Appendix to "Wage Risk and Employment Risk over the Life Cycle"

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A Appendix: Numerical Solution

Households have a finite horizon and so the model is solved numerically by backward recursion from the terminal period. At each age we solve the value function and optimal policy rule, given the current state variables and the solution to the value function in the next period. This approach is standard. The complication in our model arises from the combination of a discrete choice (to work or not) and a continuous choice (over saving). This combination means that the value function will not necessarily be concave. The discrete choice about whether to move or not is less problematic because we assume that there is no cost of moving. This means that the decision to move depends only on the relative size of the match effect in the current and new firm.

There are five state variables in this problem: age, employment status, the asset stock, the permanent component of earnings, u_{it}, and the match component, $a_{ij(t_0)}$. Age and employment status are both discrete. We also discretize both the permanent component of earnings and the distribution of possible matches, leaving the asset stock as the only continuous state variable. Since the permanent component of earnings is non-stationary, we are able to approximate this by a stationary, discrete process only because of the finite horizon of the process. We select the discrete nodes in this process to match the paths of the mean shock and the unconditional variance over the life cycle. In particular, the unconditional variance of the permanent component must increase linearly with age, with the slope given by the conditional variance of the permanent shock. Our estimates of the wage variance are for annual shocks, but the model period is one quarter. We reconcile this difference by imposing that each quarter an individual receives a productivity shock with probability 0.25, and this implies that productivity shocks occur on average once a year. This timing means that individuals who stay with the same firm expect pay to be constant over a year.

Value functions are increasing in assets A_t but they are not necessarily concave, even if we condition on labor market status in t. The non-concavity arises because of changes in labor market status in future periods: the slope of the value function is given by the marginal utility of consumption, but this is not monotonic in the asset stock because consumption can decline as assets increase and expected labor market status in future periods changes. This problem is also discussed in Lentz and Torben Tranaes (2005). By contrast, in J. P. Danforth (1979) employment is an absorbing state and so the conditional value function will be concave. Under certainty, the number of kinks in the conditional value function is given by the number of periods of life remaining. If there is enough uncertainty, then changes in work status in the future will be smoothed out leaving the expected value function concave: whether or not an individual will work in $t+1$ at a given A_t depends on the realization of shocks in $t+1$. Using uncertainty to avoid non-concavities is analogous to the use of lotteries elsewhere in the literature. In the value functions (equations 7 and 8 in the main paper, reproduced below), the choice of whether or not to work in $t+1$ is determined by the maximum of the conditional value functions in $t+1$. The value function if employed is given by:

$$V_t^e\left(A_{it}, u_{it}, a_{ij(t_0)}\right) = \max_c \left\{ \begin{array}{c} U\left(c_{it}, P_{it}=1\right) + \\ \beta\delta E_t\left[V_{t+1}^n\left(A_{it+1}, u_{it+1}, DI_{it+1}^{Elig}=1\right)\right] \\ +\beta\left(1-\delta\right)\left(1-\lambda^e\right)E_t\left[\max\left\{\begin{array}{c} V_{t+1}^n\left(A_{it+1}, u_{it+1}, DI_{it+1}^{Elig}=1\right), \\ V_{t+1}^e\left(A_{it+1}, u_{it+1}, a_{ij(t_0)}\right), \end{array}\right\}\right] \\ +\beta\left(1-\delta\right)\lambda^e E_t\left[\max\left\{\begin{array}{c} V_{t+1}^n\left(A_{it+1}, u_{it+1}, DI_{it+1}^{Elig}=1\right), \\ V_{t+1}^e\left(A_{it+1}, u_{it+1}, a_{ij(t_0)}\right), \\ V_{t+1}^e\left(A_{it+1}, u_{it+1}, a_{ij(t+1)}\right) \end{array}\right\}\right] \end{array} \right\} \tag{1}$$

Among the unemployed, for an individual who is eligible to apply for disability, the value function is given by

$$V_t^n\left(A_{it}, u_{it}, DI^{Elig}=1\right) = \max_{c,Apply}\left\{ u\left(c_{it}, P_{it}=0\right) + \beta\left\{\begin{array}{cc} V_{t+1}^A & if\ Apply=1 \\ V_{t+1}^{NA} & if\ Apply=0 \end{array}\right\}\right\} \tag{2}$$

