

Online Appendix: “Deposit Requirement in Auctions” by Xiaogang Che, Tong Li, Jingfeng Lu, and Xiaoyong Zheng

1 Appendix S1

This online appendix covers the analyses of a second-price auction with deposit and reserve price (Section 1.1), percentage deposit (Section 1.2), and deposit proportionally deducted from full payment (Section 1.3).

1.1 Second-price auction format

Given the specific auction format (second-price auction) with deposit D and reserve price r , we modify the game in Section 1 as follows: A seller sells an indivisible object to N risk-neutral bidders through a sealed-bid, second-price auction, where $1 < N < \infty$. The seller’s reservation value of the object is normalized to zero. Bidders’ private values, denoted by v_i , $i = 1, 2, \dots, N$, are independent draws from a common atomless distribution $F(\cdot)$ with density $f(\cdot) > 0$ over the support $[0, \bar{v}]$, where $\bar{v} > 0$. After the auction, each bidder i will receive an outside offer, which gives the same object (the same value v_i) but with a random price, denoted by p_i . Prices p_i , $i = 1, 2, \dots, N$, are random draws from a common atomless distribution $\Phi(\cdot)$ with density $\varphi(\cdot) > 0$ over $[0, \bar{v}]$. F and Φ are common knowledge among the seller and bidders, and they are *regular* in the sense that the hazard rates $\frac{f(\cdot)}{1-F(\cdot)}$ and $\frac{\varphi(\cdot)}{1-\Phi(\cdot)}$ are increasing. Figure 1 depicts the timing of the game, which is comprised of four stages.

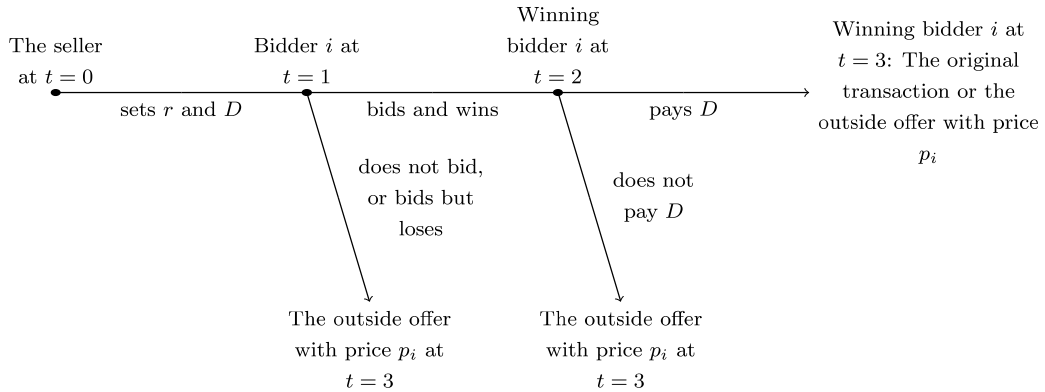


Figure 1: Timing

At stage $t = 0$, the seller announces a reserve price $r \in [0, \bar{v}]$ and a *winner-pay* deposit $D \in [0, +\infty)$. The seller commits to both the reserve price and the deposit requirement.

At $t = 1$, all N bidders observing r and D decide whether to enter and submit bids in the auction. If

a bidder chooses not to enter, the bidder then simply waits for the outside offer with a random price at $t = 3$. For convenience, a nonparticipant's bid is denoted by " $\emptyset$." If bidder i enters the auction, we denote her bid by $b_i \in [r, +\infty)$. All the entering bidders simultaneously submit their bids. The highest-bid bidder wins the auction. If there is a tie, the object goes to each winning bidder with equal probability. The full payment (the auction price) is denoted by κ , which equals r if there is only one valid bid and equals the second-highest bid otherwise. Slightly abusing the notations, we have $\kappa \equiv \max\{r, b_{-i}^{(1)}\}$, where $b_{-i}^{(1)}$ stands for the losing bidders' highest bid. All the losing bidders will face their outside offers at $t = 3$. If there is no valid bid, the auction game ends, and the seller keeps the object.

At $t = 2$, the winning bidder i decides whether to pay D . Let us denote the winner's decision by $e_i \in \{0, 1\}$, with $e_i = 1$ for "paying D " and $e_i = 0$ for "not paying D ." If the winning bidder i does not pay D , i.e., $e_i = 0$, the seller keeps the object, and the winning bidder is to face the outside offer or not make the purchase at $t = 3$. If $e_i = 1$, the winning bidder i has three options at $t = 3$: either completing the current transaction, or adopting the outside offer, or not purchasing at all. Note that we assume if $\kappa < D$ the seller, conditional on D being paid, pays $D - \kappa$ back to the winner when the final transaction goes through. Thus, the total payment to the seller is still the auction price κ .

At $t = 3$, the random price p_i for the outside offer is realized. Conditional on $e_i = 1$, the winning bidder i decides whether to complete the current transaction by paying the remaining payment $\kappa - D$, or to take the outside offer by paying price p_i , or to not make the purchase at all. If the winning bidder i does not complete the transaction, the deposit D is forfeited, and the seller keeps the object. We denote the winner's decision by o_i , where $o_i = 1$ for "taking the outside offer" and $o_i = 0$ for "completing the original transaction." For completeness, we write $o_i = NP$ for "no purchase."

For all other cases (a losing bidder or a winning bidder who does not pay the deposit D), we denote a bidder j 's decision on whether to take the outside offer at $t = 3$ by $\check{o}_j$, where $\check{o}_j = 1$ for "taking the outside offer with price p_j " and $\check{o}_j = NP$ for "no purchase."

The strategies of the seller and a bidder as well as the equilibrium concept are defined as follows: Before the bidding stage starts, the seller sets reserve price r and deposit D at $t = 0$. Given r and D , a bidder makes four decisions: first, how much to bid at the bidding stage at $t = 1$; second, conditional on winning the auction, whether to pay the deposit at $t = 2$; third, conditional on winning the auction and paying the deposit at $t = 2$, whether to take the outside offer with price p_i , or complete the original transaction by paying $\kappa - D$, or not make the purchase at all at $t = 3$; and fourth, conditional on not winning the auction

at $t = 1$ or winning at $t = 1$ but not paying the deposit at $t = 2$, whether to take the outside offer with price p_i at $t = 3$.

The bidding strategy for bidder i at $t = 1$ is a mapping from private value v_i to bid b_i ; that is, $b_i(v_i) : [0, \infty) \rightarrow [r, +\infty) \cup \{\emptyset\}$. Note that b_i is valid if and only if $b_i \geq r$. Conditional on winning at the bidding stage, the strategy of the winning bidder i regarding whether to pay the deposit is a mapping from private value v_i and full payment κ to deposit payment decision e_i at $t = 2$: $e_i(v_i, \kappa) : [0, \bar{v}] \times [r, \infty) \rightarrow \{0, 1\}$.

Contingent on paying the deposit at $t = 2$, the strategy of the winning bidder i regarding whether to complete the original transaction or opt for the outside offer or not make the purchase is a mapping from private value v_i , full payment κ , and price p_i of the outside offer to decision o_i at $t = 3$: $o_i(v_i, \kappa, p_i) : [0, \bar{v}] \times [r, \infty) \times [0, \bar{v}] \rightarrow \{0, 1\} \cup \{NP\}$. Conditional on not winning the auction or winning but not paying the deposit, the strategy of bidder i regarding whether to purchase from the outside offer is a mapping from private value v_i and price p_i of the outside offer to decision δ_i at $t = 3$: $\delta_i(v_i, p_i) : [0, \bar{v}] \times [0, \bar{v}] \rightarrow \{0, 1\} \cup \{NP\}$.

The equilibrium concept in this paper is perfect Bayesian equilibrium (PBE). Given r and D , we first characterize bidder i 's strategies concerning the outside offer at $t = 3$, i.e., $o_i(v_i, \kappa, p_i)$ and $\delta_i(v_i, p_i)$, examine the winning bidder i 's decision on paying the deposit at $t = 2$, i.e., $e_i(v_i, \kappa)$, and derive the equilibrium bidding strategy $b_i(v_i)$ at $t = 1$. Finally, we characterize the seller's optimal choices on r and D at $t = 0$.¹

1.1.1 Decision on taking the outside offer at $t = 3$

If bidder i does not submit a valid bid, or submits a valid bid but does not win the auction, or does not pay D conditional on winning, the only choice left at $t = 3$ is the outside offer with random price p_i . The characterization of the bidder i 's strategy $\delta_i(\cdot, \cdot)$ is clear: takes the offer if $v_i \geq p_i$; otherwise, chooses not to make the purchase.

Next, we consider the bidder i 's strategy $o_i(\cdot, \cdot, \cdot)$ conditional on winning and paying the deposit. The winning bidder i has three choices at $t = 3$: takes the outside offer with price p_i , or completes the original transaction with payment $\kappa - D$, or not make the purchase at all. The following is the optimal decision

¹In the following analysis, we restrict our attention to (weakly) undominated strategies. This helps rule out many uninteresting equilibria that generally arise in second-price auctions. For example, there may exist an equilibrium that a bidder submits $\bar{v}$ regardless of her true value and other bidders bid zero. Then, the winning bidder makes decisions on paying the deposit and taking the outside offer. However, this equilibrium is not admissible as bidding $\bar{v}$ is weakly dominated by submitting $b_i \leq v_i$, given bidding zero from other bidders. See discussion on all Nash equilibria of second-price auction by Blume and Heidhues (2004).

$o_i(v_i, \kappa, p_i)$ for the winning bidder i :

$$o_i(v_i, \kappa, p_i) = \begin{cases} 0 & \text{if } [e_i = 1 \text{ and } v_i \geq \kappa - D \text{ and } p_i \geq \kappa - D]; \\ 1 & \text{if } [e_i = 1 \text{ and } v_i \geq \kappa - D \text{ and } p_i < \kappa - D], \\ & \text{or } [e_i = 1 \text{ and } v_i < \kappa - D \text{ and } p_i \leq v_i]; \\ NP & \text{if } [e_i = 1 \text{ and } v_i < \kappa - D \text{ and } p_i > v_i]. \end{cases} \quad (\text{A1})$$

When $v_i \geq \kappa - D$, the winning bidder i completes the original transaction if $p_i \geq \kappa - D$ and takes the outside offer if $p_i < \kappa - D$. However, when $v_i < \kappa - D$,² the winning bidder takes the outside offer if $p_i \leq v_i$; otherwise, he chooses not to make the purchase.

