

**Appendix: Not for Publication**

Currency Misalignments and Optimal Monetary Policy:  
A Reexamination

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## Appendix A

### Model Equations

#### A1.a. Households

The representative household in the home country maximizes

$$(A1) \quad U_t(h) = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1-\sigma} C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi} N_{t+j}(h)^{1+\phi} \right] \right\}, \quad \sigma > 0, \phi \geq 0$$

$C_t(h)$  is the consumption aggregate. We assume Cobb-Douglas preferences:

$$(A2) \quad C_t(h) = (C_{Ht}(h))^{\frac{\nu}{2}} (C_{Ft}(h))^{1-\frac{\nu}{2}}, \quad 0 \leq \nu \leq 2.$$

In turn,  $C_{Ht}(h)$  and  $C_{Ft}(h)$  are CES aggregates over a continuum of goods produced in each country:

$$(A3) \quad C_{Ht}(h) = \left( \int_0^1 C_{Ht}(h, f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \quad \text{and} \quad C_{Ft}(h) = \left( \int_0^1 C_{Ft}(h, f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}}.$$

$N_t(h)$  is an aggregate of the labor services that the household sells to each of a continuum of firms located in the home country:

$$(A4) \quad N_t(h) = \int_0^1 N_t(h, f) df.$$

Households receive wage income,  $W_t(h)N_t(h)$ , aggregate profits from home firms,  $\Gamma_t$ . They pay lump-sum taxes each period,  $T_t$ . Each household can trade in a complete market in contingent claims (arbitrarily) denominated in the home currency. The budget constraint is given by:

$$(A5) \quad P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega_{t+1}} Z(\nabla^{t+1} | \nabla^t) D(h, \nabla^{t+1}) = W_t(h) N_t(h) + \Gamma_t - T_t + D(h, \nabla^t),$$

where  $D(h, \nabla^t)$  represents household  $h$ 's payoffs on state-contingent claims for state  $\nabla^t$ .  $Z(\nabla^{t+1} | \nabla^t)$  is the price of a claim that pays one dollar in state  $\nabla^{t+1}$ , conditional on state  $\nabla^t$  occurring at time  $t$ .

In this equation,  $P_t$  is the exact price index for consumption, given by:

$$(A6) \quad P_t = k^{-1} P_{Ht}^{\nu/2} P_{Ft}^{1-(\nu/2)}, \quad k = (1-(\nu/2))^{1-(\nu/2)} (\nu/2)^{\nu/2}.$$

$P_{Ht}$  is the Home-currency price of the Home aggregate good and  $P_{Ft}$  is the Home currency price of the Foreign aggregate good. Equation (A6) follows from cost minimization. Also, from cost minimization,  $P_{Ht}$  and  $P_{Ft}$  are the usual CES aggregates over prices of individual varieties,  $f$ :

$$(A7) \quad P_{Ht} = \left( \int_0^1 P_{Ht}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}, \quad \text{and} \quad P_{Ft} = \left( \int_0^1 P_{Ft}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}.$$

Foreign households have analogous preferences and face an analogous budget constraint.

Because all Home households are identical, we can drop the index for the household and use the fact that aggregate per capita consumption of each good is equal to

the consumption of each good by each household. The first-order conditions for consumption are given by:

$$(A8) \quad P_{Ht} C_{Ht} = \frac{v}{2} P_t C_t,$$

$$(A9) \quad P_{Ft} C_{Ft} = \left(1 - \frac{v}{2}\right) P_t C_t,$$

$$(A10) \quad C_{Ht}(f) = \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} C_{Ht} \quad \text{and} \quad C_{Ft}(f) = \left(\frac{P_{Ft}(f)}{P_{Ft}}\right)^{-\xi} C_{Ft},$$

$$(A11) \quad \beta \left( C(\nabla^{t+1}) / C(\nabla^t) \right)^{-\sigma} (P_t / P_{t+1}) = \ddot{Z}(\nabla^{t+1} | \nabla^t).$$

In equation (A11), we explicitly use an index for the state at time  $t$  for the purpose of clarity.  $\ddot{Z}(\nabla^{t+1} | \nabla^t)$  is the normalized price of the state contingent claim. That is, it is defined as  $Z(\nabla^{t+1} | \nabla^t)$  divided by the probability of state  $\nabla^{t+1}$  conditional on state  $\nabla^t$ .

Note that the sum of  $Z(\nabla^{t+1} | \nabla^t)$  across all possible states at time  $t + 1$  must equal  $1/R_t$ , where  $R_t$  denotes the gross nominal yield on a one-period non-state-contingent bond. Therefore, taking a probability-weighted sum across all states of equation (A11), we have the familiar Euler equation:

$$(A12) \quad \beta R_t E_t \left[ \left( C(\nabla^{t+1}) / C(\nabla^t) \right)^{-\sigma} (P_t / P_{t+1}) \right] = 1.$$

Analogous equations hold for Foreign households. Since contingent claims are (arbitrarily) denominated in Home currency, the first-order condition for Foreign households that is analogous to equation (A11) is:

$$(A13) \quad \beta \left( C^*(\nabla^{t+1}) / C^*(\nabla^t) \right)^{-\sigma} (E_t P_t^* / E_{t+1} P_{t+1}^*) = \ddot{Z}(\nabla^{t+1} | \nabla^t).$$

Here,  $E_t$  refers to the home currency price of foreign currency exchange rate.<sup>1</sup>

As noted above, we will assume at this stage that labor input of all households is the same, so  $N_t = N_t(h)$ .

### A1.b. Firms

Each Home good,  $Y_t(f)$  is made according to a production function that is linear in the labor input. These are given by:

$$(A14) \quad Y_t(f) = A_t N_t(f).$$

Note that the productivity shock,  $A_t$ , is common to all firms in the Home country.  $N_t(f)$  is a CES composite of individual home-country household labor, given by:

$$(A15) \quad N_t(f) = \left( \int_0^1 N_t(h, f)^{\frac{\eta_t-1}{\eta_t}} dh \right)^{\frac{\eta_t}{\eta_t-1}},$$

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<sup>1</sup> We offer an apology to the reader here. We want to stick to CGG's notation, who use  $e_t$  for the log of the nominal exchange rate. Consistency requires us to use  $E_t$  to refer to the level of the nominal exchange rate, so we have used the distinct but similar notation  $E_t$  to be the conditional expectation operator.

where the technology parameter,  $\eta_t$ , is stochastic and common to all Home firms.

