Online Appendix to
"Disaster Risk and Business Cycles"
Not for publication

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1 Introduction

This appendix presents (1) some comparisons of alternative disaster dynamics; (2) the computation method used to solve the model; and (3) data sources.

2 Disaster Dynamics for Alternative Calibrations

This section reports the impulse response of consumption to a disaster realization, i.e. figure 1 of the paper, for the alternative approaches to modeling disasters discussed in section IVB, and also reports the equivalent of tables 2 and 3 of the paper (business cycle statistics and asset returns). All these parametrizations use the same calibration for permanent shocks as in the paper, which explains that the long-run response is identical (and matches the data).

Figure 1 shows that in the absence of transitory productivity shocks, consumption actually rises on impact, as output has not yet fallen significantly and agents cut back sharply on investment which is now risky. Clearly, this response does not match the data. Next, figure 2 depicts the response when there are no depreciation shocks. Consumption falls too much initially and then rises, contrary to the data. The intuition is that, in the absence of depreciation shocks, capital is fairly safe. Hence, investment falls by less on impact than in the benchmark calibration (and may even rise), as agents save, because they anticipate a further decline in productivity and output. Finally, figure 3 shows the response without the doubling of the initial shock. The model has difficulty matching the early part of the consumption response. (Increasing the size of shocks for all time periods does not solve this problem, as the response becomes too negative in the medium-run.)

In terms of business cycle and asset pricing properties, the model without transitory shocks, perhaps surprisingly, does almost as well as the benchmark model. Intuitively, the fact that consumption does not decline immediately when a disaster starts is not critical, as long as the stochastic discount factor (marginal utility) is very high when the disaster starts. This implies that ex-ante people do not want to invest when the probability of disaster is large. On the other hand, the model without depreciation

		$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	Data	0.99	0.54	2.75	0.93	0.48	0.61	0.74
2	Benchmark	0.93	0.69	2.96	0.69	0.32	0.87	0.75
3	No transitory shock $\mu_\varphi = \sigma_\varphi = 0$	0.99	0.79	3.66	0.87	0.10	0.86	0.77
4	No depreciation shock $\mu_\xi = \sigma_\xi = 0$	2.30	2.41	7.76	2.59	-0.66	0.92	0.82
5	No doubling of initial productivity shock	0.88	0.60	2.29	0.52	0.57	0.89	0.77

Table 1: **Robustness: Business cycle statistics.**

		$E(R_f)$	$E(R_b)$	$E(R_e - R_b)$	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e - R_b)$
1	Data	—	0.19	1.84	—	0.82	8.20
2	Benchmark	0.16	0.28	2.24	1.00	0.74	7.67
3	No transitory shock $\mu_\varphi = \sigma_\varphi = 0$	0.42	0.48	1.90	0.50	0.36	6.95
4	No depreciation shock $\mu_\xi = \sigma_\xi = 0$	0.37	0.37	0.20	0.44	0.34	0.82
5	No doubling of initial shock	0.36	0.41	1.99	0.46	0.35	6.49

Table 2: **Robustness: Financial Statistics.**

shocks implies a very large volatility of quantities, but most importantly the sign of the response of investment, employment and output to an increase in disaster probability is flipped: investment goes up with p, regardless of the IES. As a result, this model implies procyclical risk premia. Finally, the model without the doubling of the initial productivity shock has implications inbetween the benchmark model and the RBC model, as the mechanism is weakened.

3 Computational Method

A previous version of this paper used a standard value function iteration - policy function iteration algorithm to solve a discretized version of this model (see the previous appendix on my web site). Given the larger state space, the model is now solved using projection methods (see Aruoba et al. (2006) and Caldara et al. (2011) for an introduction and implementation with Epstein-Zin preferences).

