Online Appendix

The Evolution of U.S. Retail Concentration\textsuperscript{1}
Dominic A. Smith\textsuperscript{2} & Sergio Ocampo\textsuperscript{3}

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\textsuperscript{1}Any views expressed are those of the authors and not those of the U.S. Census Bureau. The Census Bureau’s Disclosure Review Board and Disclosure Avoidance Officers have reviewed this information product for unauthorized disclosure of confidential information and have approved the disclosure avoidance practices applied to this release. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Numbers 1179 and 1975 (CBDRB-FY19-P1179-R7207, CBDRB-FY20-P1975-R8604 and CBDRB-FY23-P1975-R10585).

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A Concentration Decomposition

We calculate the HHI for the retail sector at a time \( t \), as the sales-weighted average of the product-HHIs:

\[
HHI^t = \sum_{j=1}^{J} s_j^t HHI_j^t.
\] (A.1)

The HHI for a given product \( j \), \( HHI_j^t \), can be decomposed into the contribution of local and cross-market concentration. The decomposition starts from the probability that two dollars \((x, y)\) spent on a product during some time period are spent at the same firm \( (i) \), which gives the HHI at the national level:

\[
HHI_j^t \equiv P(i_x = i_y; j, t) = \sum_i (s_i^j)^2,
\] (A.2)

where \( s_i^j \) is the share of firm \( i \) in product \( j \) during period \( t \). This probability can be divided into two terms by conditioning on the dollars being spent in the same location, \( \ell_x = \ell_y \):

\[
P(i_x = i_y; j, t) = \frac{P(i_x = i_y|\ell_x = \ell_y; j, t) P(\ell_x = \ell_y; j, t)}{P(\ell_x = \ell_y; j, t)}.\] (A.3)

When we report contribution of local and cross-market concentration for the retail sector, we report the sales-weighted average of these two terms across products.

The collocation probability is calculated as:

\[
P(\ell_x = \ell_y; j, t) = \sum_{\ell=1}^{L} (s_{\ell}^j)^2.
\] (A.4)

When we report the collocation for the retail sector, we report the sales-weighted average of collocation across products: \( \text{Collocation}_t = \sum_j s_j^t P(\ell_x = \ell_y; j, t) \).

The first component, \( P(i_x = i_y|\ell_x = \ell_y; j, t) \), is an aggregate index of local concentration, with local concentration measured as in equation (2).

\(^{4}\text{In the decomposition, each local market is weighted by the conditional probability that the two dollars are spent in location } \ell \text{ given that they are spent in the same location: } s_{\ell}^j/(1-\sum_p s_p^j). \text{ These weights give more importance to larger markets than the more usual weights } s_x - \text{the share of sales (of product } j) \text{ accounted for by location } \ell \text{ (at time } t). \text{ We present aggregated series for local concentration in Section 3 that use the latter weights. Appendix A derives these results in detail.}\)
consumers in a local market shop at the same firm. Local concentration is calculated as:

\[
P(i_x = i_y|\ell_x = \ell_y; j, t) = \sum_{\ell=1}^{L} \frac{a_{\ell}P(\ell_x = \ell|\ell_x = \ell_y; j, t)P(i_x = i_y|\ell_x = \ell, \ell_x = \ell_y; j, t)}{L_{\text{Local HHI}}} \sum_{\ell=1}^{L} \frac{(s^{jt}_{\ell})^2}{\sum_{n}(s^{jt}_{n})^2} \sum_{k=1}^{K} (s^{jt}_{k})^2.
\]

(A.5)

In the main text, when we report the local HHI for individual product categories we also report the retail sector’s average local HHI using sales weights instead of the weights implied by the decomposition to facilitate comparison to other research:

\[
\text{HHI}^\text{Local}_t = \sum_{j} s^t_j \sum_{\ell} s^{jt} \sum_{i} (s^{jt}_i)^2.
\]

(A.6)

The third component, \(P(i_x = i_y|\ell_x \neq \ell_y)\), which we call cross-market concentration, captures the probability that a dollar spent in different markets is spent at the same firm:

\[
P(i_x = i_y|\ell_x \neq \ell_y) = \sum_{\ell,n=1}^{L} \sum_{n \neq \ell} \frac{s^{jt}_{\ell} s^{jt}_{n}}{1 - \sum_{p} s^{jt}_{p}^2} \sum_{i=1}^{N} s^{jt}_{i} s^{jt}_{n}.
\]

(A.7)

The cross-market concentration between two markets (say \(\ell\) and \(n\)) is given by the product of the shares of the firms in each location (the probability that two dollars spent in different locations are spent in the same firm). The pairs of markets are then weighted by their share of sales and are summed.

The cross-market term as a whole is calculated as:

\[
P(\ell_x = \ell_y; j, t) P(i_x = i_y|\ell_x \neq \ell_y; j, t) = (1 - \sum_{\ell=1}^{L} (s^{jt}_{\ell})^2) \sum_{k=1}^{L} \sum_{\ell \neq k} \frac{s^{jt}_{k} s^{jt}_{\ell}}{1 - \sum_{p} s^{jt}_{p}^2} \sum_{i=1}^{I} s^{kjt}_{i} s^{jt}_{\ell}.
\]

This calculation is the same in the results for product category because \(1 - \sum_{m} (s^{jt}_{m})^2\) cancels in the calculation of the collocation term.

As mentioned in the main text, the collocation term determines how much purely local concentration (without cross-market linkages) can affect national concentration terms. Conversely, the collocation term determines how much can be learned about local competitive environments using national information. If it were large enough, national concentration numbers can be informative about local markets. However, the collocation term for the U.S. retail sector is low and remarkably stable across time and products. This implies that local concentration can only have a limited effect on national trends, making
Figure A.1: Share of Local Concentration Term in National Concentration

![Graph showing the share of local concentration term in national concentration from 1992 to 2012.]

Notes: The numbers are based on calculations from the Census of Retail Trade. The share of local concentration is measured as the ratio of the local concentration term in equation (5) to the national Herfindahl-Hirschman Index (HHI). The local concentration term is the product of the collocation term and local HHI. (CBDRB-FY20-P1975-R8604)

changes in national concentration mostly informative about cross-market concentration.

Figure A.1 shows the contribution of local concentration to the level national concentration by year, defined as the ratio of the local term in the decomposition to the national HHI, \( P(\ell_x=\ell_y)P(\ell_x=i_y|\ell_x=\ell_y)/P(\ell_x=i_y) \). The contribution of local concentration to national concentration is small—never above 5 percent. Moreover, the contribution of local concentration to national concentration has been falling over time as national concentration has been increasing. By 2012, local concentration accounted for just 1.7 percent of the national concentration level.

The pattern of low collocation terms and a prominent role of cross-market concentration applies across all product categories. Figure A.2 shows that the collocation term is always low, less than 2 percent, and stable over time. By the early 1990s, only furniture and groceries have contributions of over 10 percent, with the local contribution in all other products being no higher than 5.5 percent, and as low as 2 percent.
Figure A.2: Collocation across Product Categories

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Furniture</td>
<td>0.013</td>
<td>0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>Clothing</td>
<td>0.015</td>
<td>0.016</td>
<td>0.017</td>
</tr>
<tr>
<td>Sporting Goods</td>
<td>0.011</td>
<td>0.010</td>
<td>0.009</td>
</tr>
<tr>
<td>Electronics &amp; Appliances</td>
<td>0.015</td>
<td>0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>Health Goods</td>
<td>0.012</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>Toys</td>
<td>0.012</td>
<td>0.012</td>
<td>0.011</td>
</tr>
<tr>
<td>Home Goods</td>
<td>0.010</td>
<td>0.010</td>
<td>0.009</td>
</tr>
<tr>
<td>Groceries</td>
<td>0.011</td>
<td>0.011</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Notes: The numbers are collocation indexes by product weighted by market size from the Census of Retail Trade. (CBDRB-FY20-P1975-R8604)

B Additional Tables and Figures

B.1 The Role of Multi-Product Retailers

Table B.1 shows how sales for each main product category are distributed across sets of industries. This informs us of which type of establishment accounts for the sales of each product. The main subsector column refers to the NAICS subsector that most closely corresponds to the product category. The NAICS code of the subsector is indicated next to each product category. The main subsector accounts for just over half of sales on average, but this figure varies depending on the product. A larger fraction of sales of Furniture, Home Goods, and Groceries comes from establishments in their respective NAICS subsectors, while Electronics and Toys are more commonly sold by establishments in other subsectors. Over time, the share of sales accounted by the product’s own subsector has decreased for most products, with the difference captured by establishments outside of the general merchandise subsector.
Table B.1: Share of Product Category Sales by Establishment Subsector

<table>
<thead>
<tr>
<th>Product Category</th>
<th>Main Subsector</th>
<th>GM</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Furniture (442)</td>
<td>76.3</td>
<td>73.1</td>
<td>64.4</td>
</tr>
<tr>
<td>Clothing (448)</td>
<td>50.9</td>
<td>51.8</td>
<td>51.1</td>
</tr>
<tr>
<td>Sporting Goods (451)</td>
<td>55.4</td>
<td>52.2</td>
<td>54.2</td>
</tr>
<tr>
<td>Electronic &amp; Appliances (443)</td>
<td>30.3</td>
<td>31.0</td>
<td>29.5</td>
</tr>
<tr>
<td>Health Goods (446)</td>
<td>49.0</td>
<td>50.0</td>
<td>46.8</td>
</tr>
<tr>
<td>Toys (451)</td>
<td>40.7</td>
<td>27.6</td>
<td>22.0</td>
</tr>
<tr>
<td>Home Goods (444)</td>
<td>63.9</td>
<td>72.8</td>
<td>72.4</td>
</tr>
<tr>
<td>Groceries (445)</td>
<td>79.8</td>
<td>67.2</td>
<td>59.7</td>
</tr>
<tr>
<td>Average</td>
<td>55.8</td>
<td>53.2</td>
<td>50.0</td>
</tr>
</tbody>
</table>

Notes: The numbers come from the Census of Retail Trade data. GM includes stores in subsector 452. Other includes sales outside of the main subsector (indicated in parenthesis) and GM. Average is the arithmetic mean of the numbers in the column. (CBDRB-FY20-P1975-R8604)

B.2 Extended Sample

We now present results with an extended sample that covers the period 1982 to 2012. The 1982 and 1987 Censuses of Retail Trade do not include product-level sales for all the categories we consider in our main sample (1992-2012). The affected product categories, Toys and Sporting Goods, account for a relatively small share of total retail sales. Therefore, we focus on results for the retail sector as a whole which we believe are reliable for this time period.

