

Appendix to ‘Credibility of Optimal Monetary Delegation: A Comment’

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This appendix provides some additional details of the derivation of the critical discount factor in Section III of the main text.

When the government plays the reputational strategy (that is, when it is following its commitment policy), the government announces a contract $f_t^a = \omega$, with $\omega \geq \lambda\alpha y^*$. People expect inflation of zero ($\pi_t^e = 0$). The government then actually implements the contract $f_t = \lambda\alpha y^*$, i.e., the actual penalty on the central bank for creating inflation is less than the pre-announced one. It is in fact equal to the penalty that induces the central bank to deliver zero inflation. With this scheme in place, the central bank duly delivers inflation of the expected rate $\pi_t = 0$. The government’s loss is therefore

$$\tilde{L}_t^{PR}(\omega) = \lambda y^{*2} + \varphi(\lambda\alpha y^* - \omega)^2. \quad (\text{A.1})$$

When the government cheats (deviates from the commitment policy), it announces the same contract as in the commitment policy, that is $f_t^a = \omega$, and people respond by expecting zero inflation ($\pi_t^e = 0$), but then the government implements a different contract than it announced. It implements the contract that minimizes its expected loss for this period, given the announced contract and the public’s expectations of inflation. The best scheme to implement satisfies Jensen’s equation 7. The government therefore implements $f_t = \omega \frac{\varphi\Lambda}{1+\varphi\Lambda}$ and the central bank delivers inflation

$$\pi_t = \frac{\lambda\alpha y^*}{\Lambda} - \frac{\varphi\omega}{1+\varphi\Lambda}.$$

The government’s loss when it cheats is therefore

$$\begin{aligned} \tilde{L}_t^{DD}(\omega) &= \pi_t^2 + \lambda(\alpha\pi_t - y^*)^2 + \varphi\left(\omega \frac{\varphi\Lambda}{1+\varphi\Lambda} - \omega\right)^2 \\ &= \frac{\lambda y^{*2}}{\Lambda} + \frac{\omega^2 \varphi}{1+\varphi\Lambda} \end{aligned}$$

In the punishment phase of the game, the government plays the discretionary policy. The government announces an incentive scheme $f_t^{a,NCD} = \lambda\alpha y^* \frac{\Lambda(\varphi\Lambda+1)}{\varphi\Lambda^2+1}$, people expect inflation $\pi_t^e = \lambda\alpha y^* - f_t^{a,NCD} \frac{\varphi\Lambda}{\varphi\Lambda+1}$, the actual

incentive scheme is $f_t = f_t^{a,NCD} \frac{\varphi\Lambda}{\varphi\Lambda+1}$, and actual inflation turns out as expected. The government's losses are then

$$\tilde{L}_{t+1}^{NCD} = \frac{y^{*2}\lambda(\varphi\Lambda+1)\Lambda}{\varphi\Lambda^2+1}. \quad (\text{A.2})$$

The critical value of the discount factor β satisfies

$$\beta_{critical} = \frac{\tilde{L}_t^{PR}(\omega) - \tilde{L}_t^{DD}(\omega)}{\tilde{L}_{t+1}^{NCD} - \tilde{L}_{t+1}^{PR}(\omega)}.$$

The numerator of this expression can be written as

$$\tilde{L}_t^{PR}(\omega) - \tilde{L}_t^{DD}(\omega) = \lambda y^{*2} + \varphi(\lambda\alpha y^* - \omega)^2 - \frac{\lambda y^{*2}}{\Lambda} - \frac{\omega^2\varphi}{1+\varphi\Lambda};$$

which with a little manipulation becomes

$$\begin{aligned} \tilde{L}_t^{PR} - \tilde{L}_t^{DD} &= \frac{1}{\Lambda(1+\varphi\Lambda)} [\lambda\alpha y^*(1+\varphi\Lambda) - \omega\varphi\Lambda]^2 \\ &= \frac{\varphi^2\Lambda}{(1+\varphi\Lambda)} \left[\lambda\alpha y^* \left(\frac{1+\varphi\Lambda}{\varphi\Lambda} \right) - \omega \right]^2 \\ &= \frac{\varphi^2\Lambda}{(1+\varphi\Lambda)} \left[\frac{\lambda\alpha y^*}{\varphi\Lambda} + \lambda\alpha y^* - \omega \right]^2; \end{aligned}$$

