Online Appendix

# Fairness through the Lens of Cooperative Game Theory: An Experimental Approach

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## A Details for Table 4

Looking at payoffs in Table 3, we see that equal split, ESS, and the S-CEA violate the dummy player property for CF1.

R1 is more desirable than R2 in CF6, yet receives the same payoff under equal split and constrained egalitarianism. Hence these solutions violate Desirability. The Nucleolus does not contradict Desirability over CF1-CF7, but violates the axiom in general: e.g., it selects $(30, 30, 30)$ when $v(\{1, 2, 3\}) = 90$, $v(\{1, 2\}) = 10$ and $v(S) = 0$ for all other coalition $S \subset \{1, 2, 3\}$. In either case, a weaker form of desirability is usually satisfied by the nucleolus: a more desirable player does not get strictly less money (Maschler and Peleg, 1966). The situation is more complex for the core, given that it is multi-valued. For instance, the payoff vector $(80, 0, 10)$ belongs to the core of the characteristic function in the example above, but gives strictly more to player 3 than the more desirable player 2. Still, a non-empty core always contains some payoff vectors respecting desirability.[30]

Symmetry is violated by S-CEA in CF5: R1 and R2, though symmetric, receive different payoffs. The core could violate Symmetry in CF1 and CF6, but only because it is set-valued. A weaker symmetry property would hold: if $i$ and $j$ are symmetric and $x$ belongs to the core, then the modified payoff vector where $i$'s and $j$'s payoffs are switched also belongs to the core. The W-CEA satisfies Symmetry, but only vacuously in the case of CF2 and CF3, where R1 and R2 are symmetric but W-CEA is empty-valued.

[30] The nucleolus belongs to the core and respects the weaker form of desirability discussed above. It is not difficult to check that, for any three-player characteristic function where it violates desirability, one can find a nearby payoff vector in the core respecting it.

As for Monotonicity, ES violates the requirement that R3 gets strictly more in CF7 than CF1. The Shapley value and its mixtures with ES satisfy it. Being multi-valued, it is not surprising that the core violates monotonicity as well: for instance, it violates the requirement that R1 get strictly more in CF6 than CF2. The nucleolus violates that same requirement. sCEA and wCEA do not contradict Monotonicity within the context of CF1-7, but could violate the property in general. For instance, mildly increasing the worth of coalition $\{1, 2\}$ in CF6 would have no impact on the constrained egalitarian solutions, in violation of Monotonicity. Of course, one could also be interested in relaxations of monotonicity, e.g. asking that no player gets a strictly lower payoff when the worth of a coalition he belongs to increases, or asking that the sum of its members' payoffs increases. Multiple papers have investigated these notions for core selections and the nucleolus in particular (see Megiddo (1974), Young (1985) and Housman and Clark (1998) among others).

Equal split, the Shapley value, and a fortiori any convex combination of these solutions are all additive. All other solutions fail to be additive. This can be seen from Table 3 for the nucleolus and S-CEA: CF6 is equal to CF2 plus the characteristic function where where only the grand coalition creates surplus, in the amount of \$30; yet adding \$10 to each recipient's payoff in CF2 does not give the corresponding payoff for CF6. Though not additive, the core is super-additive in general, and additive on subclasses of characteristic functions; see Bloch and de Clippel (2010).

# B Proof of Observation 1

The sufficient condition follows at once from the fact that both the Shapley value and the equal split solution satisfy Efficiency, Symmetry, and Additivity.

We thus focus on the necessary condition. Additivity implies that, for any characteristic function $v$, $\sigma(2v) = \sigma(v + v) = \sigma(v) + \sigma(v) = 2\sigma(v)$. Similarly, $\sigma(v) = \sigma(\frac{v+v}{2}) = \sigma(\frac{v}{2}) + \sigma(\frac{v}{2}) = 2\sigma(\frac{v}{2})$, and hence $\sigma(\frac{v}{2}) = \frac{\sigma(v)}{2}$. Also, $\sigma(0) = 0$, by Symmetry and Efficiency. Hence $0 = \sigma(v-v) = \sigma(v)+\sigma(-v)$, and $\sigma(-v) = -\sigma(v)$. It is easy to extend such arguments to conclude that $\sigma$ must be linear

with respect to the field of rational numbers: $\sigma(\alpha v + \beta v') = \alpha\sigma(v) + \beta\sigma(w)$, for all rational numbers $\alpha$ and $\beta$, and all characteristic functions $v$ and $w$.

For each $S = \{1,2\}$, $\{1,3\}$, $\{2,3\}$, and $\{1,2,3\}$, let $v_S$ be the characteristic function such that the worth of a coalition is positive if and only if it contains $S$, in which case its worth is 1. The collection of vectors $< v_{\{1,2\}}, v_{\{1,3\}}, v_{\{2,3\}}, v_{\{1,2,3\}} >$ forms a basis of $V$ (understood as vector space over the field of rational numbers). By Symmetry and Efficiency, $\sigma(v_{\{1,2,3\}}) = (1/3, 1/3, 1/3)$, $\sigma(v_{\{1,2\}}) = (x, x, 1-2x)$, $\sigma(v_{\{1,3\}}) = (y, 1-2y, y)$, $\sigma(v_{\{2,3\}}) = (1-2z, z, z)$, for some real numbers $x, y, z$. Linearity implies that $\sigma(v_{\{1,2\}} + v_{\{1,3\}}) = (x+y, 1-2y+x, 1-2x+y)$. Notice that players 2 and 3 are symmetric in $v_{\{1,2\}} + v_{\{1,3\}}$, and hence $1-2y+x = 1-2x+y$, or $x = y$. A similar reasoning implies that $x = z$. Any game $v$ can be rewritten as $v(\{1,2\})v_{\{1,2\}} + v(\{1,3\})v_{\{1,3\}} + v(\{2,3\})v_{\{2,3\}} + (v(\{1,2,3\}) - v(\{1,2\}) - v(\{1,3\}) - v(\{2,3\}))v_{\{1,2,3\}}$. By linearity (notice that coefficients are indeed rational numbers given that $v \in V$), we conclude that

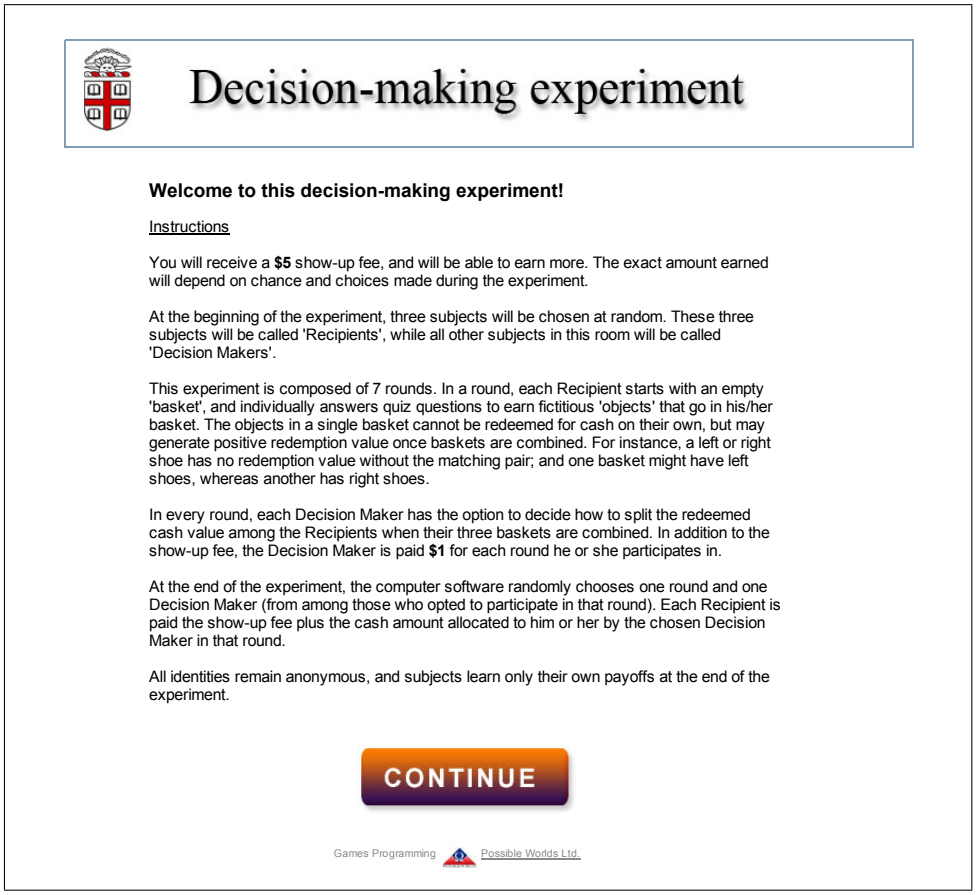
$$\sigma(v) = (x, x, 1-2x)v(\{1,2\}) + (x, 1-2x, x)v(\{1,3\}) + (1-2x, x, x)v(\{2,3\}) + (1/3, 1/3, 1/3)(v(\{1,2,3\}) - v(\{1,2\}) - v(\{1,3\}) - v(\{2,3\})).$$

It follows by simple algebra that $\sigma = \delta Sh + (1-\delta)ES$, where $\delta = 6x - 2$. $\square$

# C Instructions via computer interface

The following are successive screenshots of the experimental instructions subjects received through the experimental interface, which was accessible at each computer terminal.

## C.1 Quiz treatment



# Decision-making experiment

**Welcome to this decision-making experiment!**

Instructions

You will receive a **$5** show-up fee, and will be able to earn more. The exact amount earned will depend on chance and choices made during the experiment.

At the beginning of the experiment, three subjects will be chosen at random. These three subjects will be called 'Recipients', while all other subjects in this room will be called 'Decision Makers'.

This experiment is composed of 7 rounds. In a round, each Recipient starts with an empty 'basket', and individually answers quiz questions to earn fictitious 'objects' that go in his/her basket. The objects in a single basket cannot be redeemed for cash on their own, but may generate positive redemption value once baskets are combined. For instance, a left or right shoe has no redemption value without the matching pair; and one basket might have left shoes, whereas another has right shoes.

In every round, each Decision Maker has the option to decide how to split the redeemed cash value among the Recipients when their three baskets are combined. In addition to the show-up fee, the Decision Maker is paid **$1** for each round he or she participates in.

At the end of the experiment, the computer software randomly chooses one round and one Decision Maker (from among those who opted to participate in that round). Each Recipient is paid the show-up fee plus the cash amount allocated to him or her by the chosen Decision Maker in that round.

All identities remain anonymous, and subjects learn only their own payoffs at the end of the experiment.

CONTINUE

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# Decision-making experiment

Terminal: **1**

What happens if you are chosen to be a Recipient?

