

IX Appendix–Proofs for Lemmas and Propositions in Greenwood, Sanchez and Wang

Proof for Lemma 1. First, substitute the promise-keeping constraint (9) into the objective function to rewrite it as

$$(\pi_1 r_1 + \pi_2 r_2 - \tilde{r})k - \pi_1 w(m_1/z)^\gamma - \pi_2 w(m_2/z)^\gamma - v.$$

Next, it is almost trivial to see that optimality will dictate that $p_{12} = r_1 k$ and $p_{21} = r_2 k$, since this costlessly relaxes the incentive constraints (7) and (8). Next, drop the incentive constraint (7) from problem (P2) to obtain the auxiliary problem now displayed:

$$(P4) \quad \tilde{I}(\tau, v) \equiv \max_{p_1, p_2, m_1, k} \{(\pi_1 r_1 + \pi_2 r_2 - \tilde{r})k - \pi_1 w(m_1/z)^\gamma - v\},$$

subject to

$$(25) \quad p_1 \leq r_1 k,$$

$$(26) \quad p_2 \leq r_2 k,$$

$$(27) \quad [1 - P_{21}(m_1/k)](r_2 k - p_1) \leq r_2 k - p_2,$$

and

$$(28) \quad \pi_1(r_1 k - p_1) + \pi_2(r_2 k - p_2) = v.$$

The strategy will be to solve problem (P4) first. Then, it will be shown that (P2) and (P4) are equivalent. Problem (P4) will now be solved. To this end, note the following points:

1. The incentive constraint (27) is binding. To see why, suppose not. Then, reduce m_1 to increase the objective.
2. The constraint (26) is not binding. Assume, to the contrary, it is. Then, (27) is violated. This happens because the right-hand side is zero. Yet, the left-hand side is positive, given that $p_1 \leq r_1 k < r_2 k$, so that $[1 - P_{21}(m_1/k)](r_2 k - p_1) > 0$.

3. The constraint (25) is binding. Again, suppose not, so that $p_1 < r_1 k$. It will be shown that exists a profitable feasible deviation from any contract where this constraint is slack. Specifically, consider increasing k very slightly by $dk > 0$ while adjusting p_1 and p_2 in the following manner so that (27) and (28) still hold. Also, hold m_1 fixed. The implied perturbations for p_1 and p_2 are given by

$$\begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} = \begin{bmatrix} -[1 - P_{21}(m_1/k)] & 1 \\ \pi_1 & \pi_2 \end{bmatrix}^{-1} \times \begin{bmatrix} r_2 P_{21}(m_1/k) - (r_2 k - p_1)(m_1/k^2) dP_{21}/d(m_1/k) \\ \pi_1 r_1 + \pi_2 r_2 \end{bmatrix} dk.$$

Note that such an increase in k will raise the objective function.

It will now be demonstrated that the optimization problems (P2) and (P4) are equivalent. First, note that $\tilde{I}(\tau, v) \geq I(\tau, v)$, because problem (P4) does not impose the constraint (7). It will now be established that $\tilde{I}(\tau, v) \leq I(\tau, v)$. Consider a solution to problem (P4). It will be shown that this solution is feasible for (P2). On this, note that point 1 implies that

$$r_2 k - p_2 = [1 - P_{21}(m_1/k)](r_2 k - p_1) \leq r_2 k - p_1,$$

so that

$$p_1 \leq p_2.$$

Now, set $m_2 = 0$ in (P2), which is feasible but not necessarily optimal. Then, constraint (7) becomes $p_1 \leq p_2$, which is satisfied by the solution to (P4). Therefore, $\tilde{I}(\tau, v) \leq I(\tau, v)$.

Last, with the above facts in hand, recast the optimization problem as

$$I(\tau, v) \equiv \max_{p_2, m_1, k} \{(\pi_1 r_1 + \pi_2 r_2 - \tilde{r})k - \pi_1 w(m_1/z)^\gamma - v\},$$

subject to

$$r_2 k - p_2 = (r_2 - r_1)k[1 - P_{21}(m_1/k)], \quad \text{cf (27),}$$

and

$$r_2 k - p_2 = v/\pi_2, \quad \text{cf (28).}$$

The above two constraints collapse in the single constraint (10), by eliminating $r_2k - p_2$, that involves just m_1 and k . The problem then appears as (P3). ■