1.1.2 Decision on paying the deposit at $t = 2$

Here we examine the winning bidder i 's decision on paying deposit D at $t = 2$. If the winner pays the deposit D , i.e., $e_i = 1$, there are two possibilities: $\kappa - D \geq 0$ or $\kappa - D < 0$. In the case where $\kappa - D \geq 0$, depending on $p_i \leq \kappa - D$, the winning bidder can choose to either complete the original transaction or take the outside offer $t = 3$, as described in (A1). However, if $\kappa - D < 0$, the price p_i of the outside offer will certainly be greater than $\kappa - D$, which implies that the winner will always complete the original transaction. Given the two cases above, we can construct the winning bidder i 's payoff from paying D at $t = 2$, denoted by $\pi_D(v_i, \kappa)$, as follows:

$$\pi_D(v_i, \kappa) = \begin{cases} \int_0^{\kappa-D} (v_i - p_i) \varphi(p_i) dp_i + \int_{\kappa-D}^{\bar{v}} [v_i - (\kappa - D)] \varphi(p_i) dp_i - D & \text{if } \kappa - D \geq 0; \\ \int_0^{\bar{v}} [v_i - (\kappa - D)] \varphi(p_i) dp_i - D & \text{if } \kappa - D < 0. \end{cases} \quad (\text{A2})$$

If the winning bidder i does not pay the deposit D , i.e., $e_i = 0$, her only choice at $t = 3$ is to face the outside offer with random price p_i . In this case, the expected payoff at $t = 2$, denoted by $\pi_{ND}(v_i)$, is given by

$$\pi_{ND}(v_i) = \int_0^{v_i} (v_i - p_i) \varphi(p_i) dp_i. \quad (\text{A3})$$

It is clear that the winning bidder i is willing to pay the deposit D if and only if $\pi_D(v_i, \kappa) \geq \pi_{ND}(v_i)$. Defining $L = \pi_D(v_i, \kappa) - \pi_{ND}(v_i)$, our analysis includes the following two cases. In the case where $\kappa - D \geq 0$, $L(v_i, \kappa) = v_i - \kappa - \int_{\kappa-D}^{v_i} \Phi(p_i) dp_i$, which indicates that given κ and D , there should exist a unique threshold for bidder i 's private value, denoted by $v'_i(\kappa) \in [0, \bar{v}]$, such that $L(v_i, \kappa) > 0$ if and only if $v_i > v'_i(\kappa)$. In the case where $\kappa - D < 0$, $L(v_i, \kappa) = v_i - \kappa - \int_0^{v_i} \Phi(p_i) dp_i$, implying that there should exist a unique threshold $v''_i(\kappa) \in [0, \bar{v}]$, such that $L(v_i, \kappa) > 0$ if and only if $v_i > v''_i(\kappa)$. Summarizing the discussion above gives the

²This is an out-of-equilibrium event, as it implies that the bidder submitted a bid greater than v_i .

winning bidder's deposit strategy at $t = 2$:

$$e_i(v_i, \kappa) = \begin{cases} 1 & \text{if } [\kappa - D \geq 0 \text{ and } v_i > v'_i(\kappa)], \\ & \text{or } [\kappa - D < 0 \text{ and } v_i > v''_i(\kappa)]; \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A4})$$

1.1.3 Bidders' Equilibrium Strategies

Let us now examine the equilibrium bidding strategy at $t = 1$. We first present the following useful lemma.

Lemma 3. *It is a dominated strategy for a bidder to submit any bid greater than her private value, i.e., $b_i > v_i$.*

The proof of Lemma 3 is straightforward. Submitting $b_i > v_i$ makes the bidder win additionally when $b_{-i}^{(1)}$ is between v_i and b_i . Obviously, winning with such a price is weakly dominated by waiting for the outside offer with random price p_i . Lemma 3 allows us to focus on $b_i \leq v_i$ (which implies $\kappa \leq v_i$ and further $\kappa - D \leq v_i$, conditional on bidder i winning the auction) for the equilibrium characterization in the subsequent subgames.

Each bidder at $t = 1$ faces two choices: either bidding in the auction or waiting for the outside offer directly. In the equilibrium, the bid submitted by the bidder should make her indifferent between these two choices, specifically between “winning at her own bid and then paying the deposit” and “not submitting a bid but waiting for the outside offer directly.” We call this insight the “payoff-indifference condition,” which leads to a symmetric increasing bidding strategy, denoted by $b(v_i)$.³ Furthermore, whether a bidder after winning has a chance to default and take the outside offer depends on both the deposit and the reserve price. Given the same r and D , a bidder with a relatively low private value would not choose to default after paying the deposit; however, this may not be true if the bidder is with a sufficiently high private value. Therefore, when we utilize the “payoff-indifference condition” to construct the equilibrium bidding strategy for bidders in the following, we need to separately consider two cases in which, after winning and paying, the bidder still has a chance to default. More specifically, we pin down the details of the equilibrium bidding strategy by considering Case (I) where $r \leq D$ and Case (II) where $r \geq D$.

³The bidding strategy $b(v_i)$ is a weakly dominant strategy. In the proof of Proposition B0, we will show that submitting a bid lower than $b(v_i)$ cannot be beneficial to the bidder. Bidding higher than $b(v_i)$ would increase the probability of winning but lower the bidder's expected payoff, which encourages the bidder to wait for the outside offer instead.

Let us define two cutoff values $\check{v} \in [0, \bar{v}]$ and $\hat{v} \in [0, \bar{v}]$ satisfying the following conditions:

$$r = \int_0^{\check{v}} [1 - \Phi(p_i)] dp_i, \text{ and } D = \int_0^{\hat{v}} [1 - \Phi(p_i)] dp_i.$$

Note that if $b(\bar{v}) \leq r$, we then have $\check{v} \geq \bar{v}$, and there is no participation from bidders. If $b(\bar{v}) \leq D$, we have $\hat{v} \geq \bar{v}$, and no bidder will bid more than the deposit D .

In Case (I) where $r \leq D$, we have $\hat{v} \geq \check{v}$. Let us consider bidding strategies for the bidders separately in the valuation intervals of $v_i \in [\check{v}, \hat{v}]$ and $v_i \in (\hat{v}, \bar{v}]$.

When $v_i \in [\check{v}, \hat{v}]$, the bidder, after winning, always chooses to complete the original transaction. The “payoff-indifference condition” gives

$$\int_0^{\bar{v}} [v_i - (b - D)] \varphi(p_i) dp_i - D = \int_0^{v_i} (v_i - p_i) \varphi(p_i) dp_i,$$

where the first term on the left hand side is the expected payoff from bidding and paying the deposit D (conditional on winning) in the auction, and the right hand side is the expected payoff from waiting for the outside offer directly. Rearranging the equation gives

$$b = v_i - \int_0^{v_i} \Phi(p_i) dp_i \iff b = \int_0^{v_i} [1 - \Phi(p_i)] dp_i. \quad (\text{A5})$$

When $v_i \in (\hat{v}, \bar{v}]$, the bidder, after winning, has two options of either completing the original transaction or taking the outside offer, depending on $p_i \leq b - D$. The “payoff-indifference condition” gives

$$\int_0^{b-D} (v_i - p_i) \varphi(p_i) dp + \int_{b-D}^{\bar{v}} [v_i - (b - D)] \varphi(p_i) dp_i - D = \int_0^{v_i} (v_i - p_i) \varphi(p_i) dp_i,$$

where the first term and the second term on the left hand side present the expected payoffs from taking the outside offer if $p_i \leq b - D$ and completing the original transaction if $p_i > b - D$, respectively; the term on the right hand side is the expected payoff from waiting for the outside offer directly. Re-arranging the equation yields

$$b = v_i - \int_{b-D}^{v_i} \Phi(p_i) dp_i \iff D = \int_{b-D}^{v_i} [1 - \Phi(p_i)] dp_i. \quad (\text{A6})$$

We can then summarize bidder i 's bidding strategy at $t = 1$ in Case (I) as follows:

$$b(v_i) = \begin{cases} v_i - \int_{b(v_i)-D}^{v_i} \Phi(p_i) dp_i & \text{if } v_i > \hat{v}; \\ v_i - \int_0^{v_i} \Phi(p_i) dp_i & \text{if } v_i \in [\check{v}, \hat{v}]; \\ \emptyset & \text{if } v_i < \check{v}. \end{cases} \quad (\text{A7})$$

It is easy to check that in (A5), $\frac{db}{dv_i} = 1 - \Phi(v_i) > 0$, and in (A6), $\frac{db}{dv_i} = \frac{1-\Phi(v_i)}{1-\Phi(b-D)} > 0$ and $\frac{db}{dD} = \frac{-\Phi(b-D)}{1-\Phi(b-D)} < 0$; $b(v_i)$ is increasing in v_i and decreasing in D . Clearly, (A6) coincides with (A5) when $v_i = \hat{v}$.

In Case (II) where $r \geq D$, we have $\hat{v} \leq \check{v}$; only the bidder with $v_i \in [\check{v}, \bar{v}]$ can submit a valid bid, and after winning, the bidder has two options of either completing the original transaction or taking the outside offer, depending on $p_i \leq b - D$. The construction of the equilibrium bidding strategy is similar to (A6) but with the cutoff value $\check{v}$, which can be summarized as follows:

$$b(v_i) = \begin{cases} v_i - \int_{b(v_i)-D}^{v_i} \Phi(p_i) dp_i & \text{if } v_i \geq \check{v}; \\ \emptyset & \text{if } v_i < \check{v}. \end{cases} \quad (\text{A8})$$

Again, it is easy to check that $b(v_i)$ is increasing in v_i and decreasing in D .

Based on the discussion above, we have the following result.

