

Integrated Assessment and Macro Models of Climate Change

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AEA Continuing Education: Climate Change

January 6, 2020

Topics

1. DICE/RICE Models

- ▶ Intellectual History
- ▶ Model Details

2. Macroeconomic General Equilibrium IAMs

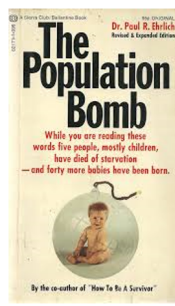
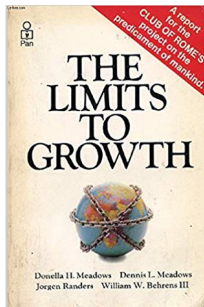
- ▶ Golosov, Hassler, Krusell, Tsyvinski (2014)
- ▶ Heterogeneity
- ▶ Uncertainty

3. PE/Simulation IAMs

4. Teaching Suggestions

Intellectual History

- ▶ After post-war boom of 1950s and 1960s, emergence of growth skepticism in 1970s
 - ▶ Fear of "limits to growth" from energy, resource scarcity



- ▶ Emerging standards of neoclassical economic growth modeling did not consider environmental processes

Intellectual History

- ▶ Nordhaus and Tobin (1972) "Is Growth Obsolete?"
- ▶ Nordhaus (1974) "Resources as a Constraint on Growth"
- ▶ Reviewed theory, data on resource endowments, consumption rates, relative price trends with labor
- Concluded resource scarcity unlikely to constrain growth
- Warned that global energy externalities, "greenhouse effect" looked like "much more serious problem"

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- Concluded resource scarcity unlikely to constrain growth
- Warned that global energy externalities, "greenhouse effect" looked like "much more serious problem"
- ▶ Acknowledged that "there is probably very little that economics alone can say" on these dangers
- ▶ Visited multi-disciplinary International Institute for Applied Systems Analysis (IIASA, Austria)

Intellectual History

- ▶ Machta (1972): *matrix representation* of global carbon cycle
- ▶ Let $M_i(t) \sim$ stock of carbon in reservoir i at time t , where $i = \{\text{Troposphere, Stratosphere, Upper oceans, Deep oceans, Short-term biosphere, Long-term biosphere, Marine biosphere}\}$
- ▶ Let $d_{ij} \sim$ annual transfer coeff. from reservoir i to reservoir j

$$M_i(t) = \sum_{j=1}^7 d_{ji} M_j(t-1)$$

→ Carbon cycle (matrix representation): $M(t) = D'M(t-1)$

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⇒ Enabled Nordhaust to incorporate carbon cycle, *constraints on greenhouse gas concentrations* into (economic) linear programming models!

Intellectual History

- ▶ Greenstone slides on BMT: "Carbon policy as an exhaustible resource problem"
 - ▶ Nordhaus (1975) - First to tackle this problem!
 - ▶ Global energy markets linear programming model
 - ▶ 6 fuels, 4 sectors, 2 regions
- ⇒ Solved for global welfare-maximizing energy allocations
subject to limits on greenhouse gas concentrations!

Table 7. Fraction of inputs which are carbon-based (fossil fuels),
by sector and period, United States

25 year period centered on:	S e c t o r :			
	Electricity	Industry	Residential	Transport
1970: 1	100%	100%	100%	100%
2	100%	100%	100%	100%
3	100%	100%	100%	100%
4	100%	100%	100%	100%
1995: 1	73%	100%	100%	100%
2	78%	100%	100%	100%
3	78%	100%	100%	100%
4	73%	100%	100%	100%
2020: 1	13%	100%	87%	100%
2	6%	100%	87%	100%
3	0	100%	75%	100%
4	0	100%	0	100%

Nordhaus (1975)

Table 8. Shadow Prices on Carbon Dioxide
Emission (1970 dollars per metric ton carbon)

Program	I. Uncontrolled	II. 200% in- crease	III. 100% in- crease	IV. 50% in- crease
1970	0.00	0.01	0.05	0.15
1995	0.00	0.07	0.57	1.80
2020	0.00	0.87	8.24	28.20
2045	0.00	21.11	46.08	47.66
2070	0.00	58.43	42.17	42.17
2095	0.00	0.00 ^a	132.88	132.88

Nordhaus (1975)

Intellectual History

- ▶ Nordhaus (1977): First presentation of **carbon taxes** (shadow prices)

