

Online Technical Appendix to “Tight Money-Tight Credit: Coordination Failure in the Conduct of Monetary and Financial Policies”

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A Model structure

The model economy is composed by the following agents: households, entrepreneurs, a financial intermediary, a capital producer, intermediate-good producers, a final-good producer, a central bank, a financial authority, and a government in charge of fiscal policy.

A.1 Households

Each period t , a representative household chooses consumption, c_t , hours worked, ℓ_t^h , and real deposits/savings, d_t , to maximize its discounted lifetime utility subject to a budget constraint. The household thus solves:

$$\max_{c_t, \ell_t^h, d_t} \mathbb{E}_T \left\{ \sum_{t=T}^{\infty} \beta^{t-T} \frac{[(c_t - hC_{t-1})^v (1 - \ell_t^h)^{1-v}]^{1-\sigma} - 1}{1 - \sigma} \right\},$$

subject to

$$c_t + d_t \leq w_t \ell_t^h + \frac{R_{t-1}}{1 + \pi_t} d_{t-1} + \text{div}_t + \mathcal{A}_t - \Upsilon_t \text{ for all } t.$$

In the utility function, β is the subjective discount factor, $h \in [0, 1]$ determines the degree of dependence on external habits on aggregate past consumption, C_{t-1} , $\sigma > 0$ is the coefficient of relative risk aversion, $v \in (0, 1)$ is the labor share parameter that ensures that labor equals $\frac{1}{3}$ at the non-stochastic steady state, and \mathbb{E}_t is the expectations operator conditional on the information available at date t . In the budget constraint, the household's uses of income in the left-hand side are assigned to buy consumption goods and make bank deposits. The sources of income on the right-hand-side derive from wage income, where w_t is the real wage rate, from the real return on deposits carried over from the previous period, where $1 + \pi_t = P_t/P_{t-1}$ is the gross inflation rate from period $t-1$ to t (P_t is the price of final goods at date t) and R_{t-1} is the gross nominal interest rate paid on one-period nominal deposits, which is also the central bank's policy instrument, and from real profits paid by monopolistic firms (div_t) plus transfers from entrepreneurs (\mathcal{A}_t) net of lump-sum transfers from government (Υ_t).

The first order conditions are

$$\lambda_t = v (c_t - hC_{t-1})^{v-1} (1 - \ell_t^h)^{1-v} [(c_t - hC_{t-1})^v (1 - \ell_t^h)^{1-v}]^{-\sigma}, \quad (1)$$

$$\lambda_t w_t = (1 - v) (c_t - hC_{t-1})^v (1 - \ell_t^h)^{-v} [(c_t - hC_{t-1})^v (1 - \ell_t^h)^{1-v}]^{-\sigma}, \quad (2)$$

$$\lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \frac{R_t}{1 + \pi_{t+1}} \right\}, \quad (3)$$

where λ_t is the Lagrange multiplier of the budget constraint.

A.2 Entrepreneurs

There is a continuum of risk-neutral entrepreneurs, indexed by $e \in [0, 1]$. Each entrepreneur purchases the stock of capital, $k_{e,t}$, at a relative price, q_t , using her own net worth, $n_{e,t}$, and one-period-maturity debt, $b_{e,t}$. The budget constraint in real terms for these capital purchases is:

$$q_t k_{e,t} = b_{e,t} + n_{e,t}.$$

At date $t + 1$, entrepreneurs rent capital services to intermediate goods producers at a real rental rate, z_{t+1} , and sell the capital stock that remains after production to a capital producer. As in BGG, the return gained by an individual entrepreneur is affected by an idiosyncratic shock $\omega_{e,t+1}$, with $E(\omega_{e,t+1}) = 1$ and $\text{Var}(\omega_{e,t+1}) = \sigma_{\omega,t+1}^2$. Hence, the real returns of an individual entrepreneur e at time $t + 1$ are $\omega_{e,t+1} r_{t+1}^k k_{e,t}$, where r_{t+1}^k is the aggregate gross real rate of return per unit of capital, which is given by

$$r_{t+1}^k \equiv \frac{z_{t+1} + (1 - \delta)q_{t+1}}{q_t}, \quad (4)$$

where δ is the rate of capital depreciation.

Idiosyncratic productivity $\omega_{e,t+1}$ is an i.i.d. random variable across time and types, with a continuous and once-differentiable c.d.f., $F(\omega_{e,t+1})$, over a non-negative support. Following Christiano, Motto and Rostagno (2014), $\omega_{e,t+1}$ features risk shocks, which are represented by the time-varying standard deviation $\sigma_{\omega,t+1}$, with a long-run average $\bar{\sigma}_\omega$. An increase of $\sigma_{\omega,t+1}$ worsens financial conditions because it implies that $F(\omega_{e,t+1})$ widens, so a larger share of entrepreneurs is likely to default.

Entrepreneurs participate in the labor market by offering one unit of labor each period at the real wage rate w_t^e ,¹ and face a probability of exiting the economy given by $1 - \gamma$. This assumption prevents entrepreneurs from accumulating enough wealth to be fully self-financed. Aggregate net worth in period t is thus given by

$$n_t = \gamma v_t + w_t^e. \quad (5)$$

The value of v_t in the first term on the right-hand side of (5) is the aggregate equity from capital holdings of entrepreneurs who survive at date t (defined below). Those who exit at t transfer their wages to new entrepreneurs entering the economy, consume part of their equity, such that $c_t^e = (1 - \gamma)\varrho v_t$ for $\varrho \in [0, 1]$, while the rest, $\mathcal{A}_t = (1 - \gamma)(1 - \varrho)v_t$, is transferred to households as a lump-sum payment.

¹This assumption is useful because, as noted by BGG, it is necessary for entrepreneurs to start off with some net worth in order to allow them to begin operations.

A.3 The lender and the financial contract

The financial intermediary takes deposits from households, paying on them the risk-free nominal interest rate R_t . Deposits are used to fund loans to entrepreneurs, so in real terms $d_t = b_t$ at all times, where $b_t = \int b_{e,t} de$ is the aggregate amount of real debt at time t . Nominal loan contracts are made before the entrepreneurs' returns are realized and these returns are not observable by the intermediary, but can be verified at a cost. Similar to BGG, the optimal credit contract is modeled following the costly-state-verification setup of Townsend (1979). Similar to Carrillo and Poilly (2013) and Christiano *et al.* (2014), we assume that debt contracts between the intermediary and entrepreneurs are denominated in nominal terms. This assumption allows the model to feature a Fisherian debt-deflation channel that is important for the amplification of financial frictions. For convenience, we express the returns of entrepreneurs and the lender in real terms, though we specify in the lender's participation constraint that the relevant opportunity cost of funds is the nominal interest rate R_t . Finally, we add to this framework a financial subsidy on the lender's participation constraint that is used as the instrument of financial policy.

At time t , when the financial contract is signed, the idiosyncratic shock $\omega_{e,t+1}$ is unknown to both the entrepreneur and the lender. At $t + 1$, if $\omega_{e,t+1}$ is higher than a threshold value $\bar{\omega}_{e,t+1}$, the entrepreneur repays her debt plus interests, $r_{e,t+1}^L b_{e,t}$, where r_t^L is the gross real interest rate that they pay under repayment. In contrast, if $\omega_{e,t+1}$ is lower than $\bar{\omega}_{e,t+1}$, the entrepreneur declares bankruptcy and gets nothing, while the lender audits the entrepreneur, pays the monitoring cost, and gets to keep any income generated by the entrepreneur's investment. The monitoring cost is a proportion $\mu \in [0, 1]$ of the entrepreneur's returns, i.e. $\mu \omega_{e,t+1} r_{t+1}^k q_t k_{e,t}$. The threshold value $\bar{\omega}_{e,t+1}$ satisfies:

$$\bar{\omega}_{e,t+1} r_{t+1}^k q_t k_{e,t} = r_{e,t+1}^L b_{e,t}. \quad (6)$$

The type sub-index can be dropped without loss of generality to characterize the optimal contract. Here, it is also useful to define the following quantities:

$$\begin{aligned} \Gamma(\bar{\omega}) &= \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega + \int_0^{\bar{\omega}} \omega f(\omega) d\omega, \\ \mu G(\bar{\omega}) &= \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega, \end{aligned}$$

where $\int_{\bar{\omega}}^{\infty} f(\omega) d\omega \equiv \text{Prob}(\omega > \bar{\omega})$ denotes the probability that ω is greater than the threshold value, and $\int_0^{\bar{\omega}} \omega f(\omega) d\omega \equiv E(\omega \mid \omega < \bar{\omega})$ is the partial expectation of ω in the interval $[0, \bar{\omega}]$.

The return earned by one entrepreneur, denoted by $\mathcal{R}_{e,t}$, is 0 if $\omega_{t+1} \leq \bar{\omega}_{t+1}$, and $\omega_{t+1} r_{t+1}^k q_t k_t - r_{t+1}^L b_t$ if $\omega_{t+1} > \bar{\omega}_{t+1}$. Using equation (6), the expected returns of entrepreneurs can be written as

$$\begin{aligned}\mathcal{R}_{e,t} &= \mathbb{E}_t \left\{ \int_{\bar{\omega}_{t+1}}^{\infty} (\omega_{t+1} - \bar{\omega}_{t+1}) r_{t+1}^k q_t k_t f(\omega) d\omega \right\} \\ &= \mathbb{E}_t \left\{ \left[\int_{\bar{\omega}_{t+1}}^{\infty} \omega_{t+1} f(\omega) d\omega - \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega) d\omega \right] r_{t+1}^k q_t k_t \right\} \\ &= \mathbb{E}_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1})] r_{t+1}^k q_t k_t \right\}.\end{aligned}\quad (7)$$

In turn, if $\omega_{t+1} \leq \bar{\omega}_{t+1}$ the lender receives the gross income made by the bankrupt entrepreneur and pays monitoring costs, so it gets $(1 - \mu)\omega_{t+1} r_{t+1}^k q_t k_t$. If $\omega_{t+1} > \bar{\omega}_{t+1}$, the lender receives $\bar{\omega}_{t+1} r_{t+1}^k q_t k_t$ irrespective of the realization of ω_{t+1} . Without an intervention by the financial authority, the expected returns of the lender, denoted by $\mathcal{R}_{L,t}$, are thus equal to:

$$\begin{aligned}\mathcal{R}_{L,t} &= \mathbb{E}_t \left\{ \left[(1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f(\omega) d\omega + \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega) d\omega \right] r_{t+1}^k q_t k_t \right\} \\ &= \mathbb{E}_t \left\{ \left[\int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f(\omega) d\omega + \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega) d\omega - \mu \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f(\omega) d\omega \right] r_{t+1}^k q_t k_t \right\} \\ &= \mathbb{E}_t \left\{ [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] r_{t+1}^k q_t k_t \right\}.\end{aligned}\quad (8)$$

To introduce a role for financial policy, we assume that the financial authority announces that for every loan given by the lender, the government provides a subsidy (a tax if negative) of $\tau_{f,t} r_{t+1}^k q_t k_t$ to the financial intermediary. The subsidy (tax) is financed (rebated) as part of the lump-sum net transfers to households. Since the entrepreneurs' risk is idiosyncratic, and thus can be perfectly diversified, participation by the lender requires the return on making loans to be equal to the risk-free interest rate paid on deposits. The following participation constraint must be satisfied for every realization of r_{t+1}^k :

$$(1 + \tau_{f,t}) [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] r_{t+1}^k q_t k_t \geq r_t b_t, \quad (9)$$

where $r_t \equiv \frac{R_t}{1 + \pi_{t+1}}$ is the gross ex-post real interest rate, which reflects the assumption that loans are embedded in nominal debt contracts.

The optimal contract sets an amount of capital expenditures, $q_t k_t$, and a threshold, $\bar{\omega}_{t+1}$, such that the expected return of entrepreneurs is maximized subject to the lender's participation constraint holding for each value that r_{t+1}^k can take.² For convenience, and following BGG, we rewrite the problem in terms of the leverage ratio,

$$x_t \equiv \frac{q_t k_t}{n_t}, \quad (10)$$

²The contract has an equivalent representation in terms of a loan amount and an interest rate. The loan size follows from the fact that net worth is pre-determined when the contract is signed and $q_t k_t = b_t + n_t$, and the interest rate is given by condition (6).

and an auxiliary variable,

$$\tilde{r}_t \equiv \frac{r_{t+1}^k}{r_t}. \quad (11)$$

The optimal contract solves the following problem:

$$\begin{aligned} \max_{x_t, \bar{\omega}_t} \quad & \text{E}_t \{ [1 - \Gamma(\bar{\omega}_{t+1})] \tilde{r}_t x_t \}, \quad \text{subject to} \\ & (1 + \tau_{f,t}) [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] \tilde{r}_t x_t \geq x_t - 1, \end{aligned}$$

where we have used the fact that $b_t = q_t k_t - n_t$. Notice that the lender's participation constraint does not involve any expectation term, since it must hold for every state \tilde{r}_t . The Lagrangian of the optimal contract problem is

$$\mathcal{L}_t = \text{E}_t \{ [1 - \Gamma(\bar{\omega}_{t+1})] \tilde{r}_t x_t \} + \Lambda_t \{ (1 + \tau_{f,t}) [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] \tilde{r}_t x_t - x_t + 1 \},$$

where Λ is the Lagrange multiplier. The first-order conditions are:

$$x : \text{E}_t \{ [1 - \Gamma(\bar{\omega}_{t+1})] \tilde{r}_t \} + \Lambda_t \{ [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] \tilde{r}_t (1 + \tau_{f,t}) - 1 \} = 0, \quad (12)$$

$$\bar{\omega} : \Lambda_t [\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})] \tilde{r}_t (1 + \tau_{f,t}) = \text{E}_t \{ \Gamma'(\bar{\omega}_{t+1}) \tilde{r}_t \}, \quad (13)$$

$$\Lambda : [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] \tilde{r}_t x_t (1 + \tau_{f,t}) = x_t - 1. \quad (14)$$

Similar to BGG, the above three equations define the equilibrium in the credit market as follows:

$$\text{efp} \equiv \text{E}_t \left\{ \frac{r_{t+1}^k}{r_t} \right\} = \frac{s(x_t)}{1 + \tau_{f,t}},$$

where $s(\cdot) \geq 1$ is a function with $\frac{\partial s(\cdot)}{\partial x_t} > 0$ for $n_{t+1} < q_t k_t$. This equation defines the external finance premium and is the key feature of the accelerator mechanism: entrepreneurs with a high self-financing ratio (or high net worth) are less likely to default, implying a lower premium on external funds, which reduces the cost of credit. The financial subsidy modifies the standard setting by reducing the external finance premium in equilibrium, which helps to close the financial wedge between the returns of capital and the returns of bonds.

Analytical expressions for the financial contract Christiano *et al.* (2014) assume that ω distributes log-normally with $\text{E}(\omega) = 1$ and a time-varying variance $\text{Var}(\omega)_t = \sigma_{\omega,t}^2$.³ This implies that an auxiliary variable $\zeta = \ln(\omega)$ has a normal distribution, with mean and variance given by $\mu_{\zeta,t}$ and $\sigma_{\zeta,t}^2$. The relationships between $\text{E}(\omega)$, $\text{Var}(\omega)_t$, $\mu_{\zeta,t}$ and $\sigma_{\zeta,t}^2$ are the following:

$$\begin{aligned} \text{E}(\omega) &= \exp \left(\mu_{\zeta,t} + \frac{\sigma_{\zeta,t}^2}{2} \right), \\ \text{Var}(\omega)_t &= \text{E}(\omega)^2 [\exp(\sigma_{\zeta,t}^2) - 1]. \end{aligned}$$

³As we did before, we are dropping the type sub-index e without loss of generality.

Solving for $\mu_{\zeta,t}$ and $\sigma_{\zeta,t}^2$, we obtain

$$\begin{aligned}\mu_{\zeta,t} &= \ln E(\omega) - \frac{\sigma_{\zeta,t}^2}{2} = -\frac{\sigma_{\zeta,t}^2}{2}, \text{ given that } E(\omega) = 1, \\ \sigma_{\zeta,t}^2 &= \ln \left[1 + \frac{\text{Var}(\omega)_t}{E(\omega)^2} \right] = \ln(1 + \sigma_{\omega,t}^2) \approx \sigma_{\omega,t}^2.\end{aligned}$$

The p.d.f. of the log-normally distributed variable ω is given by

$$f(\omega) = \frac{1}{\omega \sigma_{\zeta,t} \sqrt{2\pi}} \exp\left(-\frac{(\ln \omega - \mu_{\zeta,t})^2}{2\sigma_{\zeta,t}^2}\right).$$

while its c.d.f. is

$$F(\omega) = \int_0^\omega f(k) dk = \Phi_N\left(\frac{\ln \omega - \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right),$$

where $\Phi_N(\cdot)$ is the c.d.f. of a standard normal distribution.