Profits are given by:

$$(A16) \quad \Gamma_t(f) = P_{Ht}(f)C_{Ht}(f) + E_t P_{Ht}^*(f)C_{Ht}^*(f) - (1 - \tau_t)W_t N_t(f).$$

In this equation,  $P_{Ht}(f)$  is the home-currency price of the good when it is sold in the Home country.  $P_{Ht}^*(f)$  is the foreign-currency price of the good when it is sold in the Foreign country.  $C_{Ht}(f)$  is aggregate sales of the good in the home country:

$$(A17) \quad C_{Ht}(f) = \int_0^1 C_{Ht}(h, f) dh.$$

$C_{Ht}^*(f)$  is defined analogously. It follows that  $Y_t(f) = C_{Ht}(f) + C_{Ht}^*(f)$ .

There are analogous equations for  $Y_t^*(f)$ , with the foreign productivity shock given by  $A_t^*$ , the foreign technology parameter shock given by  $\eta_t^*$ , and foreign subsidy given by  $\tau_t^*$ .

### A1.c Equilibrium

Goods market clearing conditions in the Home and Foreign country are given by:

$$(A18) \quad Y_t = C_{Ht} + C_{Ht}^* = \frac{\nu}{2} \frac{P_t C_t}{P_{Ht}} + \left(1 - \frac{\nu}{2}\right) \frac{P_t^* C_t^*}{P_{Ht}^*} = k^{-1} \left( \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right) (S_t^*)^{-\nu/2} C_t^* \right),$$

$$(A19) \quad Y_t^* = C_{Ft} + C_{Ft}^* = \left(1 - \frac{\nu}{2}\right) \frac{P_t C_t}{P_{Ft}} + \frac{\nu}{2} \frac{P_t^* C_t^*}{P_{Ft}^*} = k^{-1} \left( \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t \right).$$

We have used  $S_t$  and  $S_t^*$  to represent the price of imported to locally-produced goods in the Home and Foreign countries, respectively:

$$(A20) \quad S_t = P_{Ft} / P_{Ht},$$

$$(A21) \quad S_t^* = P_{Ht}^* / P_{Ft}^*.$$

Equations (A11) and (A13) give us the familiar condition that arises in open-economy models with a complete set of state-contingent claims when PPP does not hold:

$$(A22) \quad \left( \frac{C_t}{C_t^*} \right)^\sigma = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}} (S_t^*)^{-\nu/2} S_t^{(\nu/2)-1}.$$

Total employment is determined by output in each industry:

$$(A23) \quad N_t = \int_0^1 N_t(f) df = A_t^{-1} \int_0^1 Y_t(f) df = A_t^{-1} (C_{Ht} V_{Ht} + C_{Ht}^* V_{Ht}^*),$$

where

$$(A24) \quad V_{Ht} \equiv \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df, \quad \text{and} \quad V_{Ht}^* \equiv \int_0^1 \left( \frac{P_{Ht}^*(f)}{P_{Ht}^*} \right)^{-\xi} df.$$

### A2 Price and Wage Setting

Households are monopolistic suppliers of their unique form of labor services. Household  $h$  faces demand for its labor services given by:

$$(A25) \quad N_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t,$$

where

$$(A26) \quad W_t = \left( \int_0^1 W_t(h)^{1-\eta_t} dh \right)^{\frac{1}{1-\eta_t}}.$$

The first-order condition for household  $h$ 's choice of labor supply is given by:

$$(A27) \quad \frac{W_t(h)}{P_t} = (1 + \mu_t^W)(C_t(h))^\sigma (N_t(h))^\phi, \text{ where } \mu_t^W \equiv \frac{1}{\eta_t - 1}.$$

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units.)

Because all households are identical, we have  $W_t = W_t(h)$  and  $N_t = N_t(h)$ .

Since all households are identical, we have from equation (A27):

$$(A28) \quad W_t / P_{Ht} = (1 + \mu_t^W) C_t^\sigma N_t^\phi S_t^{1-(v/2)}.$$

We adopt the following notation. For any variable  $K_t$ :

$\dot{K}_t$  is the value under flexible prices.

$\bar{K}_t$  is the value of variables under globally efficient allocations. In other words, this is the value for variables if prices were flexible, and optimal subsidies to monopolistic suppliers of labor and monopolistic producers of goods were in place. This includes a time-varying subsidy to suppliers of labor to offset the time-varying mark-up in wages in equation (A27).

### *Flexible Prices*

Home firms maximize profits given by equation (A16), subject to the demand curve (A10). They optimally set prices as a mark-up over marginal cost:

$$(A29) \quad \dot{P}_{Ht}(f) = \dot{E}_t \dot{P}_{Ht}^*(f) = (1 - \tau_t)(1 + \mu^P) \dot{W}_t / A_t, \text{ where } \mu^P \equiv \frac{1}{\xi - 1}.$$

When optimal subsidies are in place:

$$(A30) \quad \bar{P}_{Ht}(f) = \bar{E}_t \bar{P}_{Ht}^*(f) = \bar{W}_t / A_t.$$

From (A27), (A29), and (A30), it is apparent that the optimal subsidy satisfies

$$(A31) \quad (1 - \tau_t)(1 + \mu^P)(1 + \mu_t^W) = 1.$$

Note from (A29) that all flexible price firms are identical and set the same price. Because the demand functions of Foreign residents have the same elasticity of demand for Home goods as Home residents, firms set the same price for sale abroad:

$$(A32) \quad E_t \dot{P}_{Ht}^* = \dot{P}_{Ht} \text{ and } E_t \bar{P}_{Ht}^* = \bar{P}_{Ht}$$

From (A31), using (A28), we have:

$$(A33) \quad \dot{P}_{Ht} = \dot{E}_t \dot{P}_{Ht}^* = \dot{W}_t / ((1 + \mu^W) A_t) \text{ and } \dot{P}_{Ft}^* = \dot{E}_t^{-1} \dot{P}_{Ft} = \dot{W}_t^* / ((1 + \mu^{*W}) A_t^*).$$

We can conclude:

$$(A34) \quad \dot{S}_t^{*-1} = \dot{S}_t.$$

Because  $\dot{P}_{Ht}(f)$  is identical for all firms, (A23) collapses to

$$(A35) \quad \dot{Y}_t = A_t \dot{N}_t.$$

### *PCP*

We assume a standard Calvo pricing technology. A given firm may reset its prices with probability  $1 - \theta$  each period. We assume that when the firm resets its price, it will be able to reset its prices for sales in both markets. We assume the PCP firm sets a single price in its own currency, so the law of one price holds.