Proof for Lemma 2. Suppose that the solution dictates that $m_1/k \leq 1/\epsilon$. Then, from (P3) it is clear that the optimal solution will dictate that $m_1/k = 0$. This happens because $P_{21}(m_1/k) = 0$ for all $m_1/k \leq 1/\epsilon$, yet monitoring costs are positive for all $m_1/k > 0$. Next, by substituting (10) into (P3) it is easy to deduce that the intermediary's profit function can be written as

$$(29) \quad \begin{aligned} I(\tau, v) &= \left[\frac{\pi_1 r_1 + \pi_2 r_2 - \tilde{r}}{\pi_2 (r_2 - r_1)} - 1 \right] v = \frac{r_1 - \tilde{r}}{\pi_2 (r_2 - r_1)} v \\ &\leq 0 \text{ as } r_1 \leq \tilde{r}. \end{aligned}$$

Therefore profits are negative if $r_1 < \tilde{r}$ and $v > 0$. Hence, a contract will not be offered when $m_1/k \leq 1/\epsilon$. ■

Proof for Lemma 3. Clear from equation (17). ■

Proof for Lemma 4. Necessity: From problem (P3) it is clear that the intermediary will incur a loss when $\pi_1 r_1 + \pi_2 r_2 - \tilde{r} \leq 0$ and $v > 0$, for any $\tau \in \mathcal{A}(w)$, because $m_1/k > 1/\epsilon$ by Lemma 2.

Sufficiency: Suppose that $\pi_1 r_1 + \pi_2 r_2 - \tilde{r} > 0$ for some $\tau \in \mathcal{T}$. By equation (17) it is immediate that the intermediary will issue a loan $k > 0$. By construction it will earn zero profits on this loan. Recall that the derivation of (17), discussed in the text, used a solution for v . This solution is

$$(30) \quad \begin{aligned} v = V(\tau) &= (\psi\gamma + \gamma - \psi)^{(\psi\gamma + \gamma - \psi)/(\psi\gamma - \psi)} \left(\frac{1}{\psi}\right)^{-\psi/(\psi\gamma - \psi)} \left(\frac{1}{\psi\gamma + \gamma}\right)^{(\psi\gamma + \gamma)/(\psi\gamma - \psi)} \\ &\quad \times (\pi_1 r_1 + \pi_2 r_2 - \tilde{r})^{\frac{\gamma\psi + \gamma}{\gamma\psi - \psi}} [\pi_2 (r_2 - r_1)]^{\frac{-\gamma}{\gamma\psi - \psi}} (\pi_1 w)^{\frac{-\psi\gamma}{\gamma\psi - \psi}} (z/\epsilon)^{\frac{\psi\gamma}{\gamma\psi - \psi}}. \end{aligned}$$

Therefore, the firm will earn positive rents too. ■

Proof for Lemma 5. To begin with let $k = K(w; \tau, z)$ represent capital stock that will be employed by a type- τ firm when productivity in the financial sector is z and the wage rate is $w > \omega$. Similarly, let $m_1/z = M(w; \tau, z)$ denote the amount of monitoring services,

relative to z , that will be devoted to this project. Given this notation, the expected demand for labor by both the firm and intermediary for a project of type τ is

$$(31) \quad L(w; \tau, z) \equiv (\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}) \left[\frac{(1-\alpha)}{w} \right]^{\frac{1}{\alpha}} K(w; \tau, z) + \pi_1 M(w; \tau, z)^\gamma.$$

Note that this demand is only specified for $\tau \in \mathcal{A}(w)$ and $w > \omega$, where ω is defined by (22). In order to characterize $L(w; \tau, z)$, the properties of its components $K(w; \tau, z)$ and $M(w; \tau, z)$ must be developed when $\tau \in \mathcal{A}(w)$ and $w > \omega$. First, take $K(w; \tau, z)$. On this, rewrite equation (17) as

$$(32) \quad \begin{aligned} k &= K(w; \tau, z) \\ &= \Upsilon (\pi_1 \Theta_1 + \pi_2 \Theta_2 - \tilde{r} w^{(1-\alpha)/\alpha})^{(\gamma+\psi)/(\psi\gamma-\psi)} \left[\frac{\epsilon^\psi}{\pi_2 (\Theta_2 - \Theta_1)} \right]^{\gamma/(\psi\gamma-\psi)} z^{\gamma/(\gamma-1)} w^{-1/[\alpha(\gamma-1)]}, \end{aligned}$$

where $\Theta_i \equiv \alpha(1-\alpha)^{(1-\alpha)/\alpha} \theta_i^{1/\alpha}$ and $\Upsilon \equiv (\psi\gamma + \gamma - \psi)^{\gamma/(\psi\gamma-\psi)} \left(\frac{1}{\psi}\right)^{-\psi/(\psi\gamma-\psi)} \left(\frac{1}{\psi\gamma+\gamma}\right)^{(\gamma+\psi)/(\psi\gamma-\psi)}$.