Figure B.1 presents measured concentration indexes for different definitions of local markets and the retail sector as a whole going back to 1982. We use the store-level NAICS codes imputed by Fort and Klimek (2018) to identify retail establishments prior to 1992. Relative to Figure 1 we also include a measure of local concentration where markets are defined by Metropolitan Statistical Areas (MSA). There are more MSAs than commuting zones (about 900 vs 722) and MSAs do not partition the U.S., omitting rural areas. In practice, the measured concentration level for MSAs is similar to that of commuting zones.

Extending our sample to 1982 does not change the main result of increasing national and local concentration. All measures show sustained increases between 1982 and 2002. Looking at the full sample highlights the change in the rate of increase of national concentration after 1997 which contrasts with the slow increase during the 1980s.

Finally, we extend the decomposition exercise of Figure A.1 to 1982. The results, shown in Figure B.2, show a stark decrease in the contribution of local concentration to national
concentration. Even though the role of local concentration was never large (always below 6 percent), the share of national concentration attributed to local concentration fell sharply during the 1990s, ending at roughly 2 percent in 2002.

Figure B.1: National and Local Concentration

Notes: The data are from the Census of Retail Trade. The Herfindahl-Hirschman Index (HHI) for four different geographic definitions of local markets and national concentration are plotted. The local HHI is aggregated using each location’s share of national sales within a product category. The numbers are sales weighted averages of the corresponding HHI in the product categories. (CBDRB-FY20-P1975-R8604)
B.3 Non-Store Retailer Additional Details

The penetration of non-store retailers varies widely across products. As Figure B.3 shows, the sales share of non-store retailers is highest in Electronics and Appliances, with an initial share of 7.5 percent in 1992 and a share of 20.9 percent in 2012. The initial differences were large, with only two categories (Electronics and Sporting Goods) having a share of more than 5 percent. By 2012, non-store retailers accounted for more than 15 percent of sales in five of the eight major categories. Despite this widespread increase, not all products are sold online. By 2012, only 0.7 percent of Groceries sales and 3 percent of Home Goods sales were accounted for by non-store retailers. These two categories account for almost half of all retail sales, which explains the overall low sales share of non-store retailers.

Because the share of national sales going to non-store retailers is quite small these retailers have almost no impact on national concentration numbers until the 2000s. In Table B.2, we show national concentration numbers both with and without non-store retailers. The trends are similar with non-store retailers slightly lowering the increases in national concentration.

![Figure B.2: Share of Local Concentration Term in National Concentration](image)

**Notes:** The numbers are from the Census of Retail Trade. The share of local concentration is measured as the ratio of the local concentration term in equation (5) to the national Herfindahl-Hirschman Index (HHI). We aggregate the local concentration terms across the product categories using their sales shares. (CBDRB-FY20-P1975-R8604)
Table B.2: National HHI with and without Non-Store Retailers

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</tr>
</thead>
<tbody>
<tr>
<td>Excluding Non-store</td>
<td>0.0082</td>
<td>0.0103</td>
<td>0.0128</td>
<td>0.0184</td>
<td>0.0311</td>
<td>0.0418</td>
<td>0.0426</td>
</tr>
<tr>
<td>Including Non-store</td>
<td>0.0083</td>
<td>0.0103</td>
<td>0.0124</td>
<td>0.0175</td>
<td>0.0286</td>
<td>0.0373</td>
<td>0.0391</td>
</tr>
</tbody>
</table>

Notes: The numbers come from the Census of Retail Trade data. The numbers in the first row are the same numbers as in Figure 1 and represent the National HHI excluding non-store retailers. The numbers in the second row include establishments in NAICS 4541. (CBDRB-FY20-P1975-R8604 and CBDRB-FY23-P1975-R10585)

B.4 Top 4 Firm Shares

Here we show that the average market share of the four largest firms in each product and local market increases steadily during our sample for all geographies larger than a zip code. We find the largest increases in commuting zone and metropolitan statistical areas. The average share of the four largest firms for zip codes increases until 2002 and then decrease

Figure B.3: Non-Store Retailers Share across Product Categories

Notes: The numbers are the national sales shares of non-store retailers by product category from the Census of Retail Trade microdata. (CBDRB-FY20-P1975-R8604)
over the next 10 years.

Table B.3: Average Top 4 Firm Share

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Commuting Zone</td>
<td>0.35</td>
<td>0.37</td>
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<td>0.38</td>
<td>0.41</td>
<td>0.42</td>
<td>0.42</td>
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<tr>
<td>MSA</td>
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<td>County</td>
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<td>0.45</td>
<td>0.47</td>
<td>0.47</td>
<td>0.46</td>
</tr>
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<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
<td>0.70</td>
<td>0.68</td>
</tr>
</tbody>
</table>

*Notes:* Results come from the Census of Retail Trade. The market share of the 4 firms with the greatest sales in each product category and location in each year are summed. These results are then aggregated using a weighted average of the sales share of each product and location in a year. (CBDRB-FY20-P1975-R8604)

### B.5 10 Year Concentration Changes

We now turn to the distribution of changes in concentration across markets. We find that the increases in concentration have been broad based. Almost 60 percent of dollars spent in 2012 are spent in markets that have increased concentration since 2002 (Figure B.4d). In just 10 years, 23 percent of markets have increases in concentration of over 5 percentage points (Figure B.4b). These changes are significant. For comparison, the Department of Justice considers a 2 percentage point increase in the local HHI potential grounds for challenging a proposed merger (Department of Justice and Federal Trade Commission, 2010).

Figures B.4a and B.4c show that the changes in concentration were even more widespread between 1992 to 2002. Over 69 percent of markets, accounting for 72 percent of retail sales, increased their concentration. In both the 1992–2002 and 2002–2012 decades, the majority of retail sales occurred in markets with relatively small increases in concentration (between 0 and 5 percentage point increases in the market’s HHI). These markets account for 66 percent of retail sales in 2002 and 55 percent in 2012.
Figure B.4: Changes in Concentration across Markets

Notes: The numbers are based on calculations from the Census of Retail Trade. The top panels show the fraction of markets, commuting zone/product category pairs, with changes in concentration of a given size. The bottom panels weight markets by the value of sales in the product category. The columns report changes for the decades 1992 to 2002 and 2002 to 2012. (CBDRB-FY20-P1975-R8604)

B.6 Industry-Based Results

This subsection provides more details on our industry based results. Figure B.5 shows national and local concentration for eight retail subsectors (3-digit NAICS). Local concentration is defined at the commuting zone level. The increasing trends we
documented for national concentration in the retail sector are present in all subsectors, but the increase is particularly strong for general merchandisers (NAICS 452) at both the national and at the local level. The general merchandise subsector includes department stores, discount general merchandisers, and supercenters. Over time a small number of firms have come to dominate this format. Similar patterns arise in local concentration. Figure B.5b shows local concentration for the major subsectors, calculated as a weighted average of the industries comprising each subsector. Local industry concentration levels are higher than national and they also increase.
Figure B.5: National and Local Concentration Across Industries

(a) National Concentration

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Furniture (442)</td>
<td>0.005</td>
<td>0.016</td>
<td>0.033</td>
</tr>
<tr>
<td>Clothing (448)</td>
<td>0.062</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td>Toys/Sporting Goods (451)</td>
<td>0.103</td>
<td>0.072</td>
<td>0.038</td>
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<tr>
<td>Electronics &amp; Appliances (443)</td>
<td>0.202</td>
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<tr>
<td>Health Goods (446)</td>
<td>0.220</td>
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<td>Home Goods (444)</td>
<td>0.248</td>
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<td>0.243</td>
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<tr>
<td>Groceries (445)</td>
<td>0.139</td>
<td>0.080</td>
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<tr>
<td>General Merchandisers (452)</td>
<td>0.340</td>
<td>0.340</td>
<td>0.431</td>
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(b) Local Concentration

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Furniture (442)</td>
<td>0.030</td>
<td>0.080</td>
<td>0.139</td>
</tr>
<tr>
<td>Clothing (448)</td>
<td>0.110</td>
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</tr>
<tr>
<td>Toys/Sporting Goods (451)</td>
<td>0.193</td>
<td>0.143</td>
<td>0.219</td>
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<td>Electronics &amp; Appliances (443)</td>
<td>0.193</td>
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<td>0.243</td>
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<td>Health Goods (446)</td>
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<td>Home Goods (444)</td>
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<tr>
<td>Groceries (445)</td>
<td>0.561</td>
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<td>0.561</td>
</tr>
<tr>
<td>General Merchandisers (452)</td>
<td>0.561</td>
<td>0.504</td>
<td>0.561</td>
</tr>
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</table>

Notes: The data are from the Census of Retail Trade. Numbers are the national and local (commuting zone) Herfindahl-Hirschman Index (HHI) for various industries weighted by market size. Concentration is calculated using 6-digit NAICS codes and aggregated to the 3-digit NAICS using each industry’s share of sales. (CBDRB-FY20-P1975-R8604)
B.7 The Relationship Between National and Local HHIs

This subsection presents the levels of the national HHI in the counterfactual economy described in Section 4. Table B.4 presents the underlying numbers for Figure 6.