TABLE 1—ENERGY CONSUMPTION, CARBON EMISSIONS, AND CARBON EMISSION TAXES

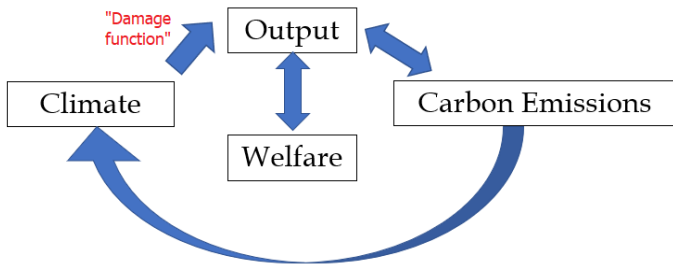
	1970	1980	2000	2020	2040	2100
	(Actual)					
Energy Consumption, United States, 10^{15} btu/yr						
Uncontrolled CO_2	{ 71 }	76.	92.	155.	250	395.
100 percent increase CO_2		76.	92.	142.	160.	405.
Global Carbon Emissions, 10^9 tons/yr						
Uncontrolled CO_2	{ 4.0 }	6.9	10.7	18.4	40.1	45.4
100 percent increase CO_2		6.9	10.7	16.6	16.0	4.9
Carbon Emission Tax (\$/ton)						
Uncontrolled CO_2	{ .00 }	.00	.00	.00	.00	.00
100 percent increase CO_2		.14	1.02	8.04	67.90	87.15

Notes: Carbon emissions are tons of carbon dioxide, carbon weight, while carbon taxes are calculated dual variables in the efficient program, and have the dimension of 1975 dollars per ton carbon weight of emission. Source is Nordhaus (1976).

- ▶ *Cost-effective* carbon prices for arbitrary climate targets
- ▶ What would social welfare-maximizing policy (target) be?

Intellectual History

- ▶ Nordhaus (1980): First optimizing integrated assessment climate-economy model



- ▶ Addition of *damage function* closes feedback loop
- ⇒ Can perform internally consistent cost-benefit analysis!
- ⇒ Can solve for dynamically *optimal* carbon prices

Nordhaus DICE Model (1992-present)

- ▶ Dynamic Integrated model of Climate and the Economy
- ▶ First version published in *Science* in 1992
 - ▶ Rejected from economics journals!
- ▶ Note: Setup modified slightly for presentation here today

Nordhaus DICE Model (1992-present)

- ▶ Global GDP Y_t produced using capital K_t , labor L_t (population), technology A_t :

$$Y_t = (1 - D(T_t)) \cdot A_t(K_t^\alpha L_t^{1-\alpha})$$

- ▶ Climate impacts summarized by damage function $D(T_t)$
 - ▶ $T_t \sim$ mean global atmospheric surface temperature change
- ▶ Damages expressed as % GDP-*equivalent* loss
 - ▶ Actual GDP impacts + value of non-market impacts
- ▶ Aggregate global damages $D(T_t) \sim$ Sum of impact estimates across regions and sectors (more later)

Nordhaus DICE Model (1992-present)

- ▶ Industrial carbon emissions E_t :

$$E_t = (1 - \mu_t) \cdot \sigma_t \cdot Y_t$$

- ▶ $\sigma_t \sim$ Baseline emissions intensity (exog.)
- ▶ $\mu_t \sim$ Abatement rate: Society can reduce emissions by fraction $\mu \in [0, 1]$ at (total) cost:

$$\Lambda_t = \pi_t \cdot \theta_{1t} (\mu_t)^{\theta_2} \cdot Y_t$$

- ▶ $\theta_{1t}, \theta_2 \sim$ abatement cost parameters
- ▶ $\pi_t \sim$ mark-up in case of incomplete global participation

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- ▶ $\theta_{1t}, \theta_2 \sim$ abatement cost parameters
- ▶ $\pi_t \sim$ mark-up in case of incomplete global participation
- ▶ Fossil fuel scarcity *can* be captured through constraint:

$$CCum \geq \sum E_t$$

Nordhaus DICE Model (1992-present)

- ▶ Carbon cycle: Industrial emissions E_t + land-based emissions E_t^{Land} (exog.) enter atmosphere
- ▶ Three reservoirs: Atmosphere AT , biosphere/upper ocean UP , lower ocean LO

$$\begin{pmatrix} M_{AT,t} \\ M_{UP,t} \\ M_{LO,t} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{21} & 0 \\ \phi_{12} & \phi_{22} & \phi_{32} \\ 0 & \phi_{23} & \phi_{33} \end{pmatrix} \begin{pmatrix} M_{AT,t-1} \\ M_{UP,t-1} \\ M_{LO,t-1} \end{pmatrix} + \begin{pmatrix} E_t + E_t^{Land} \\ 0 \\ 0 \end{pmatrix}$$

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⇒ Radiative forcing:

$$F_t = \eta \left\{ \log_2 \left(\frac{M_{AT,t}}{M_{AT,1750}} \right) \right\} + F_t^{EX}$$

where $F_t^{EX} \sim$ exogenous forcings such as aerosols

Nordhaus DICE Model (1992-present)

- Climate representation based on Schneider-Thompson model

$$\begin{pmatrix} T_t \\ T_{LO,t} \end{pmatrix} = \begin{pmatrix} (1 - \xi_1 \xi_2 - \xi_1 \xi_3) & \xi_1 \xi_3 \\ (1 - \xi_4) & \xi_4 \end{pmatrix} \begin{pmatrix} T_{t-1} \\ T_{LO,t} \end{pmatrix} + \begin{pmatrix} \xi_1 F_t \\ 0 \end{pmatrix}$$

