

Technical Appendix for:
 "Distorted gravity:
 the Intensive and Extensive Margins of International Trade"

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Proposition (intensive and extensive margins: general case) *The elasticity of substitution (σ) has a negative effect on both the elasticity of trade flows with respect to variable trade costs (ζ), and on the elasticity of trade flows with respect to fixed costs (ξ):*

$$\text{if } \zeta_{ij} \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} \text{ and } \xi_{ij} \equiv -\frac{d \ln X_{ij}}{d \ln f_{ij}}, \text{ then } \frac{\partial \zeta_{ij}}{\partial \sigma} \leq 0 \text{ and } \frac{\partial \xi_{ij}}{\partial \sigma} < 0$$

when:

1. there are only two countries.
2. there are N countries, and all foreign firms face the same fixed export cost into j .
3. countries are small.
4. fixed export costs are not too different across countries (e.g. less than 400 times apart with 3 countries).

Proof. I now relax the simplifying assumption $\frac{\partial \theta_j}{\partial \tau_{ij}} = \frac{\partial \theta_j}{\partial f_{ij}} = 0$ made in proposition 2. In this general case, the elasticity of trade flows with respect to variable trade costs is no longer the same for all country pairs. For exports from i to j , it is given by,

$$\zeta_{ij} \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \gamma \left(1 - \frac{A_i}{\sum_k A_k} \right)$$

with $A_k = (Y_k/Y) \times (w_k \tau_{kj})^{-\gamma} \times f_{kj}^{-\left(\frac{\gamma}{\sigma-1}-1\right)}$. In the absence of the simplifying assumption above, it does depend on the elasticity of substitution (σ),

$$\frac{\partial \zeta_{ij}}{\partial \sigma} = -\left(\frac{\gamma}{\sigma-1} \right)^2 \frac{\sum_{k \neq i} \ln(f_{ij}/f_{kj}) A_i A_k}{(\sum_k A_k)^2}$$

The elasticity of trade flows with respect to fixed costs is also different for different country pairs. For exports from i to j , it is given by,

$$\xi_{ij} \equiv -\frac{d \ln X_{ij}}{d \ln f_{ij}} = \left(\frac{\gamma}{\sigma-1} - 1 \right) \left(1 - \frac{A_i}{\sum_k A_k} \right)$$

It does depend on the elasticity of substitution in a more complicated way than in the simpler case presented in proposition 2,

$$\frac{\partial \xi_{ij}}{\partial \sigma} = - \left(\frac{\gamma}{\sigma - 1} \right)^2 \frac{\sum_{k \neq i} A_k \left(A_i \ln(f_{ij}/f_{kj})^{\frac{\gamma}{\sigma-1}-1} + \sum_l A_l \right)}{(\sum_k A_k)^2}$$

The sign of both derivatives ($\frac{\partial \zeta_{ij}}{\partial \sigma}$ and $\frac{\partial \xi_{ij}}{\partial \sigma}$) is negative under only mild assumptions. I prove this result in several special cases of interest, and provide conditions under which it is true in general.

1. **Two country case:** it is natural to assume in a two country case that the fixed entry cost into one's own domestic market is smaller than the entry cost into the foreign market,

$$f_{ij} > f_{jj}, \text{ for } i \neq j$$

This mild assumption is consistent with the observed empirical fact that not all domestic firms export, whereas few if any firm sells abroad and not at home. In that case, the elasticity of substitution (σ) has a negative impact on the elasticity of trade flows with respect to variable costs (ζ),

$$\frac{\partial \zeta_{ij}}{\partial \sigma} = - \left(\frac{\gamma}{\sigma - 1} \right)^2 \frac{\ln(f_{ij}/f_{jj}) A_i A_j}{(A_i + A_j)^2} < 0$$

and the elasticity of substitution (σ) has a negative impact on the elasticity of trade flows with respect to variable costs (ξ),

$$\frac{\partial \xi_{ij}}{\partial \sigma} = - \left(\frac{\gamma}{\sigma - 1} \right)^2 \frac{A_j \left(A_i \ln(f_{ij}/f_{jj})^{\frac{\gamma}{\sigma-1}-1} + \sum_k A_k \right)}{(\sum_k A_k)^2} < 0$$