We need an analytical expression for $\Gamma(\bar{\omega}) = \bar{\omega} \int_{\bar{\omega}}^\infty f(\omega) d\omega + \int_0^{\bar{\omega}} \omega f(\omega) d\omega$. The first term is simply given by $\bar{\omega} [1 - F(\bar{\omega})]$, while the second term refers to the partial expectation $E(\omega \mid \omega < \bar{\omega})$. To obtain this term, we can use the partial expectation operator for $E(\omega \mid \omega \geq \bar{\omega})$, which is given by

$$\begin{aligned}E(\omega \mid \omega \geq \bar{\omega}) &= \exp\left(\mu_{\zeta,t} + \frac{\sigma_{\zeta,t}^2}{2}\right) \Phi_N\left(\frac{\mu_{\zeta,t} + \sigma_{\zeta,t}^2 - \ln \bar{\omega}}{\sigma_{\zeta,t}}\right), \\ &= \Phi_N\left(\frac{-\ln \bar{\omega} + \frac{\sigma_{\zeta,t}^2}{2}}{\sigma_{\zeta,t}}\right), \text{ since } \mu_{\zeta,t} = -\frac{\sigma_{\zeta,t}^2}{2}, \\ &= \Phi_N\left(-\frac{\ln \bar{\omega} + \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right).\end{aligned}$$

Notice that it must be the case that $E(\omega \mid \omega < \bar{\omega}) + E(\omega \mid \omega \geq \bar{\omega}) = 1$, and so $E(\omega \mid \omega < \bar{\omega}) = 1 - \Phi_N\left(-\frac{\ln \bar{\omega} + \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right)$. Therefore,

$$\begin{aligned}\Gamma(\bar{\omega}) &= \bar{\omega} [1 - F(\bar{\omega})] + 1 - \Phi_N\left(-\frac{\ln \bar{\omega} + \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right), \\ &= \bar{\omega} \left[1 - \Phi_N\left(\frac{\ln \bar{\omega} - \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right)\right] + 1 - \Phi_N\left(-\frac{\ln \bar{\omega} + \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right), \\ &= \bar{\omega} \left[\Phi_N\left(-\frac{\ln \bar{\omega} - \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right)\right] + \Phi_N\left(\frac{\ln \bar{\omega} + \mu_{\zeta,t}}{\sigma_{\zeta,t}}\right),\end{aligned}$$

since for a standard normally distributed variable z we have that $1 - \Phi_N(\bar{z}) = \Phi_N(-\bar{z})$; the latter states that $\text{Prob}(z \geq \bar{z})$, which is the term on the left-hand side, is equivalent to $\text{Prob}(z < -\bar{z})$, which is the term on the right-hand side. Similarly, for $\mu G(\bar{\omega}) = \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega$ we have that

$$\begin{aligned}\mu G(\bar{\omega}) &= \mu \left[1 - \Phi_N \left(-\frac{\ln \bar{\omega} + \mu_{\zeta,t}}{\sigma_{\zeta,t}} \right) \right], \\ &= \mu \left[\Phi_N \left(\frac{\ln \bar{\omega} + \mu_{\zeta,t}}{\sigma_{\zeta,t}} \right) \right].\end{aligned}$$

The derivatives of Γ and G with respect to $\bar{\omega}$ are given by:

$$\begin{aligned}\frac{\partial \Gamma(\bar{\omega})}{\partial \bar{\omega}} \equiv \Gamma'(\bar{\omega}) &= 1 - F(\bar{\omega}) = \Phi_N \left(-\frac{\ln \bar{\omega} - \mu_{\zeta,t}}{\sigma_{\zeta,t}} \right), \text{ and} \\ \frac{\partial G(\bar{\omega})}{\partial \bar{\omega}} \equiv G'(\bar{\omega}) &= \bar{\omega} f(\bar{\omega}).\end{aligned}$$

Finally, we assume that the log of the standard deviation of ω_t follows an autoregressive process:

$$\ln(\sigma_{\omega,t}) = (1 - \rho_{\sigma_\omega}) \ln(\bar{\sigma}_\omega) + \rho_{\sigma_\omega} \ln(\sigma_{\omega,t-1}) + \sigma_\epsilon \epsilon_t.$$

Entrepreneurs' equity Aggregate equity from capital holdings v_t is defined by the realized returns of entrepreneurs at time t :

$$\begin{aligned}v_t &= [1 - \Gamma(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1}, \\ &= r_t^k q_{t-1} k_{t-1} - \Gamma(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1}.\end{aligned}$$

If we use the lender's participation constraint, the latter becomes:

$$\begin{aligned}v_t &= r_t^k q_{t-1} k_{t-1} - \left[\mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} + \frac{r_{t-1} b_{t-1}}{1 + \tau_{f,t}} \right], \\ &= r_t^k q_{t-1} k_{t-1} [1 - \mu G(\bar{\omega}_t)] - \frac{r_{t-1} b_{t-1}}{1 + \tau_{f,t}}.\end{aligned}$$

A.4 Capital producer

Capital producers operate in a perfectly competitive market. At the end of period $t - 1$, entrepreneurs buy the capital stock to be used in period t , i.e. k_{t-1} , from the capital producers. Once intermediate goods are sold and capital services paid, entrepreneurs sell back to the capital producers the remaining un-depreciated stock of capital. The representative capital producer then builds new capital stock, k_t , by combining investment goods, i_t , and un-depreciated capital, $(1 - \delta) k_{t-1}$. The capital producer's problem is:

$$\max_{i_t} E_T \sum_{t=T}^{\infty} \beta^{t-T} \frac{\lambda_t}{\lambda_T} \{q_t [k_t - (1 - \delta) k_{t-1}] - i_t\}, \text{ subject to} \quad (15)$$

$$k_t = (1 - \delta) k_{t-1} + \left[1 - \Phi \left(\frac{i_t}{i_{t-1}} \right) \right] i_t, \text{ for all } t.$$

Since households own the firms that produce capital, profits are discounted at the rate $\beta^{t-T} \frac{\lambda_t}{\lambda_T}$ for $t \geq T$, where λ_t is the Lagrange multiplier of the household's budget constraint. The function $\Phi \left(\frac{i_t}{i_{t-1}} \right)$ denotes adjustment costs in capital formation. We consider an *investment* adjustment cost, according to which the capital producer uses a combination of old investment goods and new investment goods to produce new capital units (see Christiano, Eichenbaum and Evans, 2005), where $\Phi \left(\frac{i_t}{i_{t-1}} \right) = (\eta/2) (i_t/i_{t-1} - 1)^2$. The first-order conditions of this problem imply that at equilibrium the relative price of capital, q_t , satisfies the following condition:

$$i_t : q_t = \left[\phi_{1,t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} q_{t+1}}{\lambda_t q_t} \phi_{2,t} \right\} \right]^{-1} \text{ where} \quad (16)$$

$$\phi_{1,t} = 1 - \Phi \left(\frac{i_t}{i_{t-1}} \right) - \Phi' \left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \quad \text{and} \quad \phi_{2,t} = \left(\frac{i_{t+1}}{i_t} \right)^2 \Phi' \left(\frac{i_{t+1}}{i_t} \right)$$

A.5 Final goods producers

Final goods, y_t , are used for consumption and investment, and produced in a competitive market by a representative producer who combines a continuum of intermediate goods indexed by $j \in [0, 1]$, via the CES production function

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (17)$$

where $y_{j,t}$ denotes the overall demand addressed to the producer of intermediate good j and θ is the elasticity of substitution among intermediate goods. The representative producer chooses $y_{j,t}$ to maximize its profits subject to its production technology, i.e.

$$\max_{y_{j,t}} P_t y_t - \int_0^1 P_{j,t} y_{j,t} dj, \quad \text{subject to (17),}$$

where $P_{j,t}$ denotes the price of the intermediate good produced by firm j and P_t is the general price index. Profit maximization yields standard demand functions

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} y_t.$$

In turn, the zero-profit condition implies that the general price index is given by

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (18)$$

A.6 Intermediate goods producers

Intermediate goods producers engage in monopolistic competition and produce differentiated goods using labor and capital services, namely $\ell_{j,t}$ and $k_{j,t-1}$, from the producer of good j at date t , respectively. Total labor input in each firm, $\ell_{j,t}$, results from combining household labor, $\ell_{j,t}^h$, and entrepreneurial labor, $\ell_{j,t}^e \equiv 1$, with a Cobb-Douglas function $\ell_{j,t} = [\ell_{j,t}^h]^\Omega [\ell_{j,t}^e]^{1-\Omega}$. Each intermediate good is also produced with a Cobb-Douglas technology

$$y_{j,t} = \ell_{j,t}^{1-\alpha} k_{j,t-1}^\alpha. \quad (19)$$

The cost function $\mathbb{S}(y_{j,t})$ associated with production of $y_{j,t}$ follows from a standard cost-minimization problem:

$$\mathbb{S}(y_{j,t}) = \min_{\ell_{j,t}^h, z_t} \{w_t \ell_{j,t}^h + z_t k_{j,t-1} + w_t^e, \text{ subject to (19)}\}. \quad (20)$$

The marginal cost is therefore $mc_{j,t} \equiv \partial \mathbb{S}(\cdot) / \partial y_{j,t}$, and the labor and capital demands satisfy

$$w_t = mc_{j,t} \Omega (1 - \alpha) \frac{y_{j,t}}{\ell_{j,t}^h}, \quad (21)$$

$$w_t^e = mc_{j,t} (1 - \Omega) (1 - \alpha) \frac{y_{j,t}}{\ell_{j,t}^e}, \quad (22)$$

$$z_t = mc_{j,t} \alpha \frac{y_{j,t}}{k_{j,t-1}}. \quad (23)$$

Because all intermediate producers have the same technology, it follows that $mc_{j,t} = mc_t$. Aggregate expressions for the input demand curves are thus the following:

$$\begin{aligned} \ell_t^h w_t &= mc_t \Omega (1 - \alpha) Y_t^{\text{supply}}, \\ w_t^e &= mc_t (1 - \Omega) (1 - \alpha) Y_t^{\text{supply}}, \\ k_{t-1} z_t &= mc_t \alpha Y_t^{\text{supply}}, \end{aligned}$$

where $Y_t^{\text{supply}} \equiv \int \ell_{j,t}^{1-\alpha} k_{j,t-1}^\alpha dj$, $\ell_t^h \equiv \int \ell_{j,t}^h dj$, $\int \ell_{j,t}^e dj = 1$, and $k_t \equiv \int k_{j,t} dj$. We discuss further details about aggregation later in this subsection.

Price setting Intermediate goods producers face a nominal rigidity in their pricing decision in the form of Calvo (1983)'s staggered pricing mechanism. At each date t , each producer adjusts its price optimally with a constant probability $1 - \vartheta$, and with probability ϑ it can only adjust its price following a passive indexation rule $P_{j,T} = \iota_{t,T} P_{j,t}$, where $t < T$ is the period of last re-optimization and $\iota_{t,T}$ is a price-indexing rule, defined as $\iota_{t,T} = (1 + \pi_{T-1})^{\vartheta_p} (1 + \pi)^{1-\vartheta_p} \iota_{t,T-1}$ for $T > t$ and $\iota_{t,t} = 1$. The coefficient $\vartheta_p \in [0, 1]$ measures the degree of past-inflation indexation of

intermediate goods prices and π is the inflation rate at steady state. In order to remove the steady-state distortion caused by intermediate-goods producers' monopolistic power, we assume that the government provides a subsidy τ_p so that aggregate output reaches the level of the flexible-price economy at the steady state.

Let $P_{j,t}^*$ denote the nominal price optimally chosen at time t and $y_{j,t,T}$ denote the demand for good j in period $T \geq t$ for a firm that last re-optimized its price in period t . Producer j selects $P_{j,t}^*$ to maximize the expected present discounted value of profits (again discounting using the household's stochastic discount factor), taking as given the demand curve for its product:

$$P_{j,t}^* = \arg \max_{P_{j,t}} \left\{ \begin{array}{l} \text{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} \frac{\lambda_T}{\lambda_t} \left[\frac{\iota_{t,T} P_{j,t}}{P_T} y_{j,t,T} - (1 - \tau_p) mc_T y_{j,t,T} \right] \right\} \\ \text{subject to } y_{j,t,T} = \left(\frac{\iota_{t,T} P_{j,t}}{P_T} \right)^{-\theta} y_T \end{array} \right\}. \quad (24)$$

The first order condition of this problem is:

$$P_t^* = \frac{\theta}{\theta - 1} \frac{\text{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} \frac{\lambda_T}{\lambda_t} (1 - \tau_p) mc_T \left(\frac{\iota_{t,T}}{P_T} \right)^{-\theta} y_T \right\}}{\text{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} \frac{\lambda_T}{\lambda_t} \left(\frac{\iota_{t,T}}{P_T} \right)^{1-\theta} y_T \right\}}, \text{ or}$$

$$p_t^* \equiv \frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} (1 - \tau_p) \frac{F_{1,t}}{F_{2,t}},$$

where

$$F_{1,t} = \text{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} \frac{\lambda_T}{\lambda_t} mc_T \left(P_t \frac{\iota_{t,T}}{P_T} \right)^{-\theta} y_T \right\},$$

$$F_{2,t} = \text{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\vartheta)^{T-t} \frac{\lambda_T}{\lambda_t} \left(P_t \frac{\iota_{t,T}}{P_T} \right)^{1-\theta} y_T \right\}.$$

Recursive expressions for the above relations can be stated as:⁴

$$F_{1,t} = mc_t y_t + \beta\vartheta \text{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\iota_{t,t+1}}{1 + \pi_{t+1}} \right)^{-\theta} F_{1,t+1} \right\},$$

$$F_{2,t} = y_t + \beta\vartheta \text{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\iota_{t,t+1}}{1 + \pi_{t+1}} \right)^{1-\theta} F_{2,t+1} \right\}.$$

⁴To find the recursive expression for $F_{1,t}$, notice that $F_{1,t+1} = \text{E}_{t+1} \left\{ \sum_{T=t+1}^{\infty} (\beta\vartheta)^{T-t-1} \frac{\lambda_T}{\lambda_{t+1}} mc_T \left(P_{t+1} \frac{\iota_{t+1,T}}{P_T} \right)^{-\theta} y_T \right\}$, and since $\iota_{t,T} = \prod_{i=1}^{T-t} (1 + \pi)^{1-\gamma_p} (1 + \pi_{T-i})^{\vartheta_p}$, and $\iota_{t+1,T} = \prod_{i=1}^{T-t-1} (1 + \pi)^{1-\vartheta_p} (1 + \pi_{T-i})^{\vartheta_p}$, it follows that $\iota_{t,T} = \iota_{t+1,T} (1 + \pi)^{1-\vartheta_p} (1 + \pi_t)^{\vartheta_p} = \iota_{t+1,T} \iota_{t,t+1}$. A similar reasoning applies for $F_{2,t}$.

To support the efficient (flexible price) production level at the steady state, the production subsidy must be equal to the inverse of the price markup, so $1 - \tau_p = (\theta - 1) / \theta < 1$. Despite this adjustment, the sticky prices still create a dynamic distortion in the form of price dispersion (see Yun, 1996). To see why, we need to equate the aggregate supply of intermediate goods, defined as $Y_t^{\text{supply}} \equiv \int_0^1 \ell_{j,t}^{1-\alpha} k_{j,t-1}^\alpha dj$, to the aggregate demand for intermediate goods, defined as $Y_t^{\text{demand}} \equiv \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} y_t dj$.

To obtain a tractable expression for Y_t^{supply} , notice that the ratio $\frac{k_{j,t-1}}{\ell_{j,t}}$ is the same for all intermediate firms. Using the input demand functions (21)-(23), we can rewrite the capital-to-labor ratio as a function of aggregate wages and the rental rate of capital, i.e.

$$\begin{aligned} \frac{k_{j,t-1}}{\ell_{j,t}} &= \frac{k_{j,t-1}}{(\ell_{j,t}^h)^\Omega (\ell_{j,t}^e)^{1-\Omega}}, \\ &= \frac{s_t \alpha \frac{y_{j,t}}{z_t}}{\left[s_t \Omega (1 - \alpha) \frac{y_{j,t}}{w_t} \right]^\Omega \left[s_t (1 - \Omega) (1 - \alpha) \frac{y_{j,t}}{w_t^e} \right]^{1-\Omega}}, \\ &= \frac{\alpha}{[\Omega(1 - \alpha)]^\Omega [(1 - \Omega)(1 - \alpha)]^{1-\Omega}} \frac{w_t^\Omega (w_t^e)^{1-\Omega}}{z_t}. \end{aligned}$$

Therefore, $\frac{k_{j,t-1}}{\ell_{j,t}} = \frac{k_{t-1}}{\ell_t}$, since it does not depend on j . We can rewrite the aggregate supply of intermediate goods as follows:

$$\begin{aligned} Y_t^{\text{supply}} &= \int_0^1 \ell_{j,t}^{1-\alpha} k_{j,t-1}^\alpha dj, \\ &= \int_0^1 \ell_{j,t} \left(\frac{k_{j,t-1}}{\ell_{j,t}} \right)^\alpha dj, \\ &= \left(\frac{k_{t-1}}{\ell_t} \right)^\alpha \ell_t, \text{ where } \ell_t = \int_0^1 \ell_{j,t} dj, \text{ so} \\ &= k_{t-1}^\alpha \ell_t^{1-\alpha}. \end{aligned}$$

In turn, the aggregate demand for intermediate goods can be stated in terms of price dispersion, i.e.

$$\begin{aligned} Y_t^{\text{demand}} &= \int_0^1 y_{j,t} dj, \\ &= \int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} y_t dj, \\ &= y_t \Delta_t, \end{aligned}$$

where $\Delta_t = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} dj \geq 1$ denotes the efficiency cost of price dispersion.⁵ Finally, the equilibrium in the intermediate-goods market yields:

$$\begin{aligned} Y_t^{\text{demand}} &= Y_t^{\text{supply}} \\ y_t \Delta_t &= k_{t-1}^\alpha \ell_t^{1-\alpha}. \end{aligned}$$

Recursive expressions for the general price index and price dispersion Using the optimal prices selected by intermediate producers, the general price index can be written in a recursive way. First, notice that a proportion $1 - \vartheta$ of intermediate producers has a price level equal to P_t^* , while a proportion $(1 - \vartheta)\vartheta$ has a price level equal to $\iota_{t-1,t} P_{t-1}^*$, and so on. In this case, the aggregate price level is

$$\begin{aligned} P_t^{1-\theta} &= \int_0^1 P_{j,t}^{1-\theta} dj, \\ &= (1 - \vartheta) P_t^{*1-\theta} + (1 - \vartheta)\vartheta (\iota_{t-1,t} P_{t-1}^*)^{1-\theta} + (1 - \vartheta)\vartheta^2 (\iota_{t-2,t} P_{t-1}^*)^{1-\theta} + \dots, \end{aligned}$$

which can be rewritten as

$$P_t^{1-\theta} = (1 - \vartheta) P_t^{*1-\theta} + \vartheta (\iota_{t-1,t} P_{t-1}^*)^{1-\theta}.$$

Dividing the latter by $P_t^{1-\theta}$ yields:

$$\begin{aligned} 1 &= (1 - \vartheta) (p_t^*)^{1-\theta} + \vartheta \left(\frac{\iota_{t-1,t}}{1 + \pi_t} \right)^{1-\theta}, \\ &= (1 - \vartheta) (p_t^*)^{1-\theta} + \vartheta \left[\frac{(1 + \pi_{t-1})^{\vartheta_p} (1 + \pi)^{1-\vartheta_p}}{1 + \pi_t} \right]^{1-\theta}, \end{aligned} \quad (25)$$

where π is the central bank's inflation target.

Following similar steps, the recursive expression for price dispersion is:

$$\begin{aligned} \Delta_t &= \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} dj, \\ &= (1 - \vartheta) p_t^{*-\theta} + (1 - \vartheta)\vartheta \left(\frac{\iota_{t-1,t} P_{t-1}^*}{P_t}\right)^{-\theta} + (1 - \vartheta)\vartheta^2 \left(\frac{\iota_{t-2,t} P_{t-1}^*}{P_t}\right)^{-\theta} + \dots, \\ &= (1 - \vartheta) p_t^{*1-\theta} + \vartheta \left[\frac{(1 + \pi_{t-1})^{\vartheta_p} (1 + \pi)^{1-\vartheta_p}}{1 + \pi_t} \right]^{-\theta} \Delta_{t-1}. \end{aligned}$$

⁵Showing that Δ is bounded below by 1 is easy using Jensen's inequality. First, let $p_j = \frac{P_j}{P}$ denote the relative price of firm j , where we have dropped the time subindex for simplicity. Then, notice that the general price index can be rewritten as $\bar{p} \equiv \int_0^1 f(p_j) dj = 1$, where $f(p_j) = (1/p_j)^{\theta-1}$. Next, consider the convex function $g(u) = u^{\theta/(\theta-1)}$, with $\theta/(\theta-1) \geq 1$ for $\theta \geq 1$. The convex transformation of f using g is given by $h(p_j) \equiv g \circ f = (1/p_j)^\theta$. Jensen's inequality states that the convex transformation of the mean of a function is less or equal than the mean of the transformed function, i.e. $g(\bar{p}) \leq \int_0^1 h(p_j) dj$. Since $g(\bar{p}) = 1$, it follows that $1 \leq \int_0^1 (p_j)^{-\theta} dj$.