Table B.4: Multi-Market Firms Counterfactual

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Actual HHI</td>
<td>0.0128</td>
<td>0.0184</td>
<td>0.0311</td>
<td>0.0418</td>
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<tr>
<td>Counterfactual HHI 1992</td>
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<td>0.0158</td>
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<tr>
<td>Counterfactual HHI 1997</td>
<td>—</td>
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<td>0.0239</td>
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<td>Counterfactual HHI 2002</td>
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<td>0.0311</td>
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<td>0.0340</td>
</tr>
<tr>
<td>Counterfactual HHI 2007</td>
<td>—</td>
<td>—</td>
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<td>0.0418</td>
<td>0.0404</td>
</tr>
</tbody>
</table>

Notes: The numbers are based on calculations from the Census of Retail Trade. The table reports the level of the national HHI and its counterfactual levels preserving the market structure of each Census year. The counterfactual values are computed by preserving the rank of firms according to the initial market structure, assigning to the firm with the $r^{th}$ largest shares in the initial year the market share of the $r^{th}$ firm in each subsequent year. (CBDRB-FY23-P1975-R10585)
C Comparison to Rossi-Hansberg, Sarte, and Trachter (2020)

This section compares our results to those in Rossi-Hansberg, Sarte, and Trachter (2021) (hereafter RST) for the retail sector and explains the factors contributing to the differences between our papers. Unlike us, they find a reduction in the local HHI for the retail sector between 1990 and 2014. RST present results for many sectors of the economy. In what follows we discuss only their results in the retail sector. However, our discussion of aggregation methods is relevant for all sectors.

There are three key differences between our paper and RST’s that each partially explains the opposite results regarding local concentration. First, we use different data sources: while RST use the National Establishment Time Series (NETS), this paper uses confidential data from the CRT and the LBD. Second, we have different definitions of markets: this paper defines markets by product based on NAICS-6 classification of establishments, while RST define markets by industry based on SIC-8 or SIC-4 classification of establishments. Third, we differ in the methodology used to aggregate markets. This paper aggregates market-level concentration using contemporaneous weights, and we report the change in this (aggregate) index of local concentration. In contrast, RST aggregate the change in market-level concentration using end-of-period weights and report this (aggregate) change.

We argue that the CRT is likely to provide better data for the study of concentration in local markets, and we show that changing from NETS to CRT data alone explains a third of the discrepancy in the change of local concentration (while controlling for market definition and aggregation methodology). Another third of the difference in estimates is explained by the definition of product markets (by changing detailed SIC-8 industries to more aggregated SIC-4 industries). The proper definition of a product market (SIC-8, SIC-4, NAICS-6, product category) can depend on the question being asked. We argue in Section 2.3 that product categories are the proper way to study retail markets. The final third of the difference in estimates is explained by the aggregation methodology. We argue that the method used by RST is biased toward finding decreasing local concentration, and we show that their method could find evidence of decreasing concentration in a time series, even when concentration is not changing in the cross-section. This occurs when markets become less concentrated as they grow. Below we expand upon these differences and their implications for the measurement of local concentration.

Data sources The baseline results in RST are based on the NETS, a data product from Walls and Associates that contains information on industry, employment, and sales by
establishments. These data have been shown to match county-level employment counts relatively closely (Barnatchez, Crane, and Decker, 2017), but the data do not match the dynamics of businesses Crane and Decker (2020). The results in this paper are based on the CRT, a data set assembled and maintained by the U.S. Census Bureau covering all employer retail establishments.

Both the NETS data and the CRT use the establishment’s reported industry and sales when available and both have some degree of imputation for establishments that do not report. However, the CRT can often impute using administrative records from the IRS.\(^5\) Beyond this, the two data sets differ in other two relevant aspects. First, the CRT contains sales by product category for the majority of sales, while the NETS contains only industry, allowing us to define markets by product categories and account for cross-industry competition by general merchandisers (see Section 2.3). Second, the NETS includes non-employer establishments, while the CRT does not. According to official estimates, non-employer establishments account for about 2 percent of retail sales in 2012 (Economy-Wide Key Statistics: 2012 Economic Census of the United States).\(^6\) On the whole, the CRT provides a more accurate picture of activity in the retail sector.

**Definition of product markets** We adopt a different definition than RST for what constitutes a product market. Each definition of product market has its own pros and cons, and researchers may choose one over the other depending on the specific context. We define markets by a combination of a geographical location and a product category that we construct using the detailed data on sales provided by the CRT, along with the (NAICS-6) industry classification of establishments (see Section 2.3). As we mentioned above, doing this treats multi-product retailers as separate firms, ignoring economies of scope, in favor of putting all sales in a product category in the same market. However, we also present results defining markets by industry and find that the same patterns of higher national and local concentration arise, but with stronger magnitudes. See Section 3.4.

In contrast, RST define markets by the establishment’s industry, using both SIC-8 and SIC-4 codes. Some examples of SIC-8 codes are department stores, discount (53119901); eggs and poultry (54999902); and Thai restaurants (58120115).\(^7\) SIC-8 codes may be overly

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5Response to the CRT is required by law. Single-unit establishments are randomly sampled for sales in the CRT, while the non-sampled units have their sales imputed. See [http://dominic-smith.com/data/CRT/crt_sample.html](http://dominic-smith.com/data/CRT/crt_sample.html) for more details.


7NETS allows for 914 retail SIC-8 codes. A full list is available at [https://www.dnb.com/content/dam/english/dnb-solutions/sales-and-marketing/sic_8_digit_codes.xls](https://www.dnb.com/content/dam/english/dnb-solutions/sales-and-marketing/sic_8_digit_codes.xls). RST indicate that many SIC-8 codes are rarely used (data appendix), but without access to the NETS data, we cannot assess the relative significance of each code for economic activity.
detailed for retail product markets, to the point that many retailers will sell multiple types of goods. For example, calculating concentration in eggs and poultry (54999902) would miss the fact that many eggs and poultry are sold by chain grocery stores (54119904) and discount department stores (53119901). This suggests that aggregating to less detailed codes may provide a better definition of product markets. To that end, RST present results for SIC-4 codes. When concentration is calculated using SIC-4 codes, the decrease in local concentration is much smaller, a 8 percentage point decrease instead of a 17 percentage point decrease.\(^8\)

Incidentally, the SIC-4 codes are quite similar to the NAICS-6 codes available in the CRT, except restaurants are included in the SIC definition of retail but not in NAICS.\(^9\) This makes the concentration measures based on each classification more closely comparable. Yet, even in this setting (NETS SIC-4 versus CRT NAICS-6) there are still significant differences between our studies. We will go back to this comparison when we discuss Figure C.2 and Table C.1 below.

**Aggregation methodology** The final difference comes from how we aggregate the market-level changes in concentration into an aggregate index of local concentration. We compute the local HHI index by first computing the HHI for each pair of product category \((j)\) and location \((\ell)\). Then we aggregate across locations, weighting each market (location-product) HHI by the market’s share of the product’s national sales. Doing this provides a measure of the average local HHI for each product. Finally, we aggregate across products, weighting by the product’s share of national retail sales, to obtain an average local HHI. Every step in the aggregation maintains the interpretation of the HHI as a probability, which also makes the levels of the HHI comparable across time. We do this for each period \((t)\) and report the time series for this index. The average local HHI is then given by

\[
HHI_t = \sum_j s_j^t \sum_\ell s^j_\ell \cdot HHI_{j\ell t}, \quad \text{where } HHI_{j\ell t} = \sum_i \left( s^{j\ell t}_i \right)^2.
\]

RST use a different methodology. Instead of computing concentration in the cross-section, they calculate the change in concentration between \(t\) and some initial period and

---

\(^8\)The change from SIC-8 to SIC-4 has little effect on concentration outside of retail (RST Data Appendix). The numbers are read off graphs for the change in retail sector concentration for zip codes between 1990 and 2012.

\(^9\)In the results in the main text, we exclude automotive dealers, gas stations, and non-store retailers because of concerns related to ownership data and defining which markets they serve (see Section 2 for further discussion). This has little impact on the estimates for local concentration.
then aggregate these changes weighting by the period $t$ share of employment of each industry $(j)$ in total retail employment. Their index for the change in concentration is given by\textsuperscript{10}

$$\Delta HHI_t^{RST} = \sum_{j, \ell} s_{j, \ell}^t \Delta HHI_{j, \ell},$$ \hspace{1cm} (C.2)

where $s_{j, \ell}^t$ is the sales share of industry $j$ and location $\ell$ in the country at time $t$\textsuperscript{11} and $\Delta HHI_{j, \ell}$ is the change in the revenue-based HHI in industry $j$ and location $\ell$ between the base period and time $t$.