There are 7 rounds. You begin each round with an empty basket. You can earn (fictitious) objects that accumulate in your basket by answering quiz questions. As an example, you may earn a 'left shoe' if you correctly answer a question, a 'right glove' if you correctly answer another, etc.

Here is an example of a question a Recipient could face in a given round:

**Question**
Who was the 2012 U.S. vice-presidential Republican nominee?

- Mitt Romney
- Ron Paul
- Joe Biden
- Paul Ryan

All quiz questions are multiple choice. Quiz questions could also be logic puzzles, math questions, etc. You will be informed whether your answer is correct. You may have multiple opportunities within a round to earn objects.

At the end of each round, each Recipient sees his own basket and the two other Recipients' baskets. They also learn the cash amounts that can be generated by combining baskets. Here is an example (note that both the objects and their values may differ from round to round).

**In this round, Recipients earned the following baskets of objects:**
Recipient 1: two left shoes, one left glove, and one leg of a tripod.
Recipient 2: one right shoe, two right gloves, and one leg of a tripod.
Recipient 3: one right shoe and one leg of a tripod.

**The redemption value of two or more baskets is calculated by summing up the value of different object combinations. These values are:**
Each matching pair (left and right) of shoes is worth $15.
Each matching pair (left and right) of gloves is worth $10.
Three legs of a tripod are together worth $5.
No other combination of objects has any value.

**Therefore, the values of all the possible basket combinations are:**
Recipients 1 and 2's baskets together have a value of $25.
Recipients 1 and 3's baskets together have a value of $15.
Recipients 2 and 3's baskets together have a value of $0.
Recipients 1, 2 and 3's baskets together have a value of $45.

CONTINUE

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# Decision-making experiment

Terminal: **1**

What if you are a Decision Maker?

At the end of each round, each Recipient has a basket of objects which they earned by answering quiz questions. These baskets must be redeemed in combination for cash. As a Decision Maker, you have the opportunity to decide how to split the proceeds among those Recipients when all three baskets are combined. You are paid $**1** for each round in which you choose to fill in the corresponding payoffs for the Recipients.

So that you do not have to calculate the redemption value of baskets, we present you those values immediately. The next screen gives an example of what you will see in a given round, and how you input your decision.

To ensure anonymity, neither your identity nor the Recipients' identities are revealed. In addition, Recipients are only identified in each round by a randomly drawn number (1, 2, or 3). That number is re-drawn in each round.

CONTINUE

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# Decision-making experiment

Terminal: **1**

Example screen for the Decision Maker

**Given the objects each Recipient earned by answering quiz questions correctly, we have the following results:**

- If only Recipients 1 and 2's baskets are combined, then Recipients 1 and 2 could share $25.
- If only Recipients 1 and 3's baskets are combined, then Recipients 1 and 3 could share $15.
- If only Recipients 2 and 3's baskets are combined, then Recipients 2 and 3 could share $0.
- If all three baskets are combined, then Recipients 1, 2 and 3 could share $45.

We request that all three baskets be combined since doing so generates the most money to be shared. However, you can decide how to split the proceeds as you see fit.

**Your options will appear in a moment.**

○ I do not wish to participate in this round, and understand that I will not receive $1 for this round.

○ I wish to participate in this round, and earn $1 for my decision. All three baskets must be redeemed together, and all proceeds must be distributed.

CONTINUE

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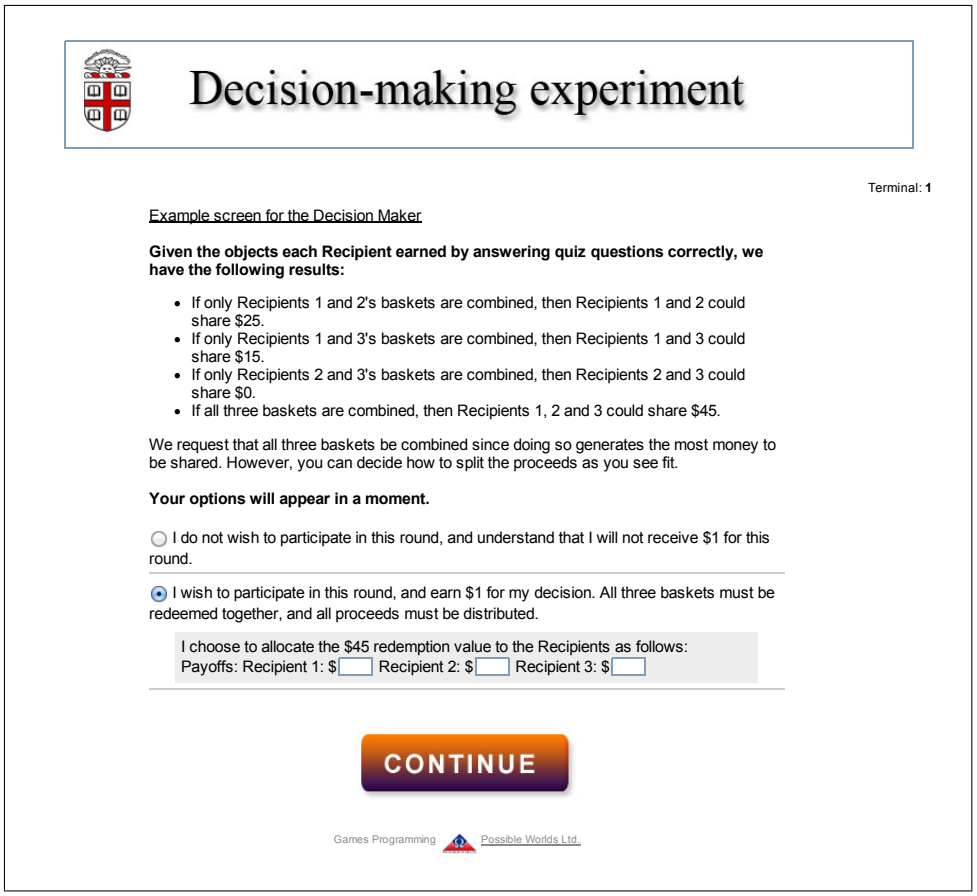
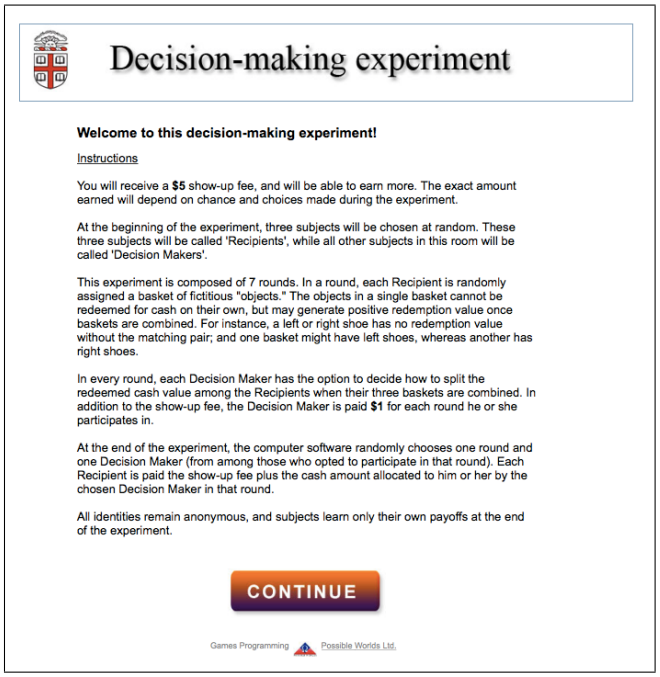
In the screen above, the value of different object combinations appear sequentially, with a small delay between lines. The options seen in the screen below appear only when selecting the second radio button. The input boxes for payoffs can be accessed in any order. Once two payoff boxes are filled out, the third is automatically filled with the remaining amount. Entries are required to be nonnegative.



## C.2 No-Quiz treatment



Decision-making experiment

**Welcome to this decision-making experiment!**

Instructions

You will receive a **$5** show-up fee, and will be able to earn more. The exact amount earned will depend on chance and choices made during the experiment.

At the beginning of the experiment, three subjects will be chosen at random. These three subjects will be called 'Recipients', while all other subjects in this room will be called 'Decision Makers'.

This experiment is composed of 7 rounds. In a round, each Recipient is randomly assigned a basket of fictitious "objects." The objects in a single basket cannot be redeemed for cash on their own, but may generate positive redemption value once baskets are combined. For instance, a left or right shoe has no redemption value without the matching pair; and one basket might have left shoes, whereas another has right shoes.

In every round, each Decision Maker has the option to decide how to split the redeemed cash value among the Recipients when their three baskets are combined. In addition to the show-up fee, the Decision Maker is paid **$1** for each round he or she participates in.

At the end of the experiment, the computer software randomly chooses one round and one Decision Maker (from among those who opted to participate in that round). Each Recipient is paid the show-up fee plus the cash amount allocated to him or her by the chosen Decision Maker in that round.

All identities remain anonymous, and subjects learn only their own payoffs at the end of the experiment.

CONTINUE

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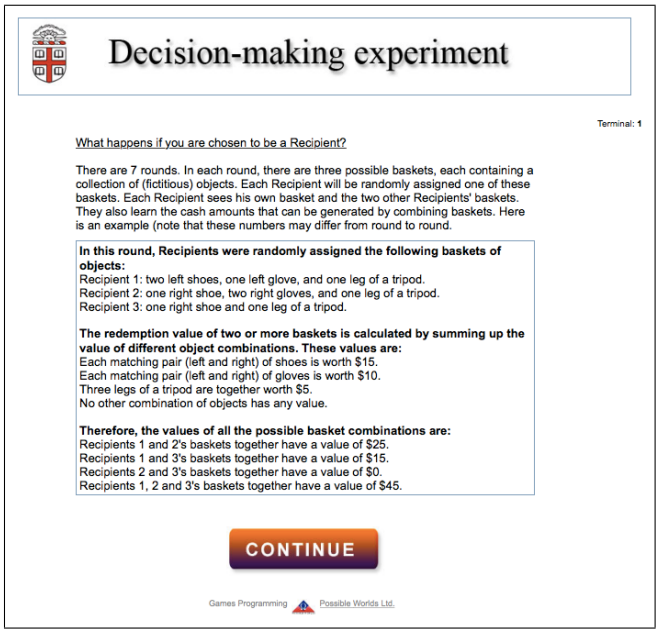

Decision-making experiment

Terminal: **1**

What happens if you are chosen to be a Recipient?

There are 7 rounds. In each round, there are three possible baskets, each containing a collection of (fictitious) objects. Each Recipient will be randomly assigned one of these baskets. Each Recipient sees his own basket and the two other Recipients' baskets. They also learn the cash amounts that can be generated by combining baskets. Here is an example (note that these numbers may differ from round to round.