The firm's objective is to maximize its value, which is equal to the value at state-contingent prices of its entire stream of dividends. Given equation (A11), it is apparent that the firm that selects its price at time  $t$ , chooses its reset price,  $P_{Ht}^0(f)$ , to maximize

$$(A36) \quad E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[ P_{Ht}^0(f) (C_{Ht+j}(f) + C_{Ht+j}^*(f)) - (1 - \tau_t) W_{t+j} N_{t+j}(f) \right],$$

subject to the sequence of demand curves given by equation (A10) and the corresponding Foreign demand equation for Home goods. In this equation, we define

$$(A37) \quad Q_{t,t+j} \equiv \beta^j (C_{t+j} / C_t)^{-\sigma} (P_t / P_{t+j}).$$

The solution for the optimal price for the Home firm for sale in the Home country is given by:

$$(A38) \quad P_{Ht}^0(z) = \frac{\xi}{\xi - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1 - \tau_t) W_{t+j} P_{Ht+j}^{\xi} (C_{Ht+j} + C_{Ht+j}^*) / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} P_{Ht+j}^{\xi} (C_{Ht+j} + C_{Ht+j}^*)}.$$

Under the Calvo price setting mechanism, a fraction  $\theta$  of prices remain unchanged from the previous period. From equation (A7), we can write:

$$(A39) \quad P_{Ht} = \left[ \theta (P_{Ht-1})^{1-\xi} + (1 - \theta) (P_{Ht}^0)^{1-\xi} \right]^{1/(1-\xi)},$$

### LCP

The same environment as the PCP case holds, with the sole exception that the firm sets its price for export in the importer's currency rather than its own currency when it is allowed to reset prices. The Home firm, for example, sets  $P_{Ht}^*(f)$  in Foreign currency. The firm that can reset its price at time  $t$  chooses its reset prices,  $P_{Ht}^0(f)$  and  $P_{Ht}^{0*}(f)$ , to maximize

$$(A40) \quad E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[ P_{Ht}^0(z) C_{Ht+j}(f) + E_t P_{Ht}^{0*} C_{Ht+j}^*(f) - (1 - \tau_t) W_{t+j} N_{t+j}(f) \right].$$

The solution for  $P_{Ht}^0(z)$  is given by:

$$(A41) \quad P_{Ht}^0(z) = \frac{\xi}{\xi - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1 - \tau_t) W_{t+j} P_{Ht+j}^{\xi} C_{Ht+j} / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} P_{Ht+j}^{\xi} C_{Ht+j}}$$

We find for export prices,

$$(A42) \quad P_{Ht}^{0*}(z) = \frac{\xi}{\xi-1} \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} (P_{Ht+j}^*)^\xi C_{Ht+j}^* / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} E_{t+j} (P_{Ht+j}^*)^\xi C_{Ht+j}^*}.$$

Equation (A39) holds in the LCP case as well. However, the law of one price does not hold. For export prices, we have:

$$(A43) \quad P_{Ht}^* = \left[ \theta (P_{Ht-1}^*)^{1-\xi} + (1-\theta) (P_{Ht}^{0*})^{1-\xi} \right]^{1/(1-\xi)}$$

### *Subsidies*

As in CGG, we will assume that subsidies to monopolists are not set at their optimal level except in steady-state. That is, instead of the efficient subsidy given in equation (A31), we have:

$$(A44) \quad (1-\tau)(1+\mu^P)(1+\mu^W) = 1.$$

Here,  $\mu^W$  is the steady-state level of  $\mu_t^W$ . We have dropped the time subscript on the subsidy rate  $\tau_t$  because it is not time-varying.

## Appendix B

### Log-linearized Model

In this section, we present log-linear approximations to the models presented above.

In this section, we present all of the equations of the log-linearized model, but we separate out those that are used in the derivation of the loss function (which do not involve price setting or wage setting) and those that are not.

#### *Equations used for derivation of loss functions*

We define the log of the currency misalignment as the average of the difference between Foreign and Home prices:

$$(B1) \quad m_t \equiv \frac{1}{2}(e_t + p_{Ht}^* - p_{Ht} + e_t + p_{Ft}^* - p_{Ft}).$$

In the flexible-price and PCP models,  $m_t = 0$ .

We also define the export premium as the average by which consumer prices of imported goods exceeds the average of locally produced goods:

$$(B2) \quad z_t \equiv \frac{1}{2}(p_{Ht}^* + p_{Ft} - p_{Ft}^* - p_{Ht})$$

In all three models, to a first order,  $\ln(V_{Ht}) = \ln(V_{Ht}^*) = \ln(V_{Ft}) = \ln(V_{Ft}^*) = 0$ . That allows us to approximate equation (A23) and its foreign counterpart as:

$$(B3) \quad n_t = y_t - a_t, \quad \text{and}$$

$$(B4) \quad n_t^* = y_t^* - a_t^*.$$

The market-clearing conditions, (A18) and (A19) are approximated as:

$$(B5) \quad y_t = \frac{\nu}{2}c_t + \frac{2-\nu}{2}c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*,$$

$$(B6) \quad y_t^* = \frac{\nu}{2}c_t^* + \frac{2-\nu}{2}c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*.$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for Home and Foreign households, equation (A22), is given by:

$$(B7) \quad \sigma c_t - \sigma c_t^* = m_t + \frac{\nu-1}{2}s_t - \left(\frac{\nu-1}{2}\right)s_t^*.$$

We define “relative” and “world” log output by:

$$(B8) \quad y_t^R \equiv \frac{1}{2}(y_t - y_t^*)$$

$$(B9) \quad y_t^W \equiv \frac{1}{2}(y_t + y_t^*).$$

For use later, it is helpful to use equations (B5)-(B7) to express  $c_t$ ,  $c_t^*$ ,  $s_t$ , in terms of  $y_t$  and  $y_t^*$  and the price deviations,  $m_t$  and  $z_t$ .

$$(B10) \quad c_t = \frac{\nu-1}{D}y_t^R + y_t^W + \frac{\nu(2-\nu)}{2D}m_t$$



$$(B11) \quad c_t^* = -\left(\frac{\nu-1}{D}\right)y_t^R + y_t^W - \frac{\nu(2-\nu)}{2D}m_t,$$

where  $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$ .

