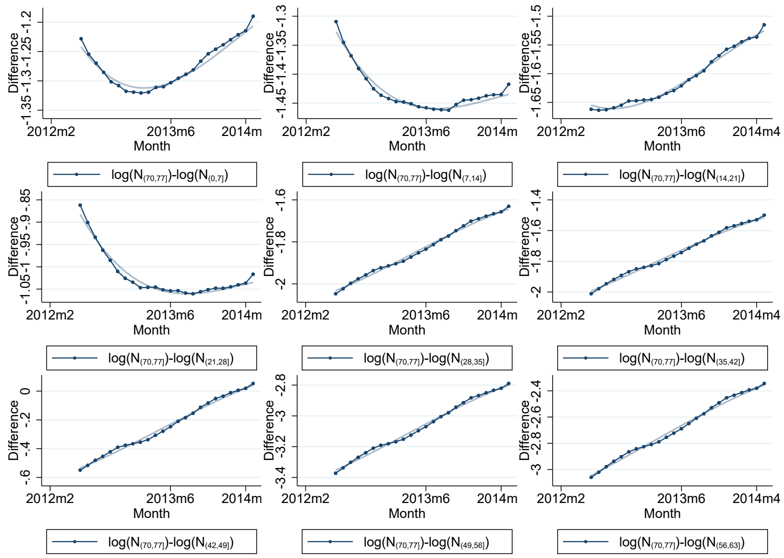
C.5 Number of Households on the Cadastro Único Registry Over Time



Note: This figure shows the raw number of households on the Cadastro Único Registry over time. The timing of the reform is indicated by the gray, shaded region.

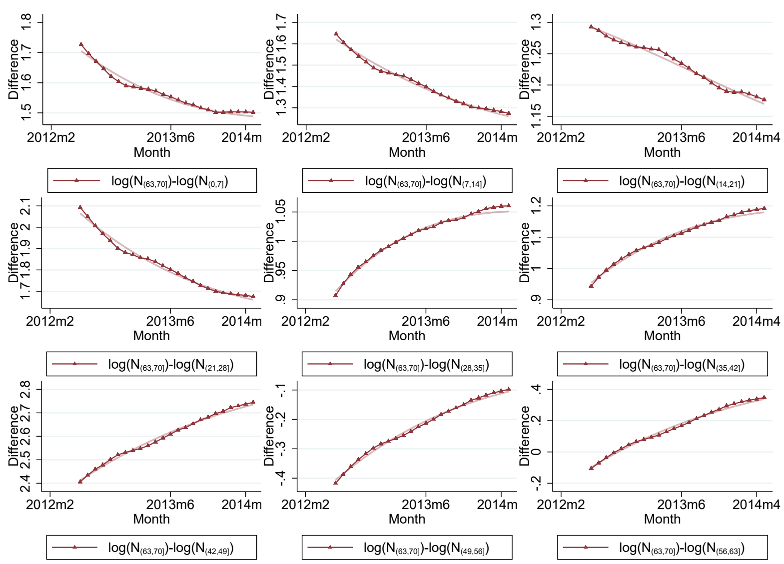
Figure 16: Number of Households on the Cadastro Único Registry

C.6 Pre-Reform Differences



Note: This figure shows how the difference between the log number of people reporting incomes in R$(70,77], denoted $\log\left(N_{(70,77]}\right)$, and the log number of people reporting in R$$(x-7,x]$, denoted $\log\left(N_{(x-7,x]}\right)$, varied prior to the reform in June, 2014. Each plot also includes the cubic trend estimated from Equation 16.