**In this round, Recipients were randomly assigned the following baskets of objects:**
Recipient 1: two left shoes, one left glove, and one leg of a tripod.
Recipient 2: one right shoe, two right gloves, and one leg of a tripod.
Recipient 3: one right shoe and one leg of a tripod.

**The redemption value of two or more baskets is calculated by summing up the value of different object combinations. These values are:**
Each matching pair (left and right) of shoes is worth $15.
Each matching pair (left and right) of gloves is worth $10.
Three legs of a tripod are together worth $5.
No other combination of objects has any value.

**Therefore, the values of all the possible basket combinations are:**
Recipients 1 and 2's baskets together have a value of $25.
Recipients 1 and 3's baskets together have a value of $15.
Recipients 2 and 3's baskets together have a value of $0.
Recipients 1, 2 and 3's baskets together have a value of $45.

CONTINUE

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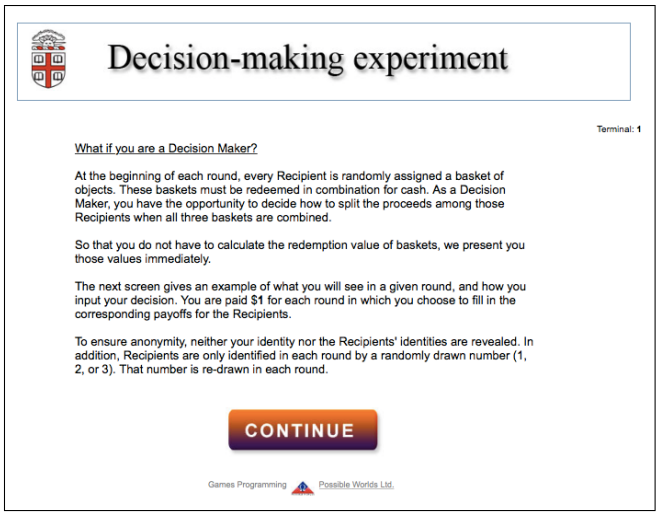

Decision-making experiment

Terminal: 1

What if you are a Decision Maker?

At the beginning of each round, every Recipient is randomly assigned a basket of objects. These baskets must be redeemed in combination for cash. As a Decision Maker, you have the opportunity to decide how to split the proceeds among those Recipients when all three baskets are combined.

So that you do not have to calculate the redemption value of baskets, we present you those values immediately.

The next screen gives an example of what you will see in a given round, and how you input your decision. You are paid $1 for each round in which you choose to fill in the corresponding payoffs for the Recipients.

To ensure anonymity, neither your identity nor the Recipients' identities are revealed. In addition, Recipients are only identified in each round by a randomly drawn number (1, 2, or 3). That number is re-drawn in each round.

CONTINUE

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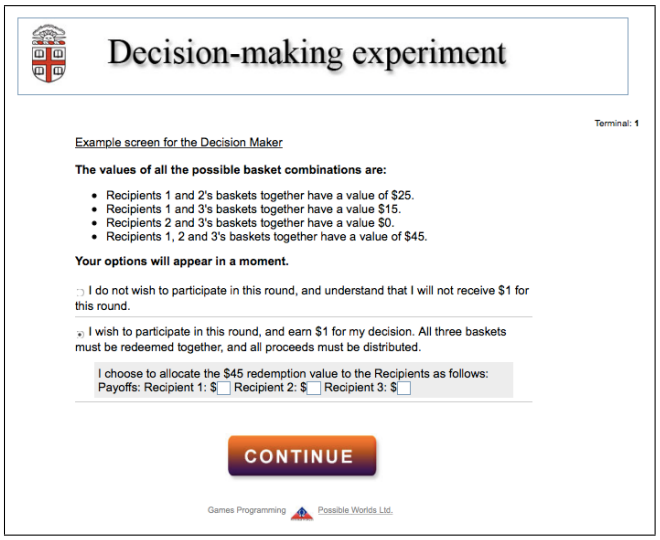
The input screen has the same features as in the other treatments.



Decision-making experiment

Terminal: 1

Example screen for the Decision Maker

**The values of all the possible basket combinations are:**

- Recipients 1 and 2's baskets together have a value of $25.
- Recipients 1 and 3's baskets together have a value $15.
- Recipients 2 and 3's baskets together have a value $0.
- Recipients 1, 2 and 3's baskets together have a value of $45.

**Your options will appear in a moment.**

I do not wish to participate in this round, and understand that I will not receive $1 for this round.

I wish to participate in this round, and earn $1 for my decision. All three baskets must be redeemed together, and all proceeds must be distributed.

I choose to allocate the $45 redemption value to the Recipients as follows:
Payoffs: Recipient 1: $ Recipient 2: $ Recipient 3: $

CONTINUE

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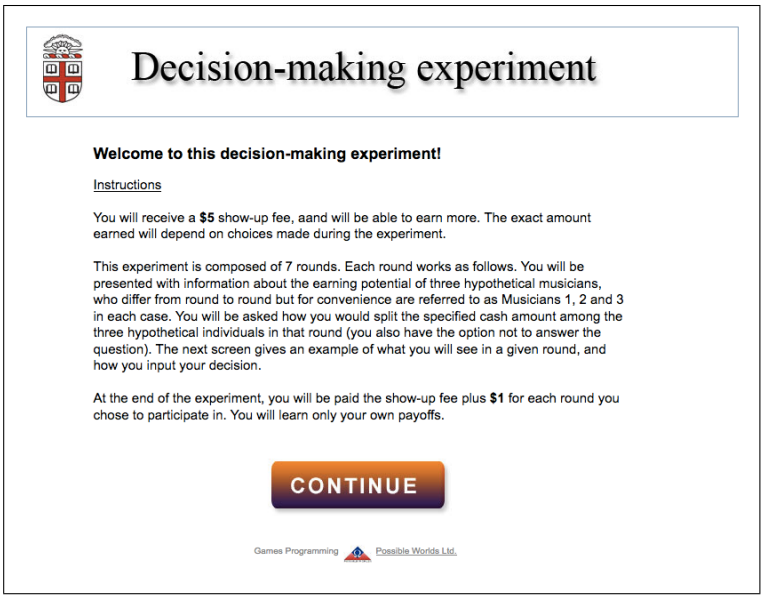
## C.3 Vignettes treatment

The input screen has the same features as in the other treatments.



Decision-making experiment

**Welcome to this decision-making experiment!**

Instructions

You will receive a **$5** show-up fee, aand will be able to earn more. The exact amount earned will depend on choices made during the experiment.

This experiment is composed of 7 rounds. Each round works as follows. You will be presented with information about the earning potential of three hypothetical musicians, who differ from round to round but for convenience are referred to as Musicians 1, 2 and 3 in each case. You will be asked how you would split the specified cash amount among the three hypothetical individuals in that round (you also have the option not to answer the question). The next screen gives an example of what you will see in a given round, and how you input your decision.

At the end of the experiment, you will be paid the show-up fee plus **$1** for each round you chose to participate in. You will learn only your own payoffs.

CONTINUE

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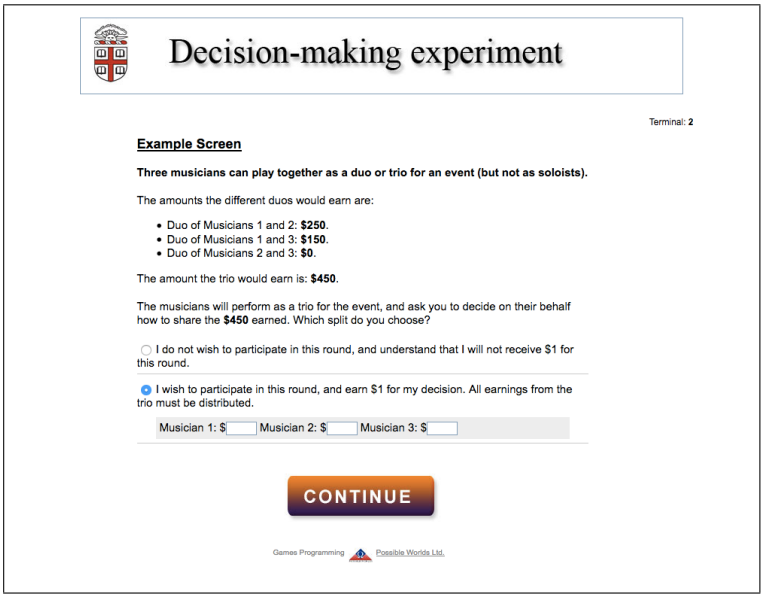

Decision-making experiment

Terminal: **2**

**Example Screen**

**Three musicians can play together as a duo or trio for an event (but not as soloists).**

The amounts the different duos would earn are:

- Duo of Musicians 1 and 2: **$250**.
- Duo of Musicians 1 and 3: **$150**.
- Duo of Musicians 2 and 3: **$0**.

The amount the trio would earn is: **$450**.

The musicians will perform as a trio for the event, and ask you to decide on their behalf how to share the **$450** earned. Which split do you choose?

I do not wish to participate in this round, and understand that I will not receive $1 for this round.

I wish to participate in this round, and earn $1 for my decision. All earnings from the trio must be distributed.

Musician 1: $ Musician 2: $ Musician 3: $

CONTINUE

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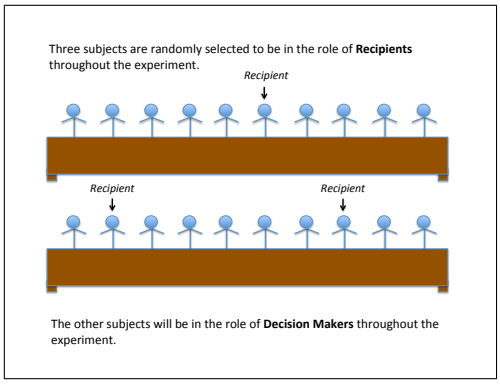
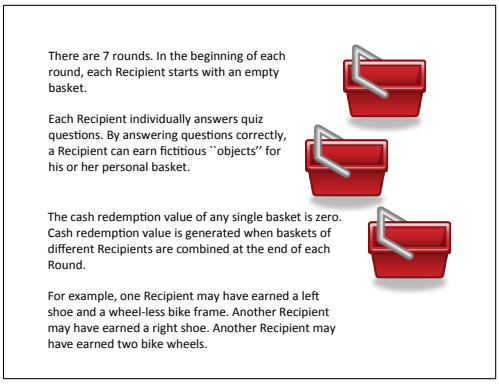
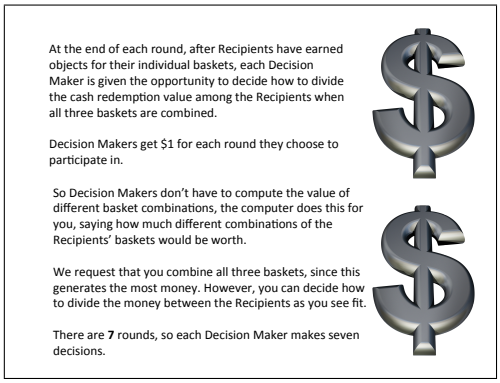
# D Instructions projected on screen

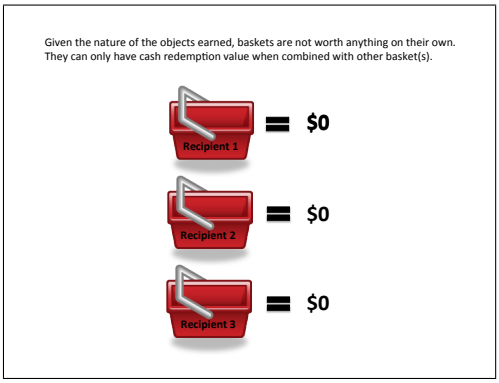
After subjects read the instructions on the computer, the session supervisor in the Quiz and No-Quiz treatments gives a graphical presentation of the instructions. The screenshots should be read left to right, starting at the top.