We can further simplify these by defining:

$$(B12) \quad c_t^R \equiv \frac{1}{2}(c_t - c_t^*)$$

$$(B13) \quad c_t^W \equiv \frac{1}{2}(c_t + c_t^*)$$

Then

$$(B14) \quad c_t^R = \frac{\nu-1}{D}y_t^R + \frac{\nu(2-\nu)}{2D}m_t$$

$$(B15) \quad c_t^W = y_t^W.$$

And, solving for the terms of trade, we find:

$$(B16) \quad s_t = \frac{2\sigma}{D}y_t^R + z_t - \frac{(\nu-1)}{D}m_t,$$

$$(B17) \quad s_t^* = -\frac{2\sigma}{D}y_t^R + z_t + \frac{(\nu-1)}{D}m_t.$$

Under a globally efficient allocation, the marginal rate of substitution between leisure and aggregate consumption should equal the marginal product of labor times the price of output relative to consumption prices. To see the derivation more cleanly, we insert the shadow real wages in the efficient allocation,  $\bar{w}_t - \bar{p}_{Ht}$  and  $\bar{w}_t^* - \bar{p}_{Ft}^*$  into equations (B18) and (B19) below. So, the efficient allocation would be achieved in a model with flexible wages and optimal subsidies. These equations then can be understood intuitively by looking at the wage setting equations below ((B22)-(B23), and (B24)-(B25)) assuming the optimal subsidy is in place. But, to emphasize, they do not depend on a particular model of wage setting, and are just the standard efficiency condition equating the marginal rate of substitution between leisure and aggregate consumption to the marginal rate of transformation.

$$(B18) \quad a_t = \bar{w}_t - \bar{p}_{Ht} = \left(\frac{\sigma}{D} + \phi\right)\bar{y}_t^R + (\sigma + \phi)\bar{y}_t^W - \phi a_t,$$

$$(B19) \quad a_t^* = \bar{w}_t^* - \bar{p}_{Ft}^* = -\left(\frac{\sigma}{D} + \phi\right)\bar{y}_t^R + (\sigma + \phi)\bar{y}_t^W - \phi a_t^*.$$

### *Equations of wage and price setting*

The real Home and Foreign product wages, from equation (A28), are given by:

$$(B20) \quad w_t - p_{Ht} = \sigma c_t + \phi n_t + \frac{2-\nu}{2}s_t + \mu_t^W,$$

$$(B21) \quad w_t^* - p_{Ft}^* = \sigma c_t^* + \phi n_t^* + \left(\frac{2-\nu}{2}\right)s_t^* + \mu_t^{*W}.$$

We can express  $w_t - p_{Ht}$ , and  $w_t^* - p_{Ft}^*$  in terms of  $y_t$  and  $y_t^*$  and the exogenous disturbances,  $a_t$ ,  $a_t^*$ ,  $\mu_t^W$ , and  $\mu_t^{*W}$ :

$$(B22) \quad w_t - p_{Ht} = \left( \frac{\sigma}{D} + \phi \right) y_t^R + (\sigma + \phi) y_t^W + \frac{D - (\nu - 1)}{2D} m_t + \frac{2 - \nu}{2} z_t - \phi a_t + \mu_t^W,$$

$$(B23) \quad w_t^* - p_{Ft}^* = - \left( \frac{\sigma}{D} + \phi \right) y_t^R + (\sigma + \phi) y_t^W - \left( \frac{D - (\nu - 1)}{2D} \right) m_t + \frac{2 - \nu}{2} z_t - \phi a_t^* + \mu_t^{*W},$$

### Flexible Prices

We can solve for the values of all the real variables under flexible prices by using equations (B3), (B4), (B10), (B11), (B16), (B22) and (B23), as well as the price-setting conditions, from (A33):

$$(B24) \quad \dot{w}_t - \dot{p}_{Ht} = a_t,$$

$$(B25) \quad \dot{w}_t^* - \dot{p}_{Ft}^* = a_t^*.$$

### PCP

Log-linearization of equations (A38) and (A39) gives us the familiar New Keynesian Phillips curve for an open economy:

$$(B26) \quad \pi_{Ht} = \delta(w_t - p_{Ht} - a_t) + \beta E_t \pi_{Ht+1},$$

where  $\delta = (1 - \theta)(1 - \beta\theta) / \theta$ .

We can rewrite this equation using (B22) and (B18) as:

$$(B27) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta E_t \pi_{Ht+1} + u_t,$$

where  $u_t = \delta \mu_t^W$ .

Similarly for foreign producer-price inflation, we have:

$$(B28) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

### LCP

Equation (B26) holds in the LCP model as well. But in the LCP model, the law of one price deviation is not zero. We have:

$$(B29) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D - (\nu - 1)}{2D} m_t + \frac{2 - \nu}{2} z_t \right] + \beta E_t \pi_{Ht+1} + u_t,$$

$$(B30) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D - (\nu - 1)}{2D} \right) m_t + \frac{2 - \nu}{2} z_t \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

In addition, from (A42) and (A43), we derive:

$$(B31) \quad \pi_{Ht}^* = \delta(w_t - p_{Ht}^* - e_t - a_t) + \beta E_t \pi_{Ht+1}^* = \delta(w_t - p_{Ht} - m_t - z_t - a_t) + \beta E_t \pi_{Ht+1}^*$$

We can rewrite this as

$$(B32) \quad \pi_{Ht}^* = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D + \nu - 1}{2D} \right) m_t - \frac{\nu}{2} z_t \right] + \beta E_t \pi_{Ht+1}^* + u_t.$$

Similarly, we can derive:

$$(B33) \quad \pi_{Ft} = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D + \nu - 1}{2D} m_t - \frac{\nu}{2} z_t \right] + \beta E_t \pi_{Ft+1} + u_t^*.$$

Consider equations (B29)-(B33). If  $z_t = 0$  in these equations, then  $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^*$ . This in turn implies  $z_{t+1} = 0$ . By induction, if the initial

condition  $z_0 = 0$  holds, it follows that  $z_t = 0$  in all periods in the LCP model, or, in other words,  $s_t^* = -s_t$ . That is, the relative price of Foreign to Home goods is the same in both countries. We emphasize that this is true for a first-order approximation in the LCP model.