while the denominator gives

$$\tilde{L}_{t+1}^{NCD} - \tilde{L}_{t+1}^{PR} = y^{*2}\lambda \left(\frac{\Lambda + \varphi\Lambda^2}{1 + \varphi\Lambda^2} \right) - \lambda y^{*2} - \varphi(\lambda\alpha y^* - \omega)^2;$$

which with some manipulation becomes

$$\begin{aligned} \tilde{L}_{t+1}^{NCD} - \tilde{L}_{t+1}^{PR} &= \frac{y^{*2}\lambda^2\alpha^2}{1+\varphi\Lambda^2} - \varphi(\lambda\alpha y^* - \omega)^2 \\ &= \varphi \left[\frac{y^{*2}\lambda^2\alpha^2}{\varphi(1+\varphi\Lambda^2)} - (\lambda\alpha y^* - \omega)^2 \right] \\ &= \varphi \left[\frac{y^*\lambda\alpha}{\sqrt{\varphi(1+\varphi\Lambda^2)}} + (\lambda\alpha y^* - \omega) \right] \left[\frac{y^*\lambda\alpha}{\sqrt{\varphi(1+\varphi\Lambda^2)}} - (\lambda\alpha y^* - \omega) \right]. \end{aligned}$$

Now all this can be put back together. The expression for the critical β can be written as

$$\begin{aligned} \beta_{critical} &= \frac{\tilde{L}_t^{PR} - \tilde{L}_t^{DD}}{\tilde{L}_{t+1}^{NCD} - \tilde{L}_{t+1}^{PR}} \\ &= \frac{\varphi\Lambda}{(1+\varphi\Lambda)} \frac{[A - \omega']^2}{[B - \omega'][B + \omega']}, \end{aligned} \quad (\text{A.3})$$

in which $\omega' \equiv \omega - \lambda\alpha y^*$, $A \equiv \frac{\lambda\alpha y^*}{\varphi\Lambda}$, $B \equiv \frac{\lambda\alpha y^*}{\sqrt{\varphi(1+\varphi\Lambda^2)}}$.

We want to choose ω to minimize the critical value. We are looking at values of ω' that lie in the range $(0, B)$. That is equivalent to looking at values of ω that are at least as great as in the Jensen solution and which go up to the value at which the punishment for cheating becomes zero, i.e., where the loss due to discretion equals the loss under reputation. At the minimum critical value,

$$-\frac{2}{A - \omega'} + \frac{1}{B - \omega'} - \frac{1}{B + \omega'} = 0.$$

Multiplying through by $(A - \omega')(B - \omega')(B + \omega')$ and tidying up gives

$$\omega' = B^2/A,$$

and the value of the function at the minimum point is

$$\begin{aligned} \widehat{\beta}^{D*} &= \min_{\omega' \in (0, B)} \left[\frac{\tilde{L}_t^{PR} - \tilde{L}_t^{DD}}{\tilde{L}_{t+1}^{NCD} - \tilde{L}_{t+1}^{PR}} \right] \\ &= \frac{\varphi\Lambda}{(1 + \varphi\Lambda)} \frac{\left[A - \frac{B^2}{A} \right]^2}{\left[B - \frac{B^2}{A} \right] \left[B + \frac{B^2}{A} \right]} \\ &= \frac{\varphi\Lambda}{(1 + \varphi\Lambda)} \frac{A^2 - B^2}{B^2} \\ &= \frac{\varphi\Lambda}{(1 + \varphi\Lambda)} \left(\frac{\lambda^2 \alpha^2 y^{*2}}{\varphi^2 \Lambda^2} - \frac{\lambda^2 \alpha^2 y^{*2}}{\varphi(1 + \varphi\Lambda^2)} \right) \frac{\varphi(1 + \varphi\Lambda^2)}{\lambda^2 \alpha^2 y^{*2}} \\ &= \frac{1}{\Lambda(1 + \varphi\Lambda)}. \end{aligned}$$

Since this value is less than $1/\Lambda$ this proves that the critical β under delegation with any announcement is less than under simple discretion.