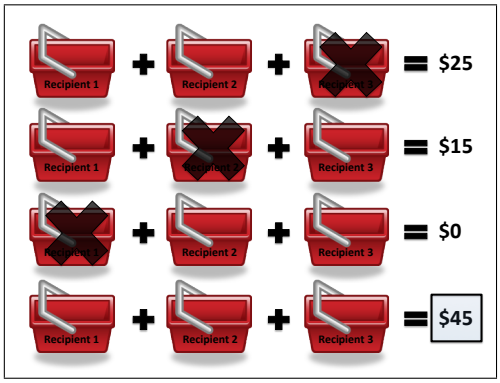
## D.1 Quiz treatment

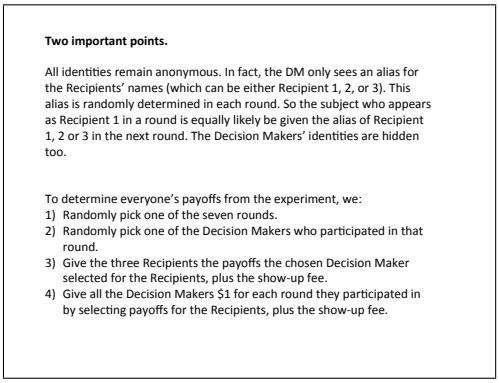


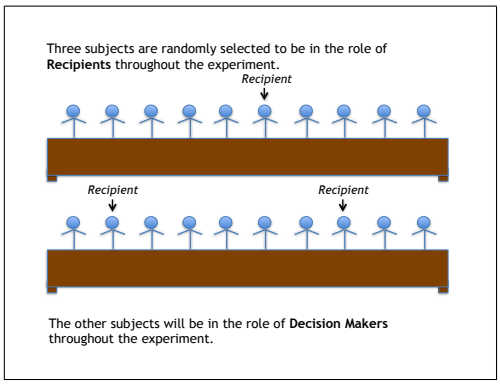



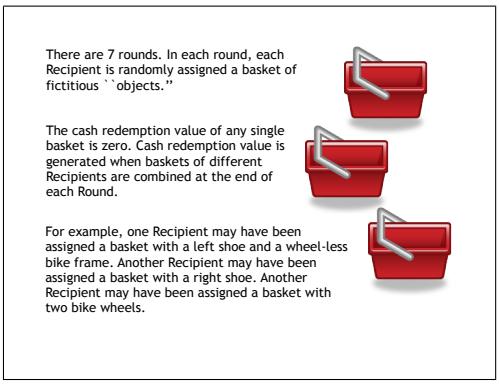



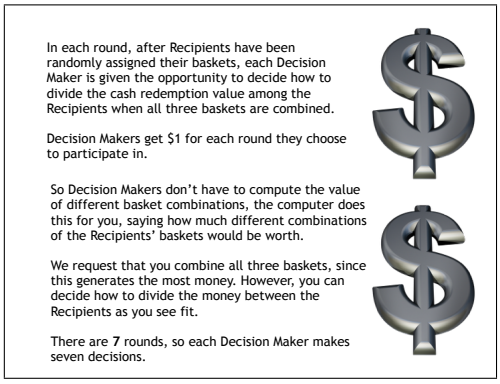



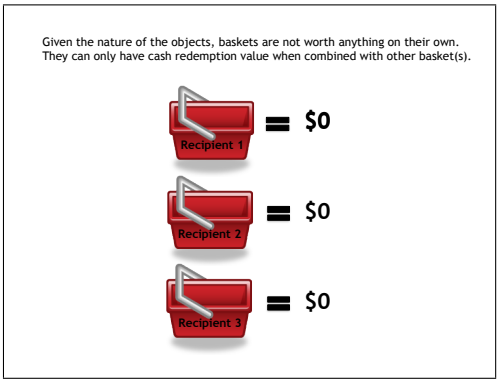



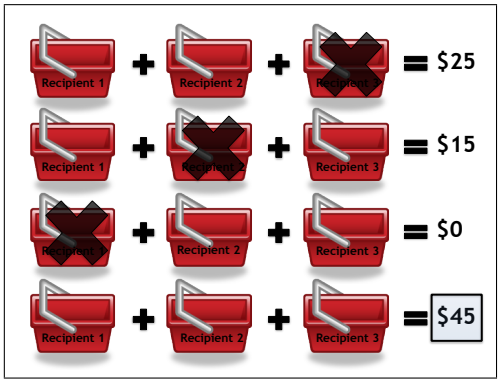
## D.2 No-Quiz treatment

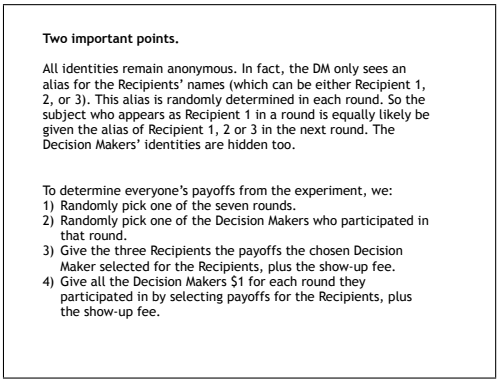


# E Latin square design

Below we detail the ordering of characteristic functions in the six sessions. Round entries identify the characteristic function using the scheme from Table 1. Notice each possible pair from CF1-6 is adjacent in some session.

| | Round | | | | | | |
|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
| **Session 1** | 1 | 6 | 2 | 5 | 3 | 4 | 7 |
| **Session 2** | 2 | 1 | 3 | 6 | 4 | 5 | 7 |
| **Session 3** | 3 | 2 | 4 | 1 | 5 | 6 | 7 |
| **Session 4** | 4 | 3 | 5 | 2 | 6 | 1 | 7 |
| **Session 5** | 5 | 4 | 6 | 3 | 1 | 2 | 7 |
| **Session 6** | 6 | 5 | 1 | 4 | 2 | 3 | 7 |

Table 7: Latin square design. The row entries in the table identify the ordering of CF's used in that session.

# F Descriptive statistics for all treatments

We include again the Quiz treatment results for completeness.

| | **CF1** | **CF2** | **CF3** | **CF4** | **CF5** | **CF6** | **CF7** |
|---|---|---|---|---|---|---|---|
| **Quiz** | 40.96% | 23.81% | 18.07% | 57.83% | 65.43% | 20.73% | 100% |
| **No Quiz** | 51.43% | 38.46% | 35.58% | 50.96% | 60.58% | 39.05% | 100% |
| **Vignettes** | 25.88% | 12.94% | 11.76% | 31.76% | 35.29% | 20.24% | 100% |

Table 8: Percent of payoff allocations that are 'equal splits'.

| Quiz Treatment | **CF1** | **CF2** | **CF3** | **CF4** | **CF5** | **CF6** | **CF7** |
|---|---|---|---|---|---|---|---|
| **Recipient 1** | \$24.30 | \$17.70 | \$19.07 | \$34.02 | \$10.51 | \$27.71 | \$20 |
| | (0.73) | (0.73) | (0.52) | (0.70) | (0.23) | (0.70) | (0) |
| **Recipient 2** | \$24.36 | \$11.42 | \$15.22 | \$29.04 | \$11.09 | \$21.57 | \$20 |
| | (0.75) | (0.51) | (0.33) | (0.52) | (0.26) | (0.57) | (0) |
| **Recipient 3** | \$11.34 | \$10.88 | \$15.70 | \$26.94 | \$8.41 | \$20.72 | \$20 |
| | (1.05) | (0.45) | (0.47) | (0.54) | (0.33) | (0.53) | (0) |
| *Observations* | 83 | 84 | 83 | 83 | 81 | 82 | 79 |
| No-Quiz Treatment | **CF1** | **CF2** | **CF3** | **CF4** | **CF5** | **CF6** | **CF7** |
| **Recipient 1** | \$22.94 | \$18.33 | \$19.64 | \$33.84 | \$10.73 | \$28.81 | \$20 |
| | (0.78) | (0.88) | (0.58) | (0.97) | (0.22) | (1.02) | (0) |
| **Recipient 2** | \$24.06 | \$11.15 | \$15.05 | \$29.96 | \$11.09 | \$19.78 | \$20 |
| | (0.84) | (0.53) | (0.45) | (0.87) | (0.24) | (0.57) | (0) |
| **Recipient 3** | \$13.00 | \$10.53 | \$15.31 | \$26.20 | \$8.18 | \$21.41 | \$20 |
| | (1.00) | (0.50) | (0.42) | (0.92) | (0.38) | (0.61) | (0) |
| *Observations* | 105 | 104 | 104 | 104 | 104 | 105 | 104 |
| Vignettes Treatment | **CF1** | **CF2** | **CF3** | **CF4** | **CF5** | **CF6** | **CF7** |
| **Recipient 1** | \$248.46 | \$216.87 | \$222.32 | \$373.53 | \$114.89 | \$330.58 | \$200 |
| | (6.38) | (8.84) | (5.90) | (8.19) | (2.70) | (11.92) | (0) |
| **Recipient 2** | \$256.11 | \$89.33 | \$141.07 | \$295.88 | \$116.18 | \$182.32 | \$200 |
| | (7.15) | (4.87) | (3.86) | (5.04) | (2.64) | (6.26) | (0) |
| **Recipient 3** | \$95.44 | \$93.80 | \$136.61 | \$230.59 | \$68.93 | \$187.10 | \$200 |
| | (8.32) | (5.47) | (3.26) | (7.87) | (3.71) | (7.04) | (0) |
| *Observations* | 85 | 85 | 85 | 85 | 85 | 84 | 85 |

Table 9: Average amounts allocated to Recipients per characteristic function, in each treatment (after dropping the 5/7/0 outliers in the Quiz/No-Quiz/Vignettes treatments), with standard errors in parentheses.