- where $T_{LO,t} \sim$ ocean temperature
- Captures heat exchange, warming delays

Nordhaus DICE Model (1992-present)

- Social welfare: Present value of population-weighted utility over per capita consumption $c_t \equiv \frac{C_t}{L_t}$

$$W = \max \sum_{t=0}^{\infty} \beta^t L_t U(c_t)$$

Nordhaus DICE Model (1992-present)

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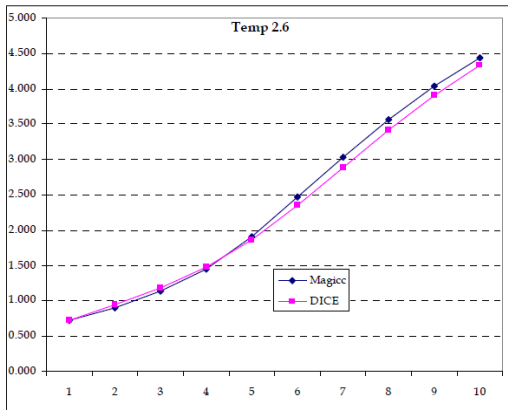
- Maximize W by choice of investment I_t , abatement μ_t s.t.:

$$\begin{aligned} Y_t &= C_t + I_t + \Lambda_t(\mu_t) \\ K_{t+1} &= (1 - \delta_k)K_t + I_t \end{aligned}$$

plus other constraints, equations of the model

DICE Quantification Example: Carbon & Climate system

- ▶ Some parameters set based on literature review
 - ▶ E.g., equilibrium climate sensitivity 2.9°C (v2013) based on Knutti and Hegerl (2008)
- ▶ Remaining calibrated to match general circulation climate model output, e.g.:



DICE: Quantification

- ▶ Population projections: United Nations
 - ▶ Baseline emissions intensity σ_t : Historical data for 12 regions
 - ▶ Land-based emissions: IPCC
 - ▶ Etc. For details, see DICE model "Lab Notes"
-
- ▶ Nordhaus (2017): **Out-of-sample Forecasting Ability?**
 - ▶ Compare 1992 DICE model predictions for 2015 to data!

	DICE 1992	DICE-2016R	Change 1992 to 2016
	2015	2015	[%]
Major driving variables			
Economic			
Population (billions)	6,868	7,403	8%
Per capita GDP (2010\$)	11,293	14,183	26%
Consumption per capita (2020\$)	9,195	10,501	14%
Geophysical			
Other Forcings (W/m2)	0.89	0.50	-44%
CO2/output ratio (tCO2/000 2010\$)	0.607	0.350	-42%
Outcome variables			
Industrial Emissions (GTCO2 per year)	42.3	35.7	-15%
Output (trillions 2010\$)	77.6	105.0	35%
Atmospheric concentration C (ppm)	399	400	0%
Atmospheric concentrations (GtC)	849	851	0%
Atmospheric Temperature (°C)	1.16	0.85	-27%
Total forcings (W/m2)	3.04	2.46	-19%
Social cost of carbon (\$/tCO2 2010\$)	4.54	30.98	582%

DICE Quantification: Damages

- Damage function:

$$1 - D(T_t) = \frac{1}{1 + \alpha_1 T_t + \alpha_2 (T_t)^{\alpha_3}}$$

- Quantification has changed over time:
 - Early DICE extrapolated from few available U.S. estimates
 - Nordhaus and Boyer (2000): Review literature, construct regional-sectoral estimates of damages from 2.5°C warming

Nordhaus and Boyer (2000)

Region	Agric.	Othr vuln. markets (energy)	Coastal	Health	Time Use	Settle- ments Ecosystms	Catastr.
US	0.03%	0.00%	0.10%	0.02%	-0.28%	0.10%	0.94%
Euro	0.03%	0.00%	0.46%	0.02%	-0.43%	0.25%	1.09%
OHI	-0.05%	-0.32%	0.09%	0.02%	-0.35%	0.10%	1.11%
Russia	-0.82%	-0.80%	0.05%	0.02%	-0.75%	0.05%	1.12%
EE/FSU	0.03%	0.00%	0.01%	0.02%	-0.36%	0.10%	0.94%
Japan	0.02%	0.00%	0.27%	0.02%	-0.31%	0.25%	1.07%
China	0.02%	0.32%	0.08%	0.09%	-0.26%	0.05%	1.04%
India	0.32%	0.29%	0.09%	0.40%	0.30%	0.10%	1.57%
MidEast	0.35%	0.20%	0.04%	0.23%	0.24%	0.05%	0.96%
SSA	0.67%	0.32%	0.02%	1.00%	0.25%	0.10%	1.78%
LA	0.42%	0.28%	0.10%	0.32%	-0.04%	0.10%	1.30%
OthAsia	0.52%	0.21%	0.09%	0.32%	-0.04%	0.10%	1.23%

DICE Quantification: Damages

- Damage function:

$$1 - D(T_t) = \frac{1}{1 + \alpha_1 T_t + \alpha_2 (T_t)^{\alpha_3}}$$

- Quantification has changed over time:
 - Early DICE extrapolated from few available U.S. estimates
 - Nordhaus and Boyer (2000): Review literature, construct regional-sectoral estimates of damages from 2.5°C warming
- 2017-DICE: Statistical aggregate of select literature estimates
 - Sets $\alpha_1 = 0$, $\alpha_2 = 0.00236$, $\alpha_3 = 2 \Rightarrow D(3^\circ\text{C}) \sim 2.1\%$ loss

A Few Degrees of Warming: So What?