The key difference between the methodologies is that RST do not account for the size of a market in the initial period. This is shown in equation C.3, which subtracts the two measures of concentration from each other. After canceling terms, the difference between the two measures is

$$\Delta HHI - \Delta HHI^{RST} = \sum_{j, \ell} \left( s_{j, \ell}^t - s_{j, \ell}^0 \right) \cdot HHI_{m, \ell}. \hspace{1cm} (C.3)$$

RST will weight markets that increase in size over time by more in the initial period, while those that decrease will be weighted less relative to our measure. As markets grow, they typically become less concentrated resulting in RST weighting markets with decreasing concentration more than markets with increasing concentration.\textsuperscript{12}

Figure C.1 shows that this methodology can find decreasing concentration in a time series, even when concentration is not changing in the cross-section. Consider three firms (A, B, and C) that operate in two markets and have the same size. In the first period ($t - 1$), firms A and B operate in market 1 and firm C operates in market 2. Consequently, the HHI is 0.5 and 1 for each market, respectively, and the aggregate (cross-sectional) HHI is $2/3$. In period $t$, market 1 shrinks and market 2 grows, with firm B changing markets. This change does not affect the cross-sectional distribution of local (market-specific) concentration, but it does imply an increase in concentration in market 1 and a decrease in market 2. Despite there being no changes in the cross-sectional HHI, RST’s methodology would report a decrease in local concentration ($\Delta HHI = -1/6$), driven by the decrease in market 2’s HHI (which happens to be the largest market in period $t$).

\textsuperscript{10}Equation C.2 is taken from RST, with notation adjusted to match the notation in this paper.

\textsuperscript{11}RST weight markets by their employment share \( e_{j, \ell}^t \) instead of their sales share \( s_{j, \ell}^t \). However, their data appendix shows this has no effect on the results.

\textsuperscript{12}A similar point is made in Appendix E of Ganapati (2021) using LBD data.
Figure C.1: Example of RST Methodology

<table>
<thead>
<tr>
<th>Period t-1</th>
<th>Period t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market 1 - HHI=1/2</td>
<td>Market 1 - HHI=1.0</td>
</tr>
<tr>
<td>Firm A</td>
<td>Firm A</td>
</tr>
<tr>
<td>![Diagram for Market 1]</td>
<td>![Diagram for Market 1]</td>
</tr>
<tr>
<td>(\Delta HHI = 1/2)</td>
<td>(\Delta HHI = 1/2)</td>
</tr>
<tr>
<td>Firm B</td>
<td>Firm B</td>
</tr>
<tr>
<td>![Diagram for Market 2]</td>
<td>![Diagram for Market 2]</td>
</tr>
<tr>
<td>Market 2 - HHI=1.0</td>
<td>Market 2 - HHI=1/2</td>
</tr>
<tr>
<td>Firm C</td>
<td>Firm C</td>
</tr>
<tr>
<td>(\Delta HHI = -1/2)</td>
<td>(\Delta HHI = -1/2)</td>
</tr>
<tr>
<td>![Diagram for Cross-Section]</td>
<td>![Diagram for Cross-Section]</td>
</tr>
<tr>
<td>Cross-Section HHI=2/3</td>
<td>Cross-Section HHI=2/3</td>
</tr>
<tr>
<td>RST Weighted (\Delta HHI=-1/6)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The figure shows how market and cross-sectional concentration indices are computed under our methodology (difference in cross-section Herfindahl-Hirschman Index (HHI)) and that of Rossi-Hansberg et al. (2021). The economy has two markets and three firms. Firms are of the same size. Markets change size from period \(t-1\) to period \(t\), but the cross-sectional distribution of markets and concentration does not change. The weighting methodology used by Rossi-Hansberg et al. (2021) puts more weight on market 2, which increases size between \(t-1\) and \(t\) and has a reduction in concentration. The result is a decrease in aggregate concentration when changes are measured according to this methodology, while cross-section HHI does not change.

Quantifying differences Figure C.2 quantifies the role of each of the differences highlighted above for the change in local concentration between 1992 and 2012.\(^{13}\) To make the comparison clear, we define markets by industry throughout the exercise.\(^{14}\)

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\(^{13}\)RST use 1990 as the base year instead of 1992. This is unlikely to matter as RST find small changes in concentration between 1990 and 1992.

\(^{14}\)To be precise, we define a market either by an SIC-8, an SIC-4, or a NAICS-6 industry in a given location. Our preferred definition of markets by product categories implies a change in the level of the HHI that makes the comparison with the results in RST less transparent.
Figure C.2: RST Comparison

The figure shows various estimates for the change in local HHI between 1992 and 2012. The estimates vary according to the data source, industry definition, and aggregation methodology. The lowest estimate corresponds to Rossi-Hansberg et al. (2021)’s estimate using SIC-8 industries, and the second lowest estimate corresponds to using SIC-4 industries. The second highest estimate corresponds to using Census of Retail Trade microdata and NAICS-6 industries (which are similar to SIC-4 industries), and the highest estimate computes indices under our aggregation methodology instead of that of Rossi-Hansberg et al. (2021). (CBDRB-FY20-P1975-R8604)

Overall, Figure C.2 shows that the difference in the estimated change of local HHI is explained in roughly equal parts by the three differences highlighted above: data source (CRT versus NETS), industry definition (NAICS-6 versus SIC-8), and aggregation methodology. We discuss each step in more detail below.

The lowest estimate for the change in local concentration (a decrease of 0.17 points in local HHI) corresponds to RST’s baseline estimate using NETS data and SIC-8 for industry classification. Once industries are aggregated to the SIC-4 level (to improve comparability across establishments), the estimate increases by 9 percentage points, still implying a reduction of 8 percentage points in the local HHI. The next estimate reproduces RST’s methodology using microdata from the CRT. Changing from NETS to CRT data implies a further increase in the estimate of 6.5 percentage points, with the
overall change suggesting a minor decrease of local HHI of 1.5 percentage points.\textsuperscript{15} Next we change the weighting methodology to ours (as explained above). Doing so increases the estimated change of local concentration again (by 9.5 percentage points), implying an overall increase of local HHI of 8 percentage points.\textsuperscript{16}

Table C.1 provides a more detailed account of the estimates presented in Figure C.2 and also includes estimates of changes in local concentration for intermediate census years (1997, 2002, and 2007). In the first panel, national concentration, we compare the numbers in RST (Figure 1b) to numbers calculated for NAICS-based measures (including all 6-digit industries in NAICS) and product-based measures. In all three cases, national concentration is increasing significantly. Despite differences in the initial levels of concentration (column 1), the national HHI increases by two to three times.\textsuperscript{17}

The second panel of Table C.1 compares concentration measured at the zip code level using RST’s weighting methodology as described above. We also provide results for the set of establishments that are included in the product-based results in the paper. Using their methodology, we find evidence for slight decreases in local concentration of 1 to 2 percentage points whether markets are aggregated using sales or employment weights. These decreases are much less severe than the 17 percentage point decrease in RST.

The final panel of Table C.1 compares concentration measured at the zip code level using our aggregation method. This method finds significant increases in local concentration across both NAICS samples. Local HHI increased between 7.1 and 8.5 percentage points; that is, the average dollar in 2012 is spent in a more concentrated market than the average dollar in 1992.

\textsuperscript{15}Part of this difference could be explained in theory by the inclusion of restaurants in SIC-4; however, the industry by industry results in RST’s Figure 7 suggest that this is not the case because they find diverging trends in most retail industries.

\textsuperscript{16}These numbers use all retail firms, including those that were dropped for the main sample in the paper. Concentration numbers are calculated for zip codes and aggregated according to each zip code’s share of employment.

\textsuperscript{17}The level of concentration is not provided in RST.
Table C.1: Comparison of Concentration to RST

### National Concentration

<table>
<thead>
<tr>
<th>Weight</th>
<th>Level</th>
<th>Change from 1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>RST</td>
<td>Emp.</td>
<td>N/A</td>
</tr>
<tr>
<td>NAICS Based</td>
<td>Sales</td>
<td>0.029</td>
</tr>
<tr>
<td>Product Based</td>
<td>Sales</td>
<td>0.013</td>
</tr>
</tbody>
</table>

### Zip Code Concentration: End-of-Period Weights

<table>
<thead>
<tr>
<th>Weight</th>
<th>Level</th>
<th>Change from 1992</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RST</td>
<td>Emp.</td>
<td>-0.070</td>
</tr>
<tr>
<td>NAICS Based</td>
<td>Emp.</td>
<td>-0.022</td>
</tr>
<tr>
<td>Sales</td>
<td>N/A</td>
<td>-0.023</td>
</tr>
<tr>
<td>Paper Sample</td>
<td>Emp.</td>
<td>-0.020</td>
</tr>
<tr>
<td>Sales</td>
<td></td>
<td>-0.024</td>
</tr>
<tr>
<td>Product Based</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

### Zip Code Concentration: Current Period Weights

<table>
<thead>
<tr>
<th>Weight</th>
<th>Level</th>
<th>Change from 1992</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RST</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>NAICS Based</td>
<td>Emp.</td>
<td>0.507</td>
</tr>
<tr>
<td>Sales</td>
<td>0.498</td>
<td>0.018</td>
</tr>
<tr>
<td>Paper Sample</td>
<td>Emp.</td>
<td>0.524</td>
</tr>
<tr>
<td>Sales</td>
<td>0.530</td>
<td>0.022</td>
</tr>
<tr>
<td>Product Based</td>
<td>Sales</td>
<td>0.2637</td>
</tr>
</tbody>
</table>

**Notes:** The numbers come from the Census of Retail Trade and Rossi-Hansberg et al. (2021) (RST). Numbers from RST are taken from retail series in Figure 2. The level column contains the 1992 level of concentration. The formula for changes in concentration using end-of-period weights does not depend on the initial 1992 level as shown in RST, and consequently the level column does not apply to these calculations. NAICS-based measures concentration calculated including all NAICS industries. Paper sample uses only establishments included in the sample for the product-based results. Retail in RST is defined using SIC codes that include restaurants. (CBDRB-FY19-P1179-R7207, CBDRB-FY20-P1975-R8604)
D Concentration and markups

In this appendix, we reproduce standard results relating the HHI with firms’ profitability when firms compete à la Cournot. We then extend those results to competition with differentiated goods and discuss aggregation across markets. Finally, we present implied changes in retail markups using the relationship between them and the HHI.

