Homeownership, Community Interactions, and Segregation

Unpublished Appendix: *A More General Model of Contracting between a Household and a Real Estate Company*

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The model presented in the text generates a moral hazard problem for a household in its choice of civic effort, under a host of simplifying specifications. In particular, we have assumed that (*i*) feasible tenure contracts have to be *linear* in the observable outcome – the future price of a home, and that (*ii*) this price is a *deterministic* function of unobservable effort. In what follows, we briefly present a more general model of contracting between a household and a real estate company, and demonstrate that our qualitative results are robust to such generalizations. So consider the following principal-agent model:

A long-lived risk-neutral principal (real estate company) owns a durable good H (housing unit). An agent (household) lives for two periods and benefits from consuming the flow services of H only in the first period. In that period, the agent can put in effort to add value to H, which benefits her *and* increases the future price of H (this price going to the principal). Both players consume a divisible *numeraire* commodity in all periods, and discount the future by $\delta \in (0, 1)$.

The agent's effort (a) can be high (e) or low (n). It is not observable by the principal and so cannot be contracted upon. The agent's current benefit from effort net of cost is $[q(a) - a]$. The future price of H is a random variable P distributed on $[0, \infty)$ with a distribution function $G(p|a)$, where $G(p|e)$ first-order stochastically dominates $G(p|n)$. A feasible contract is a pair $\{\beta, \alpha(p)\}$, where $\beta \in \Re$ is the 'up-front payment' that the agent makes to the principal, and $\alpha(p)$ is the 'rebate' that the principal gives back to the agent in the second period if the realized future price of H is p. The rebate function $\alpha(p)$ can be any real-valued function, but it is required to satisfy the *limited liability clause*: $\alpha(p) \geq 0$ for all p. A contract $\{\beta, \alpha(p)\}$ is thus a specific 'complete contract with limited liability' under moral hazard.

Under the contract $\{\beta, \alpha(\cdot)\}$, if the agent (with current income y and future income w) chooses effort level a and borrows an amount b from the credit market, her utility is:

$$u(a, b; \beta, \alpha(\cdot) \mid y) = \begin{cases} [y-\beta+b]+[q(a)-a]+\delta\int_0^{\infty}[w-(1+r_B)b+\alpha(p)]dG(p \mid a) & \text{if } y-\beta+b \geq 0, \\ -\infty & \text{otherwise,} \end{cases}$$

where r_B is the borrowing rate which exhibits the following imperfection: $1+r_B > 1/\delta$. The principal's utility is $\pi(a; \beta, \alpha(\cdot)) = \beta + \delta\int [p - \alpha(p)]dG(p|\, a)$.

Let effort $a^*(\beta, \alpha(\cdot)|\, y)$ and debt level $b^*(\beta, \alpha(\cdot)|\, y)$ maximize $u(\cdot|\, y)$ under a contract $\{\beta, \alpha(\cdot)\}$. Assume the existence of the following 'incentive problem': (*i*) if $\alpha(p) = 0$ for all p, then $\alpha^* = n$ for all y, and (*ii*) if $\alpha(p) = p$ for all p, then $\alpha^* = e$ for all y. Assume further that the players have the right to the default rental contract $\{\beta = \rho, \alpha(p) = 0\ \forall\ p\}$. This contract pins down the players' reservation payoffs: for the agent, $u^0(y) = [y - \rho] + [q(n) - n] + \delta w$; and for the principal, $\pi^0 = \rho + \delta\int p.dG(p|\, n)$.

Consider the model where the agent makes a take-it-or-leave-it contract offer. Here the optimal contract solves the following problem: maximize $u(a, b; \beta, \alpha(\cdot)|\, y)$ with respect to a, b, β, and $\alpha(\cdot)$, subject to: (*i*) $a = a^*(\beta, \alpha(\cdot)|\, y)$ and $b = b^*(\beta, \alpha(\cdot)|\, y)$ [*incentive compatibility*]; (*ii*) $\pi(a^*(\cdot); \beta, \alpha(\cdot)) \geq \pi^0$ [*individual rationality*]; and (*iii*) $\alpha(p) \geq 0$ for all p [*limited liability*].

The solution to the above problem will have the following features. There will exist a threshold income level $y^*(\rho)$ such that, if the agent's current income y is less than the threshold, the optimal contract will be the default contract and $a^* = n$; and if $y > y^*$, the optimal contract will generate $a^* = e$.

Thus there will be a wealth effect on incentives. The moral hazard problem will not be 'solved' for all agents (*i.e.*, for all y) due to (*i*) the incentive problem, (*ii*) the agents' desire to smooth consumption, (*iii*) the limited liability clause, and (*iv*) credit market imperfections. A similar result will arise in the alternative model where the principal makes a take-it-or-leave-it contract offer.

The above discussion clarifies the general nature of the moral hazard problem that is embedded in our model. The analysis seems to suggest the necessity of a limited liability clause to generate a wealth effect on incentives. However, consider the following 'extended' model:

The agent lives for three periods, but still consumes H only in the first period (and consumes the *numeraire* in all three periods). A feasible contract $\{\beta, \alpha(\cdot)\}$ no longer has to satisfy a limited liability clause (*i.e.*, $\alpha(p)$ can be negative for some p). The agent can borrow b_t in period $t = 1, 2$, under the same terms as before. Here, the agent will be forced to borrow in period 2 if $\alpha(p)$ is a large negative number to ensure non-negative consumption in period 2. Again, when the optimal contract is solved for, we will find a similar wealth effect on incentives. The basic reason for this result is that incentive payments are *effectively* bounded below because the agent will never accept a contract under which his income in any period will fall below zero.