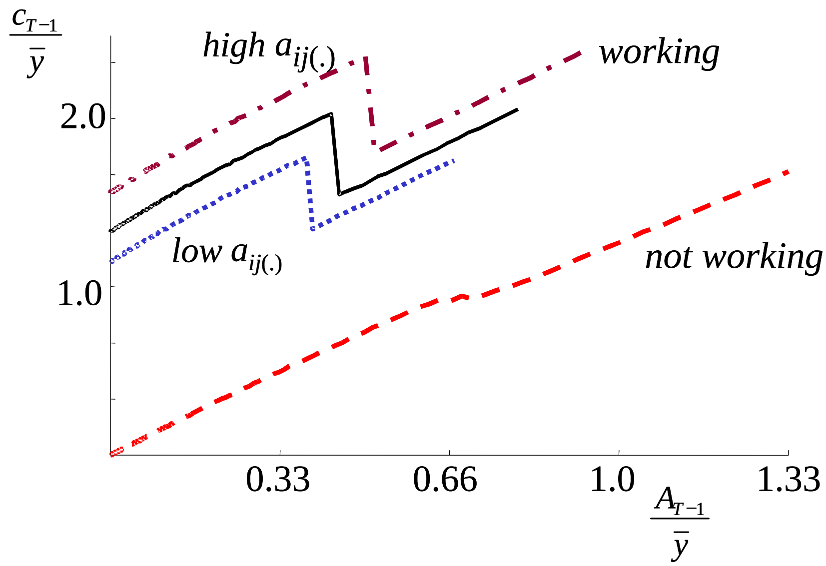
where

$$\begin{aligned} V_{t+1}^{NA} &= \begin{array}{c} \lambda^n E_t\left[\max\left\{\begin{array}{c} V_{t+1}^n\left(A_{it+1}, u_{it+1}, DI^{Elig}=1\right), \\ V_{t+1}^e\left(A_{it+1}, u_{it+1}, a_{ij(t+1)}\right) \end{array}\right\}\right] \\ +\left(1-\lambda^n\right)E_t\left[V_{t+1}^n\left(A_{it+1}, u_{it+1}, DI^{Elig}=1\right)\right] \end{array} \\ V_{t+1}^A &= S\times V_{t+1}^{DI}\left(A_{it+1}, D_{it+1}\right) + \left(1-S\right)\times E_t\left[V_{t+1}^n\left(A_{it+1}, u_{it+1}, DI^{Elig}=0\right)\right] \end{aligned}$$

In solving the maximization problem at a given point in the state space, we use a simple golden search method. We solve the model and do the calibration assuming this process is appropriate. We then check that the results in our baseline case are unaffected when we use a global optimizing routine, simulated annealing. It is worth stressing that there are parameter values for which the techniques we used do not work. In particular, as the variance of shocks gets sufficiently low, the non-concavities in the expected value functions become problematic.

To give a clear example of this, we show the solution without retirement and so the life cycle ends at age 62. The same qualitative pictures are observed with retirement. Figure 1 shows consumption as a function of assets in the period preceding the end of life, $T-1$, for workers and non-workers, and for different firm types, conditioning on individual productivity. The sharp decline in consumption when working at a given firm in $T-1$ arises at the asset stock which induces the individual not to work in the *next* period, T. Because the individual is not working in period T, lifetime income is lower and consumption falls in both periods. On the other hand, since leisure is higher in the next period, overall welfare is higher: the value function is monotonically increasing in assets. The extent of the fall depends on the degree to which consumption and leisure are substitutes. If we look at the solution in earlier time periods or the solution with retirement included, these sharp drops are smoothed out. This is partly because the fall in income associated with a change in employment in one period in the future can be smoothed out over several periods. It is also partly because uncertainty smooths the discreteness: a marginal increase in asset holdings in period t will only change employment in $t+1$ in particular states and so has less of an impact on consumption in period t than if employment in $t+1$ changed in all states.

Figure 1: Consumption as a Function of Current Assets Conditional on Current Period Work Status



B Appendix: Deriving Moments for the Variance of Wages

In our preferred model, wages are given by

$$\ln w_{it} = d_t + x_{it}'\psi + u_{it} + e_{it} + a_{ij(t_0)}$$

where $u_{it} = u_{it-1} + \zeta_{it}$ is the permanent component, e_{it} the measurement error, and $a_{ij(t_0)}$ is the match effect. Thus wage growth is

$$\Delta \ln w_{it} = \Delta d_t + \Delta x_{it}'\psi + \zeta_{it} + \Delta e_{it} + \xi_{it} M_{it}$$

where $\xi_{it} = \left(a_{ij(t)} - a_{ij(t_0)}\right)$ denotes the change in the match effect for those who switched employment. The latent indices associated to working and moving are:

$$\begin{aligned} P_{it}^* &= z_{it}'\gamma + \pi_{it} \\ M_{it}^* &= k_{it}'\theta + \mu_{it} \end{aligned}$$

for all t. Define workers in period t by $(P_{it} = 1) \equiv (P_{it}^* > 0)$ and similarly those who have changed workplace since the previous year by $(M_{it} = 1) \equiv (M_{it}^* > 0)$.