Cournot competition with a homogeneous good. Consider a market with \( N \) producers of a homogeneous good competing à la Cournot. Firms are heterogeneous in their marginal cost, \( c_i \). The (inverse) demand for the good is captured by a price function \( P(Q) \), where \( Q \equiv \sum_i q_i \) is the total amount of the good being produced. The problem for each producer is

\[
\max_{q_i} P(Q)q_i - c_i q_i \tag{D.1}
\]

The producer’s optimal choice is characterized by a markup over marginal cost rule,

\[
P(Q) = \left[1 - \frac{s_i}{\varepsilon}\right]^{-1} c_i, \tag{D.2}
\]

where \( \varepsilon^{-1} \equiv -Q/\partial P/\partial Q \) is the elasticity of demand and \( s_i \equiv Pq_i/PQ \) the producer’s market share.

The producer’s rule implies a relationship between aggregate markups (or gross margins) and the HHI. To see this, consider the market’s aggregate profits:

\[
\Pi \equiv \sum_{i=1}^{N} (P(Q)q_i - c_i q_i) = PQ \sum_{i=1}^{N} \left( s_i + \left[1 - \frac{s_i}{\varepsilon}\right] s_i \right) = PQ \frac{\text{HHI}}{\varepsilon} \tag{D.3}
\]

Then we can get the expression linking markups to the HHI. The markups or gross-margins are defined as the ratio of total (variable) cost to revenue, \( \mu \equiv \sum_i c_i q_i / PQ \).

\[
PQ - \sum_{i=1}^{N} c_i q_i = PQ \frac{\text{HHI}}{\varepsilon} \mu = \left[1 - \frac{\text{HHI}}{\varepsilon}\right]^{-1}. \tag{D.4}
\]

Cournot competition with differentiated goods. Consider a market with \( N \) producers of a differentiated goods competing à la Cournot. Firms are heterogeneous in their marginal cost, \( c_i \). The demand for the goods comes from homothetic preferences.
described by a homogeneous-of-degree-1 function, so that we write the aggregate quantity as $Q = F(q_1, \ldots, q_N)$. The demand for an individual good ($q_i$) is characterized by $p_i / P = F_i (q_i / Q)$, where $p_i$ is good $i$’s price, $P$ is the ideal price index (i.e., $PQ = \sum_i p_i q_i$), and $F_i (\cdot)$ is the $i^{th}$ partial derivative of $F$.

The problem for each producer is

$$\max_{q_i} p_i (q_i, Q, P) q_i - c_i q_i$$

(D.5)

The producer’s optimal choice is characterized by

$$p_i \left[ \left( \frac{q_i}{p_i} \frac{\partial p_i}{\partial q_i} \right) \left( \frac{Q}{q_i} \frac{\partial Q}{\partial q_i} \right) + \left( \frac{P}{p_i} \frac{\partial P}{\partial q_i} \right) \left( \frac{Q}{Q} \frac{\partial Q}{\partial q_i} \right) + 1 \right] = c_i.$$  

(D.6)

In the above expressions we assume that the producer ignores second order effects of their choices on the price through the change in other firm’s prices ($\partial p_i / \partial P$, $\partial p_i / \partial q_j$). This effect is zero when the aggregate quantity is a CES aggregator.

We can express the optimal pricing rule in terms of the elasticity of demand ($\varepsilon_i^{-1} \equiv -q_i / p_i \partial p_i / \partial q_i$) and the producer’s market share ($s_i \equiv p_i q_i / PQ$). To do this we first establish two results from the producer’s demand, $p_i / P = F_i (q_i / Q)$:

$$\frac{Q \partial p_i}{p_i \partial Q} = -\frac{q_i \partial p_i}{p_i \partial q_i} = \frac{1}{\varepsilon_i} \quad \& \quad \frac{P \partial p_i}{p_i \partial P} = 1.$$  

(D.7)

With these results we can simplify the expression to

$$p_i \left[ \frac{\varepsilon_i - 1}{\varepsilon_i} + \left( \frac{1}{\varepsilon_i} + \frac{Q \partial P}{P \partial Q} \right) \left( \frac{q_i \partial Q}{Q \partial q_i} \right) \right] = c_i.$$  

(D.8)

Finally, we assume that the expenditure in the market is fixed so that $Q / P \partial P / \partial Q = 1$ and we derive a final result relating the elasticity of $Q$ with the producer’s market share. We start from the definition of $Q$ and consider the total differential, focusing on $dq_i \neq 0$,

$$1 = F \left( \frac{q_1}{Q}, \ldots, \frac{q_N}{Q} \right)$$

$$0 = F_i \left( \frac{q_i}{Q} \right) \frac{dq_i}{Q} - \sum_{j=1}^{N} F_j \left( \frac{q_j}{Q} \right) \frac{q_j}{Q^2} dQ$$

$$0 = F_i \left( \frac{q_i}{Q} \right) dq_i - \left( \sum_{j=1}^{N} F_j \left( \frac{q_j}{Q} \right) \frac{q_j}{Q} \right) dQ$$

$$\frac{dQ}{dq_i} = \frac{p_i}{P},$$  

(D.9)

where the sum in the second to last line is equal to 1 because of Euler’s theorem and we replace for the relative price form the producer’s demand. With this result in hand, it
follows that \( q_i/Q\partial Q/\partial q_i = s_i \). Replacing in the producer’s optimal pricing rule we obtain

\[
p_i = \frac{\xi_i}{\xi_i - 1} [1 - s_i]^{-1} c_i.
\]  

(D.10)

This is the same result as in Atkeson and Burstein (2008) and Grassi (2017) under CES preferences, when all firms face the same (constant) elasticity of demand (\( \xi_i = \xi \)).

We can aggregate to a relationship between profitability and the HHI at the market level as in the homogeneous good case when the elasticity is common across firms.

\[
\Pi \equiv \sum_{i=1}^{N} (p_i q_i - c_i q_i) = PQ \sum_{i=1}^{N} \left( s_i - \frac{\xi - 1}{\xi} [1 - s_i] s_i \right)
\]

\[
= PQ \left[ \frac{1}{\xi} + \frac{\xi - 1}{\xi} \text{HHI} \right].
\]  

(D.11)

Then we can get the expression linking markups to the HHI. The markups or gross-margins are defined as the ratio of total (variable) cost to revenue, \( \mu = \frac{c_i q_i}{p_i q_i}. \)

\[
PQ - \sum_{i=1}^{N} c_i q_i = PQ \left[ \frac{1}{\xi} + \frac{\xi - 1}{\xi} \text{HHI} \right]
\]

\[
\mu = \frac{\xi}{\xi - 1} [1 - \text{HHI}]^{-1}.
\]  

(D.12)

**Aggregation across markets.** We also compute the average markup across markets (\( \ell \)) for a given product of industry (j). We can compare this measure to gross margins by product or industry obtained from the ARTS. We define the average markup as the ratio between product j’s total sales and total labor costs of the product across markets (\( \ell = 1, \ldots, L \)),

\[
\mu_j \equiv \frac{\sum_{\ell=1}^{L} P^\ell_j Q^\ell_j}{\sum_{\ell=1}^{L} C^\ell_j Q^\ell_j} = \frac{\sum_{\ell=1}^{L} P^\ell_j Q^\ell_j}{\sum_{\ell=1}^{L} 1/\mu^\ell_j} \left[ \sum_{\ell=1}^{L} (\mu^\ell_j)^{-1} \theta^\ell_j \right]^{-1},
\]  

(D.13)

where \( \mu^\ell_j = P^\ell_j/C^\ell_j \) is the average markup (gross margin) of product/industry j in market \( \ell \), with \( C^\ell_j = (\sum_{\ell'=1}^{L} c_{i'\ell'} q_{i'\ell'}/q_j \) the average cost in market \( \ell \), and \( \theta^\ell_j = p_j q_j / \sum_{\ell=1}^{L} p_j q_j \) is the share of product/industry j sales accounted for by market \( \ell \).