A.7 Monetary and financial policies

We assume that the central bank and the financial authority follow log-linear rules to set their instruments. These rules change for the analysis of Tinbergen's rule, and for that of strategic interactions. For Tinbergen's rule (in Section 4.2 in the paper), we assume that the central bank follows an augmented Taylor rule for the nominal interest rate in terms of inflation and the external finance premium, and the financial authority follows a static rule for the financial subsidy:⁶

$$R_t = R \times \left(\frac{1 + \pi_t}{1 + \pi} \right)^{a_\pi} \left(\frac{\mathbb{E}_t \{r_{t+1}^k / r_t\}}{r^k / r} \right)^{-\check{a}_{rr}}, \quad (26)$$

$$\tau_{f,t} = \tau_f, \quad (27)$$

where $a_\pi > 1$ is the elasticity of R_t with respect to inflation deviations, $\check{a}_{rr} > 0$ is the elasticity of R_t with respect to deviations in the external finance premium, R is the steady-state gross nominal interest rate, and τ_f is the steady-state level of the financial subsidy. In the monetary rule, \check{a}_{rr} enters with a negative sign because an adverse risk shock pushes up the credit spread and causes a decline in investment, to which the monetary authority responds by lowering its policy interest rate to offset the investment drop. In turn, the τ_f in the financial rule is set to close the financial wedge at the non-stochastic steady state (which implies that $r^k = r$).

In the baseline regime with dual rules, used in the analysis of strategic interactions in Section 4.3 in the paper, the central bank follows a Taylor rule in terms only of inflation, and the financial authority follows a dynamic rule in terms of the external finance premium:

$$R_t = R \times \left(\frac{1 + \pi_t}{1 + \pi} \right)^{a_\pi}, \quad (28)$$

$$\tau_{f,t} = \tau_f \times \left(\frac{\mathbb{E}_t \{r_{t+1}^k / r_t\}}{r^k / r} \right)^{a_{rr}}, \quad (29)$$

where $a_{rr} > 0$ is the elasticity of $\tau_{f,t}$ with respect to deviations in the external finance premium, and τ_f is still set to close the financial wedge at the non-stochastic steady state.

A.8 Flow conditions and resource constraint

The resource constraint is determined by the combination of all the flow conditions of the various agents in the model (budget constraints, net worth, equity of entrepreneurs, firm dividends, etc.). We list below these conditions.

⁶We abstract from a term related to the output gap because our quantitative findings show that in the model we proposed, driven only by risk shocks, it is optimal for the monetary rule not to respond to the output gap (see Section 4.2 in the paper). Nevertheless, in Section B.4 in this Appendix, we present the results of including the output gap as an argument in the Taylor rule.

Government budget constraint The government's budget constraint is:

$$\Upsilon_t = g + \tau_p s_t \int_0^1 y_{j,t} dj + \tau_{f,t} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1}. \quad (30)$$

Government expenditures, g , are kept constant. The government runs a balanced budget, so that the sum of government expenditures, plus subsidies to monopolist producers, plus financial subsidies is paid for by levying lump-sum taxes in the amount Υ_t on households.

Households Aggregating across households, and assuming they use all of their revenues yields

$$c_t + d_t = w_t \ell_t^h + r_{t-1} d_{t-1} - \Upsilon_t + \mathcal{A}_t + \text{div}_t.$$

An important assumption of the model is that households place their deposits with the intermediary and nowhere else.

Entrepreneurs. Their aggregate budget constraint, net worth, and equity are given by:

$$\begin{aligned} n_t + b_t &= q_t k_t, \\ n_t &= \gamma v_t + w_t^e, \\ v_t &= [1 - \Gamma(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1}. \end{aligned}$$

Entrepreneurs' aggregate consumption is $c_t^e = (1 - \gamma) \varrho v_t$, while their transfers to households are given by $\mathcal{A}_t = (1 - \gamma)(1 - \varrho)v_t$.

The lender Households deposits fund loans to entrepreneurs, so $d_t = b_t$ at all times. We assume the lender makes no profits from its activity. The lender's participation constraint and its balance sheet are given by:

$$\begin{aligned} (1 + \tau_{f,t}) [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} &= r_{t-1} d_{t-1}, \\ d_t + (1 + \tau_{f,t}) [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} &= r_{t-1} d_{t-1} + b_t, \end{aligned}$$

The left-hand side of the balance sheet shows inflows at time t received by the lender (deposits and realized loan returns), while the right-hand side displays outflows (principal and interest from $t - 1$ deposits, and new loans). The latter implies again that $d_t = b_t$.

Final-good producer Its zero profit condition implies that $P_t y_t = \int P_{j,t} y_{j,t} dj$.

Intermediate-goods producers Aggregate dividends are given by:

$$\begin{aligned}\text{div}_t &= \int \left[\frac{P_{j,t}}{P_t} y_{j,t} - (1 - \tau_p) mc_t y_{j,t} \right] dj, \text{ or} \\ &= \int \frac{P_{j,t}}{P_t} y_{j,t} dj - (1 - \tau_p) mc_t \int y_{j,t} dj.\end{aligned}$$

The aggregate cost function, which is linear in output, can be stated in terms of aggregate input quantities:

$$mc_t \int y_{j,t} dj = w_t \ell_t^h + z_t k_{t-1} + w_t^e.$$

Capital producer Its zero profit condition implies that $q_t [k_t - (1 - \delta)k_{t-1}] = i_t$.

Adding all up We start from households' budget constraint and add entrepreneurs consumption and use the definition for \mathcal{A}_t :

$$\begin{aligned}c_t + d_t + c_t^e &= w_t \ell_t^h + r_{t-1} d_{t-1} - \Upsilon_t + (1 - \gamma)(1 - \varrho)v_t + \text{div}_t + (1 - \gamma)\varrho v_t, \text{ or} \\ c_t + d_t + c_t^e &= w_t \ell_t^h + r_{t-1} d_{t-1} - \Upsilon_t + (1 - \gamma)v_t + \text{div}_t.\end{aligned}$$

Adding net worth yields:

$$\begin{aligned}c_t + d_t + c_t^e + n_t &= w_t \ell_t^h + w_t^e + r_{t-1} d_{t-1} - \Upsilon_t + (1 - \gamma)v_t + \gamma v_t + \text{div}_t, \text{ or} \\ c_t + d_t + c_t^e + n_t &= w_t \ell_t^h + w_t^e + r_{t-1} d_{t-1} - \Upsilon_t + v_t + \text{div}_t.\end{aligned}$$

Using the definitions of n_t and v_t leads to:

$$c_t + d_t + c_t^e + q_t k_t - b_t = w_t \ell_t^h + w_t^e + r_{t-1} d_{t-1} - \Upsilon_t + [1 - \Gamma(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} + \text{div}_t.$$

Using the balance sheet of the intermediary ($d_t = b_t$), yields:

$$c_t + c_t^e + q_t k_t = w_t \ell_t^h + w_t^e + r_{t-1} d_{t-1} - \Upsilon_t + [1 - \Gamma(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} + \text{div}_t,$$

and using the intermediary's participation constraint, we obtain:

$$\begin{aligned}c_t + c_t^e + q_t k_t &= \left\{ \begin{array}{l} w_t \ell_t^h + w_t^e \\ + (1 + \tau_{f,t-1}) [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} \\ - \Upsilon_t + [1 - \Gamma(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} + \text{div}_t \end{array} \right\}, \text{ or} \\ \left\{ \begin{array}{l} c_t + c_t^e + (q_t k_t - r_t^k q_{t-1} k_{t-1}) \\ + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} \end{array} \right\} &= \left\{ \begin{array}{l} w_t \ell_t^h + w_t^e + \text{div}_t \\ - \Upsilon_t + \tau_{f,t-1} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} \end{array} \right\}.\end{aligned}$$

Using the definition for the rate of capital returns r_t^k , and the zero-profit condition of the capital producer yields:

$$\left\{ \begin{array}{l} c_t + c_t^e + [q_t k_t - z_t k_{t-1} - (1 - \delta) q_t k_{t-1}] \\ + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} \end{array} \right\} = \left\{ \begin{array}{l} w_t \ell_t^h + w_t^e + \text{div}_t - \Upsilon_t \\ + \tau_{f,t-1} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} \end{array} \right\}, \text{ or}$$

$$c_t + c_t^e + i_t + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} = \left\{ \begin{array}{l} w_t \ell_t^h + w_t^e + z_t k_{t-1} + \text{div}_t - \Upsilon_t \\ + \tau_{f,t-1} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} \end{array} \right\}.$$

Using the definition of dividends, the expression for the aggregate cost function, and the zero-profit condition of the final-good producer yields:

$$c_t + c_t^e + i_t + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} = \left\{ \begin{array}{l} w_t \ell_t^h + w_t^e + z_t k_{t-1} - \Upsilon_t \\ + \int \frac{P_{j,t}}{P_t} y_{j,t} dj - s_t \int y_{j,t} dj + \tau_p s_t \int y_{j,t} dj \\ + \tau_{f,t-1} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} \end{array} \right\}, \text{ or}$$

$$c_t + c_t^e + i_t + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} = y_t + \tau_p s_t \int y_{j,t} dj - \Upsilon_t + \tau_{f,t-1} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1}.$$

And finally, adding the government budget constraint, we get the aggregate resource constraint of the economy:

$$\left\{ \begin{array}{l} c_t + c_t^e + i_t + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} \\ + g + \tau_p s_t \int_0^1 y_{j,t} dj \end{array} \right\} = \left\{ \begin{array}{l} y_t + \tau_p s_t \int y_{j,t} dj - \Upsilon_t \\ + \tau_{f,t-1} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} \\ \Upsilon_t - \tau_{f,t-1} [\Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t)] r_t^k q_{t-1} k_{t-1} \end{array} \right\}, \text{ or}$$

$$c_t + c_t^e + i_t + g + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} = y_t.$$

■

A.9 Setup for studying strategic interactions

To conduct the quantitative analysis of strategic interaction, we construct reaction functions that return the payoff-maximizing choice of one authority's policy rule elasticity for a given value of the other authority's rule elasticity.⁷ Denote the reaction function of the monetary authority $a_\pi^*(a_{rr})$, and the reaction function of the financial authority $a_{rr}^*(a_\pi)$, both defined over discrete grids of admissible values of elasticities, such that $A_\pi = \{a_\pi^1, a_\pi^2, \dots, a_\pi^M\}$ and $A_{rr} = \{a_{rr}^1, a_{rr}^2, \dots, a_{rr}^N\}$ with M and N elements respectively. Hence, the strategy space is defined by the $M \times N$ pairs of rule elasticities. Also, denote as $\boldsymbol{\varrho}(a_{rr}, a_\pi)$ the vector of equilibrium allocations and prices of the model for a given set of parameter values (e.g. the baseline calibration) and a particular pair of policy rule elasticities (a_{rr}, a_π) . The reaction functions satisfy the following definitions:

$$a_\pi^*(a_{rr}) = \left\{ (a_\pi^s, a_{rr}^s) : a_\pi^* = \arg \max_{a_\pi \in A_\pi} \text{E} \{L_{CB}\}, \text{ s.t. } \boldsymbol{\varrho}(a_\pi^*, a_{rr}) \text{ and } a_{rr} = a_{rr}^s \right\}_{a_{rr}^s \in A_{rr}},$$

$$a_{rr}^*(a_\pi) = \left\{ (a_\pi^s, a_{rr}^s) : a_{rr}^* = \arg \max_{a_{rr} \in A_{rr}} \text{E} \{L_F\}, \text{ s.t. } \boldsymbol{\varrho}(a_\pi, a_{rr}^*) \text{ and } a_\pi = a_\pi^s \right\}_{a_\pi^s \in A_\pi}.$$

⁷In doing this, we are implicitly assuming commitment to the log-linear policy rules and abstract from studying strategic interaction under discretion (see also De Paoli and Paustian, 2017; Bodenstein, Guerrieri and LaBriola, 2014).

In these definitions, the authorities maximize the *unconditional* expectation of their payoff, which corresponds to its mean in the stochastic steady state.

A Nash equilibrium of a non-cooperative game between the policy authorities is defined by the intersection of the two reaction curves: $N = \{(a_\pi^N, a_{rr}^N) : a_\pi^N = a_\pi^*(a_{rr}^N), a_{rr}^N = a_{rr}^*(a_\pi^N)\}$. A Cooperative equilibrium is defined by a pair of policy rule elasticities that maximize a linear combination of L_{CB} and L_F , with a weight of φ on the monetary authority's payoff, subject to the constraints that the Cooperative equilibrium must be a Pareto improvement over the Nash equilibrium and the arguments of the loss functions must correspond to the equilibrium allocations and prices for the corresponding policy rule elasticities:

$$C(\varphi) = \left\{ \begin{array}{l} (a_\pi^C, a_{rr}^C) \in \arg \max_{a_\pi^s, a_{rr}^{s'} \in A_\pi \times A_{rr}} L_{coop} = E\{\varphi L_{CB} + (1 - \varphi) L_F\} \\ \text{s.t. } \boldsymbol{\varrho}(a_\pi^s, a_{rr}^{s'}), \quad E[L_{CB}] \geq E^N[L_{CB}], \quad \& \quad E[L_F] \geq E^N[L_F] \end{array} \right\},$$

where $E^N[L_{CB}]$ and $E^N[L_F]$ are the payoffs of the central bank and the financial authority in the Nash equilibrium. There can be more than one Cooperative equilibrium depending on the value of φ (i.e. the set of Cooperative equilibria corresponds to the core of the contract curve of the two authorities). For simplicity, we focus on two cases, the symmetric Cooperative equilibrium ($\varphi = 1/2$) and one with the value of φ such that the Cooperative equilibrium yields the highest level of social welfare, denoted φ^* . We also compute Stackelberg equilibrium solutions with either the monetary or the financial authority as leaders. When the monetary (financial) authority is the leader, we compute the value of a_π (a_{rr}) that maximizes L_{CB} (L_F) along the financial (monetary) authority's reaction function.

B Extensions

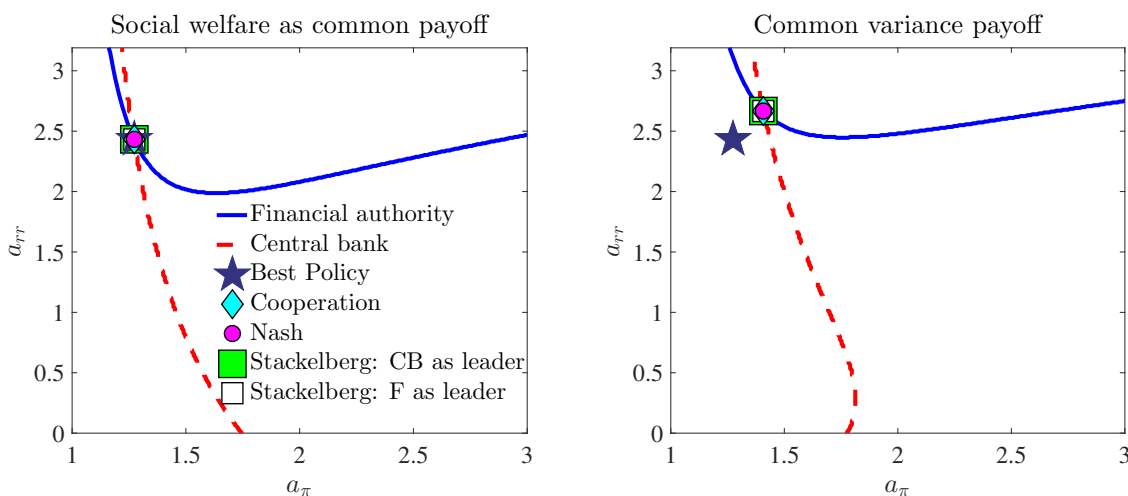
In this section, we explore some additional exercises not included in the paper, such as common payoffs across authorities, alternative calibrations, other macroeconomic shocks, and the implementation of other targeting objectives in the financial policy rule.

B.1 Common payoffs

In Figure 6 in the paper, we present the payoffs for social welfare and the symmetric cooperation (panels (a) and (d), respectively); these correspond to the common payoffs cases. In Figure 1 in this Appendix, we show the reaction curves of each of the authorities under these two scenarios. As in the different variance payoff case, presented in the paper, the reaction functions are again nonlinear. With welfare as common payoff the Cooperative, Nash, and two Stackelberg equilibria coincide. There is no coordination failure and there are no gains from policy coordination. This is

a straightforward result since the spillovers of the policy rule elasticities chosen by each authority are relevant only through their effect on the arguments of the payoff function of the other authority. Hence, the same result holds for any other specification of a common payoff function. What changes, however, is that with social welfare as the payoff, the Nash and Cooperative equilibria also match the best policy outcome that maximizes social welfare (see left-hand-side picture in Figure 1), while with other common payoff functions the Nash and Cooperative equilibria are still the same but do not match the best policy outcome. The latter is shown in the right-hand-side picture in Figure 1, where the common payoff function is $\tilde{L}_{CB} = \tilde{L}_F = -[\text{Var}(\pi_t) + \text{Var}(R_t) + \text{Var}(r_t^k/r_t) + \text{Var}(\tau_{f,t})]$.

Figure 1: Reaction curves and equilibrium outcomes with common payoffs



It is worth noting that the above result implies that assuming a common payoff is equivalent to assuming cooperation in the setup we are studying. Hence, the notion that coordination failure could be removed with the seemingly simple step of giving the two authorities the same payoff is in fact as complex as asking them to coordinate fully.

The result that Nash and Cooperative equilibria are the same with common payoff functions is not new in the monetary policy literature. For example, Dixit and Lambertini (2001) found in a monetary-union model that, when all countries and the central bank have the same objective, the ideal outcomes can be achieved in cooperative and non-cooperative games. Blake and Kirsanova (2011) show in a model with a monetary-fiscal interaction that the Nash equilibrium with a common payoff coincides with the Cooperative equilibrium, because the optimality conditions of the two problems are identical. Bodenstein *et al.* (2014) show a similar result in a model with monetary-financial interactions with open-loop Nash strategies.

B.2 Optimal weights on variance payoffs under coordination

In the strategic interaction analysis in Section 4.3 in the paper, we assumed that under coordination, the payoffs of both monetary and financial authorities have an equal weight, so the coordinated payoff was $L_{coop} = \frac{1}{2}(L_{CB} + L_F)$. However, as shown above, a more general formulation of the payoff function under coordination is a weighed sum of the payoffs of the two authorities, $L_{coop} = \varphi L_{CB} + (1 - \varphi)L_F$, where $\varphi \in [0, 1]$ is the weight assigned to the monetary authority, which must be in the $(0, 1)$ interval and must result in a Pareto improvement (i.e. both the monetary and the financial authority must be as well off as in Nash equilibrium). In the exercise below, we search for the value of φ that minimizes the distance between the welfare achieved under the coordinated game and the welfare achieved under the scenario we labeled best policy in the paper (in which a social planner chooses the policy-rule parameters to maximize welfare).

Formally, the optimal weight φ^* solves the following problem:

$$\varphi^* \in \left\{ \begin{array}{l} \arg \min_{\varphi} \left[\mathbb{W}(a_{\pi}^*, a_{rr}^*) - \mathbb{W}(a_{\pi}^C(\varphi), a_{rr}^C(\varphi)) \right]^2 \\ \text{s.t. } \varrho(a_{\pi}^s, a_{rr}^{s'}), \quad E[L_{CB}] \geq E^N[L_{CB}] \quad \& \quad E[L_F] \geq E^N[L_F] \end{array} \right\},$$

where (a_{π}^*, a_{rr}^*) denote the first-best elasticities (i.e. those chosen by the social planner when maximizing welfare), and (a_{π}^C, a_{rr}^C) are the elasticities needed for a cooperative payoff that minimizes the difference with respect to the best policy payoff.