FIGURE 17: PRE-REFORM DIFFERENCES BETWEEN $N_{(70,77]}$ AND $N_{(x-7,x]}$ FOR $x \in \{7, 14, ..., 63\}$

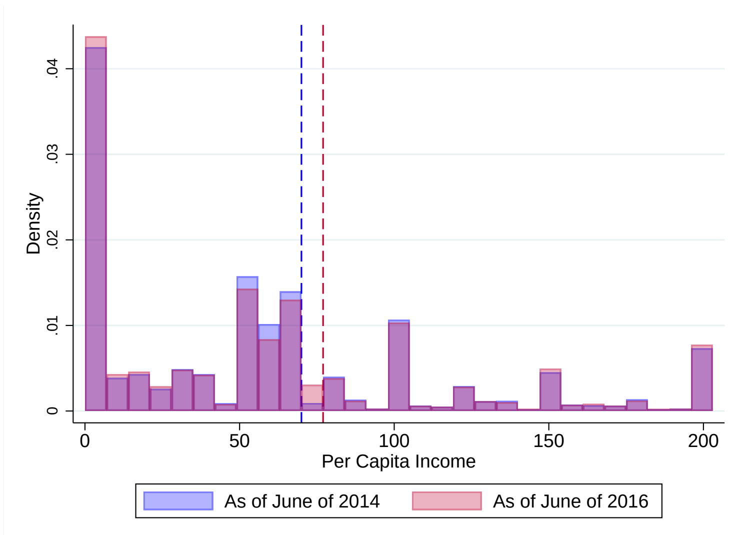


Note: This figure shows how the difference between the log number of people reporting incomes in R$(63,70], denoted $\log\left(N_{(63,70]}\right)$, and the log number of people reporting in R$$(x-7,x]$, denoted $\log\left(N_{(x-7,x]}\right)$, varied prior to the reform in June, 2014. Each plot also includes the cubic trend estimated from Equation 16.

FIGURE 18: PRE-REFORM DIFFERENCES BETWEEN $N_{(63,70]}$ AND $N_{(x-7,x]}$ FOR $x \in \{7, 14, ..., 63\}$

C.7 *Histograms of Reported Income Distribution*

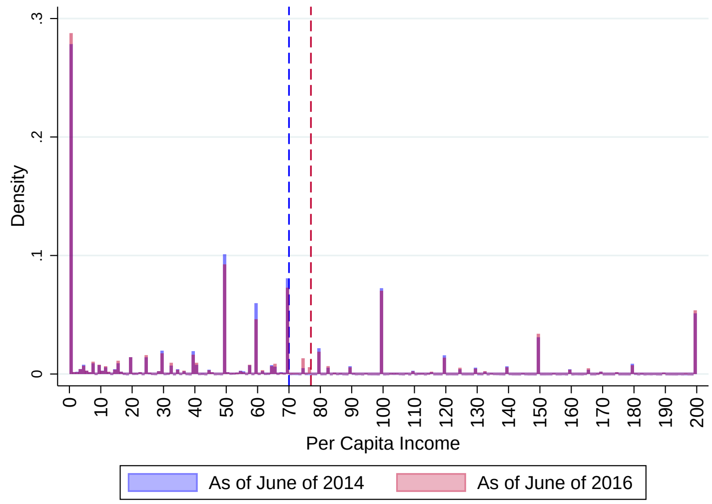
Figure 19 plots the distribution of reported incomes in June 2014 and June 2016 for single individual households split into seven increment bins. While these histograms show the pre- and post-reform distributions, they *should not* be used to make inferences about the causal impact of the reform due to significant underlying time trends in the reported income distribution.



Note: This figure shows the density of single individual households that report incomes in seven increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme-poverty threshold before June 2014 is shown with a dashed blue line and the extreme-poverty threshold after June 2014 is shown with a dashed red line.

Figure 19: Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform

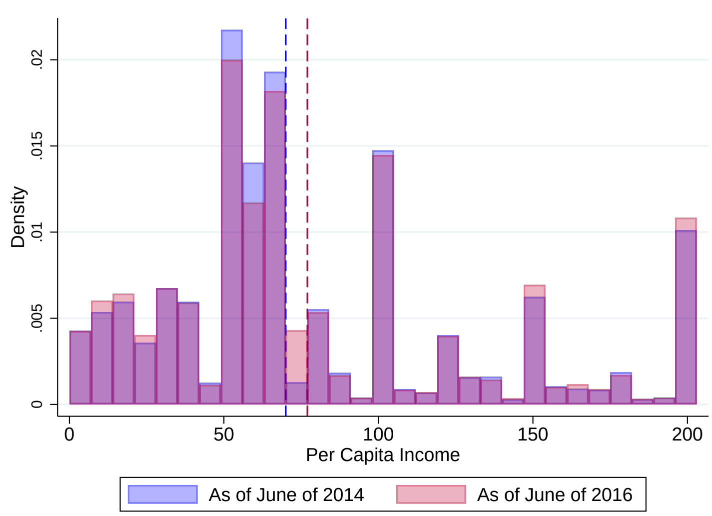
Figure 20 shows a more granular view of the same reported income distributions for single individual households split into one increment bins. There is substantial bunching at 0 mod 50 numbers and a lesser degree of bunching at 0 mod 10 numbers:



Note: This figure shows the density of single individual households that report incomes in one increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme-poverty threshold before June 2014 is shown with a dashed blue line and the extreme-poverty threshold after June 2014 is shown with a dashed red line.

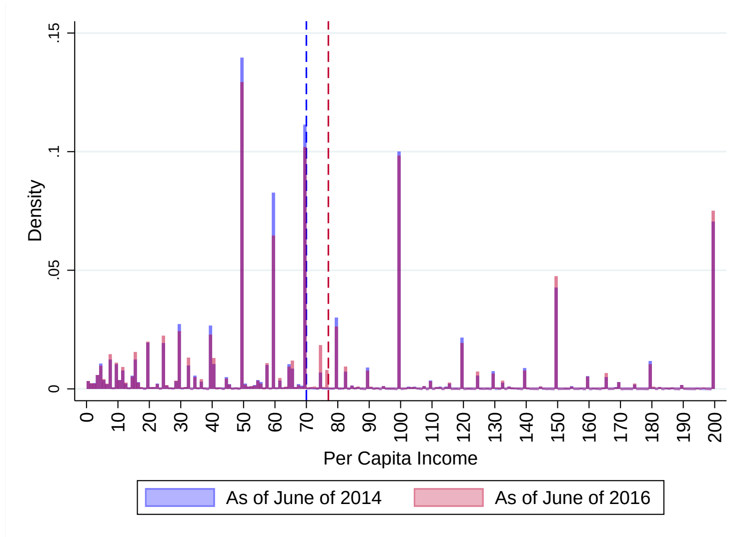
Figure 20: Granular Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform

For better visualization of the reported income distribution for per-capita incomes > 0, we repeat Figures 19 and 20 restricting to households with strictly positive per-capita incomes:



Note: This figure shows the density of single individual households that report incomes in seven increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme-poverty threshold before June 2014 is shown with a dashed blue line and the extreme-poverty threshold after June 2014 is shown with a dashed red line.

Figure 21: Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform (strictly positive incomes only)



Note: This figure shows the density of single individual households that report incomes in one increment bins in the period before the reform (June 2014) and two years after the reform (June 2016). The extreme-poverty threshold before June 2014 is shown with a dashed blue line and the extreme-poverty threshold after June 2014 is shown with a dashed red line.

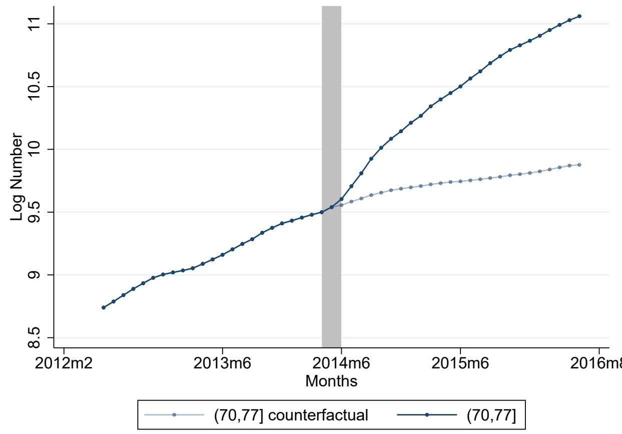
Figure 22: Granular Histogram of Single Individual Household Reported Incomes Pre- and Post-Reform (strictly positive incomes only)

C.8 Robustness: Conservative Assumption for Counterfactual Growth of R$(70,77]

Ignoring any relationships in the pre-reform period between the R$(70,77] bin and our control bins under R$63, we calculate how the R$(70,77] income bin would have grown counterfactually if it evolved according to the maximum growth of all control bins. Therefore, we calculate the following series:

$$\log(N_{(70,77],t})_{max,\mathbf{p}} = \log(N_{(70,77],0}) + \sum_{s=1}^{t} \max_{x\in\{7,\ldots,63\}} \left[\log(N_{(x-7,x],s}) - \log(N_{(x-7,x],s-1})\right] \qquad (70)$$

for $t > 0$ where $t = 0$ denotes May 2014 and $t > 0$ denotes each month in the post-reform period. The figure below plots this counterfactual series.