Using the result in (D.12) we express the markup in terms of market concentration.

\[
\mu_j = \left[ \sum_{\ell=1}^{L} \left( \frac{\theta^\ell_j}{\theta^\ell_j - 1} \right)^{-1} [1 - \text{HHI}_j^\ell] \theta^\ell_j \right]^{-1}
\]

(D.14)

If the elasticity of demand of good \( j \) is common across markets the expression simplifies to:

\[
\mu_j = \frac{\epsilon_j}{\epsilon_j - 1} [1 - \text{HHI}_j]^{-1},
\]  

(D.15)

where \( \text{HHI}_j = \sum_{\ell=1}^{L} \text{HHI}_j^\ell \theta^\ell_j \) is the sales weighted HHI of product \( j \) across markets.
E Model of Product Competition in Retail

In this Appendix, we present a model of the retail sector where retailers compete in local product markets. Consumers have traditionally chosen between nearby stores selling a given product when purchasing goods. Accordingly, we focus on a setup where competition is exclusively local. The model provides us with an explicit link between the local HHI and average retailers’ margins for each product which in the model are captured as markups.

In Appendix D, we showed that Cournot competition implies a relationship between markups and local HHIs. Now, we build upon this relationship and show how that equation fits into a fully specified model of oligopolistic competition in local product markets. We then estimate the key parameters of the model, namely elasticities of substitution for each product, to match the 1993 level of retailers’ gross margins using data from the ARTS.

We use the model to find the implied change in product margins (markups) coming from changes in local concentration. We find that increases in local concentration imply a 2.1 percentage point increase in markups between 1992 and 2012, roughly a third of the observed increase in gross margins in the ARTS during that period. The results are similar in magnitude to the ones we obtained in the case of competition in homogeneous goods described in Appendix D, see Table 5.

The model follows Grassi (2017) and Atkeson and Burstein (2008). The model economy has \( L \) locations, in each of them there are \( J \) products being transacted in local product markets. Each market has \( N_{j\ell} \) retail firms that compete with one another in product \( j \). Competition takes place at the location-product level. A perfectly competitive sector aggregates goods across firms for each product and location, aggregates products by location into location-specific retail goods, and aggregates each location’s retail output into a final consumption good. A single representative consumer demands the final consumption good and supplies labor in each location.

E.1 Technology

A retailer \( i \) selling product \( j \) in location \( \ell \) produces using a constant-returns-to-scale technology that combines labor \( (n) \) and potentially other inputs \( \{x_k\}_{k=1}^K \):

\[
y_{j\ell}^i = z_{j\ell}^{i} F \left( x_1, \ldots, x_K, n_{j\ell}^i \right),
\]

where \( z_{j\ell}^{i} \) represents the productivity of the retailer and \( F \) is homogeneous of degree 1.

The homogeneity of \( F \) implies that the retailer has a constant marginal cost of production that we denote \( c_{j\ell}^{i} \). Retailers differ in their marginal costs, reflecting productivity differences between small and large (multi-market) retailers, or differential
pricing from suppliers. Because of differences in productivity and in the prices of the inputs they require for production. In this way the model captures the cost advantages associated with large multi-market retail firms which is the most relevant aspect of large firms when thinking about local competition. Retailers maximize profits independently in each market:

$$\pi^{j\ell}_i = p^{j\ell}_i y^{j\ell}_i - c^{j\ell}_i y^{j\ell}_i,$$  \hspace{1cm} (E.2)

The demand faced by the individual retailer comes from the aggregation sector that serves the consumer. Aggregation takes place in three levels. First, a local aggregator firm that combines the output of the $N^{j\ell}$ retail firms selling product $j$ in location $\ell$. The firm operates competitively using the following technology:

$$y^{j\ell}_j = \left( \sum_{i=1}^{N^{j\ell}} (y^{j\ell}_i)^{\epsilon_j^{j\ell}-1} \right)^{\epsilon_j^{j\ell}-1}; \quad \epsilon_j > 1.$$  \hspace{1cm} (E.3)

Then, the combined product bundles, $y^{j\ell}_j$, are themselves aggregated into local retail output, $y^{j\ell}_\ell$, through the following technology:

$$y^{j\ell}_\ell = \prod_{j=1}^{J} (y^{j\ell}_j)^{\gamma^{j\ell}_j}; \quad \sum_{j=1}^{J} \gamma^{j\ell}_j = 1,$$  \hspace{1cm} (E.4)

where $\gamma^{j\ell}_j$ is the share of product $j$ in retail sales in location $\ell$.

Finally, the national retail output is created by combining local output, $y^{j\ell}_\ell$, from the $L$ locations in the country:

$$y = \prod_{\ell=1}^{L} (y^{j\ell}_\ell)^{\beta^{j\ell}_\ell}; \quad \sum_{\ell=1}^{L} \beta^{j\ell}_\ell = 1,$$  \hspace{1cm} (E.5)

where $\beta^{j\ell}_\ell$ corresponds to the share of location $\ell$ in national retail sales.

The aggregation process implies the following demand and prices:

$$y^{j\ell}_\ell = \beta^{j\ell}_\ell \frac{P^{j\ell}_\ell}{p^{j\ell}_\ell} \cdot y$$

$$P = \prod_{\ell=1}^{L} \left( \frac{p^{j\ell}_\ell}{\beta^{j\ell}_\ell} \right)^{\beta^{j\ell}_\ell}$$  \hspace{1cm} (E.6)

$$y^{j\ell}_j = \gamma^{j\ell}_j \frac{p^{j\ell}_\ell}{p^{j\ell}_j} y^{j\ell}_\ell$$

$$p^{j\ell}_\ell = \prod_{j=1}^{J} \left( \frac{p^{j\ell}_j}{\gamma^{j\ell}_j} \right)^{\gamma^{j\ell}_j}$$  \hspace{1cm} (E.7)

$$y^{j\ell}_i = \left( \frac{p^{j\ell}_i}{p^{j\ell}_j} \right)^{-\epsilon_j^{j\ell}} y^{j\ell}_j$$

$$p^{j\ell}_j = \left( \sum_{i=1}^{N} \left( \frac{p^{j\ell}_i}{p^{j\ell}_i} \right)^{1-\epsilon_j^{j\ell}} \right)^{\frac{1}{1-\epsilon_j^{j\ell}}}$$  \hspace{1cm} (E.8)
E.2 Pricing to market and average markups

Firms compete directly in the sales of each product in a given location. Firms compete à la Cournot, choosing the quantity \( y_{ij} \) in a non-cooperative fashion, taking as given the choices of other firms. Firms are aware of the effect of their choices \( (p_i^j, y_i^j) \) on the price and quantity of the product in the market they operate in \( (p_j^i, y_j^i) \). The choice of quantity implies a pricing policy for the firms according to their residual demand.

The solution to the pricing problem is the same as in Appendix D for the differentiated goods case. The optimal price satisfies equation (D.10) and depends on the firm’s share of sales in their local product market, \( s_j^i \) and the elasticity of demand they face, \( \epsilon_j \).

\[
\mu_j^i = \frac{\epsilon_j}{\epsilon_j - 1} \left[ 1 - s_j^i \right]^{-1}
\]

(E.9)

In the model, retailers with lower costs can charge lower prices and increase their market share, making it so that high-markup retailers are low-cost.

Average markups in a market satisfy equation (D.12) and average product markups (across markets) satisfy equation (D.15) from Appendix D.

\[
\mu_j = \frac{\epsilon_j}{\epsilon_j - 1} \left[ 1 - \sum_{\ell=1}^{L} s_j^\ell \text{HHI}_j^\ell \right]^{-1},
\]

(E.10)

where \( s_j^\ell \) is the share of market \( \ell \) in product’s \( j \) national sales. As markets become more concentrated, average markups increase. The sensitivity of markups to increases in concentration is larger for products with a lower elasticity of demand. Retail markups (averaging across products) are

\[
\mu = \left[ \sum_{j=1}^{J} (\mu_j)^{-1} s_j \right]^{-1},
\]

(E.11)

where \( s_j \) is the share of product \( j \) in national retail sales.

E.3 Estimation and Data

The two key ingredients for analyzing markups are firms’ market shares by product in each location, \( s_j^i \), and the elasticity of substitution for each product, \( \epsilon_j \). We obtain the shares directly from the CRT and estimate the elasticities using equation (E.10). Specifically, we use the product HHIs calculated in Section 3.2 and gross margins from the Annual Retail Trade Survey (ARTS) which are available beginning in 1993.

The ARTS provides the best source to compare our results to because it computes markups using cost of goods sold, which are the most direct data analogue to markups.
in the model. The ARTS samples firms with activity in retail, collecting data on sales and costs for each firm. The firm-level markups collected by ARTS represent an average markup across the products that the firm sells. The information available is similar to information in Compustat, but the ARTS includes activity of non-public firms that account for a significant share of retail sales.