Figure 2: Reaction curves and equilibrium outcomes with the optimal weight φ^*

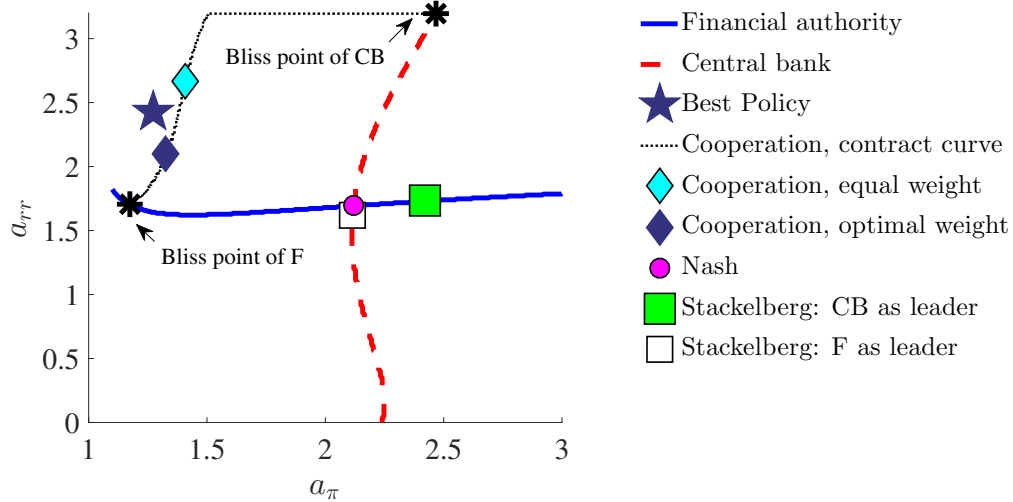


Figure 2 presents the results for the cooperative equilibrium with the optimal weight of $\varphi^* = 0.23$ obtained given the baseline calibration assumed in the paper. As intended, the equilibrium under cooperation with the optimal φ^* (blue diamond) is very close to the first-best outcome (star). Also, the best-response schedules for the common variance payoff case change slightly with the optimal φ^* with respect to the case in which $\varphi = 0.5$.

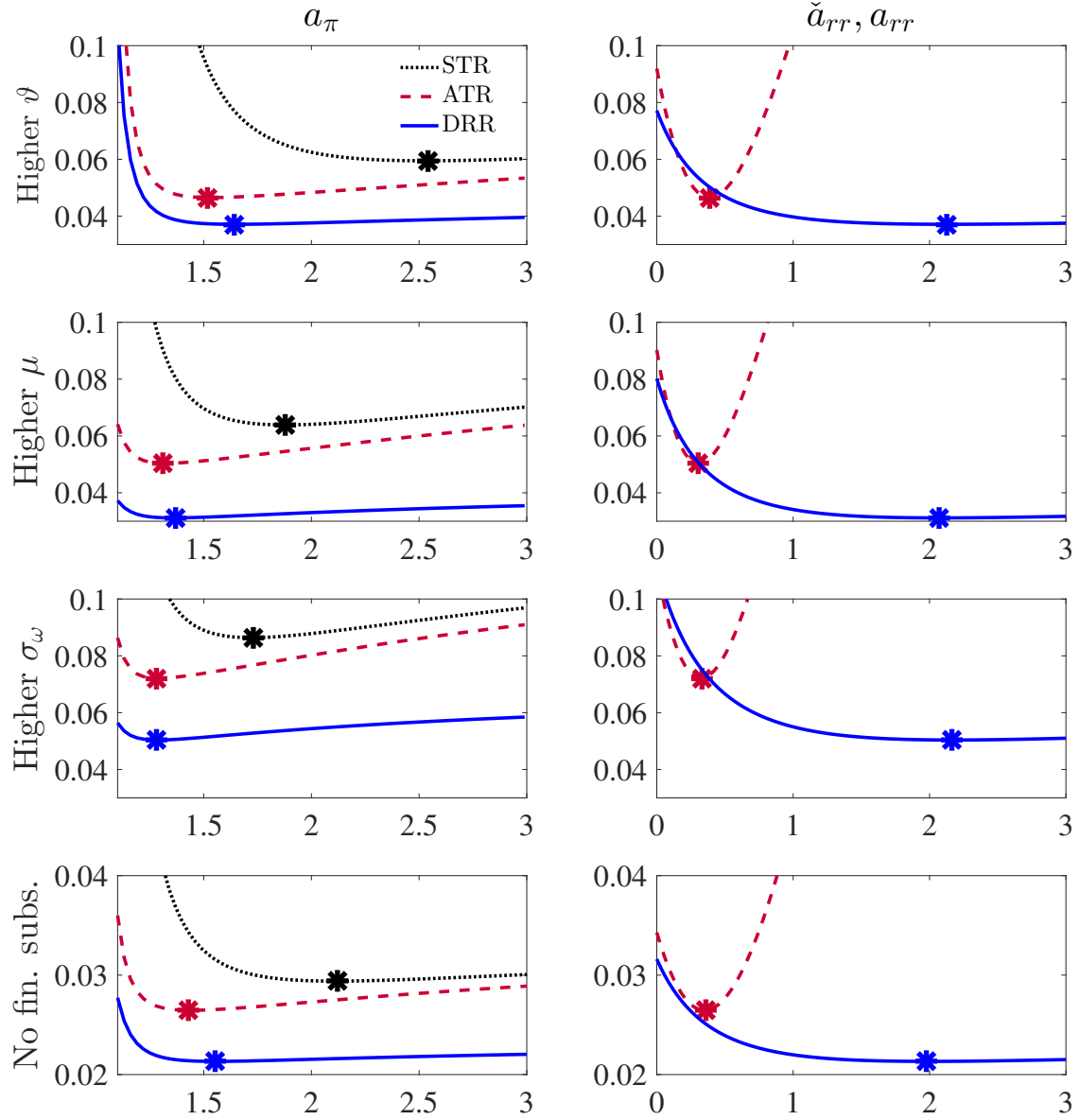
B.3 Alternative calibration for nominal rigidities, financial frictions, and profits dispersion

We now present the results of re-evaluating the quantitative implications of Tinbergen’s rule and strategic interaction under four alternative parameterizations: (a) “stickier” prices, so $\vartheta = 0.85$ v. the baseline value of 0.74, (b) larger monitoring costs, so $\mu = 0.30$ v. the baseline value of 0.12, (c) riskier entrepreneurs, so $\bar{\sigma}_\omega = 0.40$ v. the baseline value of 0.27, and (d) zero steady-state financial subsidy, so $\tau_f = 0$ v. the baseline value of 0.96%. For each alternative calibration, we re-compute the optimized policy rule elasticities that minimize welfare costs under the STR, ATR, and DRR regimes, and solve for the cooperative and non-cooperative equilibria with the different variance payoffs.

Figure 3 shows that Tinbergen’s rule remains relevant in all four cases (equivalent to Figure 3 in the paper). The DRR regime delivers welfare gains of roughly 1% and 2% relative to the ATR and STR regimes, respectively. Also, the STR regime is tight money and tight credit with respect to both the ATR and DRR regimes. In turn, the ATR regime features slightly-loose money and tight credit with respect to the best policy outcome (DRR). Overall, the results are quite similar to Figure 3 in the main paper.

In addition, Figure 4 shows that in all cases, strategic interaction’s results echo the baseline findings, with some variations. In panel (a) of Figure 4, the best response of the monetary rule elasticity features larger complementarities with the financial rule elasticity. This is the case because a higher degree of nominal rigidities implies a more aggressive response of the central bank for low values of a_{rr} , i.e. when the financial authority adopts a relatively passive rule. Thus, the monetary authority finds optimal to increase a_π more aggressively than in the baseline as a_{rr} decreases. In cases (b) and (c), making financial frictions more severe by either setting $\mu = 0.30$ or $\bar{\sigma}_\omega = 0.40$, increases the variability of the external finance premium and inflation, as Table 1 below shows. The latter implies that the policy rules have to respond to potentially larger deviations from their targets. The second row in Table 1 and panel (b) in Figure 4 show that when $\mu = 0.30$, efp becomes much more volatile than π , and thus we find a shift towards easier credit policy by the financial authority in comparison to the baseline, while monetary policy increases its strategic complementarity with financial policy. As a result, the policy regime is loose-money, loose-credit

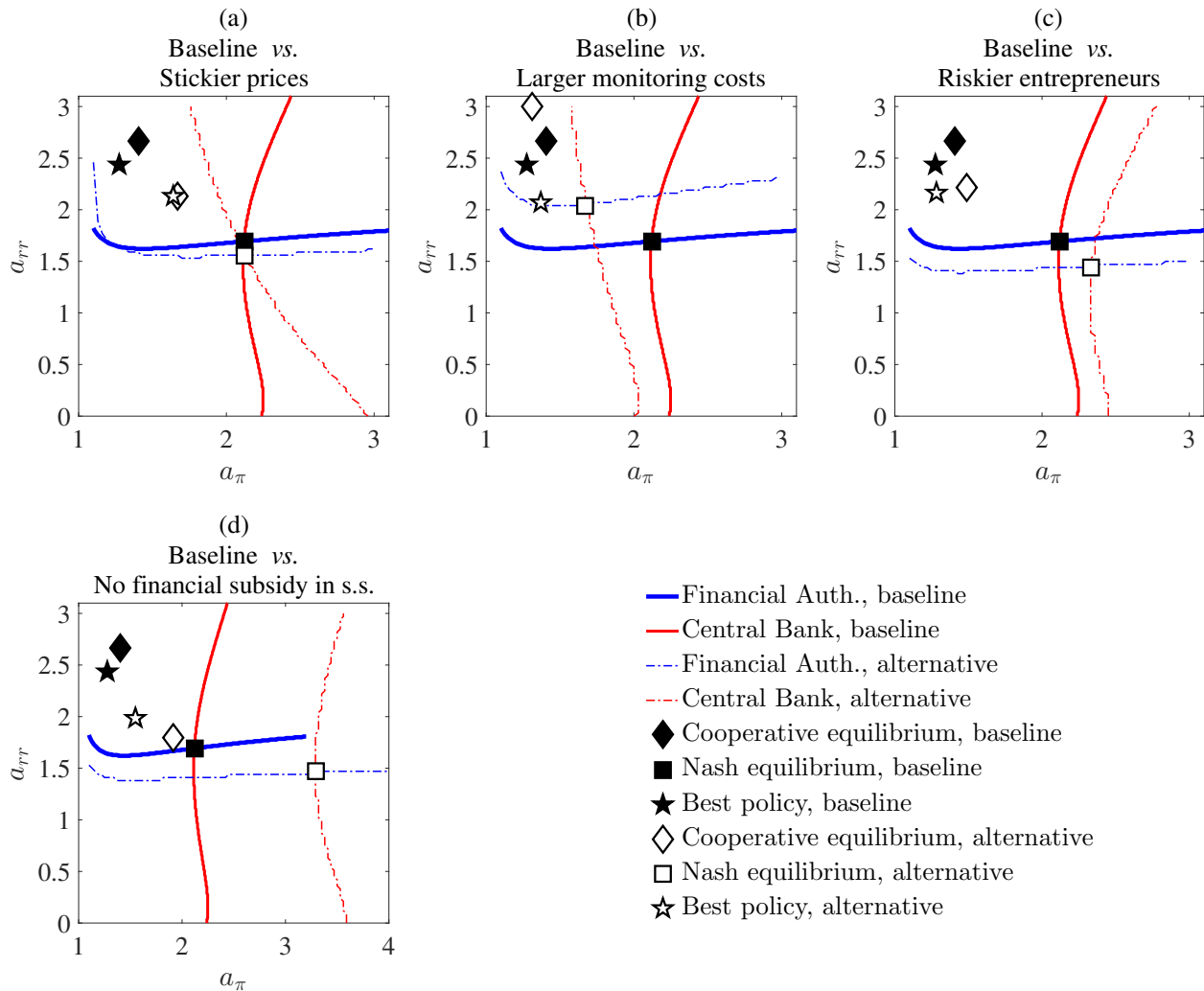
Figure 3: Baseline *v.* alternative calibrations under alternative policy regimes



Note: Asterisks show the lowest welfare cost on each curve. The *y* axis denotes *ce* cost; the left-hand-side (right-hand-side) plots show how *ce* varies as a_π (a_{rr} or \check{a}_{rr}) changes while a_{rr} or \check{a}_{rr} (a_π) is kept fixed. The *x* axis denotes different levels of the inflation coefficient, a_π , in the left-hand-side panels and different credit spread coefficients, a_{rr} or \check{a}_{rr} , in the right-hand-side panels.

in comparison to the baseline. In the case with $\bar{\sigma}_\omega = 0.40$, the variances of efp and π increase in a similar manner, and so we find just a slight shift in panel (c) of Figure 4 towards a regime of tighter-money, tighter-credit with respect to the baseline, with only moderate displacements in the reaction curves of the two policymakers.

Figure 4: Baseline v. alternative calibrations with different payoffs



Note: The solid blue and red lines correspond to the baseline calibration, while the dashed blue and red lines denote the alternative calibration.

In case (d), which removes the steady-state financial subsidy, the Nash equilibrium yields a slightly tighter financial policy rule (lower a_{rr}) and a much tighter monetary policy rule (higher a_π) than in the baseline Nash equilibrium and in the cooperative and best-policy equilibria with $\tau_f = 0$. The curvature of the reaction curves is similar to that in the baseline case, but the monetary (financial) authority's reaction curve shifts rightward (downward). The monetary authority sets a_π

Table 1: Standard deviation of inflation and the external finance premium under larger financial frictions (in percentage %)

Case	σ_π	σ_{efp}
Baseline	0.36	0.33
Larger monitoring costs	0.43	0.50
Riskier entrepreneurs	0.44	0.41

Note: For this comparison, we used the DRR regime with parameter $a_\pi = 1.27$ and $a_{rr} = 2.43$, i.e. the optimum values for the baseline case in the paper.

higher when $a_{rr} = 0$ than in the baseline because when both $a_{rr} = 0$ and $\tau_f = 0$, effectively there is no financial policy. Since in the neighborhood of $a_{rr} = 0$ the two policy rule elasticities are strategic complements, when the financial policy is completely removed the central bank cannot benefit from the effect of the financial policy response to risk shocks on inflation, and thus it finds optimal to choose a higher Taylor rule elasticity. The reaction curve of the financial authority shifts because of an analogous effect.

B.4 Monetary policy rule with an output gap argument

In the main text, we assume that the monetary policy rule does not include an output gap argument. In this subsection, we provide intuition for our assumption. In particular, we show that for the model we consider, the optimal elasticity for the output gap in the Taylor rule is equal to zero. That is, households' welfare is maximized in the model if the central bank does not answer to the output gap after a risk shock. This is the case because if the central bank does adjust the nominal interest rate to changes in the output gap after a risk shock, then the volatility of inflation increases. However, if the objective function of the central bank differs from households' welfare and is equal to a loss function that puts weight on minimizing the variance of the output gap, then the central bank would like to respond as well to output-gap fluctuations after a risk shock, which would result in a reduction in welfare.

For this exercise, we assume that the monetary policy rule takes the form

$$R_t = R \left(\frac{1 + \pi_t}{1 + \pi} \right)^{a_\pi} \left(\frac{E_t \{ r_{t+1}^k / r_t \}}{r^k / r} \right)^{-\tilde{a}_{rr}} \left(\frac{\tilde{y}_t}{\tilde{y}} \right)^{a_y}, \quad (31)$$

where \tilde{y}_t is an output gap measure that can take one of two following specifications:

1. *Full y*, in which \tilde{y}_t is equal to the economy's resource constraint, i.e. $\tilde{y}_t = c_t + i_t + c_t^e + g + \mu G(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1}$, and

2. *Absortion*, in which \tilde{y}_t is similar to the previous case but it excludes monitoring costs, i.e. $\tilde{y}_t = c_t + i_t + c_t^e + g$.

In turn, the financial policy rule takes the same form as in the paper, so

$$\tau_{f,t} = \tau_f \left(\frac{\mathbb{E}_t \{r_{t+1}^k / r_t\}}{r^k / r} \right)^{a_{rr}}.$$

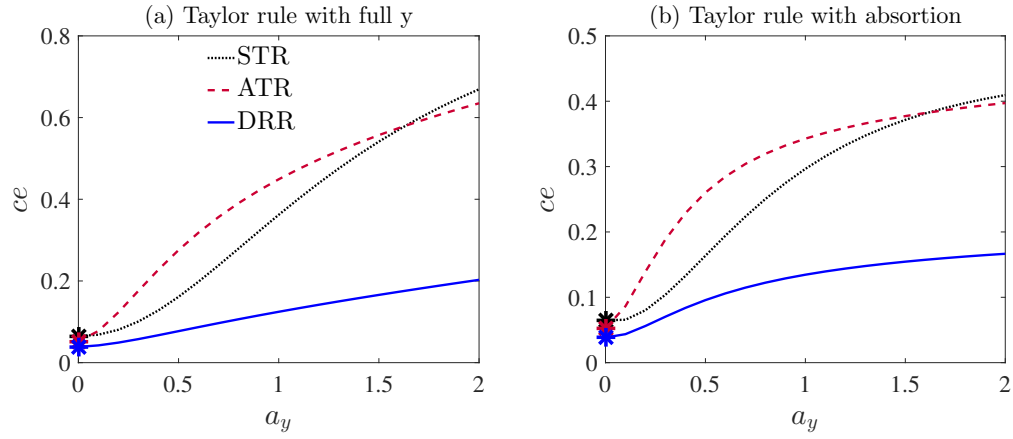
To find the welfare-maximizing elasticities, we perform a two-dimensional search over a_π and a_y for the STR regime while keeping $\check{a}_{rr} = a_{rr} = 0$, and a three-dimensional search over a_π , \check{a}_{rr} , and a_y for the ATR regime (with $a_{rr} = 0$), and over a_π , a_{rr} , and a_y for the DRR regime (with $\check{a}_{rr} = 0$). The lowest welfare costs are always obtained when $a_y = 0$. To illustrate this finding, we compute the *ce* measure and the standard deviation of output σ_y for different values of a_y , while we keep the rest of parameters equal to their optimal levels as shown in Table 2 below. The *ce* informs us about the value of a_y that minimizes welfare costs given the value of all other parameters in the model, while σ_y allow us to compute the value of a_y that minimizes the variance of each of the two alternative measures of the output gap.