Note: This figure shows the number of single individual households that report incomes in R$(70,77] along with a conservative counterfactual series for R$(70,77] in absence of the reform computed via Equation (70). The timing of the reform is indicated by the gray, shaded region.

Figure 23: Actual vs. Counterfactual Growth of R$(70,77] if R$(70,77] Grew at Rate of Fastest Growing Control Bin Each Month

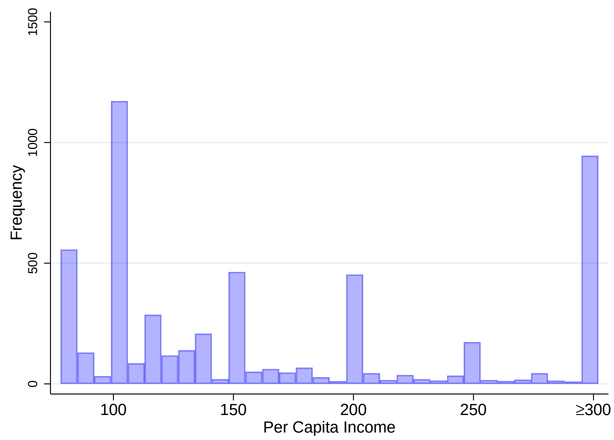
Using this series, we calculate a conservative estimate for the change in the number of households locating in R$(70,77] as a result of the reform to be 44,140 (recall that this number is $\approx 49,000$ for the cubic polynomial specification in Table 3). Using our bunching effect estimate of 27,452 from the cubic specification in Table 3, this estimate would put the jumping effect at 16,688, generating a lower MVPF bound of 0.92 (the upper bound MVPF is unchanged). Thus, even if counterfactually the R$(70,77] bin would have grown at the rate of the fastest growing control group in each month, we would still estimate a sizable jumping effect and our MVPF bounds would be very similar.

C.9 Investigating Where the Jumpers are Coming From

In this appendix, we exploit the panel nature of our data to provide descriptive evidence on where the households who jump down to bunch in R$(70,77] are coming from. First, for households reporting in R$(70,77] in June 2016, we investigate what their last reported income was prior to the reform (i.e., prior to June 2014). We focus on households reporting above R$77 prior to June 2014 and households not reporting any income prior to June 2014 so as to shed light on the extent to which the increase of $\approx$ 22,000 households in R$(70,77] (see Table 3) came from intensive margin responses (i.e., jumping down from higher reported incomes) vs. extensive margin responses (i.e., entering the program). We can also shed light on the possible size of the intensive margin responses by looking at the range of incomes these households were reporting prior to the reform.

Figure 24 shows the (frequency) distribution of reported incomes prior to reform (June 2014) for those who report in R$(70,77] in June 2016 (restricting to those reporting above R$77 prior to the reform). As can be seen, these households were reporting a range of incomes above R$77 prior to the reform with many reporting incomes at or above R$300. In total, 5,321 households locating in R$(70,77] in June 2016 reported an income above R$77 prior to reform. Interestingly, 27,844 households locating in R$(70,77] in June 2016 were not in the registry prior to the reform. While this evidence is purely descriptive, it does suggest that of many of the $\approx$ 22,000 households comprising the jumping effect we identify in Table 3 were likely extensive margin households (i.e., in absence of the reform, many of these 22,000 would not have reported an income in June 2016). One may wonder why $5,321 + 27,844 = 33,165 > 22,000$. It's worth reiterating that this analysis is descriptive - we are not determining counterfactual reported incomes in June 2016 had the reform not occurred. Some of these 33,165 households would have located in R$(63,70] in June 2016 had the reform not occurred (and are thus part of the bunching effect B rather than the jumping effect J).