Note the following things about this solution for k : (i) The level of investment in a firm, k , is continuous and strictly decreasing in w ; (ii) $k \rightarrow 0$ as $w \rightarrow \bar{W}(\tau) = [(\pi_1 \Theta_1 + \pi_2 \Theta_2)/\tilde{r}]^{\alpha/(1-\alpha)}$ (when z is finite); (iii) k is continuous and strictly increasing in z .

Now, switch attention to the second term in $L(w; \tau, z)$. A formula for $M(w; \tau, z)$ can be derived in the same manner as the one for $K(w; \tau, z)$. It is

$$(33) \quad m_1/z = M(w; \tau, z) = \Delta (\pi_1 \Theta_1 + \pi_2 \Theta_2 - \tilde{r} w^{(1-\alpha)/\alpha})^{(\psi+1)/(\psi\gamma-\psi)} w^{-1/[\alpha(\gamma-1)]} z^{1/(\gamma-1)},$$

where $\Delta \equiv \Upsilon [(\psi\gamma + \gamma - \psi)/(\psi\gamma + \gamma)]^{-1/\psi} \{\epsilon^\psi / [\pi_2 (\Theta_2 - \Theta_1)]\}^{1/(\psi\gamma-\psi)}$. Note the following things about this solution for m_1/z : (i) m_1/z is continuous and strictly decreasing in w ; (ii) $m_1/z \rightarrow 0$ as $w \rightarrow \bar{W}(\tau) \equiv [(\pi_1 \Theta_1 + \pi_2 \Theta_2)/\tilde{r}]^{\alpha/(1-\alpha)}$ (when z is finite); (iii) m_1/z is continuous and strictly increasing in z .

Thus, the demand for labor by a type- τ project has the following properties: (i) $L(w; \tau, z)$ is continuous and strictly decreasing in w ; (ii) $\lim_{w \rightarrow \bar{W}(\tau)} L(w; \tau, z) = 0$; (iii) $L(w; \tau, z)$ is continuous and strictly increasing in z . Define the function

$$(34) \quad \tilde{L}(w; \tau, z) = \begin{cases} L(w; \tau, z), & \text{for } w \leq \bar{W}(\tau), \\ 0, & \text{for } w > \bar{W}(\tau), \end{cases}$$

for $w > \omega$. The aggregate demand for labor can be expressed as $\int_{\mathcal{A}(w)} L(w; \tau, z) dF(\tau) = \int_{\mathcal{T}} \tilde{L}(w; \tau, z) dF(\tau)$. Now, determine the constant ζ by $\lim_{w \downarrow \omega} \int_{\mathcal{T}} \tilde{L}(w; \tau, \zeta) dF(\tau) = 1$, where again the lower bound on wages ω is given by (22). From the simple closed-form solutions for (32) and (33) it is easy to deduce that such a ζ must exist. To summarize, the aggregate demand for labor, $\int_{\mathcal{T}} \tilde{L}(w; \tau, z) dF(\tau)$, has the following properties:

1. $\int_{\mathcal{T}} \tilde{L}(w; \tau, z) dF(\tau)$ is continuous and strictly decreasing in w for $w \in (\omega, \infty)$;
2. $\int_{\mathcal{T}} \tilde{L}(w; \tau, z) dF(\tau)$ is continuous and strictly increasing in z for $z \in (\zeta, \infty)$;
3. $\int_{\mathcal{T}} \tilde{L}(\hat{w}; \tau, z) dF(\tau) = 0$, where $\hat{w} = \max_{\tau \in \mathcal{T}} \bar{W}(\tau)$;
4. $\int_{\mathcal{T}} \tilde{L}(w; \tau, \zeta) dF(\tau) = 1$.