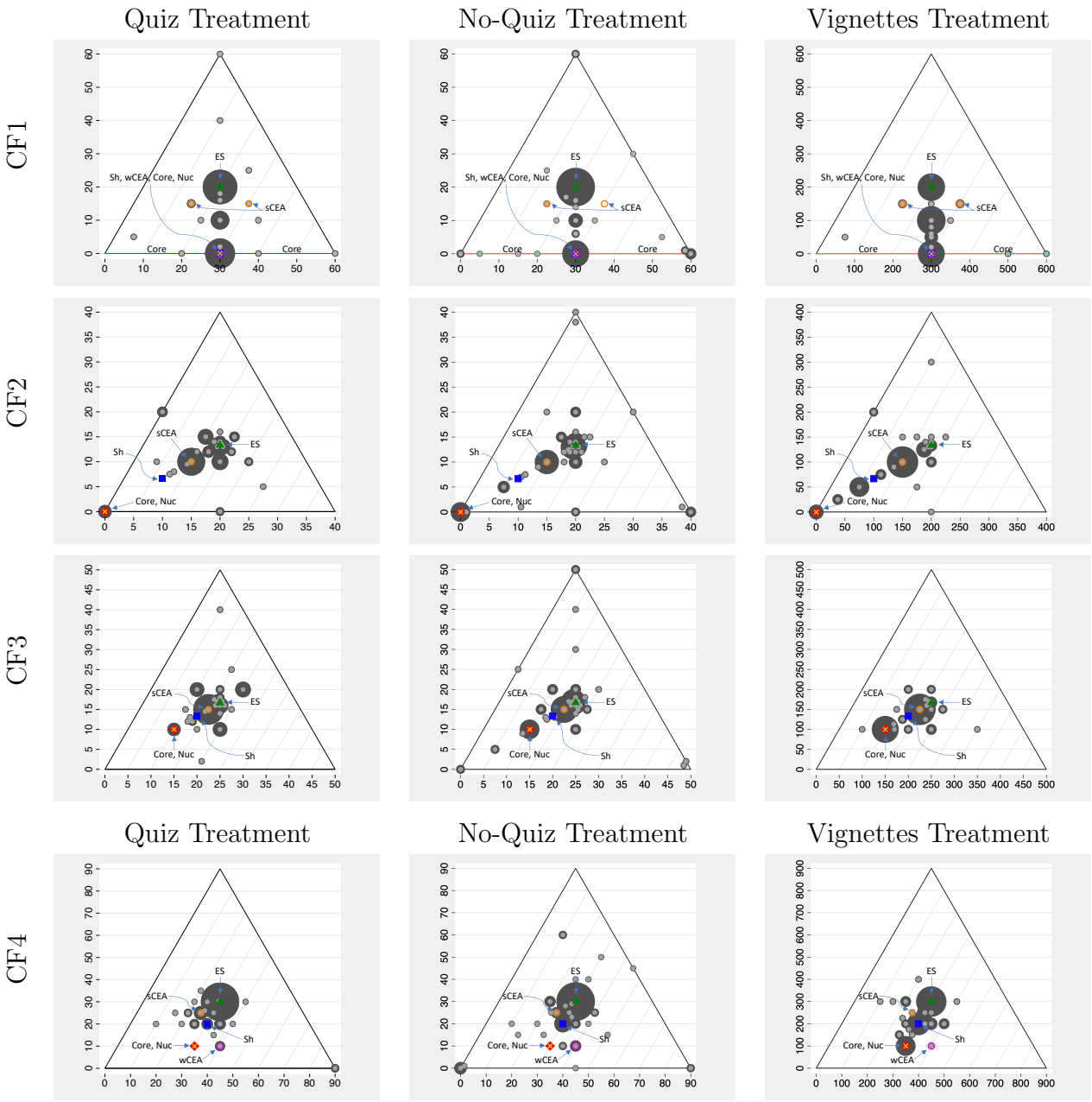



FIGURE 4: Frequency-weighted scatterplots of all choices in CF1-CF4.

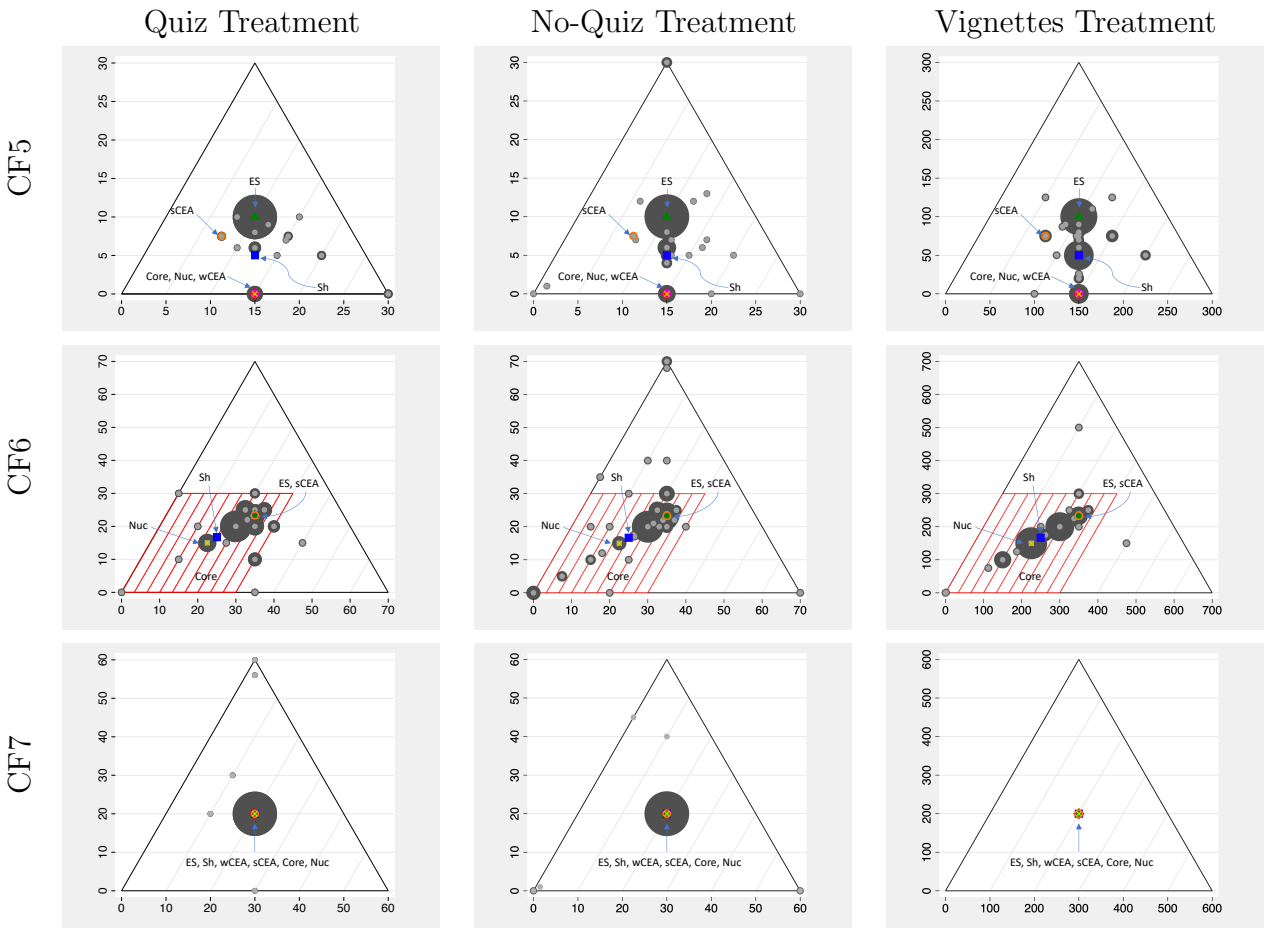



FIGURE 5: Frequency-weighted scatterplots of choices in CF5-CF7. There is no frequency-weighting for CF7 of the Vignettes treatment, as all split equally.