- ▶ Typical estimates for value of global damages from warming
 - ▶ In % of GDP-equivalent (actual GDP + non-market impacts)
- ▶ DICE-2016 (Nordhaus): $D(3^{\circ}\text{C}) \sim 2.1\%$ loss
- ▶ FUND (Tol, 2013): $D(3.5^{\circ}\text{C}) \sim 1.2\%$ (Global)
- ▶ PAGE (Hope, 2009): $D(3.5^{\circ}\text{C}) \sim 6.4\%$ (Global), 1.5% (U.S.)
- ▶ Comparison: Great Recession real GDP loss in 2009: ?

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- ▶ PAGE (Hope, 2009): $D(3.5^{\circ}\text{C}) \sim 6.4\%$ (Global), 1.5% (U.S.)
- ▶ Comparison: Great Recession real GDP loss in 2009: -2.8%

DICE Model: Results, Policy Relevance

Greenstone Lecture!

GHKT (2014, Econometrica)

- ▶ Golosov, Hassler, Krusell, and Tsyvinski: "Optimal Taxes on Fossil Fuel in General Equilibrium"
- ▶ Highly influential paper that makes two core contributions:
 1. Macro-theoretical exposition of climate-economy model
 2. Closed-form optimal carbon tax-GDP ratio given certain assumptions
- ▶ Teaching Note: For undergrads, would focus on DICE
- ▶ For Ph.D. students, would present both DICE and GHKT
 - ▶ Pedagogically useful to illustrate, e.g., decentralized equilibrium vs. social planner's problem

GHKT (2014): Setup

- ▶ Rep. household has preferences over consumption C_t :

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

- ▶ I production sectors: $i = 1, \dots, I$ producers of energy E_i , and final good producer $i = 0$, with resource constraint:

$$C_t + K_{t+1} = Y_t + (1 - \delta)K_t$$

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- ▶ Final good output Y_t produced via:

$$Y_t = F_{0,t}(K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}, S_t)$$

where $N_{0,t}$ is labor in sector $i = 0$, $E_{0,t} = (E_{0,1,t}, E_{0,2,t}, \dots, E_{0,I,t})$ is a vector of energy inputs, and S_t is atmospheric carbon stock

GHKT (2014): Setup

- ▶ Energy input $E_{i,t}$ produced via:

$$E_{i,t} = F_{i,t}(K_{i,t}, N_{i,t}, \mathbf{E}_{i,t} R_{i,t}) \geq 0$$

- ▶ For energy resources in finite supply, let $R_{i,t} \sim$ stock of resource i still left at the beginning of period t :

$$R_{i,t+1} = R_{i,t} - E_{i,t} \geq 0$$

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- ▶ Example from quantitative model: $I = 3$ energy sectors
 - ▶ Oil in finite supply $\bar{R}$ with zero extraction costs
 - ▶ Coal and clean energy produced from labor:

$$\begin{aligned} E_{coal,t} &= A_{coal,t} \cdot N_{coal,t} \\ E_{clean,t} &= A_{clean,t} \cdot N_{clean,t} \end{aligned}$$

- ▶ Final goods production uses energy *composite* E_t :

$$E_t = \left(\kappa_1 E_{oil,t}^\rho + \kappa_2 E_{coal,t}^\rho + \kappa_3 E_{clean,t}^\rho \right)^{1/\rho}$$

GHKT (2014): Assumptions

Assumption 1 $U(C_t) = \ln(C_t)$

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Assumption 2 Multiplicative damages

$$F_{0,t}(K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}, S_t) = (1 - D_t(S_t)) \tilde{F}_{0,t}(K_{0,t}, N_{0,t}, \mathbf{E}_{0,t})$$

$$1 - D_t(S_t) = \exp(-\gamma_t(S_t - \bar{S}))$$

where $\bar{S} \sim$ pre-industrial carbon concentrations

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Assumption 3 Carbon stock evolves according to:

$$S_t - \bar{S} = \sum_{s=0}^{t+T} (1 - d_s) E_{t-s}^f$$

with $d_s \sim$ fraction of carbon left s periods in future

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Assumption 4 Full capital depreciation $\delta = 1$

GHKT (2014): Optimal Allocation & Policy

- ▶ Marginal damages of carbon emissions from energy input i , $\Lambda_{i,t}^s$, in optimal allocation satisfy:

$$\Lambda_{i,t}^s \equiv E_t \sum_{j=0}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} \frac{\partial F_{0,t+j}}{\partial S_{t+j}} \frac{\partial S_{t+j}}{\partial E_{i,t}}$$