The ARTS also provides us with margins for detailed industries, which we convert to product margins using the CRT.\footnote{Appendix E.5 presents additional results based on industry-level markups as well as robustness exercises with alternative measures of product markups.} We do this in three steps. First, we compute margins for the industries, $k(j) \in K$ most closely related to each of the eight major product categories, $j$. We also compute margins for general merchandisers (NAICS 452).\footnote{For instance, we relate clothing to NAICS 448 and groceries to NAICS 445; see Appendix B.6 for a complete list.} As before, the sales of each specialized industry are all assigned to its own product category, while the sales of general merchandisers are divided across products using the share of each product category’s sales that come from general merchandisers in the CRT. Second, we estimate a scaling factor $\lambda = 0.82$ that measures how large general merchandise margins are relative to what would be implied by other industries’ margins:

$$\mu_{GM}^{ARTS} = \lambda \sum_{j} \omega_{j}^{GM} \mu_{k(j)}^{ARTS},$$

(E.12)

where $\mu_{k(j)}^{ARTS}$ is the measured margins of industry $k(j)$ in the ARTS and $\omega_{j}^{GM}$ is the share of sales of product $j$ in general merchandising from the CRT. We use the scaling factor $\lambda$ to construct product-specific margins for general merchandisers while being consistent with the measured margins from the ARTS. The margin of general merchandisers in product $j$ is then $\mu_{GM}^{j} = \lambda \mu_{k(j)}^{ARTS}$. Finally, we compute product-level margins in a model-consistent way as

$$\mu_{j} = \left( \frac{1 - \omega_{j}^{GM}}{\mu_{GM}^{ARTS}} + \frac{\omega_{j}^{GM}}{\mu_{GM}^{j}} \right)^{-1},$$

(E.13)

where $\omega_{GM}^{j}$ is the share of general merchandisers in product $j$’s sales. In this way, product-level margins incorporate the effect of competition from general merchandisers.

E.4 Changes in Concentration and Markups

We conduct two exercises with the model. First, we fit the model to match product markups in 1993 given the observed levels of local concentration in 1992, which provides us with estimates of the elasticities of substitution for each product category. Holding these
Figure E.1: Local Concentration and Markups

Notes: Diamonds mark the change in product markups between 1992 and 2012 from the Annual Retail Trade Survey and Census of Retail Trade data and a weighted average across products for the retail sector. Circles mark the change in markups implied by the change in local concentration given the model estimates for 1992.

estimates fixed, we can extend the model through 2012 and obtain the change in markups implied by the observed increase in local concentration. This exercise explains one-third of the increase in markups observed in the ARTS. Second, we can fit the model to match observed markups for each economic census year by allowing the elasticities of substitution to be time varying. To match the increase in markups, the model implies a decrease in the elasticity of substitution for most products.

The increase in local concentration implies an increase in retail markups of 2.1 percentage points between 1992 and 2012, about one-third of the 6 percentage point increase in product-level markups. Figure E.1 shows that in all but two product categories, the observed increase in markups is higher than what is implied by the rise in product-level HHI. These results are robust to alternative measures of product markups (see Appendix E.5). The changes in model markups in Figure E.1 assume that the elasticity of demand faced by firms are constant over time and vary only because of changes in local HHI. However, many changes in the competitive environment of retail can
Table E.1: Estimated Elasticities of Substitution

<table>
<thead>
<tr>
<th>Product Category</th>
<th>$\epsilon_j$</th>
<th>1992</th>
<th>2002</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Furniture</td>
<td></td>
<td>2.70</td>
<td>2.43</td>
<td>2.43</td>
</tr>
<tr>
<td>Clothing</td>
<td></td>
<td>3.07</td>
<td>2.83</td>
<td>2.48</td>
</tr>
<tr>
<td>Sporting Goods</td>
<td></td>
<td>3.73</td>
<td>3.77</td>
<td>3.20</td>
</tr>
<tr>
<td>Electronics &amp; Appliances</td>
<td></td>
<td>4.48</td>
<td>5.74</td>
<td>4.95</td>
</tr>
<tr>
<td>Health Goods</td>
<td></td>
<td>4.38</td>
<td>5.30</td>
<td>5.09</td>
</tr>
<tr>
<td>Toys</td>
<td></td>
<td>5.55</td>
<td>5.91</td>
<td>4.91</td>
</tr>
<tr>
<td>Home Goods</td>
<td></td>
<td>4.85</td>
<td>4.13</td>
<td>3.92</td>
</tr>
<tr>
<td>Groceries</td>
<td></td>
<td>5.82</td>
<td>5.39</td>
<td>6.40</td>
</tr>
</tbody>
</table>

Notes: The data are authors’ estimates of product elasticities of substitution using industry markups from the Annual Retail Trade Survey and product-level local Herfindahl-Hirschman Indexes calculated from the Census of Retail Trade. The elasticities are the solution to equation (E.10).

be reflected in changes in these elasticities rather than changes in market concentration.

Table E.1 shows the value of the elasticity of substitution needed to match the level of markups in each year. We find the lowest elasticities of substitution in Clothing and Furniture. These are categories that feature many different brands only available from a small set of retail firms, leaving more room for differentiation than in products such as Toys and Groceries, where different firms carry similar or even identical physical products.

To match the observed increase in markups, most product categories require a decrease in their elasticity of substitution. The magnitude of the decrease depends on the initial level of the elasticities as markups respond more to changes for lower elasticities. The decreasing trend for the elasticities of substitution is consistent with the findings of Bornstein (2018), Brand (2020), and Neiman and Vavra (2020), who link the decrease to the rise of store and brand loyalty/inertia. The exception to the trend of decreasing elasticities of substitution are Electronics & Appliances and Health Goods, which instead require an increase in their elasticities. Health Goods had almost no change in markups in the data, but based on the change in concentration, markups should have increased by about 5 percentage points.

E.5 Additional Markup Results

We perform the same exercises as above using various assumptions regarding the behavior of markups. Our first set of results changes only the measure of markups we use, keeping
the change in local concentration constant as measured by product-level local (commuting-zone) HHI. We find that the changes implied by the change in local product-level concentration on markups are robust to changes in the level of markups. The second set of results uses changes in industry-level concentration instead of product-level concentration. We find that these changes imply larger increases in markups for the retail sector as a whole. This follows from the fact that industry-level measures of concentration ignore the competition between general merchandisers and other retailers.

**Product-based results** We consider four alternative measures of markups. Our baseline measure constructs product-level markups combining information from the ARTS and the CRT. Our second measure assigns to each product category the markup of its main NAICS industry without adjusting for the role of general merchandisers. Our last two measures consider the possibility that markups are much lower or higher than we estimate, respectively decreasing markups by fifty percent or doubling the product-level markups we constructed in our baseline. Table E.2 reports the level of markups in 1992 under each of our alternative measures.

We estimate the implied elasticity of substitution for each product category using equation (E.10) from the model. The implied elasticities are reported in Table E.2. The level of the elasticities varies to match the level of markups under each alternative specification but the general rank stays largely unchanged between the product- and industry-based exercises.

Finally, we use the changes in product-based local HHI computed in Section 3 along with equation (E.10) to compute the change in markups implied by the model and the changes in local concentration. Table E.3 presents the implied change in markups under our four alternative measures. It is clear that the choice of the level of markups does not affect our main result regarding the effect of local concentration on markups.

**Industry-based results** We also consider how our results would change if used industry-level measures of concentration. In Section 3.4 we show that this leads to larger measured changes in local concentration at the industry level. Consequently, using industry-level measures of concentration would have led to a higher implied change in markups.

Table E.4 presents the results of our exercise using industry-level markups from the ARTS and industry-level local concentration from the CRT. The change in industry-level concentration are much larger than those in product-based measures. This is particularly true in the general merchandise subsector (NAICS 452), where the change in local concentration implies a change in markups of 89 percentage points. During this period
Table E.2: Markups Robustness: Estimated Elasticities of Substitution

<table>
<thead>
<tr>
<th>Product</th>
<th>Industry</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_j^{92}$</td>
<td>$\epsilon_j$</td>
<td>$\mu_j^{92}$</td>
</tr>
<tr>
<td>Furniture</td>
<td>1.67</td>
<td>2.7</td>
<td>1.73</td>
</tr>
<tr>
<td>Clothing</td>
<td>1.55</td>
<td>3.1</td>
<td>1.69</td>
</tr>
<tr>
<td>Sporting Goods</td>
<td>1.47</td>
<td>3.7</td>
<td>1.57</td>
</tr>
<tr>
<td>Electronics &amp; Appliances</td>
<td>1.34</td>
<td>4.5</td>
<td>1.44</td>
</tr>
<tr>
<td>Health Goods</td>
<td>1.38</td>
<td>4.4</td>
<td>1.44</td>
</tr>
<tr>
<td>Toys</td>
<td>1.43</td>
<td>5.6</td>
<td>1.57</td>
</tr>
<tr>
<td>Home Goods</td>
<td>1.32</td>
<td>4.9</td>
<td>1.37</td>
</tr>
<tr>
<td>Groceries</td>
<td>1.31</td>
<td>5.8</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Notes: The data are authors’ estimates of product elasticities of substitution using different measures of markups for each product category in 1992. Our baseline measures correspond to product-level markups. In industry $\mu$ we assign to each product category the markup of its main NAICS industry. In low $\mu$ we half the product-level markup. In low $\mu$ we double the product-level markup. Markup information comes from the Annual Retail Trade Survey. The elasticities are the solution to equation (E.10) using the measured product-level local Herfindahl-Hirschman Indexes from the Census of Retail Trade.

Table E.3: Markups Robustness: Implied changes in markups

<table>
<thead>
<tr>
<th>Product</th>
<th>Industry</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_j^{92}$</td>
<td>$\epsilon_j$</td>
<td>$\mu_j^{92}$</td>
</tr>
<tr>
<td>Furniture</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>Clothing</td>
<td>−0.02</td>
<td>−0.02</td>
<td>−0.02</td>
</tr>
<tr>
<td>Sporting Goods</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Electronics &amp; Appliances</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Health Goods</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Toys</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Home Goods</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Groceries</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Retail Sector</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Notes: The data are authors’ estimates of the changes in markups implied by the change in product-level local concentration. Our baseline measures uses product-level markups. In “Industry $\mu$” we assign to each product category the markup of its main NAICS subsector. In “Low $\mu$” we half the product-level markup. In “High $\mu$” we double the product-level markup. Markup information comes from the Annual Retail Trade Survey and product-level local Herfindahl-Hirschman Indexes from the Census of Retail Trade.