I use the characterization of the equilibrium using the recursive formulation of the two policy functions, consumption $c(k, p, z_r, x)$, employment $N(k, p, z_r, x)$, and the value function $g(k, p, z_r, x)$. We have the three first-order conditions:

$$(1-\alpha) z_r^{1-\alpha} \left(\frac{k}{N(k, p, z_r, x)} \right)^\alpha = \frac{\upsilon c(k, p, z_r, x)}{1 - N(k, p, z_r, x)}, \tag{1}$$

$$\mathbf{E} \left(M(k, p, z_r, x; p', \varepsilon', \theta', \varphi', x') R_K(k, p, z_r, x; p', \varepsilon', \theta', \varphi', x') \right) = 1, \tag{2}$$

where $\mathbf{E}$ denotes an expectation over the 5 shocks: $E_{p',\varepsilon',\theta',\varphi',x'}$, and

$$\begin{aligned} M(k,p,z_r,x;p',\varepsilon',\theta',\varphi',x') &= \beta\left(e^{\mu+\varepsilon'+\theta' x'}\right)^{-\gamma} \\ &\times\left(\frac{c(k',p',z_r',x')}{c(k,p,z_r,x)}\right)^{-\psi} \\ &\times\left(\frac{1-N(k',p',z_r',x')}{1-N(k,p,z_r,x)}\right)^{\upsilon(1-\psi)} \\ &\times\frac{g(k',p',z_r',x')^{\frac{\psi-\gamma}{1-\psi}}}{\mathbf{E}\left(\left(e^{\mu+\varepsilon'+\theta' x'}\right)^{(1-\psi)}g(k',p',z_r',x')^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{\psi-\gamma}{1-\psi}}}, \end{aligned}$$

and

$$\begin{aligned} g(k,p,z_r,x) &= c(k,p,z_r,x)^{1-\psi}(1-N(k,p,z_r,x))^{\upsilon(1-\psi)} \\ &+\beta e^{\mu(1-\psi)}\left(\mathbf{E}e^{(\varepsilon'+x'\theta')(1-\gamma)}g(k',p',z_r',x')^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{1-\psi}{1-\gamma}}, \end{aligned} \tag{3}$$

and

$$R_K(k,p,z_r,x;p',\varepsilon',\theta',\varphi',x') = e^{x'\xi'}\phi'\left(\frac{i(k,p,z_r,x)}{k}\right)\times ...$$

$$...\left(\frac{1-\delta+\phi\left(\frac{i(k',p',z_r',x')}{k'}\right)}{\phi'\left(\frac{i(k',p',z_r',x')}{k'}\right)}+\alpha\frac{y(k',p',z_r',x')}{k'}-\frac{i(k',p',z_r',x')}{k'}\right)$$

where k' is given by the law of motion:

$$k' = k'(k,p,z_r,x;p',\varepsilon',x',\theta',\varphi') = \frac{e^{x'\theta'}\left((1-\delta)k+i(k,p,z_r,x)\right)}{e^{\mu+\varepsilon'+x'\theta'}} = \frac{\left((1-\delta)k+i(k,p,z_r,x)\right)}{e^{\mu+\varepsilon'}},$$

and

$$\log z_r' = \log z_r + x'\left(\varphi'-\theta'\right),$$

and output and investment are defined as:

$$\begin{aligned} y(k,p,z_r,x) &= k^{\alpha}\left(z_r N(k,p,z_r,x)\right)^{1-\alpha}, \\ i(k,p,z_r,x) &= y(k,p,z_r,x)-c(k,p,z_r,x), \end{aligned}$$

as well as the law of motions for the discrete variables x and p.

Each of the three functions $c(k,p,z_r,x)$, $N(k,p,z_r,x)$, and $g(k,p,z_r,x)$ is approximated as a tensor product of Chebychev polynomials, i.e.

$$c(k,p,z_r,x) = \sum_{i=1}^{N_k}\sum_{j=1}^{N_z}\alpha_{i,j,p,x}T_i(k)T_j(z_r),$$

where T_i is the Chebychev polynomial of degree i, adjusted to have a domain $\left[\ \underline{k},\overline{k}\ \right]$ for k and $[1-x, 1+x]$ for z_r, and $\alpha_{i,j,p,x}$ are coefficients to be determined. Hence, for each discrete value of p and x, $c(.,p,.,x)$ is approximated by a two-dimensional Chebychev polynomial.