Proposition B0. *In the auction, bidder i adopts the weakly dominant bidding strategies (A7) or (A8), depending on $r \leq D$.*

Proof. Most of the result was already shown in the text in Section 1.1.3. The only task left is to establish the dominance of the equilibrium bidding strategy, in other words, check whether a bidder has an incentive to deviate from the bidding strategy characterized in (A7) and (A8). Given that bidder i submits $b(v_i)$ in the auction, and the payoffs are:

$$\Pi_i = \begin{cases} \pi_D(v_i, \kappa) & \text{if winning;} \\ \pi_{ND}(v_i) & \text{if not winning,} \end{cases}$$

where $\pi_D(v_i, \kappa)$ and $\pi_{ND}(v_i)$ are given by (A2) and (A3), respectively, in Section 1.1.2 above. Since $b(v_i) \geq \kappa$, we have $\pi_D(v_i, \kappa) \geq \pi_{ND}(v_i)$ and the equality holds when $b(v_i) = \kappa$. Suppose now that bidder i does not follow the equilibrium bidding strategy $b(v_i)$ (in short, b) but submits z in the auction, then we consider the following two cases:

Case (a) where $z < b$. If $b > z > \kappa$, then bidder i still wins and that the surplus $\pi_D(v_i, \kappa)$ is not affected. If $\kappa > b > z$, bidder i still loses in the auction and the surplus $\pi_{ND}(v_i)$ is from facing the outside offer. If $b > \kappa > z$, the bidder will lose and obtain $\pi_{ND}(v_i)$, where, however, he would have won and got $\pi_D(v_i, \kappa)$ if he had bid b . Thus, there is no incentive to bid lower than b .

Case (b) where $z > b$. If $z > b > \kappa$, then bidder i still wins and that the surplus $\pi_D(v_i, \kappa)$ is not affected. If $\kappa > z > b$, bidder i still loses in the auction and obtains $\pi_{ND}(v_i)$ is from facing the outside offer. If $z > \kappa > b$, by doing so it increases the probability of winning, but it is more profitable for the bidder to simply wait for the outside offer directly, as $\pi_D(v_i, \kappa)$ is now strictly less than $\pi_{ND}(v_i)$. Thus, there is no incentive to bid higher than b .

Therefore, we can conclude that b is a weakly dominate strategy for bidder i in the auction game. $\square$

Proposition B0 characterizes how bidders respond in their bids when facing a deposit requirement in the auction; a higher deposit requirement induces a higher $\hat{v}$ which lowers the possibility of ex-post default (or the chance to take the outside offer). As a consequence, bidders submit lower bids in the auction as their best responses. If the seller chooses not to demand any deposit from the winner, i.e., $D = 0$, the only solution is $b(v_i) = v_i$.⁴ We therefore have the following remark.

Remark 2. *In the auction, bidders bid truthfully when the seller does not require a deposit.*

This interesting observation indicates that the bidders submitting lower bids, i.e., lower than their private values, is not due to the existence of the post-auction outside offer but rather to the deposit requirement of the seller.

Furthermore, given the characterizations of the bidding strategies in Cases (I) and (II), the following lemma demonstrates how thresholds $\tilde{v}$ and $\hat{v}$ change with r and D .

Lemma 4. (i) *In Case (I) where $r \leq D$, $\tilde{v}$ is increasing in r but independent of D and $\hat{v}$ is increasing in D but independent of r . (ii) In Case (II) where $r \geq D$, $\tilde{v}$ is increasing in both r and D .*

Proof. Case (I) where $r - D \leq 0$. Given $r = \tilde{v} - \int_0^{\tilde{v}} \Phi(p_i) dp_i$, differentiating $\tilde{v}$ with respect to r yields

$$1 = \frac{\partial \tilde{v}}{\partial r} - \Phi(\tilde{v}) \frac{\partial \tilde{v}}{\partial r} \Leftrightarrow \frac{\partial \tilde{v}}{\partial r} = \frac{1}{1 - \Phi(\tilde{v})} > 0. \quad (\text{A9})$$

Thus, $\tilde{v}$ is increasing in r . Clearly, it is independent of D . $\hat{v}$ is given by $D = \hat{v} - \int_0^{\hat{v}} \Phi(p_i) dp_i = \int_0^{\hat{v}} [1 - \Phi(p_i)] dp_i$. Clearly, $\hat{v}$ increases with D but is independent of r ($r \leq D$).

⁴Given $r \geq 0$, if $D = 0$, then $b(\tilde{v}) \geq 0$, and the bidding strategy in (A8) can be written as $\int_b^{v_i} [1 - \Phi(p_i)] dp_i = 0$, which implies truthful bidding from bidders.

Case (II) where $r - D \geq 0$. Given $r = \check{v} - \int_{r-D}^{\check{v}} \Phi(p_i) dp_i$, differentiating $\check{v}$ with respect to r yields

$$1 = \frac{\partial \check{v}}{\partial r} - \left(\Phi(\check{v}) \frac{\partial \check{v}}{\partial r} - \Phi(r - D) \right) \Leftrightarrow \frac{\partial \check{v}}{\partial r} = \frac{1 - \Phi(r - D)}{1 - \Phi(\check{v})} > 0. \quad (\text{A10})$$

Further, differentiating $\check{v}$ with respect to D shows

$$0 = \frac{\partial \check{v}}{\partial D} - \left(\Phi(\check{v}) \frac{\partial \check{v}}{\partial D} + \Phi(r - D) \right) \Leftrightarrow \frac{\partial \check{v}}{\partial D} = \frac{\Phi(r - D)}{1 - \Phi(\check{v})} > 0. \quad (\text{A11})$$

We then have that $\check{v}$ is increasing in both r and D . $\square$

From Lemma 4, we derive the following corollary on how the equilibrium bidding strategy depends on the reserve price, which will be useful for the analysis of the optimal reserve price in the next section.

Corollary B0. *Reserve price r affects equilibrium bidding strategy $b(v_i)$ through $\check{v}$.*

Proofs of Lemmas 5 - 8

The following four lemmas help us characterize the optimal reserve price maximizing the seller's revenue for a given deposit.

Lemma 5. $\mathbb{E}_S^I[R(r, D)]$ is independent of D for any $D \geq D^{so}$.

Proof. If the seller chooses any $D > D^{so}$, $\hat{v}$ takes the corner solution $\bar{v}$, i.e., $\hat{v} = \bar{v}$, and the seller's expected revenue can be re-written as follows:

$$\mathbb{E}_S^I[R(r, D)] = N(1 - F(\bar{v}))Q(\bar{v})r + N \int_{\bar{v}}^{\bar{v}} (1 - F(x))b(x)dQ(x),$$

which does not change with D . Note that $b(x)$ does not depend on D when $D \geq D^{so}$. $\square$

Lemma 6. (i) In Case (I) where $r \leq D$, given any $D > D^{so}$, $\mathbb{E}_S^I[R(r, D)] = 0$ for all $r \in [D^{so}, D]$. (ii) In Case (II) where $r \geq D$, $\mathbb{E}_S^{II}[R(r, D)] = 0$ for all $r \geq D^{so}$.

Proof. Given the restrictions of $r \leq D$ in Case (I) and $r \geq D$ in Case (II), we show that the seller obtains zero revenue when $r \geq D^{so}$.

Case (I) where $r - D \leq 0$, seller revenue with any $D > D^{so}$ is given by

$$\mathbb{E}_S^I[R(r, D)] = N(1 - F(\bar{v}))Q(\bar{v})r + N \int_{\bar{v}}^{\bar{v}} (1 - F(x))b(x)dQ(x).$$

Further, if the seller sets $r \geq D^{so}$ (but still less than D), then $\tilde{v}$ takes the corner solution $\bar{v}$, i.e., $\tilde{v} = \bar{v}$. As a result, no one can submit a valid bid and the seller's revenue decreases to zero.

Case (II) where $r - D \geq 0$. With $r \geq D^{so}$, $\tilde{v}$ takes the corner solution $\bar{v}$, i.e., $\tilde{v} = \bar{v}$. As a result, no one can submit a valid bid and the seller's revenue decreases to zero. $\square$

Lemmas 5 and 6 help us restrict our analysis to the area of $(r, D) \in [0, D^{so}] \times [0, D^{so}]$. Given any $D \in [0, D^{so}]$, let us use $r^I(D)$ to denote the optimal reserve price in Case (I) where $r \leq D$. We first maximize (12) without taking into account the restriction of $r \leq D$; in this case, we denote the unrestricted optimal reserve price by $\tilde{r}^I(D)$. We next check whether or not $r^I(D) = \tilde{r}^I(D)$, i.e., $\tilde{r}^I(D)$ can be implemented in Case (I). We follow the same procedure to examine (13) and denote the optimal reserve price in Case (II) by $r^{II}(D)$ and the unrestricted optimal reserve price without the constraint of $r \geq D$ by $\tilde{r}^{II}(D)$. The following lemmas present the optimal choices on the reserve price in Cases (I) and (II), given $D \in [0, D^{so}]$.