# G Statistical tests and figures

## G.1 Symmetry and desirability

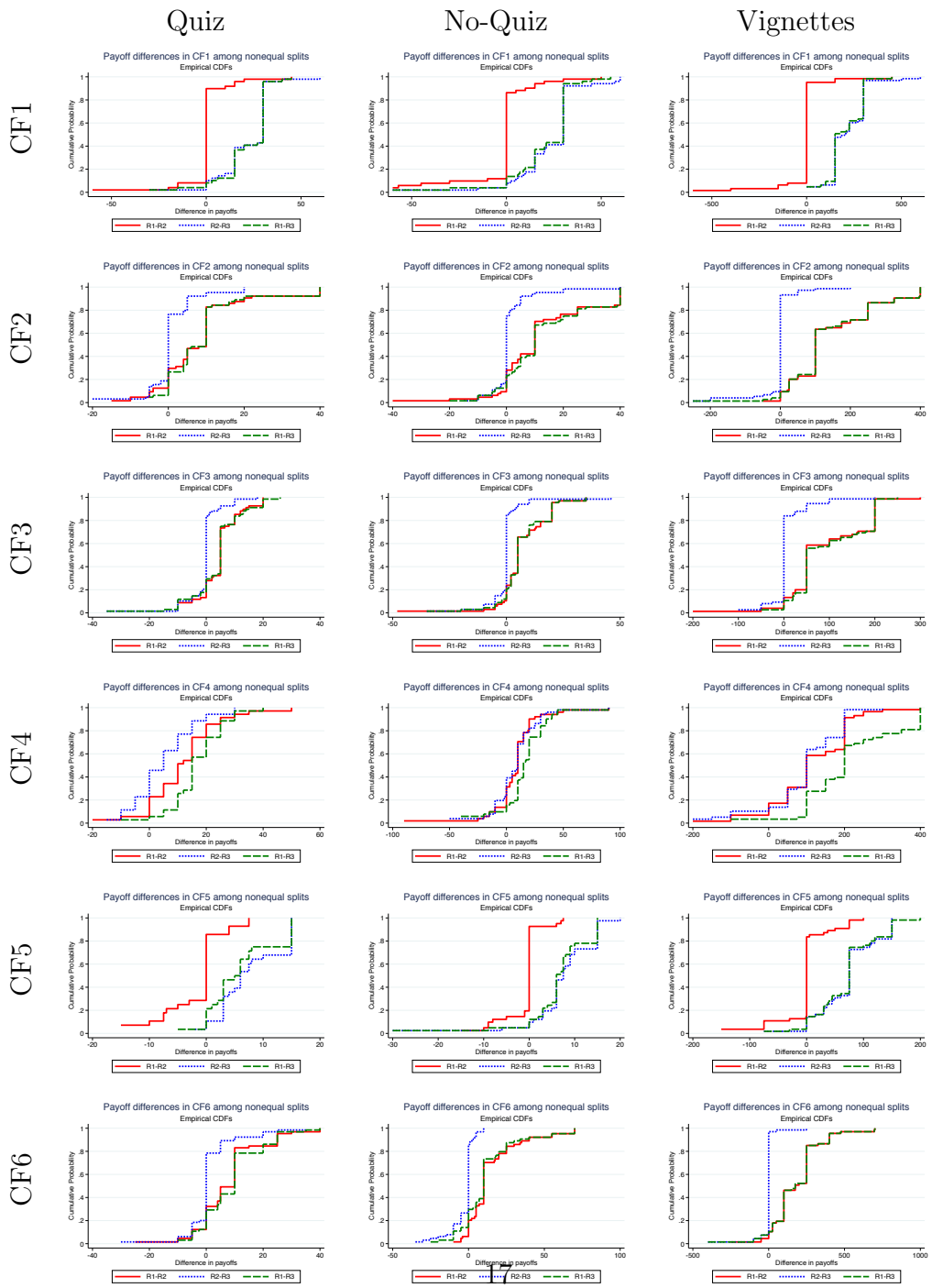



FIGURE 6: Empirical CDFs of payoff differences (among nonequal splits)

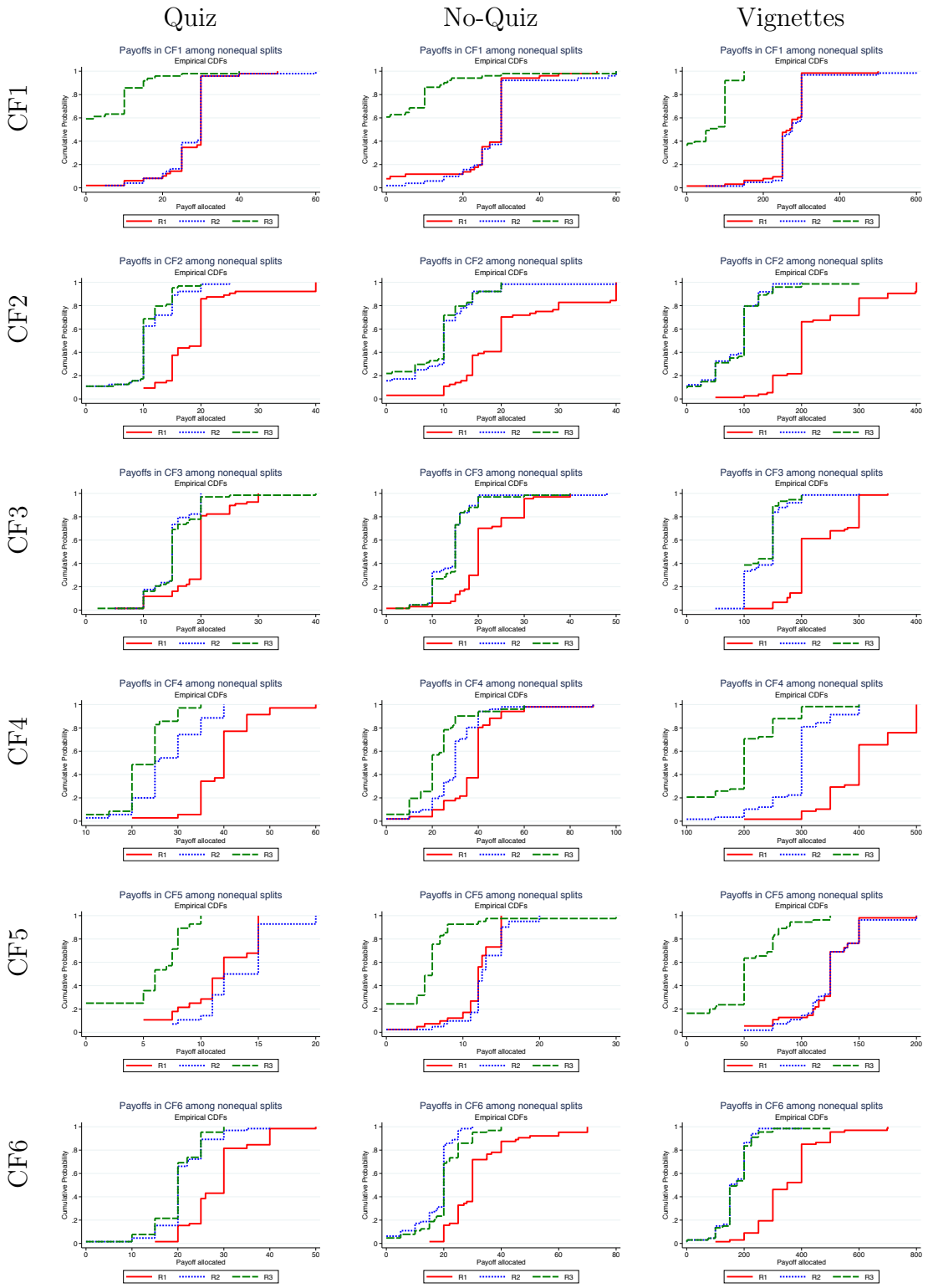



FIGURE 7: Empirical CDFs of payoffs (among nonequal splits)

The following are p-values for the desirability tests for all treatments.

| $H_0$ | Quiz | No-Quiz | Vignettes |
|---|---|---|---|
| t-test p-values | | | |
| R1-R3=0 in CF1 | 0.0000 | 0.0000 | 0.0000 |
| R2-R3=0 in CF1 | 0.0000 | 0.0000 | 0.0000 |
| R1-R2=0 in CF2 | 0.0000 | 0.0000 | 0.0000 |
| R1-R3=0 in CF2 | 0.0000 | 0.0000 | 0.0000 |
| R1-R2=0 in CF3 | 0.0000 | 0.0000 | 0.0000 |
| R1-R3=0 in CF3 | 0.0006 | 0.0000 | 0.0000 |
| R1-R2=0 in CF4 | 0.0000 | 0.0167 | 0.0000 |
| R2-R3=0 in CF4 | 0.0109 | 0.0138 | 0.0000 |
| R1-R3=0 in CF5 | 0.0001 | 0.0000 | 0.0000 |
| R2-R3=0 in CF5 | 0.0000 | 0.0000 | 0.0000 |
| R1-R2=0 in CF6 | 0.0000 | 0.0000 | 0.0000 |
| R1-R3=0 in CF6 | 0.0000 | 0.0000 | 0.0000 |
| Wilcoxon signed-ranks test p-values | | | |
| R1=R3 in CF1 | 0.0000 | 0.0000 | 0.0000 |
| R2=R3 in CF1 | 0.0000 | 0.0000 | 0.0000 |
| R1=R2 in CF2 | 0.0000 | 0.0000 | 0.0000 |
| R1=R3 in CF2 | 0.0000 | 0.0000 | 0.0000 |
| R1=R2 in CF3 | 0.0000 | 0.0000 | 0.0000 |
| R1=R3 in CF3 | 0.0000 | 0.0000 | 0.0000 |
| R1=R2 in CF4 | 0.0000 | 0.0000 | 0.0000 |
| R2=R3 in CF4 | 0.0371 | 0.0025 | 0.0000 |
| R1=R3 in CF5* | 0.0000 | 0.0000 | 0.0000 |
| R2=R3 in CF5* | 0.0000 | 0.0000 | 0.0000 |
| R1=R2 in CF6* | 0.0000 | 0.0000 | 0.0000 |
| R1=R3 in CF6* | 0.0000 | 0.0000 | 0.0000 |

The following are p-values for the symmetry tests for all treatments.

| $H_0$ | Quiz | No-Quiz | Vignettes |
|---|---|---|---|
| t-test p-values | | | |
| R1-R2=0 in CF1 | 0.9544 | 0.3865 | 0.4780 |
| R2-R3=0 in CF2 | 0.3962 | 0.2578 | 0.4222 |
| R2-R3=0 in CF3 | 0.4295 | 0.7807 | 0.2475 |
| R1-R2=0 in CF5 | 0.1167 | 0.1647 | 0.7408 |
| R2-R3=0 in CF6 | 0.3005 | 0.0091 | 0.4278 |
| Wilcoxon signed-ranks test p-values | | | |
| $H_0$ | Quiz | No-Quiz | Vignettes |
| R1=R2 in CF1 | 0.8984 | 0.9885 | 0.5625 |
| R2=R3 in CF2 | 0.4898 | 0.3570 | 0.5645 |
| R2=R3 in CF3 | 0.5237 | 0.5163 | 0.3474 |
| R1=R2 in CF5 | 0.1885 | 0.1133 | 0.9184 |
| R2=R3 in CF6 | 0.7056 | 0.0545 | 0.3125 |