Table 2: Optimal parameter combinations for the policy-rule regimes

Regime	a_π	a_{rr}	\check{a}_{rr}	a_y
Simple Taylor rule (STR)	1.75	0	0	0
Augmented Taylor rule (ATR)	1.27	0	0.36	0
Dual rules (DRR)	1.27	2.43	0	0

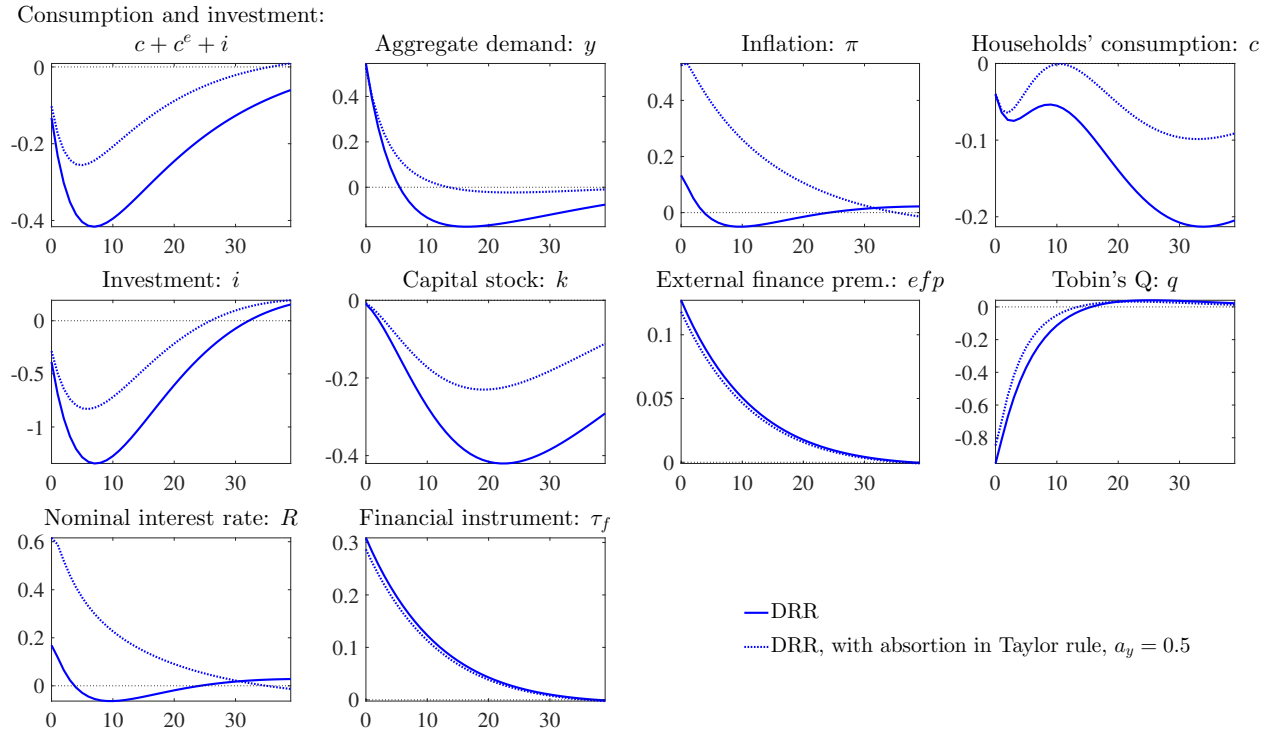
The results are presented in Figures 5 to 7. Figure 5 shows that independently of the measure of output gap considered, welfare costs are minimized when the output-gap elasticity is equal to 0. This result is in line with Schmitt-Grohé and Uribe (2007), who find that in a model with sticky prices, optimal monetary policy features a muted response to output, and that interest rate rules with a positive a_y may lead to significant welfare losses. The reason of why this is the case is presented in Figure 6, where we compare the impulse responses of selected variables to a risk shock under the baseline DRR regime, where $a_y = 0$, and an alternative DRR regime, where $a_y = 0.5$. As shown in the figure, although the responses of consumption and investment are less negative in the alternative regime, the response of inflation increases substantially. Since price dispersion is an important source of welfare costs to households, the aim of optimal monetary policy is precisely to reduce the fluctuations of inflation. Thus, the central bank finds optimal not to answer to the output gap in the face of risk shocks, given the structure of the model economy.

Figure 5: Welfare costs as a function of the output-gap elasticity



Note: The asterisks indicate the minimum welfare costs.

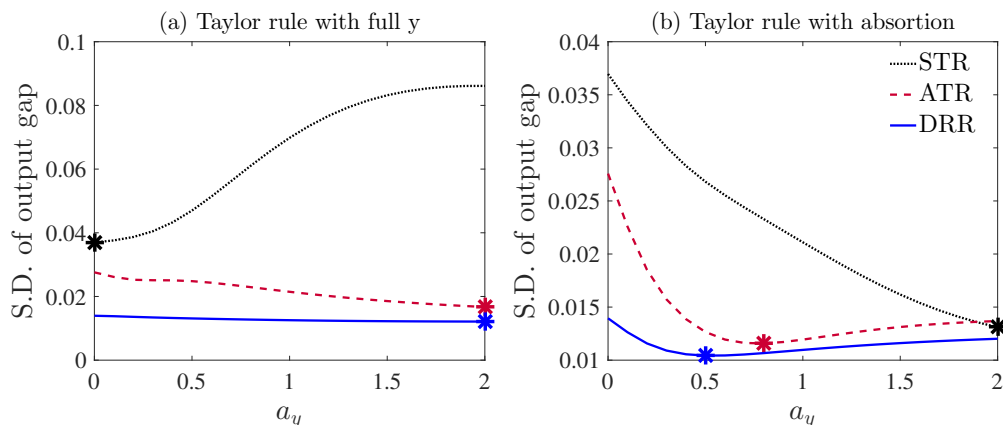
Figure 6: Impulse Response Functions to Risk Shocks for different output gap measures



Note: The x axis denotes periods while the y axis denotes percent deviations from the steady state.

Alternatively, Figures 6 and 7 also show that if the aim of the central bank is to minimize output volatility instead of maximizing welfare, then there is an incentive to adjust the nominal interest rate in response to changes in fluctuations of the output gap. Figure 7 shows that the standard deviation of output can be minimized if $a_y > 0$, at least for the ATR and DRR regimes. Nonetheless, as shown above, an $a_y > 0$ is welfare detrimental in the case of risk shocks.

Figure 7: Standard deviation of output as a function of the output-gap elasticity



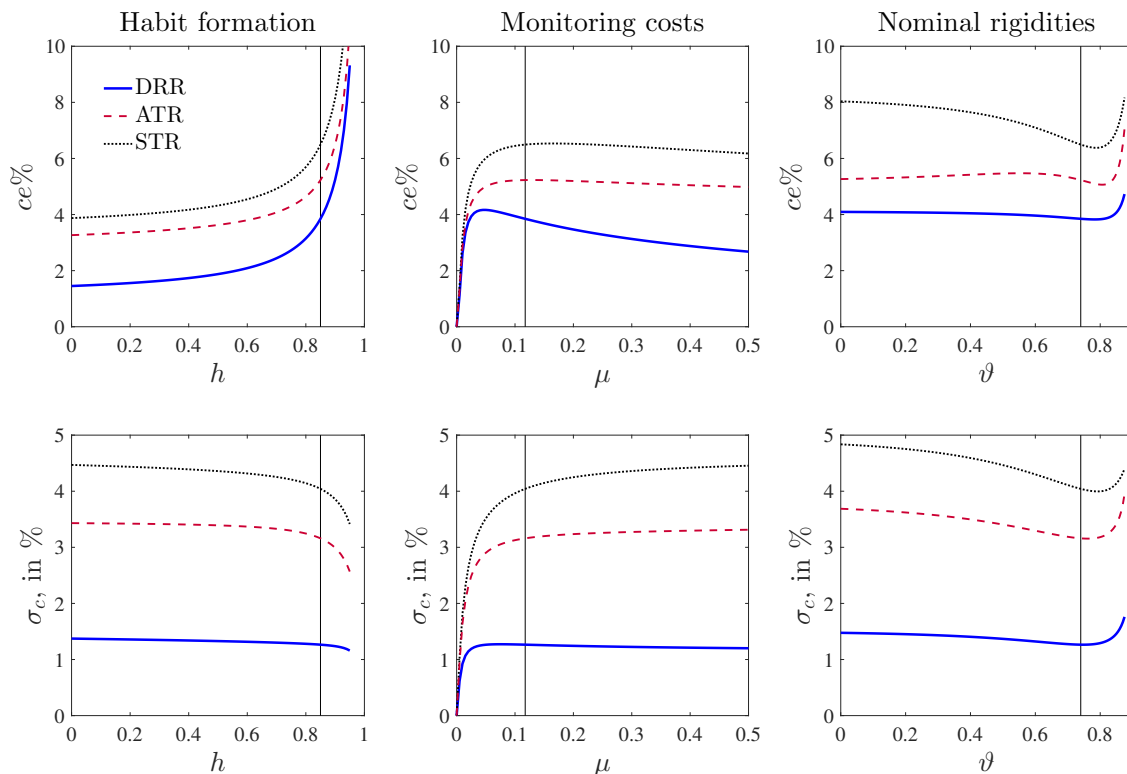
Note: The stars indicate the minimum standard deviation of output.

B.5 Sensitivity of welfare costs to habit formation, monitoring costs, and nominal rigidities

In this section, we analyze the sensitivity of welfare costs to three distortions featuring in the model: habit formation, financial frictions, and nominal rigidities. As in the paper, we measure welfare costs with compensating variations in consumption between the deterministic and stochastic steady states, i.e. ce as computed in equation (2.22) in the paper. In these exercises, we let vary the habit parameter h in the interval $[0, 0.95]$, the monitoring cost proportion μ in the interval $[0, 0.50]$, and the Calvo parameter ϑ in the interval $[0, 0.90]$. In each case, we change one parameter at a time, while we keep the remaining parameters at the benchmark calibration (i.e. $h = 0.85$, $\mu = 0.12$, and $\vartheta = 0.85$).

We perform the sensitivity analysis for the three regimes studied in the paper: DRR, ATR, STR. In all of these regimes, the economy faces a risk shock of persistence $\rho_{\sigma_\omega} = 0.97$ and standard deviation $\sigma_\epsilon = 0.1$, just as in the paper.

Figure 8: Sensitivity of welfare costs for different parameter values

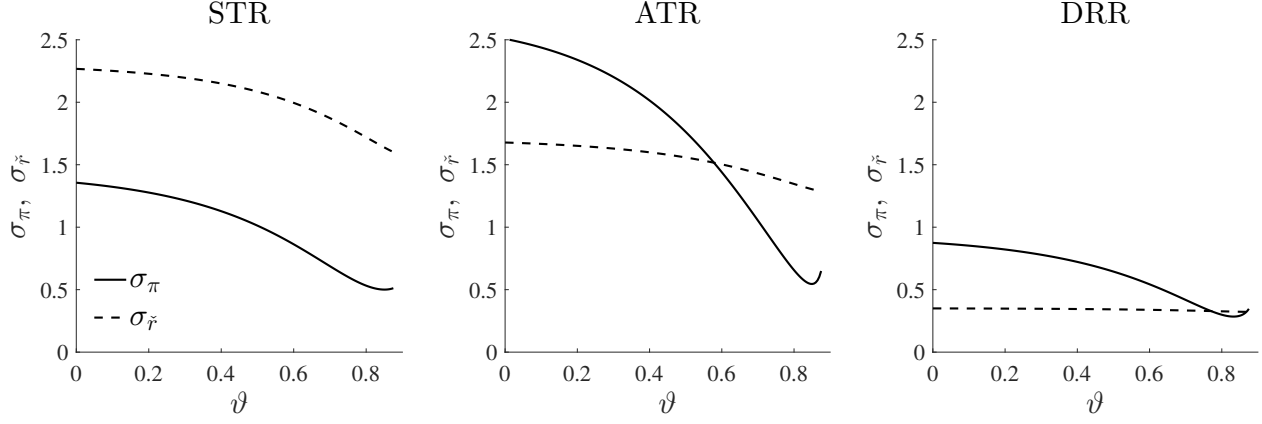


The first row of Figure 8 shows ce as a function of h , μ , and ϑ . The second row plots the standard deviation of consumption, σ_c , in the ergodic distribution of the stochastic steady state also as a function of the aforementioned parameters. The vertical lines in the figure mark the baseline calibration of the parameters. The figure shows that ce is highly non linear to changes in all the three distortions, but is more sensitive to changes in habit formation than to changes in monitoring costs or nominal rigidities. Campbell (1999) provides a rationale of why welfare costs are so sensitive to habit formation. Accordingly, when a shock hits the economy, agents may adjust in the short term through changes in prices or quantities, including consumption. But the latter is costly to households because their habits stop them from adjusting their consuming patterns quickly. As a result, the propagation of the shock lingers for several periods. When habit formation increases, it is even more difficult for households to adjust to shocks because consumption moves even slower. A symptom of the latter is that consumption volatility decreases with h (see the second row of Figure 8), but the latter relates with higher welfare costs because deviations from the long-term equilibrium take more time to dilute.

Figure 8 also suggests that welfare costs increase sharply when moral hazard pops up in the economy. Indeed, when μ equals zero (i.e. when the auditing technology is costless), the welfare costs of risk shocks are also zero, since there are no agency costs generated by moral hazard, no external finance premium, and the allocation of resources is efficient. But as soon as the auditing technology consumes resources, moral hazard appears and generates a positive external finance premium and inefficient fluctuations in output and consumption. Notice, however, that inefficient fluctuations are lower in the DRR than in the ATR and STR regimes, since both consumption volatility and welfare costs are lower in the first regime than in the last two. In the paper, we argue that the DRR helps agents to cope better with the incomplete capital market that results from positive agency costs. In contrast, in the ATR and STR regimes monetary policy alone struggles to stabilize inflation and moderate the inefficiencies associated with the incomplete capital market. Also, notice that in all three regimes welfare costs reach a plateau for different values of μ . In the DRR, ce reach its maximum when $\mu = 0.05$, in the ATR regime, it does when $\mu = 0.13$, and in the STR regime, when $\mu = 0.17$. In turn, consumption volatility stabilizes in the DRR regime, but it steadily grows with μ in the other two regimes.

Finally, Figure 8 reveals that both welfare costs and consumption volatility are lower in the DRR than in the other two regimes for any degree of nominal rigidities. Also, with the exception of the STR regime, ce and σ_c achieve their maximum values when prices are less likely to adjust (i.e. when ϑ is very high). For the STR regime, welfare costs are quite sensitive to changes in the frequency of price adjustments, and notably ce tends to be higher when ϑ is lower. A potential explanation for this puzzling result is that a more frequent adjustment in prices implies larger inflation fluctuations, which in turn brings more volatility to the monetary policy instrument, R , and the external finance premium efp . The latter is plausible because $efp \approx r^k - (R - \pi^e)$, where r^k is the real return of capital, R is the monetary policy rate, and π^e is expected inflation (see equation 11 in page 6 of this Appendix, which describes this relationship in gross rates). Figure 9 plots the volatilities of inflation and the external finance premium as a function of the Calvo parameter ϑ for each of the three regimes studied. The figure confirms that a lower ϑ increases both inflation volatility (the plain line) and the external finance premium volatility (the dashed line) in the STR regime. Interestingly, the ATR regime displays the central bank's trade-off between stabilizing inflation and the external finance premium as a function of ϑ . And finally, in the DRR regime, where the financial authority addresses financial stability, we observe that the volatility of the external finance premium is completely decoupled from that of inflation.

Figure 9: Inflation and external finance premium volatilities in the three regimes as a function of nominal rigidities



B.5.1 Decomposition of welfare costs into mean and standard deviation effects

Table 3 presents a decomposition of ce into a mean effect and a standard deviation effect for the three regimes and different intervals of parameters values. It also shows the ce mean effect that is due to the presence of agency costs, i.e. $\mu > 0$. The table presents average values of ce for the parameter intervals displayed in the first column. As a reminder, recall that ce solves the following equation

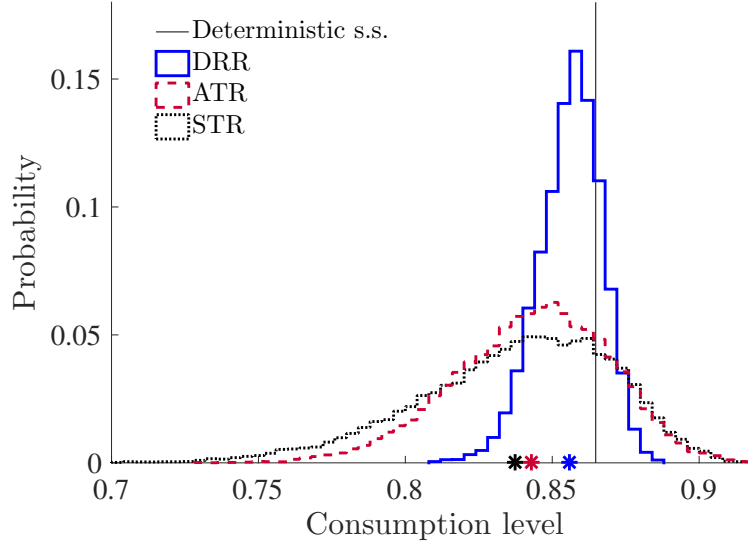
$$\mathbb{W}_{sto} = \frac{\mathcal{U}((1 - ce)c_d, \ell_d^h, (1 - ce)C_d)}{1 - \beta},$$

where the subindex d denotes the deterministic equilibrium of a variable, and \mathbb{W}_{sto} is the ergodic mean of welfare at the stochastic steady state. This level of welfare varies with the regime (i.e. with the triplet $(a_\pi, a_{rr}, \check{a}_{rr})$ configuration) and other parameter values. Variations in welfare are explained by changes in the ergodic mean and standard deviation of consumption and labor at the stochastic steady state. To visualize the latter, Figure 10 compares the histograms of the ergodic distributions of consumption at the stochastic steady state for each regime. The asterisks in the figure represent the mean of consumption in each regime (blue for the DRR, red for ATR, and black for STR). Welfare is lower in the STR regime not only because its consumption mean is lowest, but also because consumption is more volatile in that regime.

Given the particular utility function we assume in the paper and our calibrated inter-temporal elasticity of substitution $\sigma = 1$, ce is computed as follows:

$$ce = 1 - \exp \left\{ (1 - \beta) \frac{\mathbb{W}_{sto} - \mathbb{W}_d}{v} \right\},$$

Figure 10: Histograms of the ergodic distribution of consumption at the stochastic steady state



Note: To obtain these distributions, we solve the model up to a second-order approximation, and then we simulate the equilibrium aggregate dynamics for 50,000 periods assuming that the only stochastic innovations are risk shocks. Each histogram depicts a different regime configuration, corresponding to DRR, ATR, and STR. The asterisks in the x -axis indicate the mean of each distribution, where the colors match the regime, blue for DRR, red for ATR, and black for STR.