Lemma 7. *For any $D \in [r^{so}, D^{so}]$, (i) in Case (I) where $r \leq D$, $r^I(D) = \tilde{r}^I(D) = r^{so}$, and (ii) in Case (II) where $r \geq D$, $r^{II}(D) = D \geq \tilde{r}^{II}(D)$.*

Proof. Let us separately consider the following two cases. Part (i). We re-write $\mathbb{E}_S^I[R(r, D)]$ as follows:

$$\begin{aligned} \mathbb{E}_S^I[R(r, D)] &= N(1 - F(\tilde{v}))Q(\tilde{v})b(\tilde{v}) + N \int_{\tilde{v}}^{\hat{v}} (1 - F(x))b(x)dQ(x) \\ &\quad + N \int_{\tilde{v}}^{\bar{v}} (1 - F(x)) \left[(1 - \Phi(\tilde{b}(x) - D))\tilde{b}(x) + \Phi(\tilde{b}(x) - D)D \right] dQ(x). \end{aligned} \quad (\text{A12})$$

Fix D , differentiating $\mathbb{E}_S^I[R(r, D)]$ with respect to r and plugging $\frac{\partial b}{\partial v_i} = 1 - \Phi(v_i)$ into the equation yield

$$\begin{aligned} \frac{\partial}{\partial r} \frac{\mathbb{E}_S^I[R(r, D)]}{N} &= \frac{\partial}{\partial \tilde{v}} \frac{\mathbb{E}_S^I[R(r, D)]}{N} \cdot \frac{\partial \tilde{v}}{\partial r} \\ &= \left[-f(\tilde{v})Q(\tilde{v})b(\tilde{v}) + (1 - F(\tilde{v}))q(\tilde{v})b(\tilde{v}) + (1 - F(\tilde{v}))Q(\tilde{v})\frac{\partial b(\tilde{v})}{\partial \tilde{v}} - (1 - F(\tilde{v}))q(\tilde{v})b(\tilde{v}) \right] \frac{\partial \tilde{v}}{\partial r} \\ &= Q(\tilde{v}) \left[-f(\tilde{v})b(\tilde{v}) + (1 - F(\tilde{v}))\frac{\partial b(\tilde{v})}{\partial \tilde{v}} \right] \frac{\partial \tilde{v}}{\partial r} \\ &= Q(\tilde{v}) \left[1 - \Phi(\tilde{v}) - \frac{f(\tilde{v})}{(1 - F(\tilde{v}))} \left(\tilde{v} - \int_0^{\tilde{v}} \Phi(p_i)dp_i \right) \right] \frac{\partial \tilde{v}}{\partial r}, \end{aligned} \quad (\text{A13})$$

where $q(\cdot) = Q'(\cdot)$. Recall that $\frac{\partial \tilde{v}}{\partial r} = \frac{1}{1 - \Phi(\tilde{v})} > 0$ from (A9). When $\tilde{v} = 0$, the derivative $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^I[R(r, D)]}{N}$ is 0. But clearly, $\tilde{v} = 0$ cannot be optimal, because $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^I[R(r, D)]}{N} > 0$ when $\tilde{v}$ is slightly above zero. As long as F is regular (increasing hazard rate), the optimum must occur when the term in the square brackets equals

zero. We thus have at optimum

$$\begin{aligned} & \left[1 - \Phi(\check{v}) - \frac{f(\check{v})}{(1-F(\check{v}))} (\check{v} - \int_0^{\check{v}} \Phi(p_i) dp_i) \right] = 0 \\ \Leftrightarrow & \frac{(1-F(\check{v}))(1-\Phi(\check{v}))}{f(\check{v})} = \int_0^{\check{v}} [1 - \Phi(p_i)] dp_i. \end{aligned} \quad (\text{A14})$$

This gives $\check{v} = \check{v}^{so}$ and further, since $b(\check{v}) = r$, we have $r = r^{so} = \check{v}^{so} - \int_0^{\check{v}^{so}} \Phi(p_i) dp_i = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i$. The analysis above indicates that the unrestricted optimal reserve price $\tilde{r}^I(D)$ is exactly equal to r^{so} for any $D \geq 0$. Moreover, for any $D \in [r^{so}, D^{so}]$, we have $r \leq D$ and the unrestricted optimal reserve price is feasible; therefore, $r^I(D) = \tilde{r}^I(D) = r^{so}$.

Part (ii). We re-write $\mathbb{E}_S^{II}[R(r, D)]$ as follows:

$$\begin{aligned} \mathbb{E}_S^{II}[R(r, D)] &= N(1-F(\check{v}))Q(\check{v}) \left[(1-\Phi(b(\check{v})-D))b(\check{v}) + \Phi(b(\check{v})-D)D \right] \\ &+ N \int_{\check{v}}^{\bar{v}} (1-F(x)) \left[(1-\Phi(b(x)-D))b(x) + \Phi(b(x)-D)D \right] dQ(x). \end{aligned} \quad (\text{A15})$$

Fix D , differentiating $\mathbb{E}_S^{II}[R(r, D)]$ with respect to r yields

$$\begin{aligned} \frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} &= \frac{\partial}{\partial \check{v}} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} \cdot \frac{\partial \check{v}}{\partial r} \\ &= \left[-f(\check{v})Q(\check{v}) \left[(1-\Phi(b(\check{v})-D))b(\check{v}) + \Phi(b(\check{v})-D)D \right] \right. \\ &\quad \left. + (1-F(\check{v}))Q(\check{v}) \frac{\partial b(\check{v})}{\partial \check{v}} \left[(1-\Phi(b(\check{v})-D)) - \varphi(b(\check{v})-D)(b(\check{v})-D) \right] \right] \frac{\partial \check{v}}{\partial r} \\ &= Q(\check{v}) \left[-f(\check{v}) \left[(1-\Phi(b(\check{v})-D))(b(\check{v})-D) + D \right] \right. \\ &\quad \left. + (1-F(\check{v})) \frac{\partial b(\check{v})}{\partial \check{v}} \left[(1-\Phi(b(\check{v})-D)) - \varphi(b(\check{v})-D)(b(\check{v})-D) \right] \right] \frac{\partial \check{v}}{\partial r}. \end{aligned} \quad (\text{A16})$$

Recall that $\frac{\partial \check{v}}{\partial r} = \frac{1-\Phi(r-D)}{1-\Phi(\check{v})} > 0$ from (A10). When $\check{v} = 0$, the derivative $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N}$ is 0. But as long as F is regular (increasing hazard rates), $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} = 0$ should occur when the term in the big square bracket is zero. Now let us establish the following property: For any $D \in [r^{so}, D^{so}]$ and $r \geq D$, we have $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} \leq 0$, specially $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} = 0$ if and only if $D = r^{so}$ and $r = D$. This property helps us to see under conditions which the term in the big square bracket is zero.

Given $b(\check{v}) = r$, we re-write the term in the large square bracket of last line in (A16) and show that it has the same sign with the following Λ :

$$\Lambda \equiv - \left[(1 - \Phi(r - D))(r - D) + D \right] + \frac{(1 - F(\check{v}))}{f(\check{v})} \frac{\partial r}{\partial \check{v}} \left[1 - \Phi(r - D) - \varphi(r - D)(r - D) \right]. \quad (\text{A17})$$

Plugging $\frac{\partial r}{\partial \check{v}} = \frac{1 - \Phi(\check{v})}{1 - \Phi(r - D)}$ into Λ , and simplifying it shows

$$\Lambda = - \left[(1 - \Phi(r - D))(r - D) + D \right] + \frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})} \left[1 - \frac{\varphi(r - D)}{1 - \Phi(r - D)}(r - D) \right]. \quad (\text{A18})$$

Step (1). When $D = r^{so}$ and $r = D$, i.e., $\check{v} = \check{v}^{so}$, we have

$$\Lambda = -r^{so} + \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0. \quad (\text{A19})$$

Step (2). When $D = r^{so}$ and $r - r^{so} > 0$, we can then write Λ as follows:

$$\Lambda = - \left[(1 - \Phi(r - r^{so}))(r - r^{so}) + r^{so} \right] + \frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})} \left[1 - \frac{\varphi(r - r^{so})}{1 - \Phi(r - r^{so})}(r - r^{so}) \right]. \quad (\text{A20})$$

Note that $-[(1 - \Phi(r - r^{so}))(r - r^{so}) + r^{so}] < -r^{so}$, and further, since $\check{v} > \check{v}^{so}$ and the hazard rates of F and Φ are increasing, we have $\frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})}$ decreases with $\check{v}$ and $\frac{1 - \Phi(r - r^{so})}{\varphi(r - r^{so})}$ decreases with r , implying that the following inequality should hold

$$\begin{aligned} \frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})} \left[1 - \frac{\varphi(r - r^{so})}{1 - \Phi(r - r^{so})}(r - r^{so}) \right] &< \frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})} \\ &< \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})}. \end{aligned}$$

Therefore, we have $\Lambda < -r^{so} + \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0$.

Step (3). Let us consider any $D \in (r^{so}, D^{so}]$. Define $\omega = r - D \geq 0$ and denote the inverse bidding function by $b^{-1}(v, D)$. (A18) can be re-written as follows:

$$\begin{aligned} \Lambda = & - \left[(1 - \Phi(\omega))\omega + r^{so} + (D - r^{so}) \right] \\ & + \left[\frac{1 - F(b^{-1}(\omega + r^{so} + (D - r^{so}), D))}{f(b^{-1}(\omega + r^{so} + (D - r^{so}), D))} \left(1 - \Phi(b^{-1}(\omega + r^{so} + (D - r^{so}), D)) \right) \right] \left[1 - \frac{\varphi(\omega)}{1 - \Phi(\omega)}\omega \right]. \end{aligned} \quad (\text{A21})$$

We now use D' to denote the deposit when $D > r^{so}$ and D'' to denote the deposit when $D = r^{so}$. Given the same v and $D' > D''$, we have $b(v, D') < b(v, D'')$, as $b(v, D)$ is decreasing in D . Further, $b(v, D)$ is increasing in v , indicating that given the same b , we should have $v' > v''$ for $b(v', D') = b(v'', D'')$. This property implies that with the same bid $\omega + r^{so}$, we should have $b^{-1}(\omega + r^{so}, D') > b^{-1}(\omega + r^{so}, D'')$. Thus, from equation (A21), we have

$$\begin{aligned}
& \left[\frac{1 - F(b^{-1}(\omega + r^{so} + (D' - r^{so}), D'))}{f(b^{-1}(\omega + r^{so} + (D' - r^{so}), D'))} \left(1 - \Phi(b^{-1}(\omega + r^{so} + (D' - r^{so}), D')) \right) \right] \\
& < \left[\frac{1 - F(b^{-1}(\omega + r^{so}, D'))}{f(b^{-1}(\omega + r^{so}, D'))} \left(1 - \Phi(b^{-1}(\omega + r^{so}, D')) \right) \right] \\
& < \left[\frac{1 - F(b^{-1}(\omega + r^{so}, D''))}{f(b^{-1}(\omega + r^{so}, D''))} \left(1 - \Phi(b^{-1}(\omega + r^{so}, D'')) \right) \right] \\
& < \left[\frac{1 - F(b^{-1}(r^{so}, D''))}{f(b^{-1}(r^{so}, D''))} \left(1 - \Phi(b^{-1}(r^{so}, D'')) \right) \right] \\
& = \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})}.
\end{aligned} \tag{A22}$$

Since $r \geq D > r^{so}$, we have $\Lambda < -r^{so} + \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0$. We can conclude that for any $D \in [r^{so}, D^{so}]$ and $r \geq D$, $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} \leq 0$, in particular, $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} = 0$ when $D = r^{so}$ and $r = D$. This established property indicates that the unrestricted optimal reserve price $\tilde{r}^{II}(D)$ must be smaller than or equal to D , i.e., $\tilde{r}^{II}(D) \leq D$ for any $D \in [r^{so}, D^{so}]$ (the equality holds if and only if $D = r^{so}$ and $r = D$), and therefore, in Case (II) where $r \geq D$, $\tilde{r}^{II}(D)$ is not implementable and $\mathbb{E}_S^{II}[R(r, D)]$ is maximized at the boundary condition of $r^{II}(D) = D$.