Therefore by the intermediate theorem for all $z > \zeta$ there will exist a single value of w that sets labor demand equal to labor supply (or 1)—see Figure 4. ■

Proof for Proposition 1. Express the labor-market-clearing condition as

$$(35) \quad \int_{\mathcal{A}(w)} \{(\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}) [\frac{(1-\alpha)}{w}]^{1/\alpha} K(w; \tau, z) + \pi_1 M(w; \tau, z)^\gamma\} dF = 1.$$

Let the $\theta_i^{1/\alpha}$'s grow at the common rate $g^{1/\alpha}$ and z grow at rate $g^{1/(1-\alpha)}$. Recall that exists a solution to the model without growth, as demonstrated by Lemma 5. It is easy to construct a balanced growth path using this solution. The solution implies that there will be a wage rate that solves (35). Conjecture that along a balanced growth path wages, w , will grow at rate $g^{1/(1-\alpha)}$. From (32) it can be deduced that $K(w; \tau, z)$ will grow at rate $g^{1/(1-\alpha)}$. Therefore, the first term in braces in (35) will be constant. Equation (33) implies that $M(w; \tau, z)$, or the second term, will be constant too. The active set $\mathcal{A}(w)$ will not change—equation (20). Therefore, labor demand remains constant. Hence, the conjectured solution for the rate of growth in wages is true. Using (30) is easy to calculate that v will grow at rate $g^{1/(1-\alpha)}$. Last, since $M(w; \tau, z)$ is constant it must be the case that m_1 is growing at the same rate as z , or $g^{1/(1-\alpha)}$. Therefore, m_1/k will remain unchanged along a balanced growth path. ■

Proof for Proposition 2. First, point 2 in the proof of Lemma 5 established that the aggregate demand for labor is continuous and strictly increasing in z . Therefore, at a given wage rate the demand for labor rises as z moves up. In order for equilibrium in the labor market to be restored, wages must increase, since the demand for labor is decreasing in wages—point 1. Last, recall from (19) that a type- τ project will only be funded when $w < \bar{W}(\tau) = \alpha^{\alpha/(1-\alpha)}(1-\alpha)[(\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})/\tilde{r}]^{\alpha/(1-\alpha)}$. It's trivial to see that as w rises the set of $\tau \in \mathcal{T}$ satisfying this restriction, or $\mathcal{A}(w)$, shrinks; if $\tau = (\theta_1, \theta_2)$ fulfills the restriction for some wage it will meet it for all lower ones too, yet there will exist a higher wage that will not satisfy it. Furthermore, observe that $\bar{W}(\tau)$ is strictly increasing in $\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha}$. Therefore, those τ 's offering the lowest expected return will be cut first as w rises, because they have the lowest threshold wage. ■

Proof for Proposition 3. Let the $\theta_i^{1/\alpha}$'s increase by the common factor $g^{1/\alpha} > 1$. Suppose that wages increase in response by the proportion $g^{1/(1-\alpha)}$. Will the labor-market-clearing condition (35) still hold? The answer is no, because the demand for labor will fall. Take the first term behind the integral, which gives the demand for labor by a firm. From (32) it is clear that $K(w; \tau, z)$ will rise by a factor less than $g^{1/(1-\alpha)}$, when z is held fixed. Therefore, the first term in braces in (35) will decline. Turn to the second term. From (33) it is easy to see that $M(w; \tau, z)$ will drop under the conjecture solution. Therefore, wages must rise by less than $g^{1/(1-\alpha)}$, since the demand for labor is decreasing in w (as was established in the proof of Lemma 5). The active set, $\mathcal{A}(w)$, will therefore expand, because $\pi_1 r_1 + \pi_2 r_2$ increases—see (18). ■

Proof for Proposition 4. The set of projects in \mathcal{T} offering the highest expected return is given by

$$\mathcal{A}^* = \arg \max_{\tau=(\theta_1, \theta_2) \in \mathcal{T}} [\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}].$$