Table E.4: Markups Robustness: Industry Estimates

<table>
<thead>
<tr>
<th>Industry</th>
<th>$HHI_{i}^{92}$</th>
<th>$HHI_{i}^{12}$</th>
<th>$\epsilon_{i}$</th>
<th>$\mu_{i}^{92}$</th>
<th>$\Delta \mu_{i}^{\text{ARTS}}$</th>
<th>$\Delta \mu_{i}^{\text{Model}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Furniture (442)</td>
<td>0.08</td>
<td>0.14</td>
<td>2.7</td>
<td>1.73</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>Clothing (448)</td>
<td>0.10</td>
<td>0.11</td>
<td>2.9</td>
<td>1.69</td>
<td>0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>Electronics &amp; Appliances (443)</td>
<td>0.14</td>
<td>0.28</td>
<td>5.2</td>
<td>1.44</td>
<td>0.00</td>
<td>0.27</td>
</tr>
<tr>
<td>Health Goods (446)</td>
<td>0.15</td>
<td>0.25</td>
<td>5.3</td>
<td>1.44</td>
<td>0.02</td>
<td>0.20</td>
</tr>
<tr>
<td>Toys and Sporting Goods (451)</td>
<td>0.20</td>
<td>0.24</td>
<td>4.8</td>
<td>1.57</td>
<td>0.15</td>
<td>0.09</td>
</tr>
<tr>
<td>Home Goods (444)</td>
<td>0.22</td>
<td>0.30</td>
<td>15.4</td>
<td>1.37</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>Groceries (445)</td>
<td>0.16</td>
<td>0.22</td>
<td>9.1</td>
<td>1.33</td>
<td>0.05</td>
<td>0.11</td>
</tr>
<tr>
<td>General Merchandisers (452)</td>
<td>0.28</td>
<td>0.56</td>
<td>3606</td>
<td>1.39</td>
<td>-0.03</td>
<td>0.89</td>
</tr>
<tr>
<td>Retail Sector</td>
<td>1.42</td>
<td>0.05</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The data are industry-level local Herfindahl-Hirschman Indexes from the Census of Retail Trade and markups from the Annual Retail Trade Survey. The elasticities are the solution to equation (E.10). Decimal places are not shown on the $\epsilon_{i}$ for General Merchandisers due to its magnitude. High levels of $\epsilon$ imply essentially the same markups. For instance an $\epsilon_{GM} = 165.7$ implies the same level of markups up to the second decimal as the estimate we report.

General merchandiser markups were almost unchanged in ARTS. These two facts can be reconciled by the fact that general merchandisers face significant competition from retailers outside of their industry. When aggregated, these changes imply an increase in retail markups of 30 percentage points which significantly exceeds the change in margins observed in the ARTS (6.0 percentage points).

E.6 Uniform prices across locations

In this subsection, we show that retail markups are lower on average if multi-market firms engage in uniform pricing. Consider the problem of firm $i$ that sales product $j$ across various locations $\ell \in \mathcal{L}_i$ and sets a uniform price (in a Bertrand fashion). The firm’s problem is:

$$\max_{\mathcal{L}_i} \sum_{\ell \in \mathcal{L}_i} \left[ p_{i}^{\ell} y_{i}^{j\ell} - c_{i}^{j\ell} y_{i}^{j\ell} \right]$$

(E.14)

s.t. $y_{i}^{j\ell} = \left( \frac{p_{i}^{j}}{p_{j}^{\ell}} \right)^{-\epsilon_{j}} y_{j}^{\ell} \quad y_{j}^{\ell} = \gamma_{j}^{\ell} \frac{y_{j}^{\ell}}{p_{j}^{\ell}} \quad p_{j}^{\ell} = \left( \sum_{i=1}^{N} \left( p_{i}^{j\ell} \right)^{1-\epsilon_{j}} \right)^{\frac{1}{1-\epsilon_{j}}}$

Replacing the constraints:

$$\max_{\mathcal{L}_i} \sum_{\ell \in \mathcal{L}_i} \left[ \left( p_{i}^{j\ell} \right)^{1-\epsilon_{j}} - c_{i}^{j\ell} \left( p_{i}^{j\ell} \right)^{-\epsilon_{j}} \right] \left( \sum_{i=1}^{N} \left( p_{i}^{j\ell} \right)^{1-\epsilon_{j}} \right)^{-1} \gamma_{j}^{\ell} p_{j}^{\ell} y_{j}^{\ell}$$

(E.15)
The first order condition is:
\[
0 = \sum_{\ell \in \mathcal{L}_i} \left[ (1 - \epsilon_j) p_i^{j\ell} + \epsilon_j c_i^{j\ell} \right] \frac{y_i^{j\ell}}{p_i^{j\ell}} - (1 - \epsilon_j) \left[ p_i^{j\ell} - c_i^{j\ell} \right] s_i^{j\ell} y_i^{j\ell}.
\]
\[
0 = \sum_{\ell \in \mathcal{L}_i} \left[ -(\epsilon_j - 1) \left( 1 - s_i^{j\ell} \right) y_i^{j\ell} p_i^{j\ell} + (\epsilon_j - (\epsilon_j - 1) s_i^{j\ell}) c_i^{j\ell} y_i^{j\ell} \right]
\]
(E.16)
Rearranging:
\[
\left( \frac{\epsilon_j - (\epsilon_j - 1) s_i^{j\ell}}{\sum_{\ell} (\epsilon_j - 1) \left( 1 - s_i^{j\ell} \right) y_i^{j\ell}} \right) y_i^{j\ell} p_i^{j\ell} = \left( \frac{\epsilon_j - (\epsilon_j - 1) s_i^{j\ell}}{\sum_{\ell} (\epsilon_j - 1) \left( 1 - s_i^{j\ell} \right) y_i^{j\ell}} \right) \left( \frac{y_i^{j\ell}}{p_i^{j\ell}} \right) - (\epsilon_j - (\epsilon_j - 1) s_i^{j\ell}) c_i^{j\ell} y_i^{j\ell}
\]
(E.17)
If marginal cost is constant across markets then we define the markup:
\[
p_i^{j\ell} = \mu_i^{j\ell} c_i^{j\ell} \quad \mu_i^{j\ell} = \frac{\sum_{\ell} \left( \epsilon_j - (\epsilon_j - 1) s_i^{j\ell} \right) y_i^{j\ell}}{\sum_{\ell} (\epsilon_j - 1) \left( 1 - s_i^{j\ell} \right) y_i^{j\ell}}
\]
(E.18)
The firm’s markup reflects its market power across different markets, captured by the firm’s output-weighted average share, \( \hat{s}_i \). Define \( \tilde{y}_i^{j\ell} \equiv \frac{y_i^{j\ell}}{\sum_{\ell} y_i^{j\ell} / L_i} \) and \( \hat{s}_i \equiv \sum_{\ell} s_i^{j\ell} \tilde{y}_i^{j\ell} \), then:
\[
\mu_i^{j\ell} = \frac{\sum_{\ell} \left( \epsilon_j - (\epsilon_j - 1) s_i^{j\ell} \right) \tilde{y}_i^{j\ell}}{\sum_{\ell} (\epsilon_j - 1) \left( 1 - s_i^{j\ell} \right) \tilde{y}_i^{j\ell}} = \frac{\epsilon_j - (\epsilon_j - 1) \hat{s}_i}{(\epsilon_j - 1) \left( 1 - \hat{s}_i \right)}
\]
(E.19)
The firm’s uniform markup is lower than the average markup if the firm chooses prices in each market separately. To see this, define the firm’s average price in product \( j \) such that:
\[
p_i^{j\ell} y_i^{j\ell} = \sum_{\ell} \tilde{p}_i^{j\ell} \tilde{y}_i^{j\ell} \text{, where } \tilde{y}_i^{j\ell} \equiv \sum_{\ell} \tilde{y}_i^{j\ell} \text{.}
\]
It follows that \( \tilde{p}_i^{j\ell} = \sum_{\ell} \tilde{p}_i^{j\ell} \tilde{y}_i^{j\ell} \). The average markup would then be:
\[
\tilde{\mu}_i^{j\ell} \equiv \frac{\tilde{p}_i^{j\ell}}{c_i^{j\ell}} = \sum_{\ell} \frac{\tilde{p}_i^{j\ell}}{c_i^{j\ell}} \tilde{y}_i^{j\ell} = \sum_{\ell} \mu_i^{j\ell} \tilde{y}_i^{j\ell},
\]
(E.20)
which is the output-weighted average of the individual market (Bertrand) markups
\[
\mu_i^{j\ell} = \frac{\epsilon_j - (\epsilon_j - 1) s_i^{j\ell}}{(\epsilon_j - 1) \left( 1 - s_i^{j\ell} \right)}
\]
(E.21)
The average markup \( \tilde{\mu}_i^{j\ell} \) in (E.20) is higher than the uniform markup \( \mu_i^{j\ell} \) in (E.19). The result follows from Jensen’s inequality as the Bertrand markup in (E.21) is convex in the firm’s sales share.