Finally, Λ in (A18) is decreasing in r ($\leq D$), as $\check{v}$ increases with r . This indicates that the term in the big square bracket in (A16) single crosses zero when $r \leq D$. We thus have unconstrained optimum $\tilde{r}^{II}(D)$ is uniquely defined by $\Lambda = 0$. $\square$

Lemma 8. For any $D \in [0, r^{so}]$, (i) in Case (I) where $r \leq D$, $r^I(D) = D \leq \tilde{r}^I(D) = r^{so}$, and (ii) in Case (II) where $r \geq D$, $r^{II}(D) = \tilde{r}^{II}(D)$.

Proof. We examine $r^I(D)$ and $r^{II}(D)$ for any $D \in [0, r^{so}]$.

Part (i). From Lemma 7(i), we have shown that the unrestricted optimal reserve price $\tilde{r}^I(D)$ is equal to r^{so} for any $D \geq 0$. From (A14), since $\frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})}$ decreases with $\check{v}$ and $\int_0^{\check{v}} [1 - \Phi(p_i)] dp_i$ increases with

$\check{v}$, we have the following inequality for $D \in [0, r^{so})$ and $r \leq D$:

$$\begin{aligned} \frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})} - \int_0^{\check{v}} [1 - \Phi(p_i)] dp_i &> \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} - r^{so} \\ &= 0. \end{aligned} \quad (\text{A23})$$

The inequality above implies that $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^I[R(r, D)]}{N} > 0$ for any $D \in [0, r^{so})$ and $r \leq D$. Therefore, under case (I) where $r \leq D$, $\tilde{r}^I(D)$ is not feasible for any $D \in [0, r^{so})$ and $\mathbb{E}_S^I[R(r, D)]$ is maximized at the boundary condition of $r^I(D) = D$.

Part (ii). Given the fact that $\frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})}$ decreases with $\check{v}$, (A18) gives the following inequality for any $D \in [0, r^{so})$ and $D = r$, that is,

$$\begin{aligned} \Lambda &= \frac{(1 - F(\check{v}))(1 - \Phi(\check{v}))}{f(\check{v})} - D > \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} - r^{so} \\ &= 0. \end{aligned} \quad (\text{A24})$$

The inequality above implies that $\frac{\partial}{\partial r} \frac{\mathbb{E}_S^{II}[R(r, D)]}{N} > 0$ for any $D \in [0, r^{so})$ and $r = D$. Similar to proof of Lemma 7(ii), we can establish that Λ in (A18) is decreasing in r ($\leq D$). Therefore, we have that for any $D \in [0, r^{so})$ the unrestricted optimal reserve price $\tilde{r}^{II}(D)$ is greater than D and feasible in Case (II) where $r \geq D$, i.e., $r^{II}(D) = \tilde{r}^{II}(D)$. $\square$

1.1.4 Figures to illustrate the optimal D and r

We here use the following figures to illustrate the seller's choice on reserve price for any given deposit. Figure 2 presents a graphic illustration of how $r^I(D)$ varies when D changes in $[0, D^{so}]$. The X-axis is the reserve price, and the Y-axis is the deposit. The seller's choice of $r^I(D)$ is marked by bold red. For $D \in [0, r^{so})$, $r^I(D)$ is equal to D , which is located on the 45-degree line, for $D \in [r^{so}, D^{so}]$, $r^I(D)$ is equal to r^{so} , and the seller's revenue becomes independent of D when $D > D^{so}$. Figure 3 graphically illustrates how $r^{II}(D)$ varies when D changes in $[0, D^{so}]$, which is marked by bold red. For $D \in [0, r^{so})$, $r^{II}(D)$ is greater than the deposit D (below the 45-degree line), for $D \in [r^{so}, D^{so}]$, $r^{II}(D)$ is equal to D (which is located along the 45-degree line), and for any $D > D^{so}$, the seller's revenue becomes independent of D . Figure 4 graphically depicts the optimal reserve price $r^*(D)$, which achieves the overall maximized seller revenue across Cases (I) and (II). In the figure, $r^*(D)$ is marked by bold blue. For $D \in [r^{so}, D^{so}]$, $r^*(D)$ is equal to r^{so} from Case (I), and for $D \in [0, r^{so})$, $r^*(D)$ is equal to $r^{II}(D)$ from Case (II).

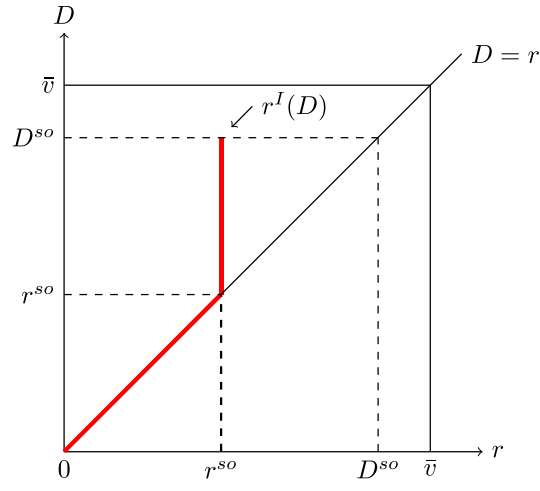


Figure 2: Case (I) where $r \leq D$

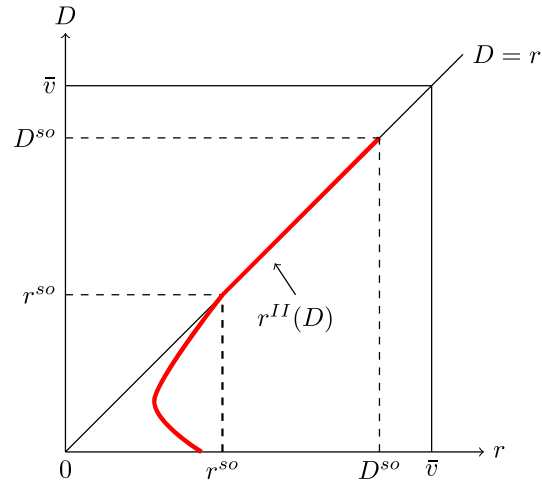


Figure 3: Case (II) where $r \geq D$

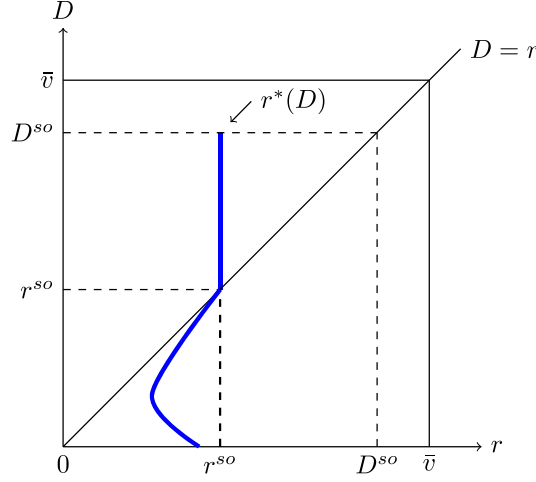


Figure 4: $r^*(D)$ for $D \in [0, D^{so}]$

1.2 Percentage deposit

Let $\gamma \in [0, 1]$ denote the percentage of the final price the seller requires as the deposit. Then, conditional on winning, the deposit amount bidder i as the winner needs to pay can be written as $D = \gamma\kappa$. Clearly, the analyses we have constructed for the outside option strategy in (A1) is not affected. We then look at the deposit strategy. Since the percentage deposit cannot be greater than the final price, the deposit strategy is the same as the case where $\kappa - D \geq 0$ in Section 1.1.2. Next let us examine the bidding strategy. Again, the construction of the equilibrium bid should make a bidder indifferent between “winning at her own bid and then paying the deposit” and “not submitting a bid but waiting for the outside offer directly,” that is,

$$\int_0^{(1-\gamma)b} (v_i - p_i) \varphi(p) dp_i + \int_{(1-\gamma)b}^{\bar{v}} [v_i - (1-\gamma)b] \varphi(p_i) dp_i - \gamma b = \int_0^{v_i} (v_i - p_i) \varphi(p_i) dp_i$$

$$\Leftrightarrow b = v_i - \int_{(1-\gamma)b}^{v_i} \Phi(p_i) dp_i.$$

Given r and γ , we write the bidding strategy as $b(v_i)$. It is easy to show that $b(v_i)$ is increasing in v_i and decreasing in γ , as $\frac{\partial b(v_i)}{\partial v_i} = \frac{1 - \Phi(v_i)}{1 - \Phi(b(v_i)(1-\gamma))(1-\gamma)} > 0$ and $\frac{\partial b(v_i)}{\partial \gamma} = \frac{-b(v_i)\Phi(b(v_i)(1-\gamma))}{1 - \Phi(b(v_i)(1-\gamma))(1-\gamma)} < 0$. Given that

$b(\check{v}) = r$, the seller's expected revenue denoted by $\mathbb{E}_S[R(r, \gamma)]$ is given by

$$\begin{aligned}\mathbb{E}_S[R(r, \gamma)] &= N(1 - F(\check{v}))Q(\check{v}) \left[(1 - \Phi(b(\check{v})(1 - \gamma)))b(\check{v}) + \Phi(b(\check{v})(1 - \gamma))\gamma b(\check{v}) \right] \\ &\quad + N \int_{\check{v}}^{\bar{v}} \int_{\check{v}}^{v_i} \left[(1 - \Phi(b(x)(1 - \gamma)))b(x) + \Phi(b(x)(1 - \gamma))\gamma b(x) \right] dQ(x) dF(v_i) \\ &= N(1 - F(\check{v}))Q(\check{v}) \left[(1 - \Phi(b(\check{v})(1 - \gamma)))b(\check{v}) + \Phi(b(\check{v})(1 - \gamma))\gamma b(\check{v}) \right] \\ &\quad + N \int_{\check{v}}^{\bar{v}} (1 - F(x)) \left[(1 - \Phi(b(x)(1 - \gamma)))b(x) + \Phi(b(x)(1 - \gamma))\gamma b(x) \right] dQ(x).\end{aligned}$$

The seller chooses r and γ to maximize $\mathbb{E}_S[R(r, \gamma)]$. We introduce notations γ^{so} and r^{so} as follows:

$$\gamma^{so} = 1 \text{ and } r^{so} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i,$$

where $\check{v}^{so}$ is determined by $\frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i$. Let us write $r^*(\gamma)$ to denote the optimal reserve price, given any γ . We can then establish the following result.