By assumption $\int_{\mathcal{A}^*} dF > 0$. Take *any* equilibrium wage w . From (20) it is immediate that if $\tau \in \mathcal{A}^*$ then $\tau \in \mathcal{A}(w)$, since $\pi_1 r_1 + \pi_2 r_2 - \tilde{r} = \alpha(1-\alpha)^{(1-\alpha)/\alpha} w^{-(1-\alpha)/\alpha} (\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}) - (\hat{r} + \delta)$ is increasing in $\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}$. Hence, $\mathcal{A}^* \subseteq \mathcal{A}(w)$ for all w . In equilibrium the wage will be a function of z , so denote this dependence by $w = W(z)$. Now, let $z \rightarrow \infty$. It

will be shown that $w = W(z) \rightarrow w^*$, where

$$(36) \quad w^* \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha) \left\{ \max_{\tau=(\theta_1, \theta_2) \in \mathcal{T}} [\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}] / \tilde{r} \right\}^{\alpha/(1-\alpha)}.$$

To see why, suppose alternatively that $w \rightarrow \tilde{w} \neq w^*$. First, presume that $\tilde{w} < w^*$. Then, by (19) all projects of type $\tau \in \mathcal{A}^*$ will be funded since their cutoff wage is $\overline{W}(\tau) = w^* > \tilde{w}$. From equations (31), (32) and (34) it is clear that $\lim_{z \rightarrow \infty} \tilde{L}^d(W(z); \tau, z) = \infty$, for $\tau \in \mathcal{A}^*$. Since, $\int_{\mathcal{A}^*} dF > 0$, this implies that $\lim_{z \rightarrow \infty} \int_{\mathcal{T}} \tilde{L}^d(W(z); \tau, z) dF = \infty$. Therefore, such an equilibrium cannot exist because the demand for labor will exceed its supply. Second, no firm can survive at a wage rate bigger than w^* , by (19). Here, $\lim_{z \rightarrow \infty} \int_{\mathcal{T}} \tilde{L}^d(W(z); \tau, z) dF = 0$. This establishes (23). Last, note that $\mathcal{A}(w^*) = \mathcal{A}^*$.

It is immediate that $\mathcal{A}^* \subseteq \lim_{w \uparrow w^*} \mathcal{A}(w)$, because $\tau \in \mathcal{A}^*$ is viable for *all* wages $w \leq w^* = \overline{W}(\tau)$ by (19). It is also true that $\lim_{w \uparrow w^*} \mathcal{A}(w) \subseteq \mathcal{A}^*$, since from (19) any project $\tau \notin \mathcal{A}^*$ requires an upper bound on wages $\overline{W}(\tau) < w^*$ to survive; that is, for any $\tau \notin \mathcal{A}^*$ there will exist some high enough wage w such that $\overline{W}(\tau) < w < w^*$. Therefore, $\lim_{w \uparrow w^*} \mathcal{A}(w) = \mathcal{A}^* = \mathcal{A}(w^*)$. This establishes point 1 of the Proposition.

To have an equilibrium it must be the case that $m_1/z < \infty$ for $\tau \in \mathcal{A}^*$, otherwise the demand for labor would be infinitely large. From equation (33) this can only happen when $\tilde{r}w^{(1-\alpha)/\alpha} \rightarrow \pi_1\Theta_1 + \pi_2\Theta_2$, or equivalently when $\pi_1r_1 + \pi_2r_2 \rightarrow \tilde{r}$. Solve problem (P3) for the optimal level of monitoring, m_1/k , and then use (30) to solve out for v to obtain

$$m_1/k = \left(\frac{\psi\gamma + \gamma - \psi}{\psi\gamma + \gamma} \right)^{-1/\psi} \left[\frac{\epsilon^\psi (\pi_1r_1 + \pi_2r_2 - \tilde{r})}{\pi_2(r_2 - r_1)} \right]^{-1/\psi}.$$

It is apparent that $\lim_{z \rightarrow \infty} m_1/k = \infty$; because $\pi_1r_1 + \pi_2r_2 \rightarrow \tilde{r}$. Consequently, a false report by a firm will be caught with certainty, or $\lim_{z \rightarrow \infty} P_{12}(m_1/k) = 1$. The contracting problem (P2) then requires $\lim_{z \rightarrow \infty} p_2 = r_2k$ and $\lim_{z \rightarrow \infty} v = 0$. A comparison of (30) and (33) leads to the conclusion that in fact $\lim_{z \rightarrow \infty} m_1/z = 0$, when $\lim_{z \rightarrow \infty} v = 0$. Using this result and (36), in conjunction with the labor-market-clearing condition, $\int_{\mathcal{T}} \tilde{L}(w; \tau, z) dF = 1$, then gives (24). ■