## G.2 Additivity

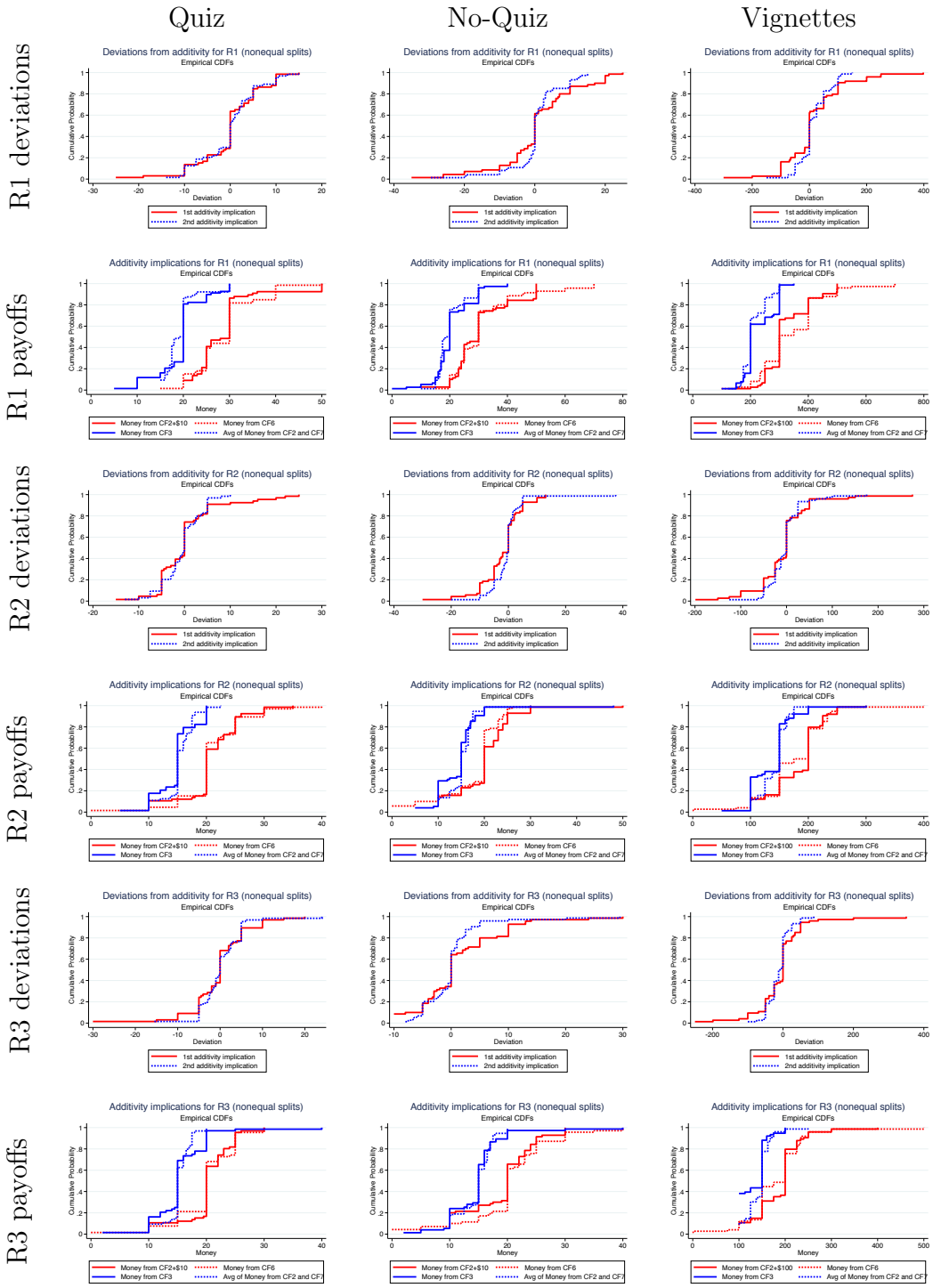



FIGURE 8: Empirical CDFs for additivity (among nonequal splits)

The following are p-values for the $1^{st}$ and $2^{nd}$ additivity implications.

| $H_0$ | Quiz | No-Quiz | Vignettes |
|---|---|---|---|
| t-test p-values | | | |
| R1: CF6-(CF2+$10)=0 | 0.9930 | 0.5458 | 0.2214 |
| R2: CF6-(CF2+$10)=0 | 0.7859 | 0.0180 | 0.3073 |
| R3: CF6-(CF2+$10)=0 | 0.7799 | 0.1468 | 0.3980 |
| R1: CF3-Avg(CF2,CF7)=0 | 0.8170 | 0.5141 | 0.0111 |
| R2: CF3-Avg(CF2,CF7)=0 | 0.3482 | 0.3889 | 0.3800 |
| R3: CF3-Avg(CF2,CF7)=0 | 0.6061 | 0.9116 | 0.0016 |
| Wilcoxon signed-ranks test p-values | | | |
| $H_0$ | Quiz | No-Quiz | Vignettes |
| R1: CF2+$10=CF6 | 0.6086 | 0.4285 | 0.3500 |
| R2: CF2+$10=CF6 | 0.2297 | 0.0320 | 0.0938 |
| R3: CF2+$10=CF6 | 0.7333 | 0.4660 | 0.1336 |
| R1: CF3=Avg(CF2,CF7) | 0.4379 | 0.1192 | 0.0078 |
| R2: CF3=Avg(CF2,CF7) | 0.3199 | 0.0688 | 0.0505 |
| R3: CF3=Avg(CF2,CF7) | 0.9080 | 0.4152 | 0.0003 |

For R2 in No-Quiz, the statistically-significant deviation with $p = 0.0180$ amounts to a discrepancy of only $1.37. In CF3 of Vignettest, there are 21 Decision Makers who allocate ($300, $100, $100), which is precisely the singleton core in that CF, and thus also the nucleolus. To satisfy the $2^{nd}$ implication of additivity, those same Decision Makers would need to allocate $0 to both R2 and R3 in CF2, since they allocate $200 to all Recipients in CF7. These subjects do treat R2 and R3 symmetrically in CF2, but only 5 subjects out of these 21 give them $0.[31] The following repeats the analysis above for the Vignettes treatment, without the 21 individuals who choose the

[31] Among the other 16 subjects, 1 subject gives them $1; 1 subject gives them $25; 6 subjects give them $50; 2 subjects give them $75; and 6 subjects give them $100. This can either be noise or a disinclination to give Recipient 1 all the money in CF2, which is theoretically different than violating the Dummy Player axiom. In fact, half of these 16 subjects do give $0 to the dummy player in CF1.

nucleolus in CF3 (even dropping the 5 among them who do satisfy Additivity). This leaves 54/55 subjects for the $1^{st}/2^{nd}$ additivity implication.

| Vignettes treatment | Paired t-test p-value | Wilcoxon test p-value |
|---|---|---|
| R1: CF2+$10=CF6 | 0.4280 | 0.6273 |
| R2: CF2+$10=CF6 | 0.5134 | 0.1627 |
| R3: CF2+$10=CF6 | 0.6280 | 0.2264 |
| R1: CF3=Avg(CF2,CF7) | 0.8679 | 0.8172 |
| R2: CF3=Avg(CF2,CF7) | 0.4315 | 0.7079 |
| R3: CF3=Avg(CF2,CF7) | 0.2014 | 0.0977 |

## G.3 Monotonicity

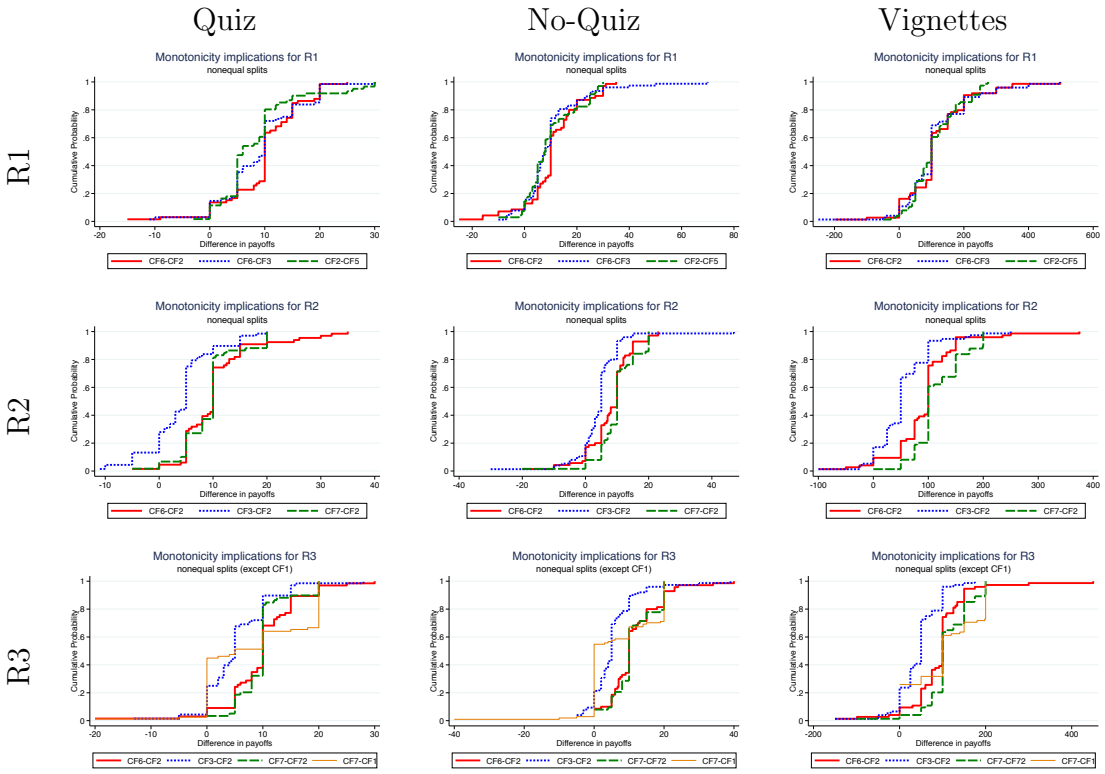



FIGURE 9: Empirical CDFs for monotonicity

The following are p-values for testing monotonicity, for all treatments.

| $H_0$ | Quiz | No-Quiz | Vignettes |
|---|---|---|---|
| t-test p-values | | | |
| R1: CF6-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R2: CF6-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF6-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R2: CF3-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF3-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R2: CF7-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF7-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R1: CF6-CF3=0 | 0.0000 | 0.0000 | 0.0000 |
| R1: CF2-CF5=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF7-CF1=0 | 0.0000 | 0.0000 | 0.0000 |
| Wilcoxon signed-ranks test p-values | | | |
| $H_0$ | Quiz | No-Quiz | Vignettes |
| R1: CF6-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R2: CF6-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF6-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R2: CF3-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF3-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R2: CF7-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF7-CF2=0 | 0.0000 | 0.0000 | 0.0000 |
| R1: CF6-CF3=0 | 0.0000 | 0.0000 | 0.0000 |
| R1: CF2-CF5=0 | 0.0000 | 0.0000 | 0.0000 |
| R3: CF7-CF1=0 | 0.0000 | 0.0000 | 0.0000 |