Wage growth is measured annually. However, all decisions by the individual are made quarterly in the model. To make these two consistent we assume that the individual receives a wage shock each quarter with 0.25 probability. Then when we observe a wage over a year we assume it is the result of aggregating wages over the quarters that the individual worked in this firm. Thus the employment Mills ratio we use is the average Mills ratio over the number of quarters the individual worked with the firm where he is currently observed. This effectively assumes that the employment and wage shocks are both independent over time.[1] Denoting the number of quarters the individual worked by Q and noting that the Mills ratio for a working quarter is $\lambda_{i(q)}^P = \frac{\phi\left(z_{i(q)}'\gamma_q\right)}{\Phi\left(z_{i(q)}'\gamma_q\right)}$ (with ϕ and Φ being the standard normal density and probability functions, respectively), for annual wages the Mills ratio

[1] We make this assumption for tractability: otherwise we would have to condition jointly on an eight-dimensional selection vector, i.e. the sequence of decisions to work in each quarter over 2 years. As we do it we still condition on each of the employment outcomes but not on the joint event.

we use is the average of the quarterly Mills ratios, i.e. $\overline{\lambda_{it}^P} = \frac{1}{Q}\sum_{q=1}^{Q}\frac{\phi\left(z_{it(q)}'\gamma_q\right)}{\Phi\left(z_{it(q)}'\gamma_q\right)}$. When computing higher-order moments we also construct $\overline{z_{it}'\gamma\lambda_{it}^P} = \frac{1}{Q}\sum_{q=1}^{Q}\left\{\frac{\phi\left(z_{it(q)}'\gamma_q\right)}{\Phi\left(z_{it(q)}'\gamma_q\right)}z_{it(q)}'\gamma_q\right\}$. This procedure is not exact aggregation, but an approximation because the dependent variable is the log of the average wage and the right-hand side is a model for the average log wage (both within the year). However, the approximation error is likely to be negligible for two reasons: first, it is proportional to the difference between the arithmetic and geometric mean of wages within a year that depends on the within-year/within-firm variance of wages, which is small. Second, any systematic error in the levels is removed by taking the growth of wages and the remaining part will be absorbed by the measurement error, leaving the estimate of the variance of the permanent effect σ_ζ unaffected.

In all calculations the mobility equation is kept constant across quarters and measures the probability that an individual changes jobs between any two years, which is what we need to correct for mobility selection.

Conditioning on working in periods t and $t-1$, we obtain:

$$\begin{aligned} E\left(\Delta\ln w_{it}|P_{it}=P_{it-1}=1\right) &= E\left(\Delta\ln w_{it}|M_{it}=0,P_{it}=P_{it-1}=1\right)\left(1-\Pr\left(M_{it}=1\right)\right) \\ &\quad +E\left(\Delta\ln w_{it}|M_{it}=1,P_{it}=P_{it-1}=1\right)\Pr\left(M_{it}=1\right) \\ &= \Delta d_t+\Delta x_{it}'\psi+G_{it} \end{aligned}$$

where

$$G_{it}=\rho_{\zeta\pi}\sigma_\zeta\overline{\lambda_{it}^P}+\rho_{\xi\pi}\sigma_\xi\overline{\lambda_{it}^P}\Phi\left(k_{it}'\theta\right)+\rho_{\xi\mu}\sigma_\xi\phi\left(k_{it}'\theta\right)+\rho_{\xi\pi_{-1}}\sigma\overline{\lambda_{it-1}^P}\Phi\left(k_{it}'\theta\right)$$

The ρ_{ls} are correlation coefficients between stochastic terms l and s. Thus, G_{it} is a "selection term" accounting for conditioning on multiple indices. The estimation of the equation above is standard (Heckman 2-step method).