So we can simplify equations (B29)-(B33):

$$(B34) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D - (\nu - 1)}{2D} m_t \right] + \beta E_t \pi_{Ht+1} + u_t,$$

$$(B35) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D - (\nu - 1)}{2D} \right) m_t \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

$$(B36) \quad \pi_{Ht}^* = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D + \nu - 1}{2D} \right) m_t \right] + \beta E_t \pi_{Ht+1}^* + u_t.$$

$$(B37) \quad \pi_{Ft} = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D + \nu - 1}{2D} m_t \right] + \beta E_t \pi_{Ft+1} + u_t^*.$$

#### Relationship to CGG's Phillips Curve

The Phillips curve in CGG's PCP model has Home inflation depending only on the Home output gap. Our model should be equivalent to theirs when there is not home bias in preferences, but equation (B27) has  $\pi_{Ht}$  depending on both  $\tilde{y}_t^R$  and  $\tilde{y}_t^W$ . This will not reduce to a function only of  $\tilde{y}_t$  except in the case of  $\sigma = 1$ .

However, it can be seen that in fact (B27) is equivalent to CGG's equation when one recognizes that CGG's definition of the output gap differs from the one used here. CGG define the output gap as the difference between  $y_t$  and what I will call  $\bar{y}_t^{CGG}$ .  $\bar{y}_t^{CGG}$  is the efficient level of Home output when the Foreign output level is taken as given. That contrasts to our definition in which  $\bar{y}_t$  is the globally efficient output level. CGG's definition is convenient because their analysis focuses on non-cooperative monetary policy, while the definition used here is more convenient because of the focus on cooperative monetary policy. But this is a matter of convenience: algebraically the equations are the same.

To see this, note that equation (B26) is the same as in CGG. When there are no deviations from the law of one price, equation (B22) can be written as:

$$(B38) \quad \begin{aligned} w_t - p_{Ht} &= \left( \frac{\sigma}{D} + \phi \right) y_t^R + (\sigma + \phi) y_t^W - \phi a_t + \mu_t^W \\ &= \left( \frac{\sigma}{D} + \phi \right) \left( \frac{y_t - y_t^*}{2} \right) + (\sigma + \phi) \left( \frac{y_t + y_t^*}{2} \right) - \phi a_t + \mu_t^W \end{aligned}$$

When there is no home bias in preferences, this simplifies to:

$$(B39) \quad w_t - p_{Ht} = \left( \frac{\sigma + 1}{2} + \phi \right) y_t + \frac{\sigma - 1}{2} y_t^* - \phi a_t + \mu_t^W$$

Then, CGG define:

$$(B40) \quad \bar{y}_t^{CGG} = \frac{2}{\sigma + 1 - 2\phi} \left[ (1 + \phi) a_t - \left( \frac{\sigma - 1}{2} \right) y_t^* \right].$$

So we can write using (B39) and (B40):

$$(B41) \quad w_t - p_{Ht} - a_t = \left( \frac{\sigma + 1}{2} + \phi \right) (y_t - \bar{y}_t^{CGG}) + \mu_t^w .$$

Substituting this expression into equation (B26) gives us CGG's version of the Phillips curve, in which inflation depends only on the Home output gap under their definitions.

## Appendix C

### C.I Derivation of Welfare Function in Clarida-Gali-Gertler model with Home Bias in preferences

The object is to rewrite the welfare function, which is defined in terms of home and foreign consumption and labor effort into terms of the squared output gap and squared inflation. We derive the joint welfare function of home and foreign households, since we will be examining cooperative monetary policy.

Most of the derivation requires only 1<sup>st</sup>-order approximations of the equations of the model, but in a few places, 2<sup>nd</sup>-order approximations are needed. If the approximation is 1<sup>st</sup>-order, I'll use the notation “ $+o(\|a^2\|)$ ” to indicate that there are 2<sup>nd</sup>-order and higher terms left out, and if the approximation is 2<sup>nd</sup>-order, I will use “ $+o(\|a^3\|)$ ”. ( $a$  is notation for the log of the productivity shock)

From equation (A1), the period utility of the planner is given by:

$$(C1) \quad v_t \equiv \frac{1}{1-\sigma} (C_t^{1-\sigma} + C_t^{*1-\sigma}) - \frac{1}{1+\phi} (N_t^{1+\phi} + N_t^{*1+\phi}).$$

Take a second-order log approximation around the non-stochastic steady state. We assume allocations are efficient in steady state, so we have  $C^{1-\sigma} = C^{*1-\sigma} = N^{1+\phi} = N^{*1+\phi}$ . The fact that  $C^{1-\sigma} = N^{1+\phi}$  follows from the fact that in steady state,  $C = N$  from market clearing and symmetry, and  $C^{-\sigma} = N^\phi$  from the condition that the marginal rate of substitution between leisure and consumption equals one in an efficient non-stochastic steady state.

We get:

$$(C2) \quad v_t = 2 \left( \frac{1}{1-\sigma} - \frac{1}{1+\phi} \right) C^{1-\sigma} + C^{1-\sigma} (c_t + c_t^*) + \frac{1-\sigma}{2} C^{1-\sigma} ((c_t)^2 + (c_t^*)^2) \\ - C^{1-\sigma} (n_t + n_t^*) - \frac{1+\phi}{2} C^{1-\sigma} ((n_t)^2 + (n_t^*)^2) + o(\|a^3\|)$$

Since we can equivalently maximize an affine transformation of (C2), it is convenient to simplify that equation to get:

$$(C3) \quad v_t = c_t + c_t^* - n_t - n_t^* + \frac{1-\sigma}{2} (c_t^2 + c_t^{*2}) - \frac{1+\phi}{2} (n_t^2 + n_t^{*2}) + o(\|a^3\|).$$

Utility is maximized when consumption and employment take on their efficient values:

$$(C4) \quad v_t^{\max} = \bar{c}_t + \bar{c}_t^* - \bar{n}_t - \bar{n}_t^* + \frac{1-\sigma}{2} (\bar{c}_t^2 + \bar{c}_t^{*2}) - \frac{1+\phi}{2} (\bar{n}_t^2 + \bar{n}_t^{*2}) + o(\|a^3\|).$$