- ▶ Show that carbon tax = $\Lambda_{i,t}^s$ decentralizes optimal allocation
 - ▶ Pigouvian tax = marginal damages evaluated at optimum

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- ▶ Show that carbon tax = $\Lambda_{i,t}^s$ decentralizes optimal allocation
 - ▶ Pigouvian tax = marginal damages evaluated at optimum
- ▶ Given assumptions (1)-(4), and if $E_t[\gamma_{t+1}] = \overline{\gamma}_t$ for all j :

$$\frac{\Lambda_t^s}{Y_t} \equiv \overline{\gamma}_t \left(\frac{\phi_L}{1-\beta} + \frac{(1-\phi_L)\phi_0}{1-(1-\phi)\beta} \right)$$

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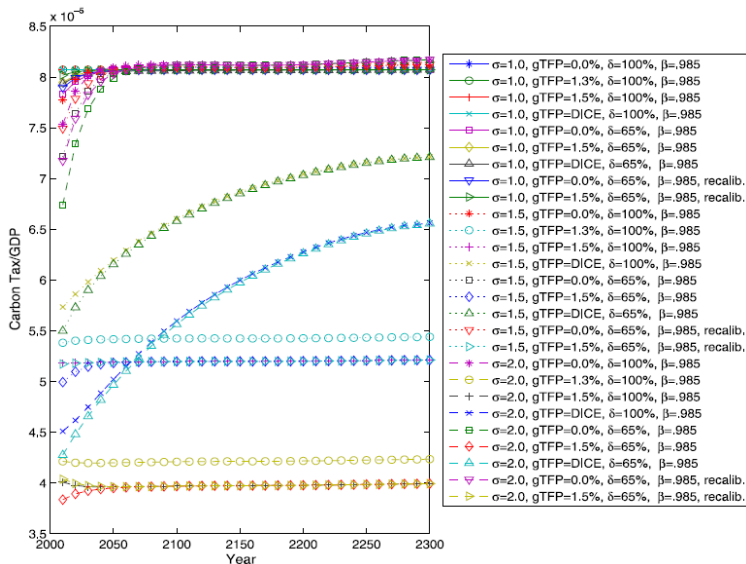
- ▶ Constant, simple optimal carbon tax-GDP ratio!
 - ▶ Does not depend on future technology, S_t , labor supply, etc.
 - ▶ Lower informational requirements than corresponding optimal emissions *quantity* (advantage for carbon tax over quota)

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- ▶ Constant, simple optimal carbon tax-GDP ratio!
 - ▶ Does not depend on future technology, S_t , labor supply, etc.
 - ▶ Lower informational requirements than corresponding optimal emissions *quantity* (advantage for carbon tax over quota)
- ▶ Intuition: Damages *proportional* to output. Marginal utility of consumption *inversely proportional* to output.
 - ▶ $\uparrow$ in Y_{2050} leads to $\uparrow$ in damages but $\downarrow$ in $U(C_{2050})$ such that *utility value* of damages remains unchanged

GHKT (2014): Sensitivity



Heterogeneity and Multi-Region Models

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Heterogeneity and Multi-Region Models

- ▶ Empirically, relevance of heterogeneity seems obvious - why even bother with a single-region model?
- ▶ Optimal carbon price still equal to global weighted *aggregate* SCC as in DICE under certain assumptions (more later)
- ▶ Of course, still many reasons for wanting to model heterogeneity!
- ▶ So why aren't multi-region models the norm?
 - ▶ Accounting for heterogeneity in a properly specified macro framework can quickly become computationally prohibitive

Heterogeneity: The Problem

- ▶ Consider a general setting where agents differ in risk tolerance, propensity to save depending on their wealth levels

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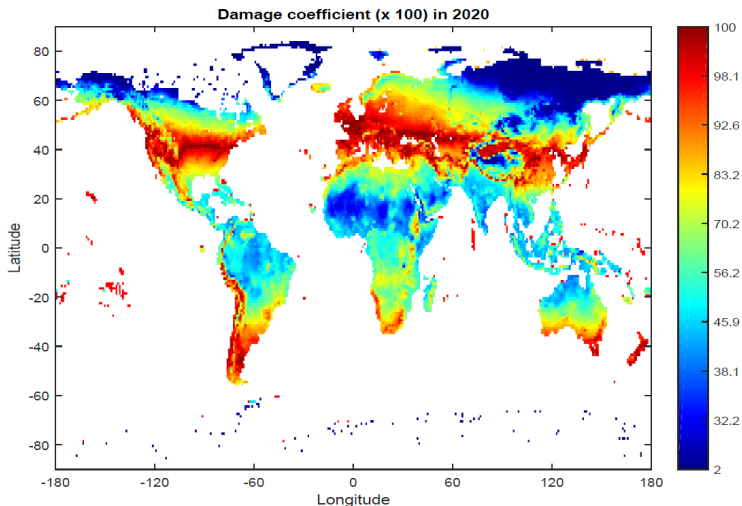
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- ▶ Key point: Every country's wealth level then becomes a state variable relevant to every other agent's decision problem at every point in time
 - ▶ Computationally difficult even without climate state variables