Proposition B1. $\frac{d\mathbb{E}_S[R(r^*(\gamma), \gamma)]}{d\gamma} \geq 0$ for any $\gamma \in [0, 1]$. In particular, $\frac{d\mathbb{E}_S[R(r^*(\gamma), \gamma)]}{d\gamma} = 0$ when $\gamma = \gamma^{so}$. As a result, the combination of γ^{so} and r^{so} maximizes the seller's revenue. Moreover, $\mathbb{E}_S[R(r^{so}, \gamma^{so})] = R^*(D^{so})$.

Proof. Our proof includes the following two steps:

Step (i). Differentiating $\mathbb{E}_S[R(r, \gamma)]$ with respect to r yields

$$\begin{aligned}\frac{\partial}{\partial r} \frac{\mathbb{E}_S[R(r, \gamma)]}{N} &= \frac{\partial}{\partial \check{v}} \frac{\mathbb{E}_S[R(r, \gamma)]}{N} \cdot \frac{\partial \check{v}}{\partial r} \\ &= Q(\check{v}) \left[-f(\check{v}) \left[(1 - \Phi(b(\check{v})(1 - \gamma)))b(\check{v}) + \Phi(b(\check{v})(1 - \gamma))\gamma b(\check{v}) \right] + (1 - F(\check{v})) \frac{\partial b(\check{v})}{\partial \check{v}} \left[(1 - \Phi(b(\check{v})(1 - \gamma))) \right. \right. \\ &\quad \left. \left. - \varphi(b(\check{v})(1 - \gamma))(1 - \gamma)^2 b(\check{v}) + \Phi(b(\check{v})(1 - \gamma))\gamma \right] \right] \cdot \frac{\partial \check{v}}{\partial r}.\end{aligned}\tag{A25}$$

When $\check{v} = 0$, the derivative is 0. But as long as F is regular (increasing hazard rate), then $\frac{\partial}{\partial \check{v}} \frac{\mathbb{E}_S[R(r, \gamma)]}{N} = 0$ should occur when the term in the large square bracket is zero. This gives the unrestricted $\check{v}(r^*(\gamma), \gamma)$ and further, the optimal reserve price $r^*(\gamma)$ is given by $b(\check{v}(r^*(\gamma), \gamma))$.

Step (ii). We then know that the impact of γ through $r^*(\gamma)$ is zero by envelope theorem. We thus have

the following equation

$$\frac{d}{d\gamma} \frac{\mathbb{E}_S[R(r^*(\gamma), \gamma)]}{N} = \frac{\partial}{\partial \gamma} \frac{\mathbb{E}_S[R(r, \gamma)]}{N} \Big|_{r=r^*(\gamma), \check{v}=\check{v}(r^*(\gamma), \gamma)}. \quad (\text{A26})$$

which is given by

$$\begin{aligned} & \frac{\partial}{\partial \gamma} \frac{\mathbb{E}_S[R(r, \gamma)]}{N} \Big|_{r=r^*(\gamma), \check{v}=\check{v}(r^*(\gamma), \gamma)} \\ &= (1 - F(\check{v}(r^*(\gamma), \gamma))) Q(\check{v}(r^*(\gamma), \gamma)) \left(\frac{-\varphi(b(\check{v}(r^*(\gamma), \gamma))(1 - \gamma))(1 - \gamma)}{\Phi(b(\check{v}(r^*(\gamma), \gamma))(1 - \gamma))} \cdot \frac{\partial b(\check{v}(r^*(\gamma), \gamma))}{\partial \gamma} \right) \\ &+ \int_{\check{v}(r^*(\gamma), \gamma)}^{\bar{v}} (1 - F(x)) \left(\frac{-\varphi(b(x)(1 - \gamma))(1 - \gamma)}{\Phi(b(x)(1 - \gamma))} \cdot \frac{\partial b(x)}{\partial \gamma} \right) dQ(x). \end{aligned} \quad (\text{A27})$$

Clearly, $\check{v}(r^*(\gamma), \gamma) < \bar{v}$ for any $\gamma \in [0, 1]$. Further, given $\frac{\partial b(v_i)}{\partial \gamma} < 0$, we therefore have $\frac{d}{d\gamma} \frac{\mathbb{E}_S[R(r^*(\gamma), \gamma)]}{N} > 0$, indicating that it is optimal for the seller to choose $\gamma^{so} = 1$. Plugging $\gamma^{so} = 1$ and $\frac{\partial b(v_i)}{\partial v_i} = \frac{1 - \Phi(v_i)}{1 - \Phi(b(v_i)(1 - \gamma))(1 - \gamma)}$ into the term in large square bracket of the last line of (A25) gives $\check{v}^{so}$ and r^{so} :

$$\begin{aligned} & -f(\check{v}^{so})b(\check{v}^{so}) + (1 - F(\check{v}^{so})) \frac{\partial b(\check{v}^{so})}{\partial \check{v}^{so}} = 0 \\ \Leftrightarrow & \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i, \end{aligned}$$

and r^{so} is determined by

$$r^{so} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i. \quad (\text{A28})$$

Note that the optimal reserve and bidding cutoff are exactly the same as in our main analysis.

Step (iii). Finally, plugging $\gamma^{so} = 1$ and r^{so} into $\mathbb{E}_S[R(r, \gamma)]$ yields

$$\begin{aligned} \mathbb{E}_S[R(r^{so}, \gamma^{so})] &= N(1 - F(\check{v}^{so}))Q(\check{v}^{so})b(\check{v}^{so}) + N \int_{\check{v}^{so}}^{\bar{v}} (1 - F(x))b(x)dQ(x) \\ &= \mathbb{E}_S^I[R(r^{so}, D^{so})] \\ &= R^*(D^{so}). \end{aligned} \quad (\text{A29})$$

The intuition of the result above is consistent with what we had before. The seller's expected revenue is maximized when the default possibility from the winner is zero. To do this, the seller should set the percentage sufficiently high (equal to 100 percent) so that the winner's default possibility is fully deterred. Moreover, at the optimal level, the percentage deposit requirement is equivalent to the fixed amount deposit requirement, generating the same maximized seller revenue. $\square$

1.3 Deposit proportionally deducted from full payment

So far, we have analyzed the deposit requirement in which the amount the winner paid will be completely deducted from the full payment. However, in practice sellers also commonly implement another type of deposit requirement where the amount paid by the winners is in addition to the full payment. For convenience, we call the former *the inclusive deposit requirement*, and the latter *the exclusive deposit requirement*. It is of interest to compare the two types of the deposit requirements, in particular, which one would benefit the seller more in terms of the expected revenue, or whether they would yield the same expected revenue to the seller. It seems that the exclusive deposit requirement is more profitable, as it is an additional fee, besides the final price, charged by the seller. However, because of this additional “cost” after winning, the bidders would strategically adjust their bids (intuitively, to submit lower bids) in the auction. Thus, the comparison is not trivial and obvious.

Instead of focusing only on these two particular deposit rules, we here provide a more general analysis. Denote the deduction proportion of the deposit D from the full payment by $\alpha \in [0, 1]$. Given α , the winner needs to pay $\kappa - \alpha D$ to complete the transaction. In other words, the winner’s total payment is $\kappa + (1 - \alpha)D$ for the object. Here, the inclusive and exclusive deposit requirements correspond to the two special cases with $\alpha = 1$ and $\alpha = 0$, respectively. In the second-price auction with $b(\tilde{v}) = r$ and $b(\hat{v}) = \alpha D$, bidders’ bidding strategies at $t = 1$ can be re-written as follows:

In Case (I) where $r \leq \alpha D$, $b(v_i)$ is given by

$$b(v_i) = \begin{cases} v_i - \int_{b(v_i) - \alpha D}^{v_i} \Phi(p_i) dp_i - (1 - \alpha)D & \text{if } v_i > \hat{v}; \\ v_i - \int_0^{v_i} \Phi(p_i) dp_i - (1 - \alpha)D & \text{if } v_i \in [\tilde{v}, \hat{v}]; \\ \emptyset & \text{if } v_i < \tilde{v}, \end{cases}$$

and the associated seller revenue $\mathbb{E}_S^I[R(r, D, \alpha)]$ is given by

$$\begin{aligned} \frac{\mathbb{E}_S^I[R(r, D, \alpha)]}{N} &= (1 - F(\tilde{v}))Q(\tilde{v})(r + (1 - \alpha)D) + \int_{\tilde{v}}^{\hat{v}} (1 - F(x))(b(x) + (1 - \alpha)D)dQ(x) \\ &\quad + \int_{\hat{v}}^{\bar{v}} (1 - F(x)) \left[(1 - \Phi(\tilde{b}(x) - \alpha D))(\tilde{b}(x) + (1 - \alpha)D) + \Phi(\tilde{b}(x) - \alpha D)D \right] dQ(x), \end{aligned}$$

where the bidding strategies are denoted by $b(v_i)$ and $\tilde{b}(v_i)$ for $v_i \in [\tilde{v}, \hat{v}]$ and $v_i \in (\hat{v}, \bar{v}]$, respectively.