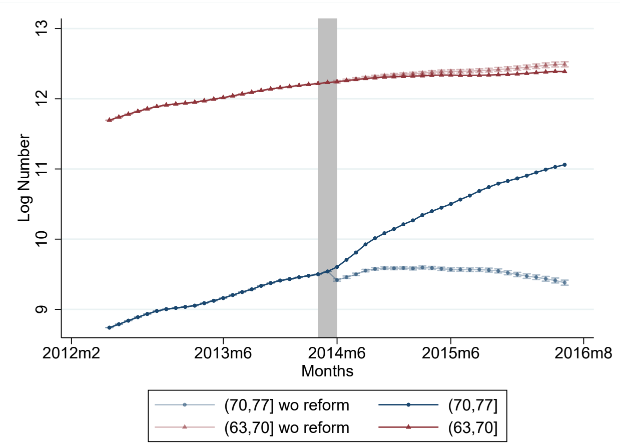
Finally, we think it is worth noting why we only present descriptive evidence on where households are jumping from. While we are able to precisely and robustly identify an increase of 22,000 households in the R$(70, 77] income bin, identifying where these households are coming from involves estimating very small decreases across many bins above R$77; ultimately, our identification strategy doesn't have the power to estimate these very small treatment effects.



Note: This figure plots the last reported income prior to June 2014 for single individual households who report an income in R$(70,77] in June 2016, restricting to households who were reporting above R$77 in June 2014.

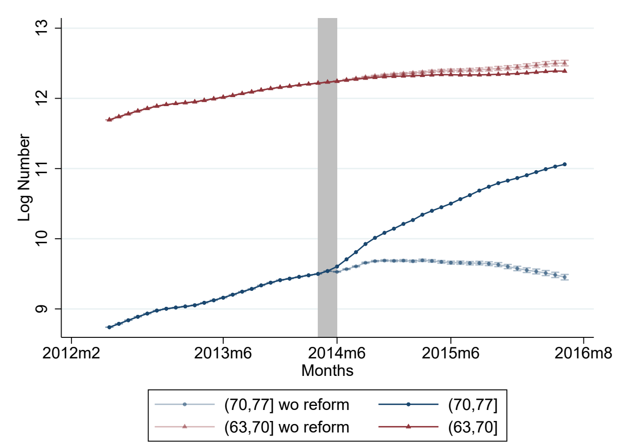
FIGURE 24: REPORTED INCOME PRIOR TO JUNE 2014 FOR THOSE REPORTING IN R$(70,77] IN JUNE 2016

C.10 Actual and Counterfactual Paths Estimated from Equation 16 with Different Polynomial Degrees



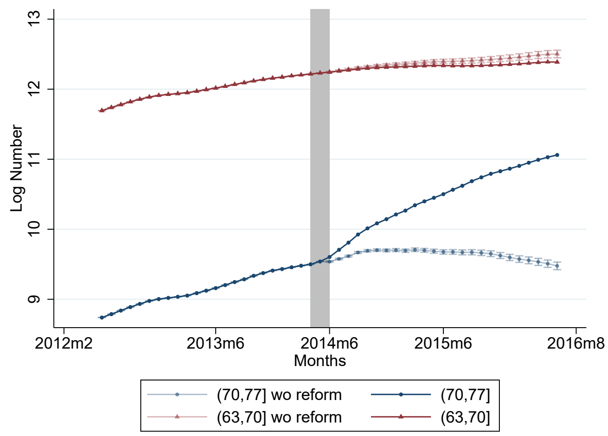
Note: This figure shows the log number of single individual households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation 16 where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 2$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

FIGURE 25: ACTUAL AND COUNTERFACTUAL PATHS FOR TREATMENT BINS, $K = 2$



Note: This figure shows the log number of single individual households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation 16 where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 4$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