To find the coefficients $\{\alpha_{i,j,p,x}\}$ for each of the three functions c, N and g, the first order conditions (1)-(3) are evaluated at the nodes of the Chebychev polynomial, and for each value of p and x. This gives a system of $3N_kN_z \times 2N_p$ equations with the same number of unknowns. This system is solved using a quasi-Newton algorithm. To help with convergence, we start with small values for N_k, N_z, N_p, and small risk aversion parameter θ, and progressively increase these parameters, using as initial guess the previous solution. This allows to solve the model for $N_c = 7$, $N_z = 4$ and $N_p = 7$ or more. Expanding the number of points beyond this does not alter the solution significantly, so in practice I use $N_c = 5, N_z = 3$, and $N_p = 6$.

Once the policy functions are found, asset prices are readily calculated using the definition of the stochastic discount factor and the expectations over all the shocks. More precisely, we can calculate the investment return using equation (14) in the main text, and long-term bond prices are calculated by iterating on the standard recursion. (I approximate bond prices, as a function of the states, with Chebychev polynoimials as well.) These two returns allow to construct the levered equity returns. Finally, VIX is calculated using the formula of the main text; for simplicity, the equity return used in this formulation is calculated assuming that firms adjusting their leverage to keep it constant each period. Because this equity return is very strongly correlated with the model benchmark equity return, the approximation is accurate.

Matlab programs are available on the AER website. Note that the programs are written with the equivalent definition of preferences (footnote 8 of the paper).

4 Data Sources

The moments of Tables 2 and 3 are calculated using standard US data sources, for the sample 1947q1-2010q4. Consumption is nondurable + services, investment is nonresidential fixed investment, and output is GDP, all from the NIPA Table 1.1.3. Hours is nonfarm business hours from the BLS productivity program (through FRED: HOABNS). The return data is from Ken French's webpage: monthly benchmark factors, aggregated to quarterly frequency, and deflated by the CPI (CPIAUCSL through FRED)). TFP is computed as output divided by labor to the power 2/3 and capital to the power 1/3. Capital is from the fixed asset tables, and is linearly interpolated within the year. To be consistent with Bloom (2009), I use his data on volatility on the 1963q1-2008q4 sample. Matlab programs and data files are available on the AER website.

References

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[3] Caldara, Dario, Jesus Fernandez-Villaverde, Juan F. Rubio-Ramírez, and Wen Yao, 2011. "Computing DSGE models with recursive preferences and stochastic volatility", Review of Economic Dynamics, forthcoming.