Heterogeneity: Approaches

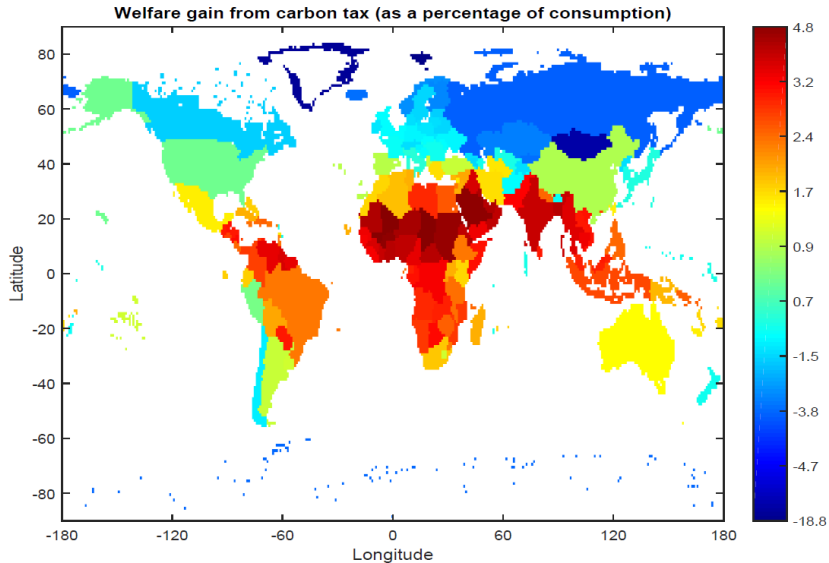
- ▶ **Regional Autarky:** Limit investment across regions, e.g.:
 - ▶ RICE Model (Nordhaus): 12-region version of DICE
 - ▶ Used to study, e.g., Copenhagen Accord, Nash carbon pricing (2011, *PNAS*), "Climate Clubs" (2015, *AER*) (later)
 - ▶ Hassler, Krusell, Olovsson (2012, *JEEA*): Multi-region GE model with trade in oil but no cross-country investment
- ▶ **Partial/Non-Equilibrium:** Exogenous savings or output, e.g.:
 - ▶ FUND Model (Anthoff and Tol): Climate damages come out of exogenous income paths for each of 16 regions
- ▶ State-of-the-art:
- ▶ **Krusell-Smith Method:** 'Approximate Aggregation'
 - ▶ Intuition: Individual decision-makers focus only on first moment of future wealth distribution in forecasting future prices

Krusell and Smith (2018)

Macroeconomic climate-economy model with 19,000 regions!



Krusell and Smith (2018)



Uncertainty

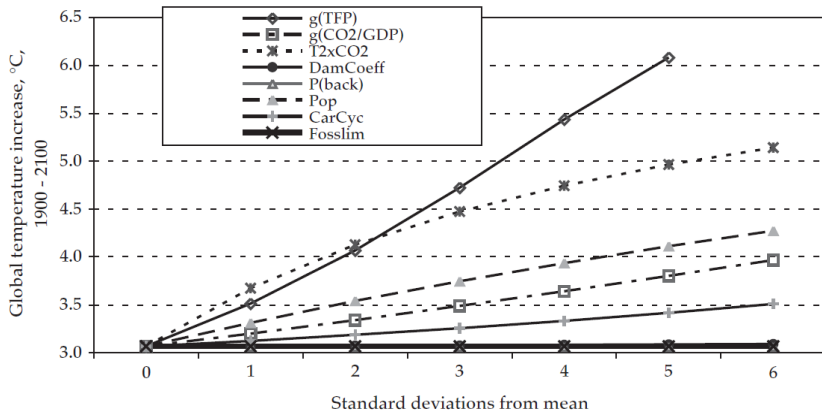
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- ▶ Con: Does not model decision-making under uncertainty
- ▶ Pro: Can consider multiple sources of uncertainty at once, readily see model sensitivity to parameters

Monte Carlo: Nordhaus (2008)

Table 7-1. Major Assumptions about Uncertain Parameters in Uncertainty Runs

Variable	Definition	Units	Mean	Standard Deviation
$g(\text{TFP})$	Rate of growth of total factor productivity	Per year	0.0092	0.0040
$g(\text{CO}_2/\text{GDP})$	Rate of decarbonization	Per year	-0.007	0.002
$T_2 \times \text{CO}_2$	Equilibrium temperature-sensitivity coefficient	$^{\circ}\text{C}$ per CO_2 doubling	3.00	1.11
DamCoeff	Damage parameter (intercept of damage equation)	Fraction of global output	0.0028	0.0013
P(back)	Price of backstop technology	\$ per ton of carbon replaced	1,170	468
Pop	Asymptotic global population	Millions	8,600	1,892
CarCyc	Transfer coefficient in carbon cycle	Per decade	0.189	0.017
Fossilim	Total resources of fossil fuels	Billions of tons of carbon	6,000	1,200