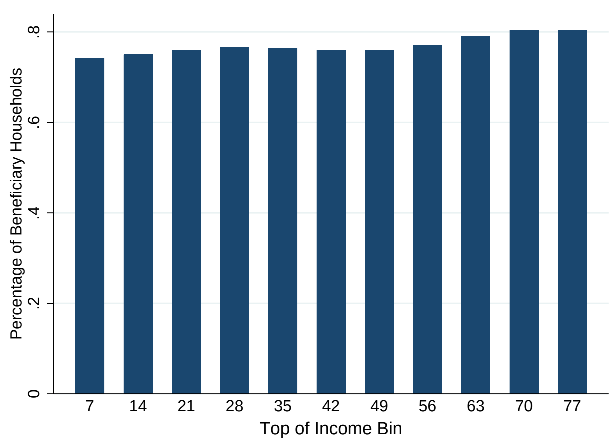
FIGURE 26: ACTUAL AND COUNTERFACTUAL PATHS FOR TREATMENT BINS, $K = 4$



Note: This figure shows the log number of single individual households reporting incomes in R$(63, 70] and R$(70, 77] over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation 16 where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 5$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

Figure 27: Actual and Counterfactual Paths for Treatment Bins, $K = 5$

C.11 Fraction of Households Receiving the Benefit by Reported Income



Note: This figure shows the fraction of households that receive the benefit in each seven increment reported income bin in June, 2016.

Figure 28: Fraction of Households Receiving Benefit by Reported Income

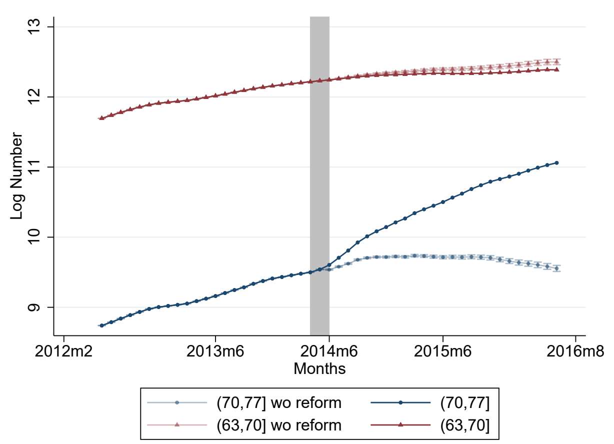
C.12 *Results Using R$(56,63] as a Treatment Bin*

Table 8: Impacts of Reform and MVPF Bounds in June 2016 Estimated from Equation 16 Using R$(56,63] as Treatment Bin

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Polynomial Degree, K	$\Delta N_{(56,63],\bar{t}}$	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	13,173	-23,591	51,879	10,418	41,461	0.82	1.04
	$(2,306)$	$(4,412)$	(197)	$(6,719)$	$(6,915)$	(0.03)	(0.03)
Cubic, $K = 3$	8,579	-25,719	49,340	17,140	32,199	0.85	1.07
	$(2,357)$	$(4,399)$	(237)	$(6,756)$	$(6,992)$	(0.03)	(0.03)
Quartic, $K = 4$	9,825	-27,331	50,968	17,507	33,462	0.85	1.07
	$(2,899)$	$(5,491)$	(260)	$(8,390)$	$(8,650)$	(0.03)	(0.04)
Quintic, $K = 5$	12,489	-26,669	50,683	14,180	36,503	0.83	1.06
	$(3,956)$	$(4,412)$	(370)	$(11,577)$	$(11,947)$	(0.05)	(0.05)

Note: Columns (1) (2), and (3) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R$(56,63], R$(63,70], and R$(70,77] for June 2016: $\Delta N_{(56,63],\bar{t}}$, $\Delta N_{(63,70],\bar{t}}$, and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated using Equation 16 with various polynomial degrees $K \in \{2, 3, 4, 5\}$. Columns (4) and (5) show the estimated bunching effect and jumping effect for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations 13 and 14. Columns (6) and (7) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations 11 and 12. Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation 16.

C.13 *Results Estimated from Nonlinear Least Squares Equation 17*



Note: This figure shows the log number of single individual households reporting incomes in R$$(63,70]$ and R$$(70,77]$ over time along with the counterfactual paths had the reform not happened. The counterfactual paths are equal to the actual number of people reporting in the given interval minus the causal impact of the reform, $\hat{\beta}_{1,x} + \hat{\beta}_{2,x}t$, estimated using Equation 17 where we set $treat_x = 1$ if $x \in \{70, 77\}$ and $K = 3$. Confidence intervals are constructed from clustered standard errors at the bin level. The timing of the reform is indicated by the gray, shaded region.