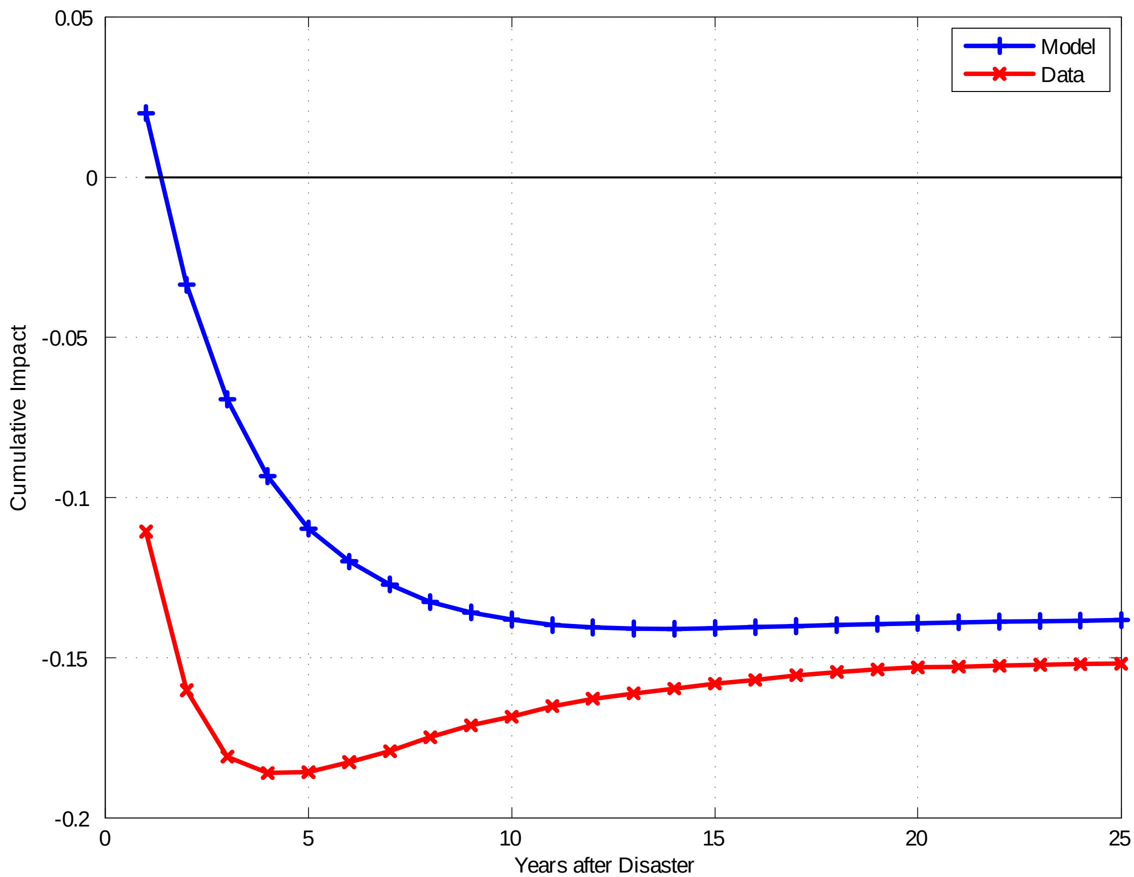

Figure 1: Response of consumption to a disaster realization, in the data and in the model without transitory productivity shocks.