Table 3: Decomposition of ce into mean and SD effects

	DDR				ATR				STR			
	ce	Mean		SD	ce	Mean		SD	ce	Mean		SD
		Total	μ			Total	μ			Total	μ	
<i>Habit formation</i>												
$h < 0.31$	1.54	1.52	1.26	0.01	3.34	3.28	0.92	0.06	3.96	3.88	0.67	0.09
$h \in [0.31, 0.63]$	1.86	1.84	1.26	0.03	3.62	3.53	0.91	0.09	4.31	4.17	0.67	0.14
$h > 0.63$	3.78	3.69	1.24	0.10	5.25	4.99	0.89	0.28	6.48	6.07	0.67	0.44
<i>Monitoring costs</i>												
$\mu < 0.16$	3.67	3.60	1.54	0.08	4.69	4.50	1.27	0.20	5.72	5.42	1.10	0.32
$\mu \in [0.16, 0.32]$	3.31	3.20	0.92	0.12	5.16	4.86	0.56	0.32	6.48	5.93	0.33	0.59
$\mu > 0.32$	2.86	2.73	0.74	0.13	5.04	4.71	0.38	0.34	6.29	5.65	0.18	0.68
<i>Nominal rigidities</i>												
$\vartheta < 0.29$	4.08	4.01	1.18	0.07	5.31	4.92	0.74	0.41	7.95	7.41	0.52	0.58
$\vartheta \in [0.29, 0.57]$	4.03	3.95	1.20	0.08	5.43	5.09	0.78	0.36	7.55	7.05	0.56	0.54
$\vartheta > 0.57$	3.92	3.82	1.24	0.10	5.32	5.04	0.89	0.30	6.74	6.30	0.67	0.47

Note: ce corresponds to the consumption equivalent welfare measure defined in equation (2.22) in the paper. A given ce means that the corresponding regime has a welfare cost equivalent to ce percent in terms of a compensating consumption variation that would be needed for an agent living in said regime to be as well off as under a scenario without risk shocks. All the numbers shown in the table are averages for the intervals shown in the first column.

where $\mathbb{W}_d = \mathcal{U}(c_d, \ell_d^h, C_d) / (1 - \beta)$ is welfare at the deterministic equilibrium. The mean effect of welfare costs, ce_{mean} , is computed as:

$$ce_{mean} = 1 - \exp \left\{ (1 - \beta) \frac{\mathbb{W}_{mean} - \mathbb{W}_d}{v} \right\},$$

where $\mathbb{W}_{mean} = \mathcal{U}(c_{sto}, \ell_{sto}^h, C_{sto}) / (1 - \beta)$, and the subindex sto denotes the ergodic mean of a variable at the stochastic steady state. The measure ce_{mean} effectively removes the contribution of volatility from the total welfare cost ce . To compute this contribution, that we label ce_{sd} , we solve the following expression:

$$ce_{sd} = 1 - \exp \left\{ (1 - \beta) \frac{\mathbb{W}_{sto} - \mathbb{W}_{mean}}{v} \right\}.$$

The difference $\mathbb{W}_{sto} - \mathbb{W}_{mean}$ measures how much welfare decreases due to the volatility of consumption and labor, once we normalize by the stochastic mean of these variables. Finally, we approximate the welfare costs caused by the presence of agency costs and monitoring activity, $ce_{mean, \mu}$, as follows:

$$ce_{mean, \mu} = ce_{mean} - ce_{mean, \text{without } \mu},$$

where $ce_{mean, \text{without } \mu}$ is computed similar to ce_{mean} but we add to consumption the resources spent in monitoring.

In Table 3, it can be verified that $ce \approx ce_{mean} + ce_{sd}$. This table offers also the following insights. First, most of ce is explained by the mean effect. Second, an important part of these costs are due to monitoring activity. Third, the standard deviation effect is of the same scale of magnitude than the costs of business cycle of Lucas (1987), i.e. these costs translate into a permanent decrease of around 0.01% to 0.68% of deterministic consumption. And fourth, as ce rises, the contribution of ce_{sd} increases relative to that of ce_{mean} .

B.6 Other shocks

We now analyze the plausibility of strategic interactions within the baseline model for three different macro shocks separately: a government spending shock, a temporary productivity shock, and a cost-push or inflation-markup shock. These shocks have been widely studied in the literature. We argue that, conditional on our model and the shocks mentioned, there is a scope for strategic interactions only for the cost-push shock.

To introduce these shocks, we assume first that government spending, g_t , is no longer constant, but fluctuates around a fixed long-term level g following a stochastic stationary process. Second, we include a common productivity index in the production function of intermediate firms, so that $y_{j,t} = \exp(A_t) \ell_{j,t}^{1-\alpha} k_{j,t-1}^\alpha$, where A_t is a stochastic stationary process with mean zero. Finally, for

the cost-push or markup shock, m_t , we assume that it has a zero mean and affects prices in the intermediate sector, so that the optimal price-setting equation in that sector changes to

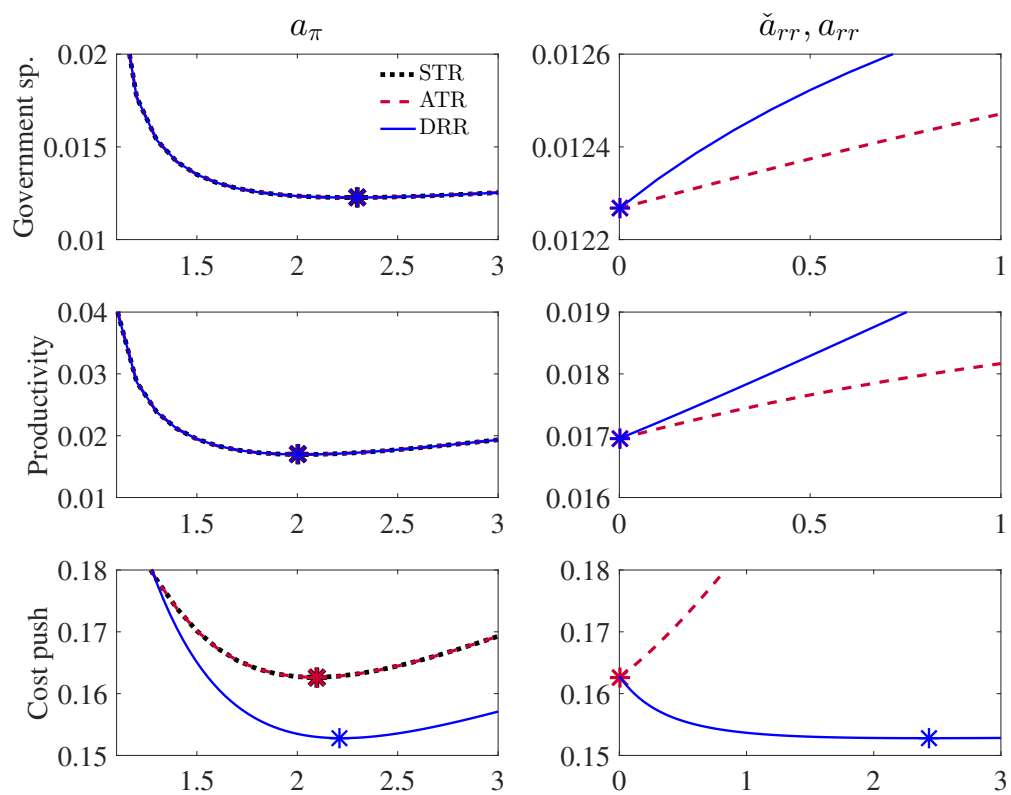
$$p_t^* = \exp(m_t) \frac{\theta}{\theta - 1} (1 - \tau_p) \frac{F_{1,t}}{F_{2,t}}.$$

For simplicity, we assume that all of the shocks follow an AR(1) process, with a persistence parameter equal to 0.8. The size of the shocks is such that it produces a 6% movement in aggregate demand at the pick. The government-spending shock is clearly an aggregate demand shock, and it will tend to move output and inflation in the same direction. In turn, the productivity and cost-push shock are supply-side shocks and they will tend to move output and prices in opposite directions. The difference between them is that after a productivity shock, output and the marginal cost of production behave in opposite directions, while after a mark-up shock they co-move. As a consequence, in the productivity shock labor does not co-move with output, while in the mark-up shock, it does. The two characteristics have relevant implications for the credit market and explains the need to use a dynamic financial instrument only for the cost-push shock.

B.6.1 Tinbergen rule with other shocks

Figure 11 shows the welfare costs as measured by ce under a given pair of policy elasticities (a_π, a_{rr}) or (a_π, \check{a}_{rr}) for the three aggregate shocks. The stars in the picture show the minimum welfare costs achieve in each case. As it is shown in the figure, for the government-spending shock and the productivity shock, maximum welfare is achieved when $\check{a}_{rr} = a_{rr} = 0$. The latter implies that for these shocks, it is optimal for both the central bank and the financial authority to do not adjust their policy instruments to changes in the external finance premium. Because of this result, for these alternative aggregate shocks the STR and ATR regimes are equivalent, and hence we refer to them jointly as the STR/ATR regime, which characterizes by $a_\pi = 2.3$ for government-spending shocks and 2 for TFP shocks. These two shocks do not seem to display a scope for strategic interactions. For the mark-up shock, the single-instrument scenario under STR and the ATR regimes are again identical, since the central bank finds optimal not to answer to the credit spread and focus on inflation deviations. The optimal parameters in the STR/ATR regime are $a_\pi = 2.1$ and $\check{a}_\pi = 0$. However, in the DRR regime, the financial authority finds optimal to adjust its instrument to efp fluctuations, so the optimal parameters in this regime are $a_\pi = 2.2$ and $a_{rr} = 2.4$. The latter results in lower welfare costs to society and very active monetary and financial policies. Therefore, for the markup shock there is scope for strategic interactions.

Figure 11: Welfare costs as policy elasticities vary in face of different shocks



Note: Asterisks show the lowest welfare cost on each curve.

B.6.2 Impulse response functions

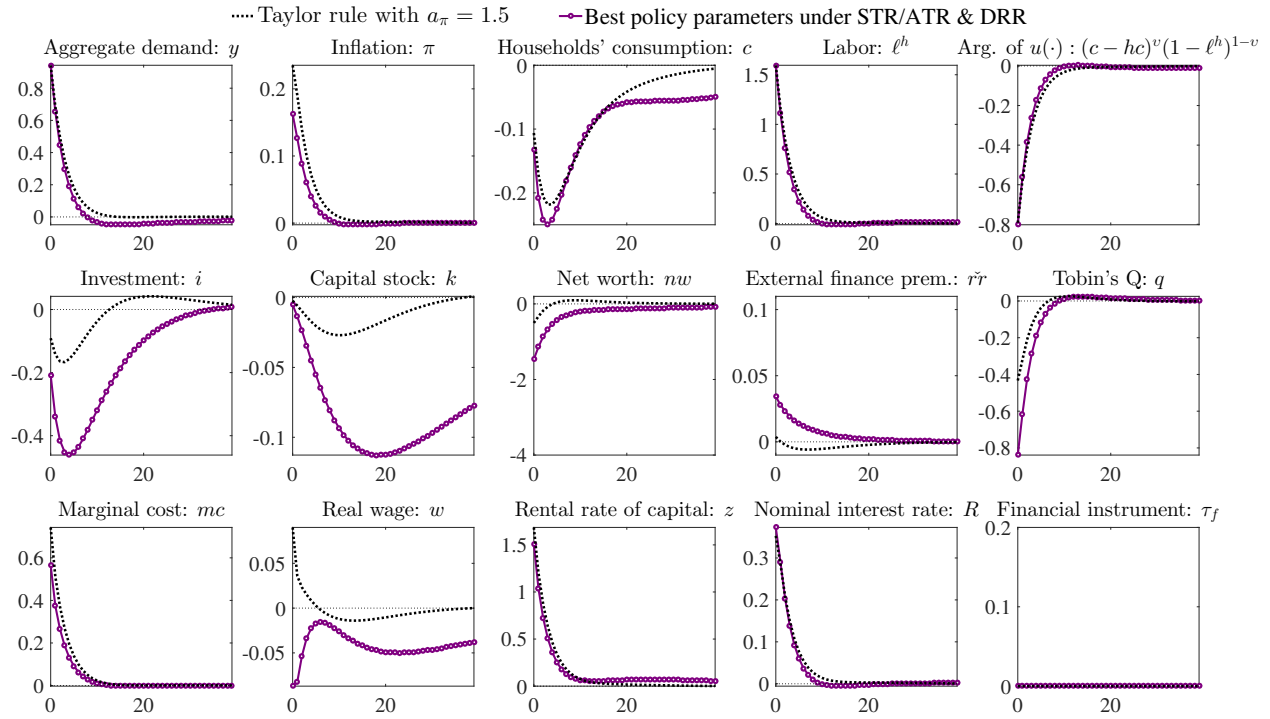
The differences between markup shocks and TFP or government spending shocks on the importance of financial policy follows from the way these shocks interact with the Bernanke-Gertler financial accelerator. To study these interactions, Figures 12 to 14 display the impulse responses of selected variables to the various shocks. The pictures highlight differences in the model's financial transmission under the three shocks, being this mechanism weaker for the TFP and government spending shocks than for markup shocks. Notice that all of these shocks have a positive effect on inflation, and a negative effect on consumption, investment, and the relative price of capital.⁸ In turn, the shock to government expenditures has a positive effect on output, while the other two have a negative effect. For financial transmission the key difference across these shocks is in the response of intermediate goods producers and their effect on credit spreads. With shocks to government expenditures or TFP, intermediate goods producers increase their demand for labor and capital, as they aim to meet the excess demand in the final good market. As a result, the rental rate of capital increases, which increases capital returns and the entrepreneurs' net worth, counteracting the downward pressure on these variables that the fall in the price of capital exerted, and thus weakening the effects of the financial accelerator. The *efp* rises around 4 basis points after the shock to government spending and 5 basis point after the shock to TFP (16 and 20 basis points in annual terms, respectively), while investment falls 0.5% and 0.8% in each case.⁹

In contrast, an increase in markups strengthens the monopolistic distortions affecting the inputs market, causing intermediate good firms to reduce their demand for inputs, so that wages and the rental rate of capital fall. The latter intensifies the reduction in entrepreneurs' net worth, strengthening the financial accelerator. In this case, *efp* rises 10 basis points (40 in annual terms) under the STR/ATR policy, and investment and output decrease 1.5% and 0.4%, respectively. In contrast, in the DRR regime an active financial policy moderates the increase in the *efp* to just 2 basis points (8 in annual terms) after the shock, and investment and output fall only 0.9% and 0.3%. This regime has a short-term cost in terms of consumption, due to the increase in lump-sum taxes that are collected to finance the financial subsidy (see Section 4.2). Still, this short-term cost pays off in the long term since consumption has a higher steady-state mean under the DRR regime than under the STR/ATR regime (about 2.2% higher). In turn, welfare costs are 100 basis points lower in terms of *ce* in the DRR regime in comparison to the STR/ATR regime, as mentioned above.

⁸Shocks to government expenditures have opposing effects on inflation. On the one hand, higher government expenditures increase aggregate demand and hence push for higher inflation. On the other hand, since agents in the model are Ricardian, government expenditures crowd out private expenditures, weakening aggregate demand. Inflation still rises with government expenditures, but by less than a one-to-one effect.

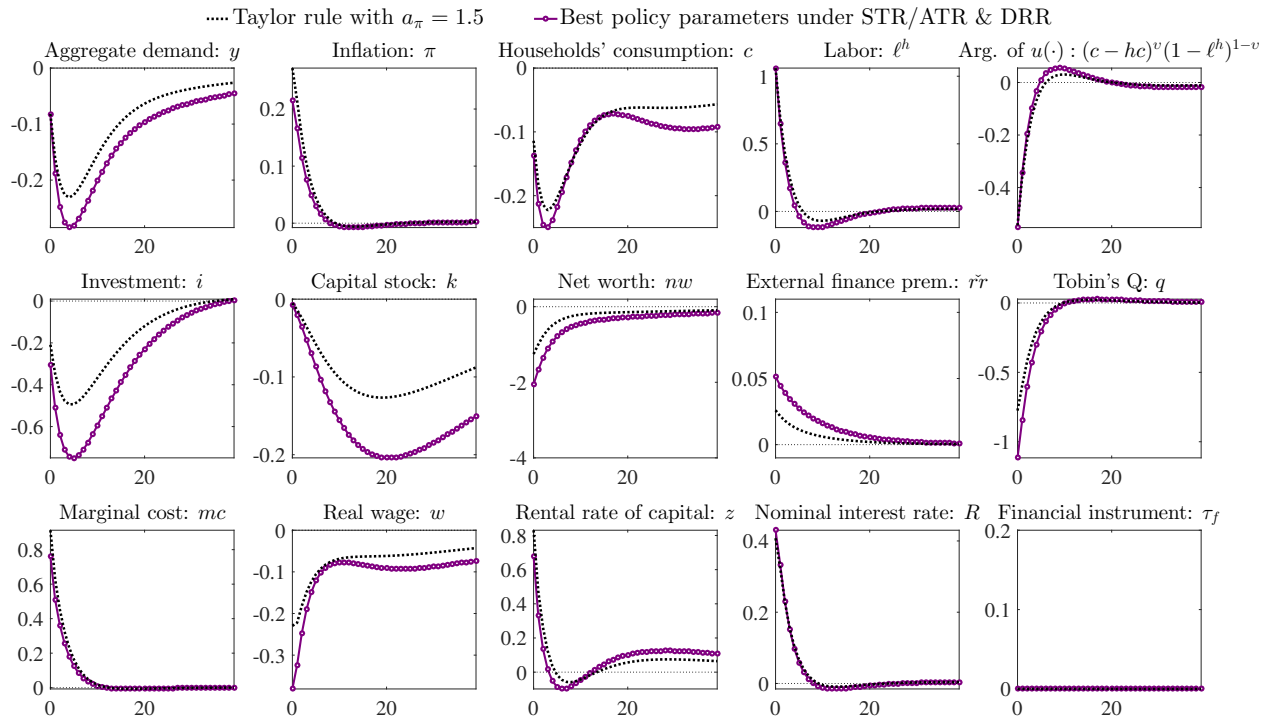
⁹These numbers are consistent with the findings in the literature showing that the financial accelerator accounts for a small share of business cycles in standard BGG models with shocks to TFP or government expenditures.

Figure 12: IRFs to a government spending shock



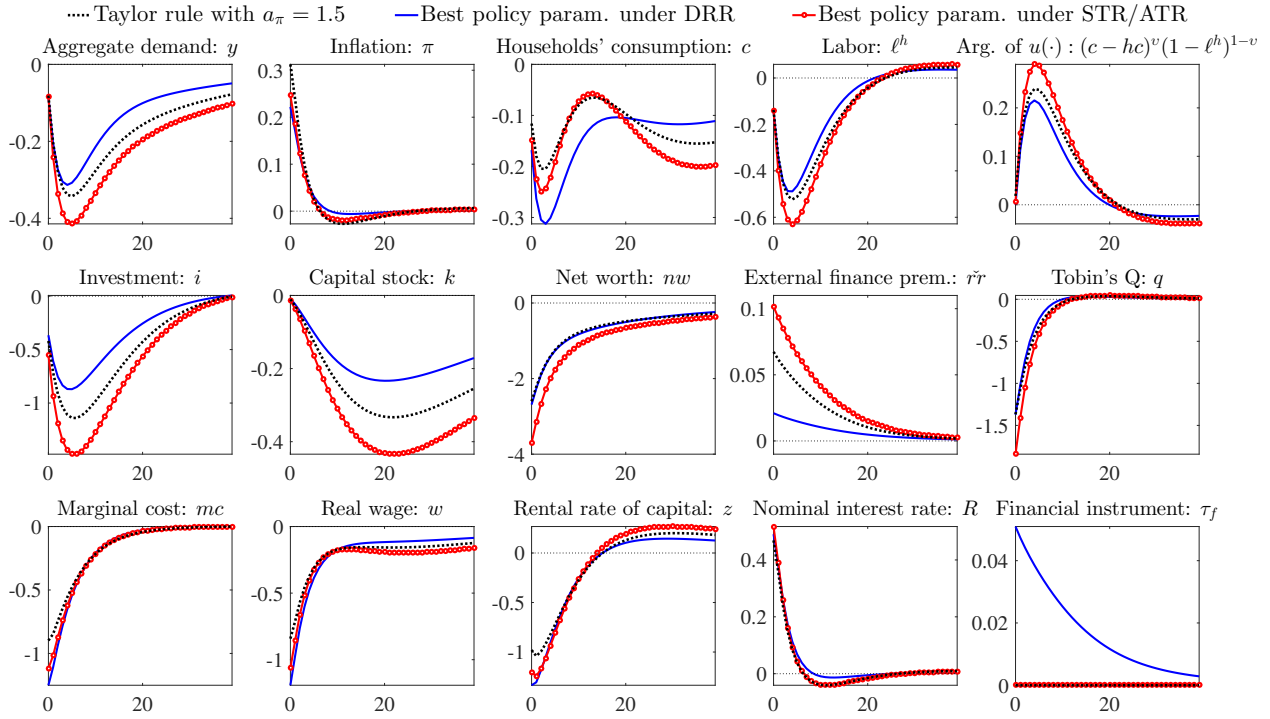
Note: The x axis denotes periods while the y axis denotes percent deviations from the steady state.