The variances of the wage shocks are identified by the restrictions imposed on the moments of residual wage growth $g_{it}\equiv\zeta_{it}+\Delta e_{it}+\xi_{it}M_{it}$. Using formulae from G. M. Tallis (1961), the first moment for job stayers and movers, respectively, is:

$$E\left(g_{it}|P_{it}=P_{it-1}=1,M_{it}=0\right) = -\rho_{\zeta\mu}\sigma_\zeta\widetilde{\lambda}_{it}^M+\rho_{\zeta\pi}\sigma_\zeta\overline{\lambda_{it}^P} \quad (3)$$

$$E\left(g_{it}|P_{it}=P_{it-1}=1,M_{it}=1\right) = \left(\rho_{\zeta\mu}\sigma_\zeta+\rho_{\xi\mu}\sigma_\xi\right)\lambda_{it}^M+\left(\rho_{\zeta\pi}\sigma_\zeta+\rho_{\xi\pi}\sigma_\xi\right)\overline{\lambda_{it}^P}+\rho_{\xi\pi_{-1}}\sigma_\xi\overline{\lambda_{it-1}^P} \quad (4)$$

where $\lambda_{it}^M=\frac{\phi\left(k_{it}'\theta\right)}{\Phi\left(k_{it}'\theta\right)}$ and $\widetilde{\lambda}_{it}^M=\frac{\phi\left(k_{it}'\theta\right)}{1-\Phi\left(k_{it}'\theta\right)}$. The parameters of the model are clearly not identified from the first moments alone. Consider then the second moment for workers who either stay or move:

$$E\left(g_{it}^2|P_{it}=P_{it-1}=1,M_{it}=0\right)=\sigma_\zeta^2\left(\begin{array}{c}1-\rho_{\zeta\pi}^2\overline{z_{it}'\gamma\lambda_{it}^P}+\rho_{\zeta\mu}^2k_{it}'\theta\widetilde{\lambda}_{it}^M \\ -2\rho_{\zeta\pi}\rho_{\zeta\mu}\overline{\lambda_{it}^P}\widetilde{\lambda}_{it}^M\end{array}\right)+2\sigma_e^2 \quad (5)$$

and

$$\begin{aligned} E\left(g_{it}^2|P_{it}=P_{it-1}=1,M_{it}=1\right) &= \sigma_\xi^2\left(\begin{array}{c}1-\frac{1}{2}\rho_{a\pi}^2\overline{z_{it}'\gamma\lambda_{it}^P}-\frac{1}{2}\rho_{a\pi}^2\overline{z_{it-1}'\gamma\lambda_{it-1}^P}-2\rho_{a\mu}^2k_{it}'\theta\lambda_{it}^M \\ +2\rho_{a\mu}\rho_{a\pi}\lambda_{it}^M\overline{\lambda_{it}^P}-2\rho_{a\mu}\rho_{a\pi}\lambda_{it}^M\overline{\lambda_{it-1}^P} \\ -\rho_{a\pi}^2\overline{\lambda_{it}^P\lambda_{it-1}^P}\end{array}\right) \\ &\quad +\sigma_\zeta^2\left(\begin{array}{c}1-\rho_{\zeta\pi}^2\overline{z_{it}'\gamma\lambda_{it}^P}-\rho_{\zeta\mu}^2k_{it}'\theta\lambda_{it}^M \\ +2\rho_{\zeta\mu}\rho_{\zeta\pi}\lambda_{it}^M\overline{\lambda_{it}^P}\end{array}\right)+2\sigma_e^2 \end{aligned} \quad (6)$$

Finally, we consider the first order autocovariance $E\left(g_{it}g_{it-1}|.\right)$. At least in principle, we could use information on those who work for three periods in a row and classify them on the basis of their mobility decisions. In practice, there are too few observations in the relevant categories to be able to get structural identification in this case. We thus assume $\Pr\left(M_t = 1,\ M_{t-1} = 1\right) \approx 0$ and consider only the restrictions on the unconditional autocovariance, namely

$$E\left(g_{it} g_{it-1}\right) = -\sigma_e^2$$

C Appendix: Tables on Employment, Mobility, Calibration

Table 1: Summary Statistics, SIPP 1993 panel

Variable	Mean	Standard deviation
Earnings	6755.03	5209.55
Typical weekly hours	37.69	17.09
Employed	0.87	0.33
Age	40.19	9.91
White	0.88	0.32
Married	0.71	0.45
Unearned income (net of transfers)	1037.62	2205.08
High education	0.55	0.50
Public sector	0.07	0.26
Northeast	0.21	0.41
North Central	0.27	0.45
South	0.26	0.44

Note: The table report quarterly averages. Earnings and unearned income are converted in real terms using the CPI (CPI=1 in 1992:10).