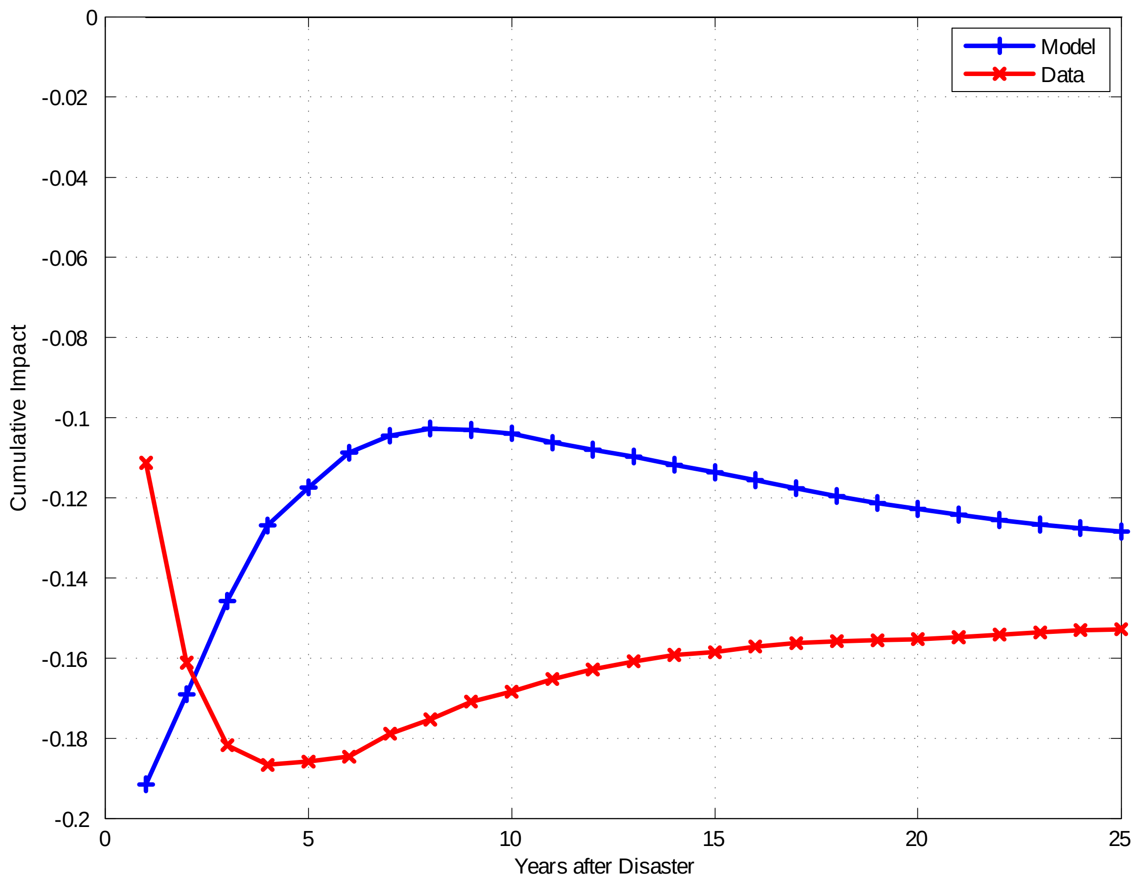

Figure 2: Response of consumption to a disaster realization, in the data and in the model without depreciation shocks.

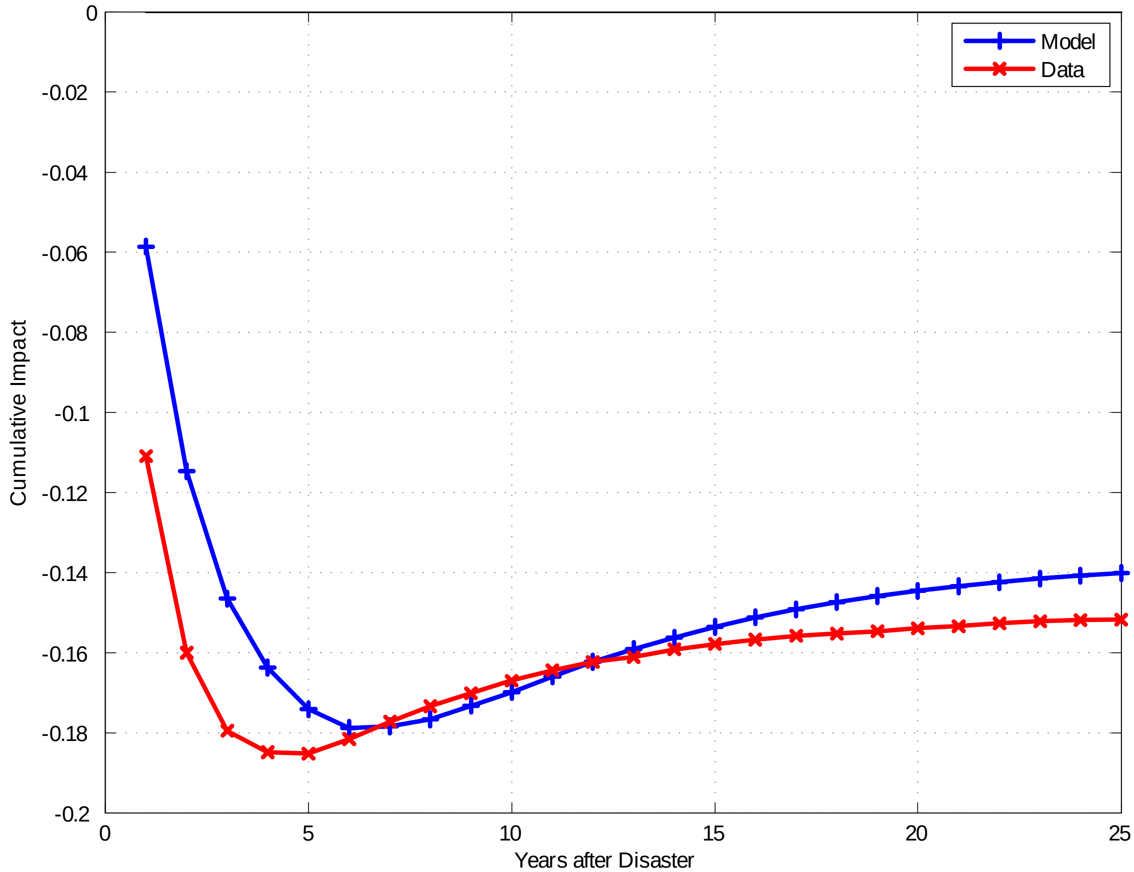

Figure 3: Response of consumption to a disaster realization, in the data and in the model without a twice larger productivity shock on impact.