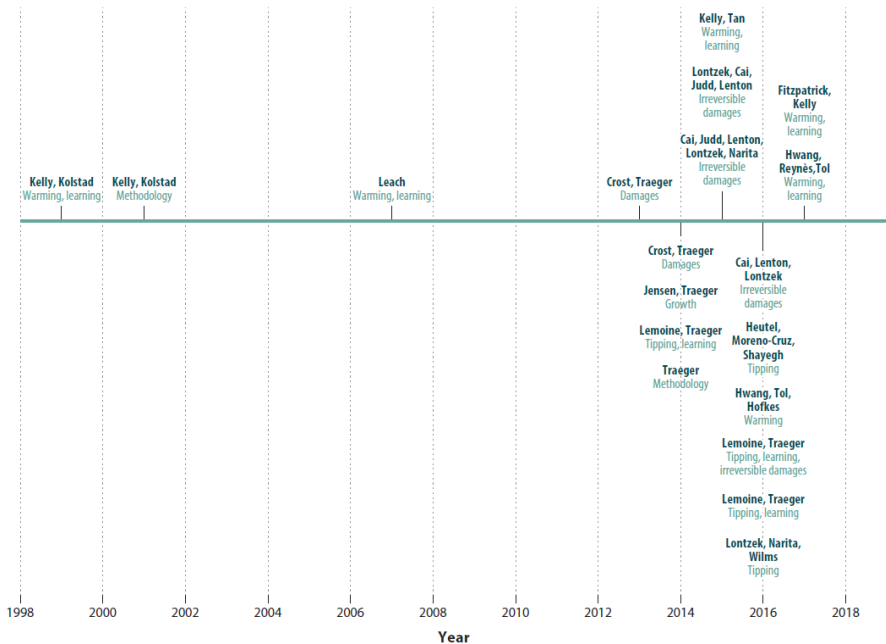
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- ▶ Approach (2): **Stochastic Dynamic Programming (SDP)**
- ▶ Pro: Agents in model aware of uncertainty, (learning)
- ▶ Con: Typically only one or two sources of uncertainty
- ▶ Next slide: Figure from Lemoine and Rudik (2017)



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- ▶ Approach (3): **Mixed Approaches**
 - ▶ Pizer (1999): Analytic approximation to household decision rules under uncertainty + Monte Carlo
 - ▶ Nordhaus and Popp (1997): Summarize Monte Carlo results in 5 "States of the World" + SDP

Uncertainty

- ▶ One current frontier: multiple sources of uncertainty
- ▶ **Computational advancements:**
- ▶ Cai, [Judd,] and Lontzek (2019, *JPE*): Develop DSICE ($S \sim$ 'stochastic') model
 - ▶ Stochastic TFP & climate tipping points
 - ▶ Also: Annual time step, Epstein-Zinn preferences

Cai, [Judd,] Lontzek (2018, JPE)

	DICE-CJL ($\psi = 0.5$)	DICE-CJL ($\psi = 1.5$)	DSICE ($\psi = 1.5, \gamma = 10$)		
			Stochastic Growth	Climate Tipping	Stochastic Growth with Tipping
2005 SCC	37	94	61	188	124
Mean SCC in 2100	180	389	357	620	461
S.d. of SCC in 2100	0	0	247	105	316
90% quantile SCC in 2100	180	389	667	662	844

Table 9: SCC (\$/tC) for three benchmark examples

⇒ Modeling climate and economic uncertainty *jointly* leads to very different SCC distribution than separate approach!

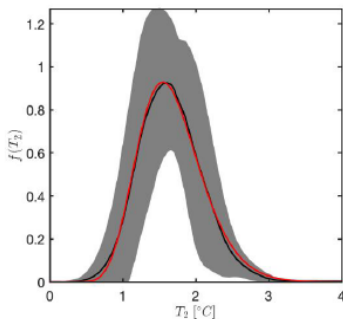
Note: "This study required the use of a few million core hours on Blue Waters, a modern supercomputer."

Uncertainty

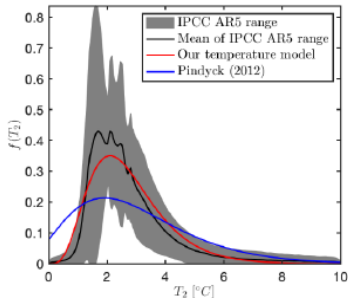
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- ▶ **Analytic advancements:**
- ▶ Traeger (2017, WP): Analytic Climate Economy Model
 - ▶ Distinguishes stochasticity from epistemological uncertainty
 - ▶ Uncertainty over carbon flows, climate sensitivity to CO_2
- ▶ van den Bremer and van der Ploeg (2018, WP)
 - ▶ Uncertainty: growth, carbon flows, climate sensitivity, damages
 - ▶ Approach: Perturbation methods (AK model)

Uncertainty: v.d. Bremer & v.d. Ploeg (2019)