Table 2: The Employment Probit for each Quarter

	High school or less				College dropout or more			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Age	0.0129	0.0107	0.0140	0.0150	0.0167	0.0140	0.0140	0.0192
	(0.0033)	(0.0033)	(0.0033)	(0.0031)	(0.0016)	(0.0016)	(0.0016)	(0.0016)
$Age^2/100$	−0.0227	−0.0205	−0.0243	−0.0252	−0.0219	−0.0188	−0.0188	−0.0252
	(0.0039)	(0.0039)	(0.0039)	(0.0036)	(0.0020)	(0.0019)	(0.0019)	(0.0019)
White	0.1207	0.1153	0.1082	0.1040	0.0557	0.0652	0.0600	0.0596
	(0.0137)	(0.0135)	(0.0134)	(0.0125)	(0.0097)	(0.0099)	(0.0097)	(0.0093)
Married	0.1410	0.1571	0.1555	0.1505	0.0579	0.0592	0.0569	0.0695
	(0.0103)	(0.0103)	(0.0103)	(0.0095)	(0.0067)	(0.0064)	(0.0064)	(0.0064)
Region dummies	28.51	34.68	34.57	38.93	11.29	12.87	7.35	11.83
	(3, 0%)	(3, 0%)	(3, 0%)	(3, 0%)	(3, 1%)	(3, 0%)	(3, 6%)	(3, 1%)
Year dummies	0.35	1.02	0.05	3.90	0.16	0.44	0.42	4.95
	(2; 84%)	(2; 60%)	(2; 98%)	(2; 14%)	(2; 93%)	(2; 80%)	(2; 81%)	(2; 8%)
Unearned income	−0.0480	−0.0424	−0.0396	−0.0468	−0.0119	−0.0116	−0.0129	−0.0116
	(0.0020)	(0.0019)	(0.0017)	(0.0020)	(0.0007)	(0.0007)	(0.0007)	(0.0007)
UI generosity	−0.0010	−0.0009	−0.0009	−0.0007	0.0001	−0.0002	−0.0001	0.0000
	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0002)	(0.0002)	(0.0002)	(0.0002)

Notes: The table reports marginal effects. Asymptotic standard errors in parentheses. For region and year dummies we report the value of the χ^2 statistics of joint significance and, in parentheses, the degrees of freedom and the p-value of the test.

Table 3: The Mobility Equation

	High school or less	College dropout or more
Age	−0.0128	−0.0148
	(0.0031)	(0.0025)
$Age^2/100$	0.0112	0.0142
	(0.0038)	(0.0031)
White	−0.0300	0.0091
	(0.0129)	(0.0091)
Married	−0.0096	−0.0177
	(0.0089)	(0.0071)
Not-for-profit	−0.0549	0.0057
	(0.0180)	(0.0144)
Industry dummies	81.20	65.66
	(4 df; p-value χ^2 0%)	(4 df; p-value χ^2 0%)
Region dummies	6.81	2.35
	(3 df; p-value χ^2 8%)	(3 df; p-value χ^2 50%)
Year dummies	35.65	76.50
	(2 df; p-value χ^2 0%)	(2 df; p-value χ^2 0%)
Unearned income	0.0027	0.0016
	(0.0006)	(0.0004)
UI generosity	0.0008	−0.0001
	(0.0004)	(0.0003)

Notes: The table reports marginal effects. Asymptotic standard errors in parentheses. For region and year dummies we report the value of the χ^2 statistics of joint significance and, in parentheses, the degrees of freedom and the p-value of the test.

Table 4: Observed and Matched Moments

Statistic	Age	High education Data	High education Model	Low education Data	Low education Model
	22-31	0.97	0.95	0.91	0.91
	32-41	0.95	0.94	0.89	0.89
Employment rate	42-51	0.92	0.92	0.82	0.84
	52-61	0.73	0.79	0.56	0.65
	22-26	1.65	1.65	1.88	1.93
	27-31	2.08	2.11	2.38	2.38
	32-36	1.93	2.30	3.03	2.62
Mean duration	37-41	2.47	2.34	2.60	2.90
	42-46	2.84	2.78	3.51	3.51
	47-51	5.41	5.96	4.18	8.08
	52-56	7.45	6.49	8.40	7.21
	57-61	5.50	4.09	4.86	4.05