Figure 29: Actual and Counterfactual Paths for Treatment Bins from Equation 17

Table 9: Impacts of Reform and MVPF Bounds in June 2016 Estimated from Equation 17

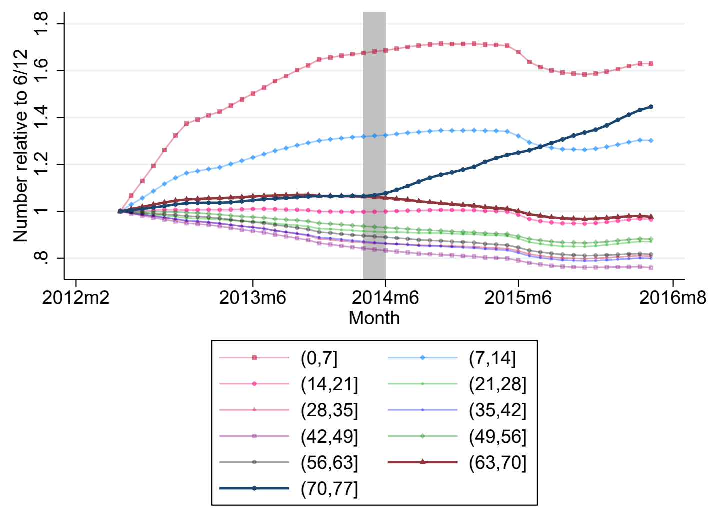
	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	-29,038	50,724	29,038	21,686	0.90	1.13
	(13,711)	(635)	(13,711)	(13,723)	(0.06)	(0.07)
Cubic, $K = 3$	-28,834	49,505	28,834	20,671	0.90	1.13
	(5,485)	(309)	(5,485)	(5,532)	(0.02)	(0.03)
Quartic, $K = 4$	-23,825	50,824	23,825	26,999	0.87	1.10
	(9,358)	(485)	(9,358)	(9,429)	(0.04)	(0.04)
Quintic, $K = 5$	-24,571	51,228	24,571	26,658	0.87	1.11
	(8,311)	(426)	(8,311)	(8,374)	(0.04)	(0.04)

Note: Columns (1) and (2) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R\$(63,70] and R\$(70,77] for June 2016: $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated from the nonlinear least squares regression given by Equation 17 with various polynomial degrees $J \in \{2, 3, 4, 5\}$. Columns (3) and (4) show the estimated bunching effect and jumping effect for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations 13 and 14. Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations 11 and 12. Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation 17.

C.14 Results for Households with Kids

The benefit schedule for families with children is substantially more complex than the benefit schedule for households without children, as discussed in detail Appendix B.2. Prior to June 2014, in addition to the guaranteed minimum income and the basic benefit, there was also a variable per-child benefit for households below the poverty threshold of R\$140 per-capita. The June 2014 reform led to changes in the levels and locations of the guaranteed minimum income kink, the basic benefit notch, and the variable benefit notch. For example, the poverty threshold was raised from R\$140 per-capita to R\$154 per-capita and the levels of the variable benefits were also increased by around 10% (see Appendix B.2 for more details).

Estimating the MVPF of the reform for households with children will require estimating the WTP for the all of the different changes to the BF schedule. Estimating the WTP for the change to the variable benefit schedule would be particularly difficult given that this benefit is made conditional on investments in children. For example, suppose parents are under-investing in their children's education and the reform increases school attendance. Then we would need to estimate the childrens' WTP for the increased education they receive as a result of the reform to the variable benefit schedule. Thus, we leave calculating the MVPF of the reform for households with children to future work. We do, however, show strong evidence that households with kids respond to the reform. In particular, Figure 30 shows *prima facie* evidence that the reform increased the number of households reporting incomes in R\$(70, 77].



Note: This figure shows the number of households with kids that report incomes in various bins. The number in each bin is normalized to 1 in June, 2012. The timing of the reform is indicated by the gray, shaded region.

