Appendices

A An Alternative Formulation of Overconfidence

As an extension of the overconfidence model in Section 3.2, in this section we explore an alternative formulation of overconfidence first proposed in Scheinkman and Xiong (2003, SX).[13] In contrast with the specification of the model presented in the main text, in which the overconfident investor perceives that the investor's private signal has lower variance than it actually does, in the SX specification the overconfident investor thinks that the invetor is observing a signal that is highly correlated with innovations in firm value, when in reality the signal is only loosely correlated with firm value innovations.[14]

This formulation has the advantage of generating momentum effects without biased-self-attribution. However, this formulation embeds multiple assumptions about the agent's information processing. In standard modeling of overconfidence,[15] as presented in Section 3, the agent receives signal which are unbiased, but imprecise, and the overconfident agent overestimates the signal precision. In this alternative formulation, the signal the agent receives is biased toward the prior, or to put it differently, it is more strongly aligned with old information than the individual realizes, and it is this bias that generates the momentum effect. While *overprecision* is well documented in the psychology literature, we are not aware of psychological evidence for the idea that people underestimate the degree to which their signals are aligned with with old information (after taking into account any overprecision).

To illustrate this alternative formulation of overconfidence, we construct a simple model like that in Section 3.2, but with a structure that captures the SX structure. As in the model in Section 3.2, true asset value θ is drawn from a common knowledge prior distribution:

$$\theta = \bar{\theta} + \epsilon_0, \tag{1}$$

where $\epsilon_0 \sim \mathcal{N}(0, 1/\tau_0)$ The timeline for the model is as follows: at date 0, the agent knows only the prior distribution, and the price $P_0 = \bar{\theta}$. At time 1 the agent observes distinct hard

[13] This specification of is also used in Alti and Tetlock (2013) and Kelley and Tetlock (2013).

[14] See pp. 1189-1190, Scheinkman and Xiong (2003)

[15] By 'standard' we mean the signal as truth plus noise, as used in the models of Kyle and Wang (1997), Odean (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Daniel, Hirshleifer and Subrahmanyam (2001), Hirshleifer and Luo (2001), and others.

(h) and soft (s) signals of the form:[16]

$$\begin{aligned} \sigma_h &= \theta + \epsilon_h \\ \sigma_s &= \bar{\theta} + \eta\epsilon_0 + \sqrt{1-\eta^2}\ \epsilon_s. \end{aligned} \tag{2}$$

At time 2, θ is revealed and $P_2 = \theta$.

ϵ_h and ϵ_s are mean zero, normally distributed with precisions τ_s, and τ_h.[17] However, SX model the investor's overconfidence as leading her to believe that the private/soft signal is:

$$\sigma_s = \bar{\theta} + \eta_C\epsilon_0 + \sqrt{1-\eta_C^2}\ \epsilon_s, \tag{3}$$

where $\eta_C > \eta$.

To see how this is distinct from the standard overconfidence setting, note that equations (2) and (3) can be rewritten as:

$$\begin{aligned} \sigma_s &= \eta\theta + (1-\eta)\bar{\theta} + \sqrt{1-\eta^2}\ \epsilon_s \\ \sigma_s &= \eta_C\theta + (1-\eta_C)\bar{\theta} + \sqrt{1-\eta_C^2}\ \epsilon_s \end{aligned}$$

In the setting in Section 3.2, overconfident investors underestimate the variance of ϵ_s. In contrast, in this setting an "overconfident" investor (with $\eta_C > \eta$) not only underestimates the signal variance, but also overestimates the extent to which σ_s is pushed away from the prior $\bar{\theta}$ and towards the true value of θ. As a result, when the agent's overconfidence is of this form, P_1 will be pushed away from θ and towards $\bar{\theta}$.

To better illustrate the importance of this assumption, consider an extreme setting where $\eta = 0$, implying $\sigma_s = \bar{\theta} + \epsilon_s$—so that the informed investor's signal is equal to the mean of the prior distribution plus pure noise. However, assume that the investor is severely overconfident, meaning that $\eta_C = 1$, implying that the investor believes that her signal is unbiased, and infinite precision — *i.e.*, $\sigma_s = \theta$. Thus, $P_1 = \bar{\theta}$, while the rational expected time 2 price, conditional on the hard signal σ_h, is:

$$\mathbb{E}^R[P_2|\sigma_h] = \mathbb{E}^R[\theta|\sigma_h] = \left(\frac{\tau_0}{\tau_0 + \tau_h}\right)\bar{\theta} + \left(\frac{\tau_h}{\tau_0 + \tau_h}\right)\sigma_h.$$

[16] Alti and Tetlock (2013) label these signals as hard and soft, rather than as public and private.

[17] In the Section 3.2 specification, the public and private signals were revealed at times 1 and 2 respectively. Here, they arrive simultaneously at time 1.

Thus,

$$\mathbb{E}^R[r_{12}|\sigma_h] = \left(\frac{\tau_h}{\tau_0 + \tau_h}\right)(\sigma_h - \bar{\theta}).$$

Thus, in a setting where the investor both underestimates the noise variance in the private signal, and underestimates the extent to which the signal is shrunk towards the prior, there will be underreaction to public information signals, and a form of public-signal linked momentum will result. However, if all signals are unbiased—i.e., the true value θ plus noise—then additional model structure is necessary to generate the observed underreaction to public information, and price momentum.

How consistent is are the psychological evidence on overconfidence with these two possible specifications? The SX specification assumes a combination of overestimating signal precision (as in the standard overconfidence approach) and a distinct second bias of believing (above and beyond the effects of any misperception of signal precision) that the realized signal is closer to the true value than it really is. We view the psychological underpinning of overconfidence (that people think they are good at generating high quality signals) and the psychological evidence of overprecision as more supportive of the first bias than the second.

B A proof that in the Model 2 setting, $\mathbb{E}[R_{2,3}|s_B] = 0$

From the equation:

$$P_2 = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \hat{\tau}_V s_V + \tau_B s_B\right),$$

$$\mathbb{E}[P_2|s_B] = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B + \hat{\tau}_V \mathbb{E}[s_V|s_B]\right). \tag{4}$$

However,

$$\mathbb{E}[s_V|s_B] = \mathbb{E}[\theta|s_B] = \frac{1}{\tau_0 + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B\right).$$

Substituting this into equation (4) yields:

$$\mathbb{E}[P_2|s_B] = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B\right)\left(1 + \frac{\hat{\tau}_V}{\tau_0 + \tau_B}\right).$$