Table 5: Comparative Statics: Varying the Job Destruction Rate δ

δ	π	Output	σ_y	Mean Duration	Mean $\Delta \ln c_t$		Median $(\Delta A/y)$			Age at max
				47-52	25-44	45-62	25-35	36-50	51-62	
High education										
0.01	0.056	0.111	0.233	8.5	0.030	0.014	0.14	0.24	0.09	62
0.028	**0.0**	**0.0**	**0.332**	**6.0**	**0.029**	**0.013**	**0.14**	**0.25**	**0.076**	**62**
0.049	−0.049	−0.087	0.399	5.0	0.028	0.012	0.10	0.24	0.066	62
0.07	−0.090	−0.153	0.448	4.4	0.027	0.012	0.071	0.24	0.058	62
Low education										
0.01	0.099	0.254	0.240	13.5	0.019	0.011	0.18	0.21	0.037	59
0.028	0.043	0.099	0.345	9.7	0.018	0.011	0.18	0.20	0.012	58
0.049	**0.0**	**0.0**	**0.420**	**8.1**	**0.017**	**0.010**	**0.10**	**0.17**	**−0.010**	**57**
0.07	−0.038	−0.085	0.473	7.2	0.016	0.010	0.08	0.15	−0.025	57

Notes: For the columns concerning the amount of assets, the denominator is average realized earnings (net of the fixed cost of work) in the education-specific baseline. Duration is measured in quarters. The baseline case is in bold.

Table 6: Comparative Statics: Varying the Standard Deviation of the Match-Specific effect σ_α

σ_α	π	Output	σ_y	Mean Duration	Mean $\Delta \ln c_t$		Median $(\Delta A/y)$			Age at max
				47-52	25-44	45-62	25-35	36-50	51-62	
High education										
0.11	-0.185	-0.185	0.274	7.8	0.025	0.013	0.10	0.22	0.065	61
0.22	**0.0**	**0.0**	**0.332**	**6.0**	**0.029**	**0.013**	**0.14**	**0.25**	**0.076**	**62**
0.33	0.130	0.180	0.363	5.6	0.032	0.012	0.19	0.26	0.086	62
Low education										
0.11	-0.149	-0.170	0.359	10.1	0.014	0.010	0.12	0.16	-0.019	56
0.22	**0.0**	**0.0**	**0.420**	**8.1**	**0.017**	**0.010**	**0.10**	**0.17**	**$-$0.010**	**57**
0.33	0.124	0.184	0.455	7.1	0.020	0.010	0.067	0.20	-0.006	58

Notes: For the columns concerning the amount of assets, the denominator is average realized earnings (net of the fixed cost of work) in the education-specific baseline. Duration is measured in quarters. The baseline case is in bold.

Table 7: Comparative Statics: Varying the Job Arrival Rate for the Unemployed λ^n

λ^n	π	Output	σ_y	Mean Duration	Mean $\Delta \ln c_t$		Median $(\Delta A/y)$			Age at max
				47-52	25-44	45-62	25-35	36-50	51-62	
High education										
0.96	0.005	0.003	0.331	6.0	0.029	0.013	0.14	0.25	0.078	62
0.82	**0.0**	**0.0**	**0.332**	**6.0**	**0.029**	**0.013**	**0.14**	**0.25**	**0.076**	**62**
0.76	-0.002	-0.002	0.331	6.0	0.029	0.013	0.14	0.25	0.075	62
0.66	-0.006	-0.006	0.337	6.3	0.029	0.013	0.15	0.25	0.074	62
Low education										
0.96	0.010	0.013	0.416	7.7	0.017	0.010	0.11	0.17	-0.001	58
0.82	0.003	0.004	0.420	8.0	0.017	0.010	0.11	0.17	-0.005	58
0.76	**0.0**	**0.0**	**0.420**	**8.1**	**0.017**	**0.010**	**0.10**	**0.17**	**$-$0.010**	**57**
0.66	-0.006	-0.007	0.424	8.3	0.017	0.010	0.10	0.17	-0.016	57

Notes: For the columns concerning the amount of assets, the denominator is average realized earnings (net of the fixed cost of work) in the education-specific baseline. Duration is measured in quarters. The baseline case is in bold.

D The Alternative Model

Consider a model in which the matching effect follows a random walk over the life of a job:

$$\begin{aligned}\alpha_{ij(t_0)t} &= \alpha_{ij(t_0)t-1} + \omega_{ij(t_0)t} \\ &= \underbrace{\alpha_{ij(t_0)}}_{\text{initial "fixed" matching component}} + \underbrace{\sum_{s=t_0+1}^{t} \omega_{ij(t_0)s}}_{\text{cumulative shocks to the initial match component}}\end{aligned}$$

The notation is as follows: $j(t_0)$ indexes a job that started in period t_0. In the first year in a new job (say t_0), the match component is $\alpha_{ij(t_0)}$. In the second year, it's $\alpha_{ij(t_0)} + \omega_{ij(t_0)t_0+1}$, and so forth.