## G.4 Cross-treatment comparisons

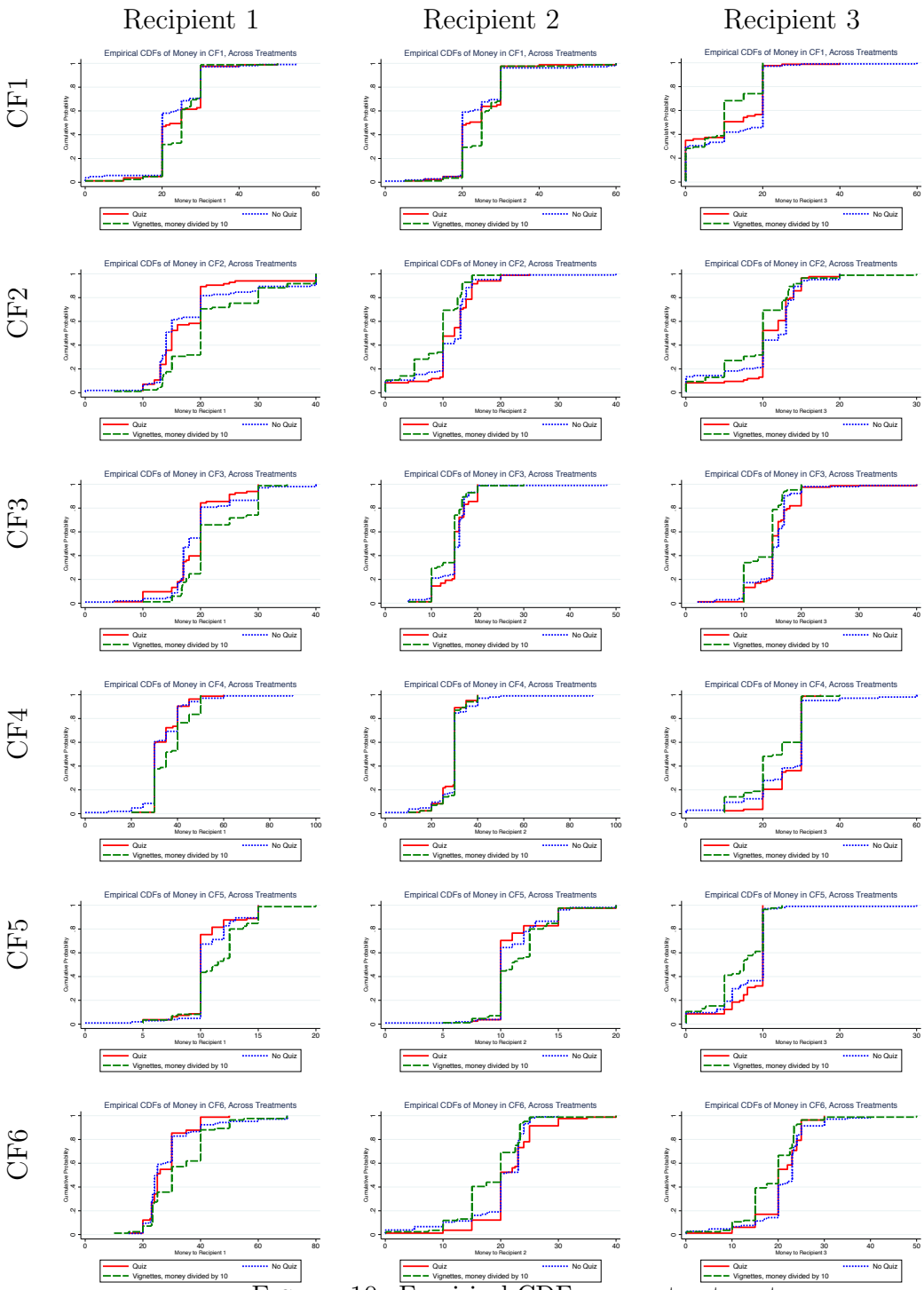



FIGURE 10: Empirical CDFs across treatments

| **Quiz Treatment** | (all) $\frac{m-\text{ES}}{ES}$ | (no D-equal splitters) $\frac{m-\text{ES}}{ES}$ | (D-equal splitters) $\frac{m-\text{ES}}{ES}$ |
|---|---|---|---|
| $\frac{\text{Sh}-\text{ES}}{ES}$ | $0.368^{***}$ (0.039) | $0.514^{***}$ (0.044) | $0.065^{*}$ (0.029) |
| constant | $-0.005$ (0.005) | $-0.012$ (0.006) | $0.008^{*}$ (0.004) |
| No. subjects | 84 | 57 | 27 |
| Observations | 1150 | 772 | 378 |
| $R^2$ | 0.3003 | 0.4245 | 0.0446 |
| **No-Quiz Treatment** | (all) $\frac{m-\text{ES}}{ES}$ | (no D-equal splitters) $\frac{m-\text{ES}}{ES}$ | (D-equal splitters) $\frac{m-\text{ES}}{ES}$ |
| $\frac{\text{Sh}-\text{ES}}{ES}$ | $0.368^{***}$ (0.047) | $0.595^{***}$ (0.064) | $0.027^{*}$ (0.011) |
| constant | 0.001 (0.006) | $-0.002$ (0.009) | $0.005^{**}$ (0.001) |
| No. subjects | 105 | 63 | 42 |
| Observations | 1460 | 874 | 586 |
| $R^2$ | 0.2085 | 0.3305 | 0.0384 |
| **Vignettes Treatment** | (all) $\frac{m-\text{ES}}{ES}$ | (no D-equal splitters) $\frac{m-\text{ES}}{ES}$ | (D-equal splitters) $\frac{m-\text{ES}}{ES}$ |
| $\frac{\text{Sh}-\text{ES}}{ES}$ | $0.613^{***}$ (0.049) | $0.733^{***}$ (0.048) | 0.054 (0.036) |
| constant | $0.009^{*}$ (0.004) | 0.008 (0.005) | 0.014 (0.007) |
| No. subjects | 85 | 70 | 15 |
| Observations | 1188 | 978 | 210 |
| $R^2$ | 0.4774 | 0.5752 | 0.0285 |

$^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

TABLE 14: Regressions of the percentage departure from equal split of allocations to R1 and R3, against percentage departure of the Shapley value from equal split. Huber-White standard errors in parentheses, clustered by Decision Maker. Estimates rounded to 3 decimal places.

For each $k = 1, 2, 3$ and each characteristic function, we test the null hypothesis that across the specified pair of treatments, the distributions of money allocated to R$k$ are the same. Because the choices in different treatments come from different subjects, we use the Kolmogorov-Smirnov test here. For the case of Vignettes, monetary allocations are divided by ten for comparison purposes.

| | Kolmogorov-Smirnov p-value | | |
|---|---|---|---|
| Quiz vs. No-Quiz | Recipient 1 | Recipient 2 | Recipient 3 |
| Money in CF1 | 0.566 | 0.646 | 0.639 |
| Money in CF2 | 0.230 | 0.754 | 0.551 |
| Money in CF3 | 0.247 | 0.404 | 0.512 |
| Money in CF4 | 0.960 | 0.989 | 0.859 |
| Money in CF5 | 0.716 | 0.832 | 0.431 |
| Money in CF6 | 0.226 | 0.232 | 0.347 |
| Quiz vs. Vignettes | Recipient 1 | Recipient 2 | Recipient 3 |
| Money in CF1 | 0.147 | 0.046 | 0.072 |
| Money in CF2 | 0.005 | 0.009 | 0.071 |
| Money in CF3 | 0.049 | 0.164 | 0.023 |
| Money in CF4 | 0.027 | 0.901 | 0.002 |
| Money in CF5 | 0.000 | 0.007 | 0.002 |
| Money in CF6 | 0.003 | 0.000 | 0.008 |
| No-Quiz vs. Vignettes | Recipient 1 | Recipient 2 | Recipient 3 |
| Money in CF1 | 0.001 | 0.000 | 0.000 |
| Money in CF2 | 0.000 | 0.000 | 0.001 |
| Money in CF3 | 0.000 | 0.002 | 0.000 |
| Money in CF4 | 0.010 | 1.000 | 0.026 |
| Money in CF5 | 0.002 | 0.027 | 0.007 |
| Money in CF6 | 0.004 | 0.004 | 0.001 |

# H Testing for demographic and order effects

Among the 89 Decision Makers in the Quiz treatment, 82 provided some demographic information in the optional exit survey.[32] 19 Decision Makers were economics concentrators (we use this term to include joint concentrations at

[32]Recipients were also given the opportunity to respond to the exit survey. We do not include their information here in order to accurately represent the population whose choices we are analyzing.

Brown with related departments, such as applied math-economics, and concentrations affiliated with the economics department, like Business, Entrepreneurship and Organizations). A wide variety of other concentrations was reported, with biology-related concentrations a popular choice. Ages ranged from 18 to 24, with a mean of 20.2. The gender distribution was 40.2% male and 59.8% female.[33] 7.2% of Decision Makers were only children, 55.4% have 1 sibling, 24.1% have 2 siblings, and 13.2% have $\geq 3$ siblings.

Among the 112 Decision Makers in the No-Quiz treatment, 106 provided some demographic information. There were 26 Economics-related concentrators. A wide variety of other concentrations was reported. Reported ages ranged from 18 to 27, with a mean of 19.9. Among the 87 reporting their gender, 51.89% are male, 47.17% female, and 0.94% report other. 16.98% of Decision Makers answering this question reported being only children, 37.74% have one sibling, 26.42% have two siblings, and 18.87% have $\geq 3$ siblings.

Among the 85 Decision Makers in the Vignettes treatment, all but one provided some demographic information. There were 17 Economics-related concentrators. A wide variety of concentrations was reported. Among those reporting, Decision Makers ranged from 18 to 28, with a mean of 20.23. Among those reporting gender, 27.71% are male, 69.88% female, and 2.41% report other. 10.98% of Decision Makers answering this question reported being only children, 52.44% have one sibling, 23.17% have two siblings, and 13.41% have $\geq 3$ siblings.

[33]Enrollment at Brown University at the time was around 47% male and 53% female according to http://colleges.usnews.rankingsandreviews.com/best-colleges/brown-university-3401.

| *All treatments* | $\frac{m-\text{ES}}{ES}$ | $\frac{m-\text{ES}}{ES}$ | $\frac{m-\text{ES}}{ES}$ |
|---|---|---|---|
| $\frac{\text{Sh}-\text{ES}}{ES}$ | $0.418^{***}$ (0.061) | $0.379^{***}$ (0.054) | $0.433^{***}$ (0.049) |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Session 2 | $-0.019$ (0.091) | – | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Session 3 | 0.090 (0.087) | – | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Session 4 | 0.121 (0.100) | – | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Session 5 | $-0.001$ (0.089) | – | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Session 6 | $-0.026$ (0.086) | – | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x No-Quiz | – | $-0.049$ (0.061) | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Vignettes | – | $0.253^{***}$ (0.063) | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Econ | – | $0.175^{**}$ (0.062) | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Econ x Age$<$ 20 | – | – | 0.030 (0.115) |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Econ x Age$\geq$ 20 | – | – | $0.200^{*}$ (0.089) |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Non-Econ x Age$\geq$ 20 | – | – | $-0.071$ (0.062) |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Female | – | $-0.078$ (0.053) | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x Only Child | – | 0.090 (0.075) | – |
| $\frac{\text{Sh}-\text{ES}}{ES}$ x #Siblings$\geq$3 | – | $-0.091$ (0.077) | – |
| constant | 0.002 (0.003) | 0.001 (0.003) | 0.001 (0.003) |
| No. subjects | 274 | 257 | 260 |
| Observations | 3798 | 3586 | 3628 |
| $R^2$ | 0.3171 | 0.3508 | 0.3238 |

$^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

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