In general, this maximum may not be attainable because of distortions. We can write  $x_t = \bar{x}_t + \tilde{x}_t$ , where  $\tilde{x}_t \equiv x_t - \bar{x}_t$ . So, we have:

(C5)

$$\begin{aligned} \nu_t = & \left[ \bar{c}_t + \bar{c}_t^* - \bar{n}_t - \bar{n}_t^* + \frac{1-\sigma}{2} (\bar{c}_t^2 + \bar{c}_t^{*2}) - \frac{1+\phi}{2} (\bar{n}_t^2 + \bar{n}_t^{*2}) \right] \\ & + \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2} (\tilde{c}_t^2 + \tilde{c}_t^{*2} + 2\bar{c}_t\tilde{c}_t + 2\bar{c}_t^*\tilde{c}_t^*) - \frac{1+\phi}{2} (\tilde{n}_t^2 + \tilde{n}_t^{*2} + 2\bar{n}_t\tilde{n}_t + 2\bar{n}_t^*\tilde{n}_t^*) \\ & + o(\|a^3\|) \end{aligned}$$

or,

$$(C6) \quad \begin{aligned} \nu_t - \nu_t^{\max} = & \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2} (\tilde{c}_t^2 + \tilde{c}_t^{*2}) - \frac{1+\phi}{2} (\tilde{n}_t^2 + \tilde{n}_t^{*2}) \\ & + (1-\sigma)(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*) - (1+\phi)(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*) + o(\|a^3\|) \end{aligned}$$

We can rewrite this as:

$$(C7) \quad \begin{aligned} \nu_t - \nu_t^{\max} = & 2\tilde{c}_t^W - 2\tilde{n}_t^W + (1-\sigma)((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2) - (1+\phi)((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2) \\ & + 2(1-\sigma)(\bar{c}_t^R\tilde{c}_t^R + \bar{c}_t^W\tilde{c}_t^W) - 2(1+\phi)(\bar{n}_t^R\tilde{n}_t^R + \bar{n}_t^W\tilde{n}_t^W) + o(\|a^3\|) \end{aligned}$$

The object is to write (C7) as a function of squared output gaps and squared inflation if possible. We need a second-order approximation of  $2\tilde{c}_t^W - 2\tilde{n}_t^W$ . But because the rest of the terms are squares and products, the 1<sup>st</sup>-order approximations that have already been derived will be sufficient.

Recalling that  $m_t = 0$  in the PCP model, we can write equations (B14)-(B15) as:

$$(C8) \quad c_t^R = \frac{\nu-1}{D} y_t^R + o(\|a^2\|),$$

$$(C9) \quad c_t^W = y_t^W + o(\|a^2\|).$$

It follows from (C8) and (C9) that:

$$(C10) \quad \bar{c}_t^R = \frac{\nu-1}{D} \bar{y}_t^R + o(\|a^2\|),$$

$$(C11) \quad \bar{c}_t^W = \bar{y}_t^W + o(\|a^2\|),$$

$$(C12) \quad \tilde{c}_t^R = \frac{\nu-1}{D} \tilde{y}_t^R + o(\|a^2\|),$$

$$(C13) \quad \tilde{c}_t^W = \tilde{y}_t^W + o(\|a^2\|).$$

Next, we can easily derive:

$$(C14) \quad \tilde{n}_t^R = \tilde{y}_t^R + o(\|a^2\|), \text{ and}$$

$$(C15) \quad \tilde{n}_t^W = \tilde{y}_t^W + o(\|a^2\|).$$

These follow as in (B3)-(B4) because  $n_t = y_t - a_t + o(\|a^2\|)$  and  $\bar{n}_t = \bar{y}_t - a_t$  (and similarly in the Foreign country.)

We need expressions for  $\bar{n}_t^R$  and  $\bar{n}_t^W$ . We have, using (B18)-(B19):

$$a_t^R = \left( \frac{\sigma}{D} + \phi \right) \bar{y}_t^R - \phi a_t^R + o(\|a^2\|),$$

$$a_t^W = (\sigma + \phi) \bar{y}_t^W - \phi a_t^W + o(\|a^2\|).$$

Using  $a_t^R = \bar{y}_t^R - \bar{n}_t^R$  and  $a_t^W = \bar{y}_t^W - \bar{n}_t^W$ , we can write these as

$$(C16) \quad \bar{n}_t^R = \frac{1-\sigma}{1+\phi} \left( \frac{(1-\nu)^2}{D} \right) \bar{y}_t^R + o(\|a^2\|), \text{ and}$$

$$(C17) \quad \bar{n}_t^W = \frac{1-\sigma}{1+\phi} \bar{y}_t^W + o(\|a^2\|).$$

Turning attention back to the loss function in equation (C7), we focus first on the terms

$$(1-\sigma) \left( (\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2 \right) - (1+\phi) \left( (\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2 \right) + 2(1-\sigma) \left( \bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W \right) - 2(1+\phi) \left( \bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W \right)$$

. These involve only squares and cross-products of  $\tilde{c}_t^R$ ,  $\tilde{c}_t^W$ ,  $\bar{c}_t^R$ ,  $\bar{c}_t^W$ ,  $\tilde{n}_t^R$ ,  $\tilde{n}_t^W$ ,  $\bar{n}_t^R$ , and  $\bar{n}_t^W$ . We can substitute from equations (C10)-(C17) into this expression. It is useful provide a few lines of algebra since it is a bit messy:

$$(C18) \quad \begin{aligned} & (1-\sigma) \left( (\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2 \right) - (1+\phi) \left( (\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2 \right) \\ & + 2(1-\sigma) \left( \bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W \right) - 2(1+\phi) \left( \bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W \right) \\ & = \left[ (1-\sigma) \left( \frac{\nu-1}{D} \right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 + 2\nu(2-\nu) \left( \frac{(1-\sigma)(\nu-1)}{D} \right)^2 \bar{y}_t^R \tilde{y}_t^R \end{aligned}$$

Now return to the  $2\tilde{c}_t^W - 2\tilde{n}_t^W$  term in equation (C7) and do a 2<sup>nd</sup>-order approximation. Start with equation (A18), dropping the  $k^{-1}$  term because it will not affect the approximation, and noting that in the PCP model,  $S_t^* = S_t^{-1}$ :

$$(C19) \quad Y_t = \frac{\nu}{2} S_t^{(2-\nu)/2} C_t + \left( \frac{2-\nu}{2} \right) S_t^{\nu/2} C_t^*$$