Assume wages are now given by:

$$\ln w_{it} = d_t + x'_{it}\psi + f_i + e_{it} + \alpha_{ij(t_0)t}$$

where f_i is an individual time-invariant random effect with variance $\sigma^2_{f(b)}$ that varies by cohort b. Define the growth residual as:

$$g_{it} = \Delta\left(\ln w_{it} - d_t - x'_{it}\psi\right)$$

Then

$$\begin{aligned}g_{it} &= \left[\left(\alpha_{ij(t)t} - \alpha_{ij(t_0)t_0}\right) - \sum_{s=t_0+1}^{t-1} \omega_{ij(t_0)s}\right] M_{it} + \xi_{ij(t_0)t}\left(1 - M_{it}\right) + \Delta e_{it} \\ &= \left[\xi_{it} - \sum_{s=t_0+1}^{t-1} \omega_{ij(t_0)s}\right] M_{it} + \xi_{ij(t_0)t}\left(1 - M_{it}\right) + \Delta e_{it}\end{aligned}$$

where $\xi_{it} = \left(\alpha_{ij(t)t} - \alpha_{ij(t_0)t_0}\right)$ is the innovation in the "initial" match component.

Assume as before that an individual moves ($M_{it} = 1$) if $\mu_{it} > -k'_{it}\theta$ and works at time t if $\pi_{it} > -z'_{it}\gamma$. Identification is based on the following assumption

$$E\left(\omega_{ij(t_0)s}\pi_{it}\right) = \begin{cases} 0 & \text{for } s \neq t \\ \sigma_{\omega\pi} & \text{for } s = t \end{cases}$$

and e is a pure measurement error term (no selection issues). The first moment of g_{it} conditional on moving is

$$\begin{aligned}E\left(g_{it} | M_{it} = 1, P_{it} = 1, P_{it-1} = 1\right) &= \sigma_\xi \rho_{\xi\mu} \overline{\lambda^P_{it}} + \sigma_\xi \rho_{\xi\pi} \overline{\lambda^P_{it}} - \sigma_\omega \rho_{\omega\pi} \overline{\lambda^P_{it-1}} \\ &\quad - \left(t - t_0 - 1\right) \sigma_\omega \rho_{\omega\mu} \lambda^M_{it} \end{aligned} \tag{7}$$

where we have made the assumption that $E\left(\omega_{ij(t_0)s}\mu_{it}\right) = \sigma_{\omega\mu}$ for $s \leq t$. The first moment of g_{it} conditional on staying is

$$E\left(g_{it} | M_{it} = 0, P_{it} = 1, P_{it-1} = 1\right) = -\sigma_\omega \rho_{\omega\mu} \widetilde{\lambda}^M_{it} + \sigma_\omega \rho_{\omega\pi} \lambda^P_{it} \tag{8}$$

The second moment of g_{it} conditional on moving or staying is

$$
\begin{aligned}
E\left(g_{it}^2 | M_{it}=1, P_{it}=1, P_{it-1}=1\right) &= \sigma_\xi^2 \left(1-\rho_{\xi\pi}^2 \overline{z_{it}'\gamma\lambda_{it}^P} - \rho_{\xi\mu}^2 k_{it}'\theta\lambda_{it}^M + 2\rho_{\xi\mu}\rho_{\xi\pi}\lambda_{it}^M \overline{\lambda_{it}^P}\right) \\
&\quad + \sigma_\omega^2 \left(\begin{array}{c} 1-\rho_{\omega\pi}^2 \overline{z_{it-1}'\gamma\lambda_{it-1}^P} - (t-t_0-1)\,\rho_{\omega\mu}^2 k_{it}'\theta\lambda_{it}^M \\ +2\rho_{\omega\mu}\rho_{\omega\pi}\lambda_{it}^M \overline{\lambda_{it-1}^P} \end{array} \right) \\
&\quad + 2\sigma_e^2 \qquad (9)
\end{aligned}
$$

and

$$
E\left(g_{it}^2 | M_{it}=0, P_{it}=1, P_{it-1}=1\right) = \sigma_\omega^2 \left(\begin{array}{c} 1-\rho_{\omega\pi}^2 \overline{z_{it}'\gamma\lambda_{it}^P} + \rho_{\omega\mu}^2 k_{it}'\theta\widetilde{\lambda}_{it}^M \\ -2\rho_{\omega\pi}\rho_{\omega\mu}\overline{\lambda_{it-1}^P}\widetilde{\lambda}_{it}^M \end{array} \right) + 2\sigma_e^2 \qquad (10)
$$