Figure 13: IRFs to a productivity shock



Note: The x axis denotes periods while the y axis denotes percent deviations from the steady state.

Figure 14: IRFs to a cost push shock



Note: The x axis denotes periods while the y axis denotes percent deviations from the steady state.

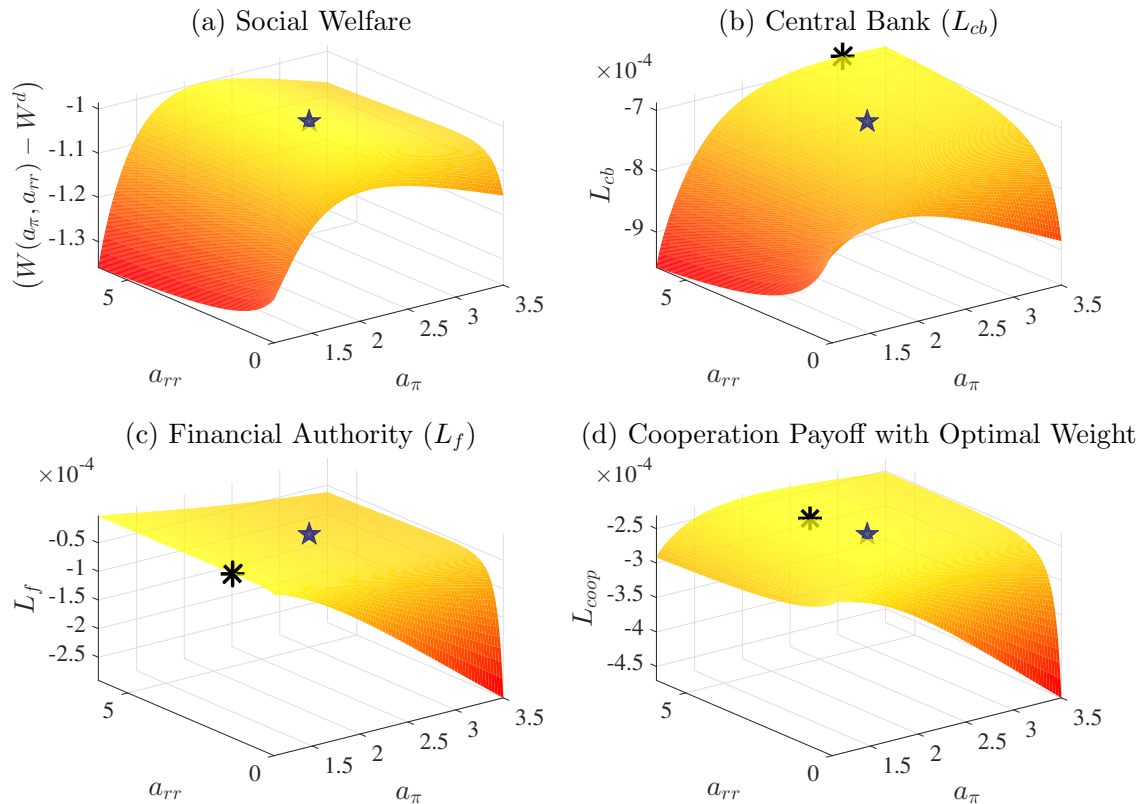
B.6.3 Strategic Interaction: Markup shocks

The following exercises correspond to the strategic interaction set-up of the paper but for the mark-up shock. We first evaluate the reaction function of each policy maker when welfare is the common payoff, then when each authority has a different variance payoff, and finally when they have a common variance payoff. Figure 15 shows that four different payoff functions we consider in this section.

Welfare as a common payoff Panel (a) in Figure 16 shows the reaction curves and the Nash, Stackelberg, and Cooperative equilibria when both policy authorities have welfare as a common payoff. The blue line is the reaction function of the financial authority and the red line is the reaction function of the monetary authority. As in the case of the risk shock, the reaction curves are non-linear. In particular, the best elasticity response of the financial authority shifts from strategic complement to strategic substitute as the a_π increases. Similarly, the central bank's best elasticity response changes from strategic complement to strategic substitute as $a_{r\tilde{r}}$ increases, although the

variation is rather moderate. As argued in the paper, when there is a common payoff, the Cooperative, Stackelberg, and Nash equilibria coincide, so there is no coordination failure and there are no gains from policy coordination. However, when social welfare is the common payoff, the Nash and Cooperative equilibria also match the best policy outcome.

Figure 15: Payoffs for the cost-push shock



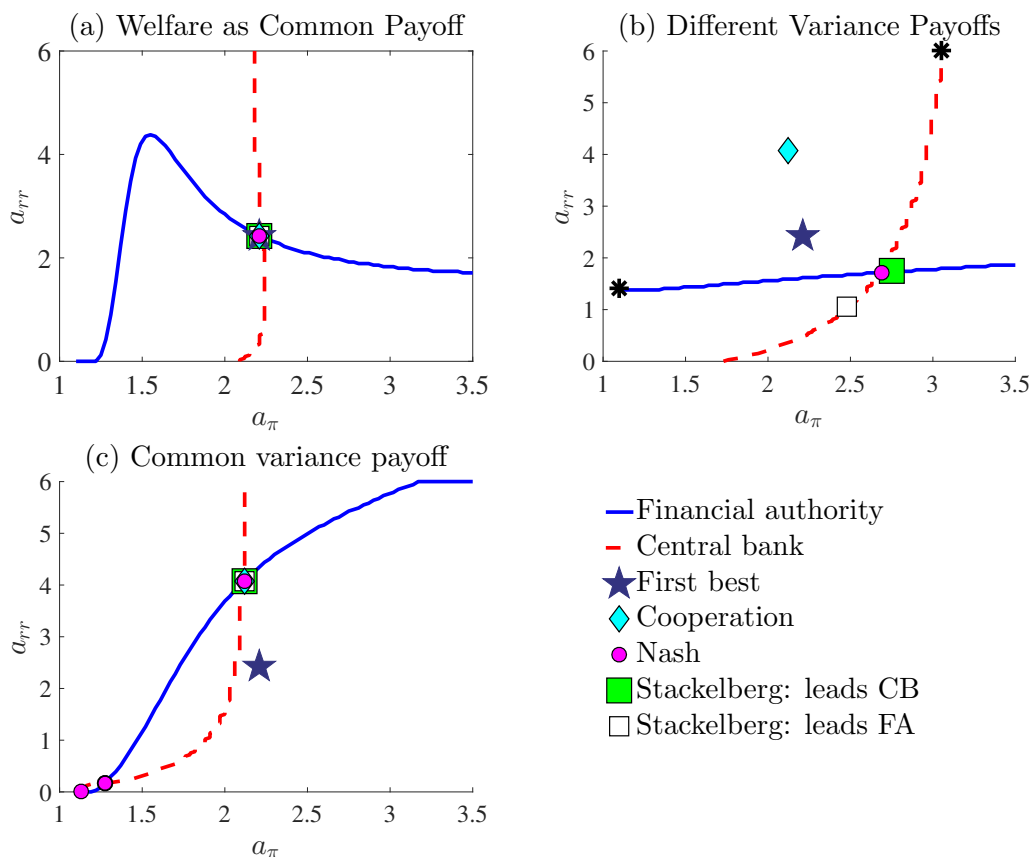
Note: The asterisks indicate the maximum for each loss function and the start indicate the maximum for the welfare or best policy case.

Different payoff functions As in the paper, we consider strategic interaction when the monetary and financial authorities have different payoff functions, given the different objective functions defined earlier. For the cooperative case, we use the optimal weight φ^* , which in this case equals 0.30, so $L_C = \varphi^* L_{CB} + (1 - \varphi^*) L_F$.

Panels (b) and (c) in Figure 15 show surface plots of the individual payoff functions of the monetary and financial authorities as functions of the two policy rule elasticities, and panel (d) shows the payoff function under cooperation with the optimal weight φ^* . As in the case of the risk shock shown in the paper, these plots are single peaked and the elasticity pairs that maximize each

payoff unconditionally (i.e. the bliss points) differ for the monetary and financial authorities, and both also differ from the best policy pair of elasticities. These differences reflect the conflict of objectives between the two authorities and their incentives for engaging in strategic behavior.

Figure 16: Reaction curves, cooperative, and Nash equilibria for the cost-push shock



Panel (b) in Figure 16 displays the reaction functions of the central bank and the financial authority when their payoff functions are given by variances payoffs defined earlier. Qualitatively, the plot is consistent with standard results when coordination failure is present, in terms of the relative location of the Nash and Cooperative equilibria and the bliss points. The reaction functions are non-linear, but now the financial authority's reaction curve only changes slightly, while that of the monetary authority implies that the central bank's best elasticity response is always a strategic complement of a_{rr} .

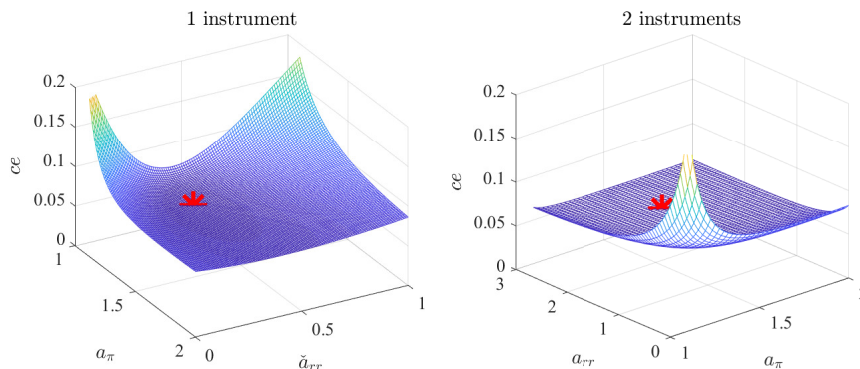
The Nash equilibrium features a higher inflation elasticity than the best policy outcome (2.65 v. 2.20), while the efp elasticity is lower than in the best policy outcome (1.45 v. 2.45). Hence, relative to the best policy pair of elasticities, the Nash equilibrium produces a tight money-tight credit regime. In turn, the Cooperative equilibrium features a slightly lower inflation elasticity than the best policy outcome (2.15 v. 2.20), and a higher efp elasticity than the best policy outcome (3.70 v. 2.45), thus featuring a slightly easy money-easy credit regime.

Since the Nash equilibrium is farthest away from the the best policy outcome, coordination failure implies welfare losses that could be moderated with policy coordination. In terms of welfare costs relative to the deterministic (and Pareto efficient) steady state, the Nash equilibrium has a 1.23 percent higher cost than the best policy, while the cost of the Cooperative equilibrium is only 0.11 percent higher than in the best policy. Hence, the cost of the coordination failure is about 1 percent in terms of a compensating consumption variation.

Common implementable payoff Finally, we consider an alternative scenario in which we use a common payoff function but formulated in terms of a loss function instead of social welfare. As in the paper, we assume that the common payoff is the optimally weighted sum of the variances of all policy instruments and targets. The bottom-left panel of Figure 16 displays the outcome of this alternative scenario. Notice that in this case there are three Nash equilibria, two of them closer to the origin, so that $(a_\pi^{N,1}, a_{rr}^{N,1}) = (1.10, 0)$ and $(a_\pi^{N,2}, a_{rr}^{N,2}) = (1.36, 0.38)$, and a third one equal to the Cooperative equilibria, so $(a_\pi^{N,3}, a_{rr}^{N,3}) = (a_\pi^C, a_{rr}^C) = (2.15, 3.70)$. A way to discriminate between the three Nash equilibria is to change the rules of the game from a one-shot setting to a leader-follower setting. The latter would provide us with a set of Stackelberg equilibria which may vary depending on the specific role taken by each policymaker.

Assume the central banks is the leader. As such, the central bank computes its potential payoffs along the best-response schedule of the financial authority (the solid-blue line in the bottom-left panel of Figure 16). Given the financial authority's best response to any possible value of a_π in the feasible set, the central bank chooses a_π^C , since the couple (a_π^C, a_{rr}^C) is the one that maximizes its payoff function \tilde{L}_{CB} . In the same vein, assume that the financial authority takes the leading role. As such, this policymaker computes its potential payoffs along the best-response schedule of the central bank (the dashed-red line in the bottom-left panel of Figure 16). Similar to the previous case, the couple that maximizes the financial authority's payoff \tilde{L}_F is (a_π^C, a_{rr}^C) . Therefore, the Nash equilibria 1 and 2 are not robust to a change in the moving order of the game, while there is a unique Stackelberg equilibrium regardless of who is taking the leading role. Finally, notice that the Cooperative and the robust Nash equilibria feature a slightly easy money-easy credit regime relative to the best policy outcome.

Figure 17: Welfare Costs under Alternative Policy Regimes with Multiple Shocks



Note: ce is the welfare cost for each pair of elasticities shown in the plots, computed as averages over the ergodic distribution of the welfare measure defined in eq. (2.19). Asterisks denote the minimum of ce (i.e. the best outcome in terms of welfare), which defines the optimized elasticities for the ATR and DRR cases.

B.7 Including all the shocks

We now perform the same exercise as the main text but including the four shocks that we analyze at the same time (risk, temporary productivity, mark-up, and government expenditure shocks). The specifications of the shocks correspond to the ones presented in Section B.6, however, the parameters that we use for the autoregression process and the standard deviations correspond to the ones estimated by Christiano *et al.* (2014). In particular, $\rho_A = 0.81$, $\rho_g = 0.94$, $\rho_{mkp} = 0.91$, and $\sigma_A = 0.0046$, $\sigma_g = 0.023$, and $\sigma_{mkp} = 0.011$. In their estimation, these four shocks explain around 90% of output fluctuations, with risk shocks being the dominant ones explaining 66% of output fluctuations. The messages are the same as in the main text: Tinbergen’s rule is quantitative relevant and there is scope for strategic interaction when all the shocks are included at the same time.

Tinbergen rule Figure 17 shows surface plots of welfare costs for a set of elasticity pairs under the ATR (labeled “1 instrument” in the left plot) and the DRR (labeled “2 instruments” in the right plot). The results for the STR regime are also included. They correspond to the cases with $\tilde{a}_{rr} = 0$ in the ATR case or with $a_{rr} = 0$ in the DRR case.

These surface plots show two key results that are also reflected in the case of only having risk shocks. First, welfare costs are large in all three policy regimes and for all the elasticity pairs considered, with ce ranging from about 4 to 17 percent. This is due to long-run effects of changes in efp and monitoring costs. Second, the curvature of the surface plots is indicative of the relevance of Tinbergen’s rule and strategic interaction. In particular, at low levels of a_π , welfare costs in the ATR show a marked U shape as \tilde{a}_{rr} varies, whereas in the DRR regime they first fall sharply as a_{rr}

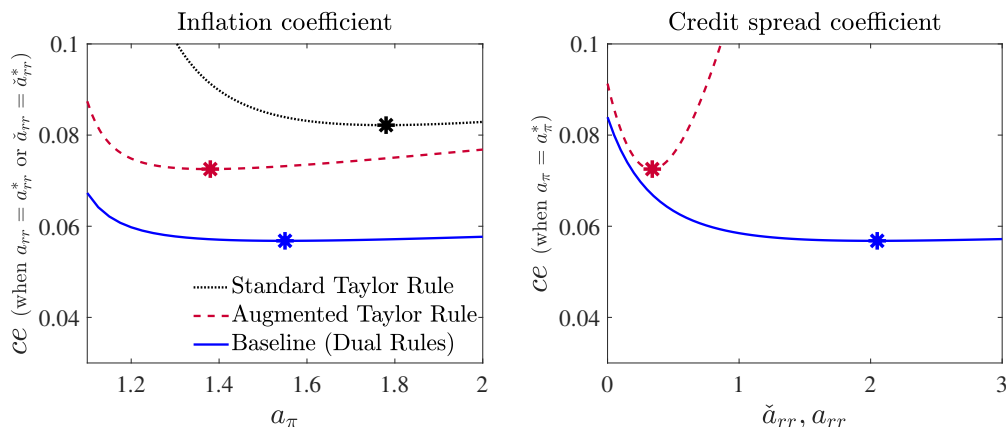
rises from 0 but then change only slightly. For arbitrarily chosen elasticities either regime can be the most desirable. Welfare costs are slightly higher with the DRR than with the ATR for a_{rr} and \check{a}_{rr} near 0 for all values of a_π , but lower if those efp elasticities are sufficiently high. Moreover, for $a_{rr} \geq 1.2$, welfare costs for $a_\pi = 1$ are nearly unchanged as a_{rr} rises in the DRR, whereas under the ATR they are sharply increasing in \check{a}_{rr} when a_π is low but moderately decreasing in \check{a}_{rr} when a_π is high.

These differences in the curvature of the two surface plots indicate that Tinbergen's rule is relevant because they show that the DRR can avoid sharply increasing welfare costs as a_{rr} rises for a given a_π , which is possible because it has separate instruments to tackle price and financial stability. The curvature also indicates that there are significant policy spillovers, which provide the incentives for strategic interaction.

The differences in the welfare costs across policy regimes are illustrated further in Figure 18. This Figure provides plots that show how ce varies as one of the elasticities changes, keeping the other fixed at its optimized value. The plot on the left is for a_π and the one on the right is for \check{a}_{rr} and a_{rr} . The dashed-red curves are for the ATR and the solid-blue curves are for the DRR, and in the left plot the dotted-black line is for the STR. In each curve, asterisks identify the value of the elasticity in the horizontal axis that yields the lowest welfare cost.

The left panel shows that, for all values of a_π considered, ce is uniformly lower under the DRR than under the ATR, and much lower than under the STR. The right panel shows that for spread elasticities below 0.3 ce does not differ much between the DRR and ATR, but for higher spread elasticities ce is much lower under the DRR. Welfare costs under the ATR rise much faster with the spread elasticity, producing a markedly U-shaped curve, while under the DRR welfare costs are nearly unchanged as the spread elasticity rises. This is again evidence of Tinbergen's rule relevance: a_{rr} in the DRR can increase with much less adverse welfare consequences than \check{a}_{rr} in the ATR because the separate financial rule of the DRR targets efp with its own instrument, and hence without affecting the instrument of monetary policy. Notice also that in all the curves in the two plots, there is always an internal solution for the value of the policy rule elasticity with the smallest welfare cost. These findings are important because they show that the different policy regimes, by having different effects on the mechanisms that drive the financial accelerator and the nominal rigidities, produce significant differences in equilibrium allocations and welfare as policy rule elasticities vary, and that as a result there is a non-trivial interaction between monetary and financial policies that yields well-defined optimized rule elasticities.