2. **N asymmetric countries, same fixed export cost into country j :** another simple case arises when the all firms face the same fixed export cost into country j , f_j^X , whereas domestic firms from in j face a fixed entry cost f_j^D , with,

$$f_j^X > f_j^D$$

Notice first that variable trade costs can take any values (satisfying the triangular inequality), with any asymmetry allowed. Second, notice that fixed entry costs into different countries can vary and take any value. I reduce the dimension of fixed entry costs from N^2 to $2N$. In that case, the elasticity of substitution (σ) has a negative impact on the elasticity of trade flows with respect to variable costs (ζ),

$$\frac{\partial \zeta_{ij}}{\partial \sigma} = - \left(\frac{\gamma}{\sigma - 1} \right)^2 \frac{\ln(f_j^X/f_j^D) A_i A_j}{(\sum_k A_k)^2} < 0$$

And the elasticity of substitution (σ) has a negative impact on the elasticity of trade flows with respect to variable costs (ξ),

$$\frac{\partial \xi_{ij}}{\partial \sigma} = - \left(\frac{\gamma}{\sigma - 1} \right)^2 \frac{A_j \left(A_i \ln(f_j^X/f_j^D)^{\frac{\gamma}{\sigma-1}-1} + \sum_k A_k \right)}{(\sum_k A_k)^2} < 0$$

3. **Small countries** ($Y_i/Y \approx 0$): I present here a formal proof that if country i is small, then $\zeta_{ij} \approx \zeta = \gamma$ and $\xi_{ij} \approx \xi = \frac{\gamma}{\sigma-1} - 1$. If country i is small, $Y_i/Y \approx 0$, then for finite trade barriers, $A_i \approx 0$. It follows that,

$$\zeta_{ij} = \gamma \left(1 - \frac{A_i}{\sum_k A_k} \right) \approx \gamma$$

and that,

$$\xi_{ij} = \left(\frac{\gamma}{\sigma-1} - 1 \right) \left(1 - \frac{A_i}{\sum_k A_k} \right) \approx \frac{\gamma}{\sigma-1} - 1$$

This is the result presented in proposition 2,

$$\frac{\partial \zeta_{ij}}{\partial \sigma} \approx 0 \text{ and } \frac{\partial \xi_{ij}}{\partial \sigma} \approx - \left(\frac{\gamma}{\sigma-1} \right)^2 < 0$$

4. **General case:** In general where different countries face different fixed entry costs into j , it is possible that for some countries (never all), $\frac{\partial \zeta_{ij}}{\partial \sigma} > 0$ and $\frac{\partial \xi_{ij}}{\partial \sigma} > 0$. But then, for all the other countries, $\frac{\partial \zeta_{ij}}{\partial \sigma} < 0$ and $\frac{\partial \xi_{ij}}{\partial \sigma} < 0$. This follows from the fact that in order to reverse the sign of $\frac{\partial \zeta_{ij}}{\partial \sigma}$, it must be that f_{ij} is smaller than f_{kj} for some k , but then by construction, $f_{kj} > f_{ij}$ for those k 's. Moreover, it requires extreme parameter restrictions to reverse the sign of $\frac{\partial \xi_{ij}}{\partial \sigma}$ even for a single i . It cannot happen if the fixed export cost from i to j is not at least $\exp(2N)$ times smaller than the fixed export cost into j for all other k countries, $i \neq k$. With 3 countries (i, j and k), this would imply that the fixed export cost from i to j is at least 400 ($e^{2 \times 3} \approx 403$) times smaller than the export cost from k to j . As the number of countries grows, the conditions become even more severe.

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