In Case (II) where $r \geq \alpha D$, $b(v_i)$ is given by

$$b(v_i) = \begin{cases} v_i - \int_{b(v_i) - \alpha D}^{v_i} \Phi(p_i) dp_i - (1 - \alpha)D & \text{if } v_i \geq \check{v}; \\ \emptyset & \text{if } v_i < \check{v}, \end{cases}$$

and the associated seller revenue $\mathbb{E}_S^{II}[R(r, D, \alpha)]$ is given by

$$\begin{aligned} \frac{\mathbb{E}_S^{II}[R(r, D, \alpha)]}{N} &= (1 - F(\check{v}))Q(\check{v}) \left[(1 - \Phi(r - \alpha D))(r - \alpha D) + D \right] \\ &\quad + \int_{\check{v}}^{\bar{v}} (1 - F(x)) \left[(1 - \Phi(b(x) - \alpha D))(b(x) - \alpha D) + D \right] dQ(x). \end{aligned}$$

Define D^{so} and r^{so} as follows

$$D^{so} = \int_0^{\hat{v}^{so}} [1 - \Phi(p_i)] dp_i \quad \text{and} \quad r^{so} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i - (1 - \alpha)D^{so},$$

where $\hat{v}^{so} = \bar{v}$ and $\check{v}^{so}$ is given by $\frac{(1 - F(\hat{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\hat{v}^{so})} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i$. Interestingly, we see that although the thresholds of $\hat{v}^{so}$ and $\check{v}^{so}$ and the optimal deposit D^{so} are the same as before, the optimal reserve price r^{so} depends on the deduction proportion rule. Furthermore, r^{so} increases in α and therefore there must exist a threshold, called $\tilde{\alpha} > 0$, such that $r^{so} = 0$. This fact indicates that if the seller wants to implement deduction proportion rule $\alpha < \tilde{\alpha}$, the optimal reserve price should be a negative value, implying that submitting negative-valued bids from bidders should be allowed.

We next examine the impacts of the deduction proportion rule on seller's revenue. In particular, we are interested in comparing seller revenues across different proportion rules, answering the question of whether there exists an optimal proportion α to the seller. Let us denote the expected seller revenues and bidding strategies associated with two proportion rules α and α' by $\mathbb{E}_S^I[R(r, D, \alpha)]$ and $\mathbb{E}_S^I[R(r, D, \alpha')]$, and $b(v_i, \alpha)$ and $b(v_i, \alpha')$, respectively, where $\alpha, \alpha' \in [0, 1]$ and $\alpha \neq \alpha'$. We can then establish the following result.

Proposition B2. *If submitting negative-valued bids is allowed, the seller's expected revenue with the optimal r^{so} and D^{so} is independent of the deduction proportion rule α .*

Proof. Our proof can be split into the following three steps: *Step (i).* We first assume that $\check{v}$, $\hat{v}$, and D are exactly the same across both deduction proportion rules α and α' , then the equilibrium bidding strategies show:

If $v_i \in [\check{v}, \bar{v}]$, the equilibrium bidding strategy implies the following fact that

$$b(v_i, \alpha) + (1 - \alpha)D = v_i - \int_0^{v_i} \Phi(p_i) dp_i = b(v_i, \alpha') + (1 - \alpha')D. \quad (\text{A30})$$

If $v_i \in (\hat{v}, \bar{v}]$, from the equilibrium bidding strategy, we have the fact that

$$\int_{\tilde{b}(v_i, \alpha) - \alpha D}^{v_i} [1 - \Phi(p_i)] dp_i = D = \int_{\tilde{b}(v_i, \alpha') - \alpha' D}^{v_i} [1 - \Phi(p_i)] dp_i. \quad (\text{A31})$$

which implies

$$\tilde{b}(v_i, \alpha) - \alpha D = \tilde{b}(v_i, \alpha') - \alpha' D. \quad (\text{A32})$$

Step (ii). Given what we have in step (i.), we can establish the equivalence between $\mathbb{E}_S^I[R(r, D, \alpha)]$ and $\mathbb{E}_S^I[R(r, D, \alpha')]$ as follows

$$\begin{aligned} & \frac{\mathbb{E}_S^I[R(r, D, \alpha)]}{N} \\ &= (1 - F(\check{v}(\alpha)))Q(\check{v}(\alpha))(r + (1 - \alpha)D) + \int_{\check{v}(\alpha)}^{\check{v}(\alpha)} (1 - F(x))(b(x, \alpha) + (1 - \alpha)D)dQ(x) \\ & \quad + \int_{\hat{v}(\alpha)}^{\bar{v}} (1 - F(x)) \left[(1 - \Phi(\tilde{b}(x, \alpha) - \alpha D))(\tilde{b}(x, \alpha) - \alpha D) + D \right] dQ(x) \\ &= (1 - F(\check{v}(\alpha'))Q(\check{v}(\alpha'))(r + (1 - \alpha')D) + \int_{\check{v}(\alpha')}^{\check{v}(\alpha')} (1 - F(x))(b(x, \alpha') + (1 - \alpha')D)dQ(x) \\ & \quad + \int_{\hat{v}(\alpha')}^{\bar{v}} (1 - F(x)) \left[(1 - \Phi(\tilde{b}(x, \alpha') - \alpha' D))(\tilde{b}(x, \alpha') - \alpha' D) + D \right] dQ(x) \\ &= \frac{\mathbb{E}_S^I[R(r, D, \alpha')]}{N}. \end{aligned} \quad (\text{A33})$$

This implies that $\mathbb{E}[R(r, D, \alpha)]$ exactly equals $\mathbb{E}[R(r, D, \alpha')]$; both settings generate the same revenue to the seller.

Step (iii). The optimal $\check{v}^{so}$, $\hat{v}^{so}$, and D^{so} do not depend on the deduction proportion rule α . Therefore, if submitting negative-value bids from bidders is allowed, the maximized revenue of the seller is independent of the deduction proportion rule α . $\square$

A bidder will strategically adjust her bid as the best response to the deduction proportion rule implemented by the seller; a higher deduction proportion induces a higher bid from the bidder, that is, $b > b'$ if $\alpha > \alpha'$. Interestingly, we show that if negative-valued bids are allowed, the seller obtains exactly the same expected revenue for all α . We call this property the “deduction proportion independence.” However, in

practice, it is unlikely (if not impossible) to allow negative-valued bids. Obviously if the seller imposes a further constraint $r \geq 0$, implementing a lower α should generate a lower revenue to the seller.

Corollary B1. *If bidders can only submit non-negative bids, the maximized seller revenue is increasing in α .*

This result above indicates that the “deduction proportion independence” breaks down if a “non-negative-bid” restriction is imposed in the auction, and this implies that the exclusive deposit requirement is worse than the inclusive deposit requirement in terms of seller revenue. This provides an explanation of why the latter is relatively more popular in practice.

2 Appendix S2

This online appendix covers all the computation details in numerical examples 1 and 2.

2.1 Numerical example 1

Taking $N = 2$ and $v_i \sim U[0, 1]$, $p_i \sim U[0, 1]$, we have the following two parts for the computations.

Case (I) where $r \leq D$. In the second-price auction, $\check{v}$ and $\hat{v}$ for any given r and D can be computed by the following two equations:

$$r = \check{v} - \frac{1}{2}\check{v}^2, \quad \text{and} \quad D = \hat{v} - \frac{1}{2}\hat{v}^2.$$

Note that the optimal r^{so} and D^{so} can be computed by $\hat{v}^{so} = 1$ and $(1 - \check{v}^{so})(1 - \check{v}^{so}) = \int_0^{\hat{v}^{so}} [1 - p_i] dp_i$.

Bidder i 's bidding strategy can be simplified as follows:

$$b(v_i, r, D) = \begin{cases} (D+1) - \left[-2v_i + v_i^2 + (D+1)^2 - D^2 \right]^{\frac{1}{2}} & \text{if } v_i > \hat{v}; \\ v_i - \frac{1}{2}v_i^2 & \text{if } v_i \in [\check{v}, \hat{v}]; \\ No & \text{if } v_i < \check{v}. \end{cases}$$

The associated seller's revenue function is given by

$$\begin{aligned} \mathbb{E}_S^I[R(r, D)] &= 2(1 - \check{v})\check{v}r + 2 \int_{\check{v}}^{\hat{v}} (1 - v_i)b(v_i, r, D)dv_i \\ &\quad + 2 \int_{\hat{v}}^1 (1 - v_i) \left[\left(1 - (\tilde{b}(v_i, r, D) - D) \right) \tilde{b}(v_i, r, D) + (\tilde{b}(v_i, r, D) - D)D \right] dv_i \\ &= 2(1 - \check{v})\check{v}r + \hat{v}^2 - \check{v}^2 - \hat{v}^3 + \check{v}^3 + \frac{\hat{v}^4}{4} - \frac{\check{v}^4}{4} \\ &\quad + 2 \int_{\hat{v}}^1 (1 - v_i) \left[\sqrt{v_i^2 - 2v_i + 2D + 1 - v_i^2 + 2v_i - D - 1} \right] dv_i, \end{aligned}$$

where $\check{v} = 1 - \sqrt{1 - 2r}$ and $\hat{v} = 1 - \sqrt{1 - 2D}$.