Finally, we assume as before

$$
E\left(g_{it}g_{it-1}|.\right) = -\sigma_e^2 \qquad (11)
$$

Notice that $\sigma_{f(b)}^2$ (the variance of the individual random effect) remains not identified from growth terms alone. Identification must come from using levels. Conditional on working,

$$
E\left(\ln w_{it}|P_{it}=1\right) = d_t + x_{it}'\psi + \left(\sigma_{f(b)}\rho_{f\pi} + \sigma_\alpha\rho_{a\pi} + \sigma_\omega\rho_{\omega\pi}\right)\overline{\lambda_{it}^P}
$$

Hence, we control for selection into work adding $\overline{\lambda_{it_0}^P}$ and the interaction of $\overline{\lambda_{it_0}^P}$ with year of birth dummies and with tenure, using only data on workers. Define next the residual in levels

$$
\varepsilon_{it} = \left(\ln w_{it} - d_t - x_{it}'\psi\right) = f_i + \alpha_{ij(t_0)t} + e_{it}
$$

From here, $\sigma_{f(b)}^2$ can be identified from using the two restrictions:

$$
\begin{aligned}
E\left(\varepsilon_{it}|P_{it}=1\right) &= \left(\sigma_{f(b)}\rho_{f\pi} + \sigma_\alpha\rho_{\alpha\pi} + \sigma_\omega\rho_{\omega\pi}\right)\overline{\lambda_{it}^P} \qquad (12) \\
E\left(\varepsilon_{it}^2|P_{it}=1\right) &= \sigma_{f(b)}^2\left(1-\rho_{f\pi}^2\overline{z_{it}'\gamma\lambda_{it}^P}\right) + \sigma_\alpha^2\left(1-\rho_{\alpha\pi}^2\overline{z_{it}'\gamma\lambda_{it}^P}\right) + \sigma_\omega^2\left(1-\rho_{\omega\pi}^2\overline{z_{it}'\gamma\lambda_{it}^P}\right) + 2\sigma_e^2 \qquad (13)
\end{aligned}
$$

As before, these moments depend partly on some of the parameters characterizing unexplained wage growth. We follow a two-step strategy. We first estimate σ_e, σ_ω, σ_α, $\rho_{\xi\pi}$ using the restrictions on unexplained wage growth (7)-(11). We then use (12) and (13) to estimate $\sigma_{f(b)}$ separately for each cohort.

D.1 Estimates of Wage Variances in the Alternative Model

The expressions above make clear that one needs data on tenure $(t-t_0)$ to identify the parameters of interest in the alternative model. The first SIPP interview contains a special module collecting labor market data, including information on tenure with the current employer. We then update tenure over subsequent interviews using information on firm mobility.

Using a similar estimation procedure with all the suitable corrections for selection for job mobility and employment, we produce estimates for the key parameters, shown in Table 8 just for the low-education group for simplicity. Thus σ_ω is the standard deviation of the permanent shock to the match-specific effect $(\omega_{ij(t_0)t})$, σ_e is the standard deviation of the measurement error (e_{it}), σ_a is the standard deviation of the initial value of the match-specific effect $(a_{ij(t_0)})$, and σ_f is the standard deviation of individual heterogeneity (f_i). The estimated standard deviation of the innovation is similar to what we obtain in our preferred model. However, we need a much higher level of heterogeneity upfront than we do in our model so as to match the cross-sectional variance of wages. As we discuss in the main text, this implies a much higher variance of wages for the young and a far too low variance for older individuals.

Table 8: Standard Deviations for the Alternative Model

Parameter	Estimate	Standard error
σ_ω	0.099	0.038
σ_e	0.084	0.021
σ_a	0.20	0.013
$\sigma_{f(b)}$	0.34	0.008

Notes: σ_f is the mean over all cohorts. The standard error of σ_f is computed under the assumption of independence of the $\sigma_{f(b)}$. Standard errors are not corrected.

References

[1] Lentz, Rasmus, and Torben, Tranaes. 2005. "Job Search and Savings: Wealth Effects and Duration Dependence." *Journal of Labor Economics*, 23(3): 467-490.

[2] Tallis G.M. 1961. "The Moment Generating Function of the Truncated Multi-normal Distribution." *Journal of the Royal Statistical Society. Series B (Methodological)*, 23(1): 223-229.