Figure 18: Welfare Costs as Policy Elasticities Vary: All Shocks Considered



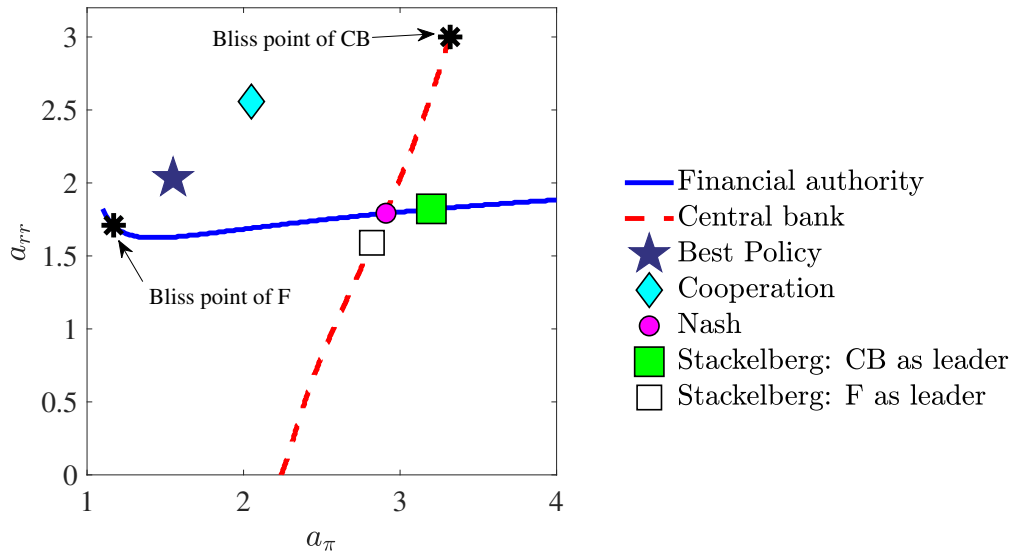
Note: Asterisks show the lowest welfare cost on each curve.

Strategic Interaction Figure 19 displays the reaction functions of the central bank (red-dashed curve) and the financial authority (blue-solid curve) and the equilibria of the various games: Cooperative with $\varphi = 0.5$ (blue rhombus), Nash (pink dot), and Stackelberg with CB (F) as leader (green and transparent squares, respectively). The plots also identify the locations of each authority's bliss point (black asterisks) and of the Best Policy elasticity pair (blue star).

The two reaction curves are convex and change slope, indicating the relevance of the incentives for strategic interaction. The financial authority's reaction curve has a more pronounced curvature than the one for the monetary authority. In the financial authority's reaction curve, for $a_\pi < 1.5$, $a_{rr}^*(a_\pi)$ falls slightly as a_π rises, while the opposite happens for $a_\pi \geq 1.5$, similarly to only having risk shocks. Hence, the financial authority shifts from treating the two elasticities as strategic complements to treating them as strategic substitutes. The monetary authority's reaction curve has analogous features but with weaker curvature, and much weaker than only having risk shocks.

Table 4 compares the outcome of Nash and Cooperative equilibria (symmetric and with the welfare-maximizing value of φ , which is $\varphi^* = 0.27$) against the Best Policy DRR. The Nash equilibrium features a higher a_π than both the two Cooperative equilibria and the DRR outcome (2.91 v. 2.05, 1.55 and 1.55, respectively). Similarly, a_{rr} in the Nash equilibrium is lower than in the Cooperative and DRR equilibria (1.79 v. 2.56, 1.97 and 2.05, respectively). Hence, relative to the DRR and the Cooperative equilibria, the Nash equilibrium is a tight money-tight credit regime: The interest rate rises too much when inflation is above target, and the financial subsidy does not rise enough when the spread is above target. Comparing Cooperative equilibria v. the DRR, the former

Figure 19: Reaction Curves and Equilibrium Outcomes: All Shocks Considered



are still tight-money regimes, but the symmetric Cooperative equilibrium is a loose-credit regime (the financial subsidy rises too much when the spread is above target). The welfare-maximizing Cooperative equilibrium, however, is a tight money-tight credit regime compared with the DRR.

In terms of welfare, the Nash equilibrium is a “third-best” outcome, in the sense that it is inferior to both the Best Policy regime and the Cooperative outcomes. The gains from policy coordination are sizable: Relative to the DRR, the Nash equilibrium implies a reduction in social welfare equivalent to a decline of 37 basis points in the ce measure of welfare. In contrast, the Cooperative equilibrium with $\varphi^* = 0.27$ implies a welfare loss of only 1 basis point. Hence, the welfare cost of coordination failure is roughly 36 basis points (or 26 if we compare v. the symmetric Cooperative equilibrium). It is worth noting that the welfare-maximizing Cooperative outcome increases the weight of the financial authority from 50 to 73 percent, which reflects how a social planner would aim to compensate for the large costs of risk shocks due to the increased efficiency losses and monitoring costs as efp rises. Note also that the decomposition of welfare costs continues to show large total welfare costs with the bulk coming from differences in averages in the stochastic steady state v. the deterministic steady state.

Table 4: Strategic Interaction Results: All Shocks Considered

Regime x v. regime y	Param. values of x		ce v.	Decomp. of ce into mean and SD eff.		
	a_π	a_{rr}	DRR	Full ce	Mean eff.	SD eff.
Nash	2.91	1.79	37bp.	6.05%	5.55%	0.50%
Cooperative ($\varphi = 0.5$)	2.05	2.56	11bp.	5.79%	5.32%	0.48%
Cooperative ($\varphi^* = 0.27$)	1.55	1.97	1bp.	5.68%	5.20%	0.48%
DRR (Best Policy)	1.55	2.05		5.68%	5.20%	0.48%

Note: ce corresponds to the consumption equivalent welfare measure defined in equation (2.19).

B.8 Other financial variables in the policy rule

In the final section, we argue that the baseline financial policy rule studied in the paper is observationally equivalent, up to a first order approximation, to rules that react to different financial variables, such as leverage or debt. We show these equivalences using a linearization of the financial contract equations (12)-(14), which read (we have dropped the expectation operator for the ease of exposition):

$$\hat{\Lambda}_t = \hat{r}_t + \hat{x}_t + f_0 \hat{\omega}_{t+1} + g_0 \hat{\sigma}_{\omega,t}, \quad (32)$$

$$\hat{\Lambda}_t + \hat{\tau}_{f,t} = f_1 \hat{\omega}_{t+1} + g_1 \hat{\sigma}_{\omega,t}, \quad (33)$$

$$\frac{1}{x-1} \hat{x}_t = \hat{r}_t + \hat{\tau}_{f,t} + f_2 \hat{\omega}_{t+1} + g_2 \hat{\sigma}_{\omega,t}, \quad (34)$$

where \hat{a}_t represents the percent deviation of variable a from its non-stochastic steady-state level, and f_i and g_i are reduced-form parameters that equal:

$$\begin{aligned} f_0 &= -\omega \frac{\Gamma_\omega}{1-\Gamma}, & g_0 &= -\sigma_\omega \frac{\Gamma_\sigma}{1-\Gamma}, \\ f_1 &= \omega \left(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} \right), & g_1 &= \sigma_\omega \left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega} \right), \\ f_2 &= \omega \frac{\Gamma_{\omega} - \mu G_\omega}{\Gamma - \mu G}, & g_2 &= \sigma_\omega \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}. \end{aligned}$$

The derivations are in Section B.8.3. Rearranging the optimal contract first-order conditions, we get:

$$\hat{r}_t + \hat{\tau}_{f,t} = \chi_x \hat{x}_t + \chi_\sigma \hat{\sigma}_{\omega,t}, \quad (35)$$

where $\chi_x \equiv \frac{f-f_2x}{(x-1)f}$, $\chi_\sigma \equiv \frac{f_2(g_1-g_0)-(f_1-f_0)g_2}{f}$, and $f \equiv f_2 + f_1 - f_0$. In the STR and ATR regimes, the reduced-form parameters χ_x and χ_σ denote the elasticity of the external finance premium to leverage and the risk shock, respectively. In the baseline calibration, they equal 0.055 and 0.040. In the DRR regime, financial policy reacts to changes in the external finance premium according to a rule that up to the first-order is

$$\hat{\tau}_{f,t} = a_{rr} \hat{r}_t.$$

The DRR regime moderates variations in the external finance premium due to variations in leverage and the risk shock by reducing the aforementioned elasticities, since $a_{rr} > 0$ and equation (35) simplifies to

$$\hat{r}_t = \frac{\chi_x}{1 + a_{rr}} \hat{x}_t + \frac{\chi_\sigma}{1 + a_{rr}} \hat{\sigma}_{\omega,t}.$$

B.8.1 Leverage in the financial policy rule

Assume instead that the financial policy rule reacts to changes in leverage and entrepreneurs' risk rather than changes in the external finance premium. Up to the first-order, the alternative financial rule is

$$\hat{\tau}_{f,t} = a_x \hat{x}_t + a_\sigma \hat{\sigma}_{\omega,t}.$$

As such, the equilibrium condition for the external finance premium changes to

$$\hat{r}_t = (\chi_x - a_x) \hat{x}_t + (\chi_\sigma - a_\sigma) \hat{\sigma}_{\omega,t}.$$

It follows that the leverage rule and the baseline rule would be observationally equivalent, up to a first-order approximation, if

$$a_x = \chi_x \frac{a_{rr}}{1 + a_{rr}} \text{ and } a_\sigma = \chi_\sigma \frac{a_{rr}}{1 + a_{rr}}.$$

Figure 20 shows that the impulse responses obtained with the baseline DRR regime rule and with the leverage/dispersion rule are exactly the same up to a first-order approximation. In the figure, we calibrate the model as in the DRR regime (so $a_{rr} = 2.43$).

B.8.2 Credit in the financial policy rule

Similar to the previous case, one can find a first-order observationally equivalent financial rule where the instrument $\tau_{f,t}$ reacts to credit. First, notice that the aggregate budget constraint of entrepreneurs implies that total credit is related to leverage as follows $b_t + n_t = q_t k_t$, or $b_t/n_t + 1 = x_t$. A log-linear approximation of this identity yields

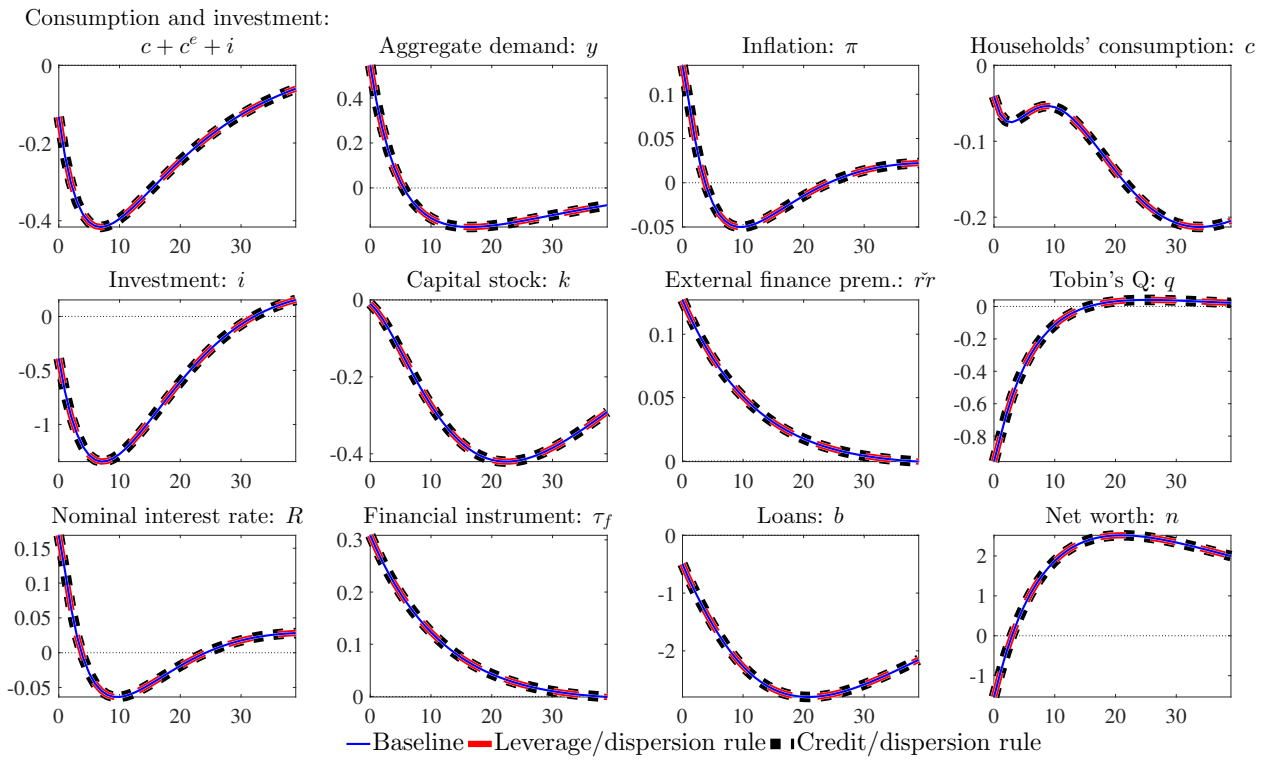
$$\hat{x}_t = \frac{x-1}{x} (\hat{b}_t - \hat{n}_t).$$

It follows that a financial rule with credit is observationally equivalent to the baseline rule if the former also reacts to entrepreneurs' net worth, and to the risk shock, i.e.¹⁰

$$\hat{\tau}_{f,t} = a_b (\hat{b}_t - \hat{n}_t) + a_\sigma \hat{\sigma}_{\omega,t}.$$

¹⁰The rule states that the financial instrument increases whenever credit increases faster than the net worth. This result might seem counterintuitive from a macroprudential perspective, since it promotes credit when it increases! However, it must be taken into account that this rule increases welfare when the economy faces a risk shock, i.e. a situation in which the lender tightens credit more of what it is efficient due to financial frictions, so capital does not grow as fast as in the efficient equilibrium.

Figure 20: IRFs to a risk shocks with first-order observationally equivalent rules



Note: The x axis denotes periods while the y axis denotes percent deviations from the steady state.

To ensure the first-order equivalence, it must be the case that

$$a_b = \chi_x \frac{x-1}{x} \frac{a_{rr}}{1+a_{rr}}, \text{ and } a_\sigma = \chi_\sigma \frac{a_{rr}}{1+a_{rr}}.$$

Figure 20 again shows that the impulse responses obtained with the credit/dispersion rule are the same to the responses obtained with the baseline rule.

B.8.3 A first-order approximation to the financial contract conditions

We rewrite below the first order conditions of the optimal financial contract (where we have dropped the expectation operator for the ease of exposition). Notice that in the first equation, we have made use of the lender's participation constraint so that $1 - [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] \check{r}_t (1 + \tau_{f,t}) = \frac{1}{x_t}$.

$$[1 - \Gamma(\bar{\omega}_{t+1})] \check{r}_t = \Lambda_t \frac{1}{x_t}, \quad (36)$$

$$\Lambda_t [\Gamma_\omega(\bar{\omega}_{t+1}) - \mu G_\omega(\bar{\omega}_{t+1})] \check{r}_t (1 + \tau_{f,t}) = \Gamma_\omega(\bar{\omega}_{t+1}) \check{r}_t, \quad (37)$$

$$[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})] \check{r}_t x_t (1 + \tau_{f,t}) = x_t - 1. \quad (38)$$

The above equations imply that at the steady state, the following relationships hold:

$$\Lambda = \frac{(1 - \Gamma) \check{r}}{1 - (\Gamma - \mu G) \check{r} (1 + \tau_f)} = (1 - \Gamma) \check{r} x = \frac{\Gamma_\omega}{(\Gamma_\omega - \mu G_\omega) (1 + \tau_f)},$$

and

$$\frac{1}{x} = 1 - (\Gamma - \mu G) \check{r} (1 + \tau_f),$$

where Γ_ω and G_ω represent the first derivatives of Γ and G with respect to $\bar{\omega}_{t+1}$, respectively.

A first-order approximation to equation (36) is:

$$\begin{aligned} \Lambda \hat{\Lambda}_t &= (1 - \Gamma) \check{r} x (\hat{r}_t + \hat{x}_t) - \omega \check{r} x \Gamma_\omega \hat{\omega}_{t+1} - \sigma_\omega \check{r} x \Gamma_\sigma \hat{\sigma}_{\omega,t}, \text{ or} \\ \hat{\Lambda}_t &= \hat{r}_t + \hat{x}_t - \omega \frac{\Gamma_\omega}{1 - \Gamma} \hat{\omega}_{t+1} - \sigma_\omega \frac{\Gamma_\sigma}{1 - \Gamma} \hat{\sigma}_{\omega,t}, \end{aligned}$$

where Γ_σ and G_σ represent the first derivatives of Γ and G with respect to $\sigma_{\omega,t}$, respectively. In turn, the first-order approximation to equation (37) is:

$$\begin{aligned} \Lambda (\Gamma_\omega - \mu G_\omega) (1 + \tau_f) (\hat{\Lambda}_t + \hat{\tau}_{f,t}) &= \left\{ \begin{array}{l} \omega \Gamma_{\omega\omega} \hat{\omega}_{t+1} + \sigma_\omega \Gamma_{\sigma\omega} \hat{\sigma}_{\omega,t} \\ -\omega \Lambda (\Gamma_{\omega\omega} - \mu G_{\omega\omega}) (1 + \tau_f) \hat{\omega}_{t+1} \\ -\sigma_\omega \Lambda (\Gamma_{\sigma\omega} - \mu G_{\sigma\omega}) (1 + \tau_f) \hat{\sigma}_{\omega,t} \end{array} \right\}, \text{ or} \\ \hat{\Lambda}_t + \hat{\tau}_{f,t} &= \omega \left(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} \right) \hat{\omega}_{t+1} + \sigma_\omega \left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega} \right) \hat{\sigma}_{\omega,t}, \end{aligned}$$

where $\Gamma_{\omega\omega}$ and $G_{\omega\omega}$ represent the second derivatives of Γ and G with respect to $\bar{\omega}_{t+1}$, and $\Gamma_{\omega\sigma}$ and $G_{\omega\sigma}$ represent the derivatives of Γ_{ω} and G_{ω} with respect to $\sigma_{\omega,t+1}$, respectively. Finally, the first-order approximation to equation (38) is:

$$\begin{aligned}
x\hat{x}_t &= \left\{ \begin{array}{l} (\Gamma - \mu G) \check{r}x (1 + \tau_f) (\hat{r}_t + \hat{\tau}_{f,t} + \hat{x}_t) \\ +\omega (\Gamma_{\omega} - \mu G_{\omega}) \check{r}x (1 + \tau_f) \hat{\omega}_{t+1} \\ +\sigma_{\omega} (\Gamma_{\sigma} - \mu G_{\sigma}) \check{r}x (1 + \tau_f) \hat{\sigma}_{\omega,t} \end{array} \right\}, \text{ or} \\
x\hat{x}_t &= \left\{ \begin{array}{l} (x - 1) (\hat{r}_t + \hat{\tau}_{f,t} + \hat{x}_t) \\ +\omega (\Gamma_{\omega} - \mu G_{\omega}) \check{r}x (1 + \tau_f) \hat{\omega}_{t+1} \\ +\sigma_{\omega} (\Gamma_{\sigma} - \mu G_{\sigma}) \check{r}x (1 + \tau_f) \hat{\sigma}_{\omega,t} \end{array} \right\}, \text{ or} \\
\frac{x}{x-1}\hat{x}_t &= \hat{r}_t + \hat{\tau}_{f,t} + \hat{x}_t + \omega \frac{\Gamma_{\omega} - \mu G_{\omega}}{\Gamma - \mu G} \hat{\omega}_{t+1} + \sigma_{\omega} \frac{\Gamma_{\sigma} - \mu G_{\sigma}}{\Gamma - \mu G} \hat{\sigma}_{\omega,t}.
\end{aligned}$$

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