- Uncertainties: Economic (growth), damage ratio, carbon cycle, climate sensitivity, e.g.:



(a) Transient climate response



(b) Equilibrium climate sensitivity

FIGURE 2. CLIMATE SENSITIVITY (PROBABILITY DENSITY FUNCTION)

Uncertainty: v.d. Bremer & v.d. Ploeg (2019)

	Asset return volatility ($\sigma_K = 12\%/ \text{year}^{1/2}$)			GDP growth volatility ($\sigma_K = 1.5\%/ \text{year}^{1/2}$)			
	base case	$\eta = 6.0$	$\gamma = 2.0$	base case	$\eta = 6.0$	$\gamma = 2.0$	$\rho = 0.1\%/ \text{year}$
Deterministic SCC (\$/tCO ₂)	11.5	11.5	8.1	11.5	11.5	8.1	25.5
Risk-adjusted SCC (\$/tCO ₂)	39.8	92.2	87.2	14.6	14.6	10.2	34.1
Economic risk mark-up	163%	492%	691%	1%	1%	1%	2%
Climate sensitivity risk mark-up	41%	112%	149%	11%	11%	9%	15%
Damage ratio risk mark-up	43%	101%	134%	15%	15%	15%	16%
Total risk mark-up	247%	705%	974%	27%	28%	25%	34%
Discount rate $r^{(0)}$ (per year)	2.9%	2.3%	2.3%	4.5%	4.5%	5.5%	3.1%
Estimates in this table are for <i>proportional</i> damages ($\theta_{ET} = 0$) and $\rho = 1.5\%/ \text{year}$ (<i>ethics-based</i> calibration), except for the last column, which considers a lower rate of impatience.							

Uncertainty: v.d. Bremer & v.d. Ploeg (2019)

TABLE 6. ESTIMATES OF THE SCC: CONVEXITY OF THE DAMAGE FUNCTION

	Proportional damages ($\theta_{ET} = 0$)	Convex damages ($\theta_{ET} = 0.28$)	Highly convex damages (AS12, $\theta_{ET} = 0.63$)
Deterministic SCC (\$/tCO ₂)	25.5	26.8	77.2
Risk-adjusted SCC (\$/tCO₂)	34.1	41.9	140.8
Economic risk mark-up	2%	1%	-1%
Carbon stock risk mark-up	0%	-1%	-1%
Climate sensitivity risk mark-up	15%	30%	61%
Damage ratio risk mark-up	16%	26%	15%
Total risk mark-up	34%	56%	82%
Discount rate $r^{(0)}$ (per year)	3.1%	3.1%	3.1%
Estimates in this table are for $\rho = 0.1\%$ /year (<i>ethics-based</i> calibration) and GDP growth volatility ($\sigma_K = 1.5\%$ /year ^{1/2}).			

PE/Simulation IAMs

- ▶ So far have focused on *macroeconomic* IAMs
 - ▶ Based on optimizing, forward-looking households
 - ▶ General equilibrium; internally and dynamically consistent
 - ▶ But: Limits feasible levels of sectoral/regional details
- ▶ PE/Simulation IAMs: Take, e.g., GDP process as exogenous
 - ▶ Allows for more disaggregated (impacts) modeling
- ▶ FUND Model (Anthoff and Tol, e.g., 2014)
 - ▶ 16 regions $\times$ 9 impact sectors
- ▶ PAGE Model (Hope, e.g., 2009)
 - ▶ 8 regions $\times$ 3 impact types

FUND Model (Anthoff and Tol, 2014)

- ▶ Instead of aggregate "damage function", models impacts in:
 - ▶ Agriculture
 - ▶ Forestry
 - ▶ Water resources
 - ▶ Energy consumption
 - ▶ Sea-level rise
 - ▶ Ecosystems
 - ▶ Health (diarrhoea, vector-borne, cardio-respiratory)
 - ▶ Tropical storms
 - ▶ Extratropical storms
- ▶ For each of 16 regions

FUND Model Damages: Example

5.7. Human health: Diarrhoea

The number of additional diarrhoea deaths $D_{t,r}^d$ in region r and time t is given by

$$D_{t,r}^d = \mu_r^d P_{t,r} \left(\frac{y_{t,r}}{y_{1990,r}} \right)^\epsilon \left(\frac{T_{t,r}}{T_{\text{pre-industrial},r}} \right)^\eta \quad (\text{HD.1})$$

where

- $P_{t,r}$ denotes population,
- r indexes region
- t indexes time,
- $y_{t,r}$ is the per capita income in region r and year t in 1995 US dollars,
- $T_{t,r}$ is regional temperature in year t , in degrees Celcius (C);
- μ_r^d is the rate of mortality from diarrhoea in 2000 in region r , taken from the WHO Global Burden of Disease (see Table HD, column 3);
- $\epsilon = -1.58$ (0.23) is the income elasticity of diarrhoea mortality
- $\eta = 1.14$ (0.51) is a parameter, the degree of non-linearity of the response of diarrhoea mortality to regional warming.