Then use equation (A22), but using the fact that  $S_t^* = S_t^{-1}$  and there are no deviations from the law of one price:

$$(C20) \quad C_t^* = C_t S_t^{\frac{1-\nu}{\sigma}}.$$

Substitute in to get:

$$(C21) \quad Y_t = \frac{\nu}{2} S_t^{\frac{2-\nu}{2}} C_t + \left( \frac{2-\nu}{2} \right) S_t^{\frac{\nu}{2} + \frac{1-\nu}{\sigma}} C_t.$$

Solve for  $C_t$ :

$$(C22) \quad C_t = Y_t \left( \frac{\nu}{2} S_t^{\frac{2-\nu}{2}} + \left( \frac{2-\nu}{2} \right) S_t^{\frac{\nu}{2} + \frac{1-\nu}{\sigma}} \right)^{-1}.$$

Then we get this 2<sup>nd</sup>-order approximation:

$$(C23) \quad c_t = y_t - \left(\frac{2-\nu}{2}\right) \left(\nu + \frac{1-\nu}{\sigma}\right) s_t - \frac{1}{2} \left(\frac{2-\nu}{2}\right) \frac{\nu}{2} (\nu-1)^2 \left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o(\|a^3\|).$$

Symmetrically,

$$(C24) \quad c_t^* = y_t^* + \left(\frac{2-\nu}{2}\right) \left(\nu + \frac{1-\nu}{\sigma}\right) s_t - \frac{1}{2} \left(\frac{2-\nu}{2}\right) \frac{\nu}{2} (\nu-1)^2 \left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o(\|a^3\|).$$

Averaging these two equations, we get:

$$(C25) \quad c_t^W = y_t^W - \frac{1}{2} \left(\frac{2-\nu}{2}\right) \frac{\nu}{2} (\nu-1)^2 \left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o(\|a^3\|).$$

Now we can take a 1<sup>st</sup>-order approximation for  $s_t$  to substitute out for  $s_t^2$ . From equation (B16), setting  $m_t = 0$  and  $z_t = 0$ , we have:

$$(C26) \quad s_t^2 = \frac{4\sigma^2}{D^2} (y_t^R)^2 + o(\|a^3\|).$$

Substituting into equation (C26), we can write:

$$(C27) \quad c_t^W = y_t^W - \frac{\nu(2-\nu)}{2} \left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 (y_t^R)^2 + o(\|a^3\|).$$

Evaluating (C27) at flexible prices, we have:

$$(C28) \quad \bar{c}_t^W = \bar{y}_t^W - \frac{\nu(2-\nu)}{2} \left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 (\bar{y}_t^R)^2 + o(\|a^3\|).$$

It follows from the fact that  $\tilde{c}_t^W = c_t^W - \bar{c}_t^W$  that

$$(C29) \quad \tilde{c}_t^W = \tilde{y}_t^W - \frac{\nu(2-\nu)}{2} \left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 \left((\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R\right) + o(\|a^3\|).$$

See section C.3 below for the second-order approximations for  $\tilde{n}_t$  and  $\tilde{n}_t^*$ :

$$(C30) \quad \tilde{n}_t = \tilde{y}_t + \frac{\xi}{2} \sigma_{p_{H^t}}^2 + o(\|a^3\|)$$

$$(C31) \quad \tilde{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2} \sigma_{p_{F^*t}}^2 + o(\|a^3\|).$$

Adding these two equations together gives us:

$$(C32) \quad 2\tilde{n}_t^W = 2\tilde{y}_t^W + \frac{\xi}{2} \sigma_{p_{H^t}}^2 + \frac{\xi}{2} \sigma_{p_{F^*t}}^2 + o(\|a^3\|)$$

Substitute expressions (C29)-(C31) along with (C18) into the loss function (C7):

$$(C33) \quad \begin{aligned} v_t - v_t^{\max} &= -\nu(2-\nu) \left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 \left((\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R\right) - \frac{\xi}{2} (\sigma_{p_{H^t}}^2 + \sigma_{p_{F^*t}}^2) \\ &+ \left[ (1-\sigma) \left(\frac{\nu-1}{D}\right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 + 2\nu(2-\nu) \left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2 \bar{y}_t^R \tilde{y}_t^R \\ &= -\left[\frac{\sigma}{D} + \phi\right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 - \frac{\xi}{2} (\sigma_{p_{H^t}}^2 + \sigma_{p_{F^*t}}^2) \end{aligned}$$



This expression reduces to CCG's when there is no home bias ( $\nu=1$ ). To see this from their expression at the top of p. 903, multiply their utility by 2 (since they take average utility), and set their  $\gamma$  equal to  $\frac{1}{2}$  (so their country sizes are equal.)

## C.2 Derivation of Welfare Function under LCP with Home Bias in Preferences

The second-order approximation to welfare in terms of logs of consumption and employment of course does not change, so equation (C6) still holds. As before, we break down the derivation into two parts. We use first-order approximations to structural equations to derive an approximation to the quadratic term

$$(1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right) + 2(1-\sigma)\left(\bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W\right) - 2(1+\phi)\left(\bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W\right)$$

Then we use second order approximations to the structural equations to derive an expression for  $2\tilde{c}_t^W - 2\tilde{n}_t^W$ .