Figure 30: Number of Households with Kids Reporting Incomes in Various Bins

C.15 Results for Households with Constant Composition

We also consider the possibility that some of the estimated behavioral impact of the reform for single individual households may be coming from households misreporting their family composition: a two adult household can receive greater benefits if they report to be two separate one adult households as benefits are paid out per-household as opposed to per-capita. For example, a household with two adults and no children that has a combined per-capita income of R\$60 is eligible for R\$70 in transfers pre-reform and R\$77 in transfers post-reform. However, if this household were to report that they were actually two single individual households with incomes of R\$60, they would *each* be eligible for R\$70 in transfers pre-reform and R\$77 in transfers post-reform. Hence, the reform may have increased incentives to misreport family composition as well as income.[11] From a theoretical perspective, Proposition 3 holds even if misreporting responses occur on the family composition margin. However, such a behavioral response may affect the validity of our identification strategy (for example, it may no longer be reasonable to assume that the distribution below R\$63 is unaffected by the reform if these new "single individual" households enter at income levels well below the threshold).

Thus, in this appendix, we present results for our sample of single individual households restricted to those who do not change their reported family composition throughout the analysis period (June 2012 to June 2016). We drop any household that reports a composition change over the analysis period. Moreover, we drop any household who enters the registry post June-2014 as we cannot tell whether these new entrants are truly new households or are pre-existing households (with multiple

11. However, the ability of households to misreport the number of members is limited as individuals must provide government issued IDs for all family members to be on the registry. Moreover, household composition is arguably more verifiable than income for many households given the large informal sector in Brazil. Thus, we suspect that households are more likely to misreport income than family composition.

adults) pretending to be separate households so as to increase the amount of benefits they receive. In other words, we restrict our sample to single individual households who were (a) on the registry prior to June 2014, and (b) do not report a change in composition over the analysis period. This reduces our sample from 1,938,653 single individual households with incomes below R$77 in June 2016 to 1,039,573 single individual households with incomes below R$77 in June 2016. Table 10 presents results for this exercise. We find very similar estimates for the MVPF lower bound and slightly smaller estimates for the MVPF upper bound. Note that the estimated bunching effect and jumping effect are smaller than in Table 3 due to the smaller sample size.

Table 10: Impacts of Reform and MVPF Bounds in June 2016 Estimated from Equation 16, Constant Composition Households

	(1)	(2)	(3)	(4)	(5)	(6)
Polynomial Degree, K	$\Delta N_{(63,70],\bar{t}}$	$\Delta N_{(70,77],\bar{t}}$	$B_{\bar{t}}$	$J_{\bar{t}}$	$MVPF_{L,\bar{t}}$	$MVPF_{U,\bar{t}}$
Quadratic, $K = 2$	-4,714	19,019	4,714	14,305	0.88	1.04
	(7,919)	(589)	(7,919)	(8,507)	(0.07)	(0.06)
Cubic, $K = 3$	-6,847	17,713	6,847	10,866	0.91	1.05
	(6,970)	(575)	(6,970)	(7,545)	(0.06)	(0.06)
Quartic, $K = 4$	-862	13,722	862	12,860	0.89	1.01
	(7,141)	(826)	(7,141)	(7,967)	(0.06)	(0.05)
Quintic, $K = 5$	-1,586	14,679	1,586	13,094	0.89	1.01
	(8,226)	(889)	(8,226)	(9,115)	(0.07)	(0.06)

Note: Columns (1) and (2) show the estimated impacts of the reform on the number of single individual households reporting incomes in bins R$(63,70] and R$(70,77] for June 2016: $\Delta N_{(63,70],\bar{t}}$ and $\Delta N_{(70,77],\bar{t}}$. Estimates are calculated from Equation 16 with various polynomial degrees $K \in \{2, 3, 4, 5\}$, restricting the sample to single individual households who do not change their reported household composition throughout the analysis period. Columns (3) and (4) show the estimated bunching effect and jumping effect for June 2016, $B_{\bar{t}}$ and $J_{\bar{t}}$, calculated using Equations 13 and 14. Columns (5) and (6) show the estimated upper and lower bounds for the MVPF for June 2016, calculated using Equations 11 and 12. Standard errors are presented in parentheses and are computed from the delta method from the clustered standard errors estimated in Equation 16.

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