Case (II) where $r \geq D$. In a second-price auction, we can compute $\check{v}$ for any given r and D by the following equation:

$$r = \check{v} - \left[\frac{1}{2}\check{v}^2 - \frac{1}{2}(r - D)^2 \right].$$

The equilibrium bidding strategy can be simplified as follows:

$$b(v_i, r, D) = \begin{cases} (D+1) - \left[(v_i - 1)^2 + 2D \right]^{\frac{1}{2}} & \text{if } v_i \geq \check{v}; \\ No & \text{if } v_i < \check{v}. \end{cases}$$

The associated seller's revenue function is given by

$$\begin{aligned} \mathbb{E}_S^{II}[R(r, D)] &= 2(1 - \check{v})\check{v} \left[\left(1 - (r - D) \right) r + (r - D)D \right] \\ &\quad + 2 \int_{\check{v}}^{\bar{v}} (1 - v_i) \left[\left(1 - (b(v_i, r, D) - D) \right) b(v_i, r, D) + (b(v_i, r, D) - D)D \right] dv_i \\ &= 2(1 - \check{v})\check{v} \left[\left(1 - (r - D) \right) r + (r - D)D \right] \\ &\quad + 2 \int_{\check{v}}^1 (1 - v_i) \left[\sqrt{2D + 1 - 2v_i + v_i^2} - D - 1 + 2v_i - v_i^2 \right] dv_i, \end{aligned}$$

where $\check{v} = 1 - \sqrt{1 - 4r + 2D - (r - D)^2}$.

Our computations show $r^{so} = 0.33333$ and $D^{so} = 0.5$.⁵ Moreover, when the seller sets $r \geq 0.5$, no valid bids can be submitted, and the seller's revenue decreases to zero. Given the deposit $D (= 0.2, 0.33333, 0.4)$, Figure 5 depicts how seller revenue functions in Cases (I) and (II) change in r . In each figure, the horizontal axes is reserve price r and the vertical axes is seller revenue $E_s(R)$. $E_s^I(R)$ is depicted by the blue curve and $E_s^{II}(R)$ by the red curve. Consistent with these properties established in Lemmas 7 and 8 and Proposition 2, our simulation results show that in Figure 5(a), given $D > r^{so}$, the maximized seller revenue is given by $E_s^I(R)$ with reserve price r^{so} . With $D = 0.33333$, both Cases (I) and (II) give the same maximized seller revenue with reserve price r^{so} in Figure 5(b). When $D = 0.2$, the maximized seller revenue is achieved under Case (II) with some reserve price less than r^{so} .

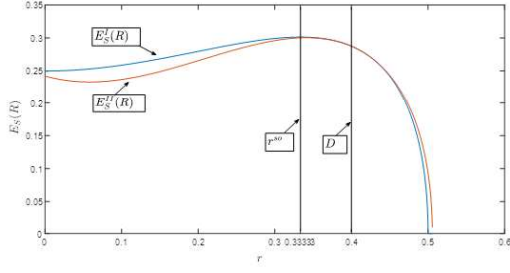
Figure 6 depicts the property characterized in Proposition 3; the seller's optimal revenue function $R^*(D)$ in (16) is increasing in D for $D \in [0, D^{so}]$ and becomes flat for any $D > D^{so}$. The global maximum seller revenue ($= 0.30157$) is achieved at $r = 0.33333$ and $D^{so} = 0.5$.

2.2 Numerical example 2

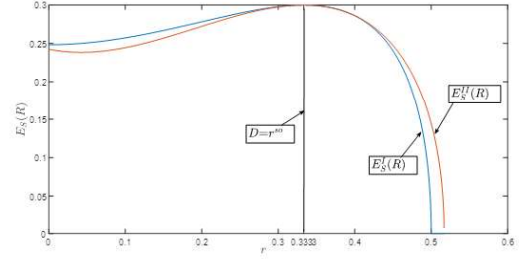
Taking $N = 2$ and $v_i \sim U[0, 1]$, and $\Phi(p_i) = p_i^\alpha$ over $[0, 1]$, we have the following two parts for the computations. First, the optimal D^{so} and r^{so} are given by

$$D^{so} = \hat{v}^{so} - \frac{(\hat{v}^{so})^{\alpha+1}}{\alpha+1} \quad \text{and} \quad r^{so} = \check{v}^{so} - \frac{(\check{v}^{so})^{\alpha+1}}{\alpha+1},$$

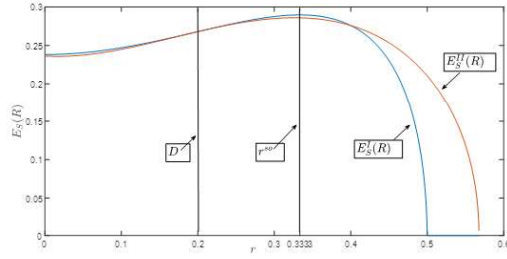
⁵Note that if no post-auction outside offer exists, i.e., $\Phi(\cdot) = 0$, the optimal reserve price is 0.5.



(a) $D = 0.4$



(b) $D = 0.33333$



(c) $D = 0.2$

Figure 5: $E_s^I(R)$ and $E_s^{II}(R)$ change in r

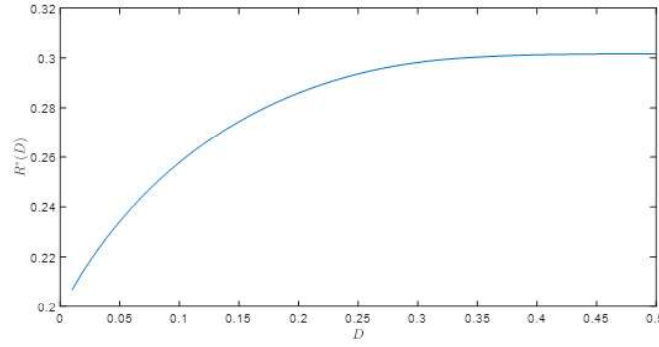


Figure 6: $R^*(D)$ changes in D

where $\hat{v}^{so} = \bar{v} = 1$ and $(1 - \check{v}^{so})(1 - (\check{v}^{so})^\alpha) = \check{v}^{so} - \frac{(\check{v}^{so})^{\alpha+1}}{\alpha+1}$. Then, we can compute the seller's optimal revenue with D^{so} and r^{so} , which is given by

$$\begin{aligned} R^* &= 2(1 - \check{v}^{so})\check{v}^{so}r^{so} + 2 \int_{\check{v}^{so}}^1 (1 - v_i)b(v_i, r^{so}, D^{so})dv_i. \\ &= 2(1 - \check{v}^{so})\check{v}^{so}r^{so} + 2 \int_{\check{v}^{so}}^1 (1 - v_i) \left(v_i - \frac{v_i^{\alpha+1}}{\alpha+1} \right) dv_i. \end{aligned}$$

When $D = 0$, it is easy to check that the equilibrium bidding is $b(v_i) = v_i$, and the optimal $r^*(D = 0)$ is given by

$$(1 - r^*) \left(1 - (1 + \alpha)(r^*)^\alpha \right) - \left(1 - (r^*)^\alpha \right) r^* = 0.$$

The seller's revenue with $D = 0$ and $r^*(D = 0)$, which corresponds to the case of $r \geq D$, can be then written as follows:

$$\mathbb{E}_S^{II}[R(r^*)] = 2(1 - r^*)r^* \left[r^* - (r^*)^{(\alpha+1)} \right] + 2 \int_{r^*}^1 (1 - v_i) \left(v_i - v_i^{(\alpha+1)} \right) dv_i.$$

We then compute how a change in α affects the sellers' choices regarding the optimal reserve price r^{so} , the optimal deposit D^{so} , and the optimal expected revenue R^* . Figure 7 illustrates how r^{so} and D^{so} change with α . Consistent with our theoretical predictions, the figures show that when α increases, which corresponds to a worse chance for the outside option, both r^{so} and D^{so} increase.

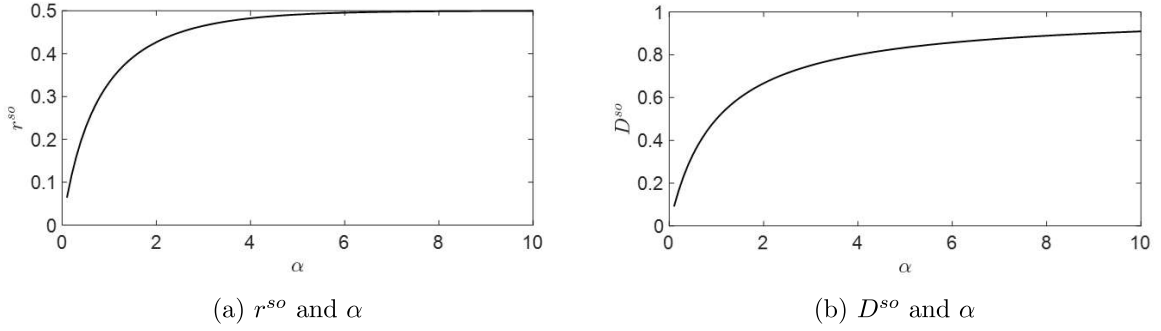


Figure 7: The impact of α on r^{so} and D^{so}

Figure 8 depicts how R^* changes with α . Again, consistent with the theoretical prediction, an increase in α generates a higher R^* , which is captured by the black curve. To illustrate the role of a deposit in the auction with a post-auction outside offer, we compute and plot how seller revenue with $D = 0$ and the associated optimal reserve price, denoted by $r^*(D = 0)$, changes with α . As depicted by the red dotted curve, seller revenue increases in α as well. Interestingly, the difference between the two revenues is not monotonic. The improvement in seller revenue after charging a deposit is small when the likelihood of having an outside option, captured by α , is either too small or too large, but a relatively large improvement is achieved when the likelihood is in the medium range.

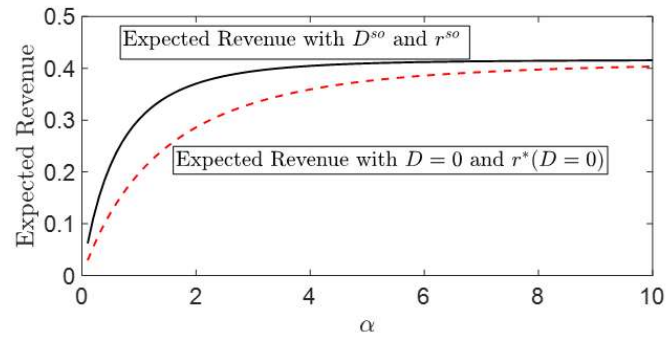


Figure 8: The relationship between R^* and α