The quadratic term involves squares and cross-products of  $\tilde{c}_t^R$ ,  $\tilde{c}_t^W$ ,  $\bar{c}_t^R$ ,  $\bar{c}_t^W$ ,  $\tilde{n}_t^R$ ,  $\tilde{n}_t^W$ ,  $\bar{n}_t^R$ , and  $\bar{n}_t^W$ . Expressions (C10)-(C11) still gives us first-order approximations for  $\bar{c}_t^R$  and  $\bar{c}_t^W$ ; equations (C14)-(C15) are first-order approximations for  $\tilde{n}_t^R$  and  $\tilde{n}_t^W$ ; and, (C16)-(C17) are first-order approximations for  $\bar{n}_t^R$  and  $\bar{n}_t^W$ . But we need to use equations (B14)-(B15) and (C10)-(C11) to derive:

$$(C34) \quad \tilde{c}_t^R = \frac{\nu-1}{D} \tilde{y}_t^R + \frac{\nu(2-\nu)}{2D} m_t + o(\|a^2\|),$$

$$(C35) \quad \tilde{c}_t^W = \tilde{y}_t^W + o(\|a^2\|).$$

With these equations, we can follow the derivation as in equation (C18). After tedious algebra, we arrive at the same result, with the addition of the terms  $\frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2} m_t^2$  and  $\frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2} m_t y_t^R$ . Note that the last term involves output levels, not output gaps. That is, we have:

$$(C36) \quad \begin{aligned} & (1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right) \\ & + 2(1-\sigma)\left(\bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W\right) - 2(1+\phi)\left(\bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W\right) \\ & = \left[ (1-\sigma)\left(\frac{\nu-1}{D}\right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 + 2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2 \bar{y}_t^R \tilde{y}_t^R \\ & + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2} m_t^2 + \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2} m_t y_t^R \end{aligned}$$

The derivation of  $2\tilde{c}_t^W - 2\tilde{n}_t^W$  is similar to the PCP model. However, one tedious aspect of the derivation is that we cannot make use of the equality  $S_t^* = S_t^{-1}$  that holds under PCP and flexible prices. We write out the equilibrium conditions for home output, and its foreign equivalent, from equations (A18) and (A19):

$$(C37) \quad Y_t = \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right) (S_t^*)^{-\nu/2} C_t^*,$$

$$(C38) \quad Y_t^* = \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t.$$

We directly take second-order approximations of these equations around the efficient non-stochastic steady state:

$$(C39) \quad \begin{aligned} y_t + \frac{1}{2} y_t^2 &= \frac{\nu}{2} c_t + \left(\frac{2-\nu}{2}\right) c_t^* + \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t - \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t^* \\ &+ \frac{1}{2} \left\{ \frac{\nu}{2} c_t^2 + \left(\frac{2-\nu}{2}\right) c_t^{*2} + \frac{\nu}{2} \left(\frac{2-\nu}{2}\right)^2 s_t^2 + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^{*2} \right\} \\ &+ \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t c_t - \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t^* c_t^* + o(\|a^3\|) \end{aligned}$$

$$(C40) \quad \begin{aligned} y_t^* + \frac{1}{2} y_t^{*2} &= \frac{\nu}{2} c_t^* + \left(\frac{2-\nu}{2}\right) c_t + \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t^* - \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t \\ &+ \frac{1}{2} \left\{ \frac{\nu}{2} c_t^{*2} + \left(\frac{2-\nu}{2}\right) c_t^2 + \frac{\nu}{2} \left(\frac{2-\nu}{2}\right)^2 s_t^{*2} + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^2 \right\} \\ &+ \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t^* c_t^* - \frac{\nu}{2} \left(\frac{2-\nu}{2}\right) s_t c_t + o(\|a^3\|) \end{aligned}$$

Averaging (C39) and (C40), we find:

$$(C41) \quad y_t^W + \frac{1}{2} \left( (y_t^R)^2 + (y_t^W)^2 \right) = c_t^W + \frac{1}{2} \left( (c_t^R)^2 + (c_t^W)^2 \right) + \frac{\nu(2-\nu)}{16} (s_t^2 + s_t^{*2}) + o(\|a^3\|).$$

Next, we can use equations (B14)-(B17) to get approximations for  $(c_t^R)^2$ ,  $(c_t^W)^2$ ,  $s_t^2$ , and  $s_t^{*2}$ . These equations are linear approximations for  $c_t^R$ ,  $c_t^W$ ,  $s_t$ , and  $s_t^*$ , but since we are looking to approximate the squares of these variables, that is sufficient. With some algebra, we find:

$$(C42) \quad \begin{aligned} c_t^W &= y_t^W - \frac{\nu(2-\nu)}{2} \left( \frac{(\nu-1)(1-\sigma)}{D} \right)^2 (y_t^R)^2 - \frac{\nu(2-\nu)}{8D^2} m_t^2 - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2} m_t y_t^R \\ &- \frac{\nu(2-\nu)}{8} z_t^2 + o(\|a^3\|) \end{aligned}$$

Note that if set  $m_t = 0$  and  $z_t = 0$  in (C42), we would arrive at the second-order approximation for  $c_t^W$  from the PCP model.

Then following the derivations as in the PCP model derivation of (C29), we can write:

$$(C43) \quad \begin{aligned} \tilde{c}_t^W = & \tilde{y}_t^W - \frac{\nu(2-\nu)}{2} \left( \frac{(\nu-1)(\sigma-1)}{D} \right)^2 \left( (\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R \right) - \frac{\nu(2-\nu)}{8D^2} m_t^2 \\ & - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2} m_t y_t^R - \frac{\nu(2-\nu)}{8} z_t^2 + o(\|a^3\|) \end{aligned}$$

As shown in section B.3, we can make the following second-order approximation:

$$(C44) \quad 2\tilde{n}_t^W = 2\tilde{y}_t^W + \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{Ht}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{F^*t}}^2 + \frac{2-\nu}{2} \sigma_{p_{Ft}}^2 \right] + o(\|a^3\|).$$

We then can substitute (C43), and (C44), along with (C36) into the loss function (C7). We find:

$$(C45) \quad \begin{aligned} v_t - v_t^{\max} = & 2\tilde{y}_t^W - \nu(2-\nu) \left( \frac{(\nu-1)(\sigma-1)^2}{D} \right) \left( (\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R \right) - \frac{\nu(2-\nu)}{4D} m_t^2 \\ & - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2} m_t y_t^R - \frac{\nu(2-\nu)}{4} z_t^2 \\ & - 2\tilde{y}_t^W - \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{Ht}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{F^*t}}^2 + \frac{2-\nu}{2} \sigma_{p_{Ft}}^2 \right] \\ & + \left[ (1-\sigma) \left( \frac{\nu-1}{D} \right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 + 2\nu(2-\nu) \left( \frac{(1-\sigma)(\nu-1)}{D} \right)^2 \bar{y}_t^R \tilde{y}_t^R \\ & + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2} m_t^2 + \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2} m_t y_t^R \\ = & - \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 - \frac{\nu(2-\nu)}{4D} m_t^2 - \frac{\nu(2-\nu)}{4} z_t^2 \\ & - \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{Ht}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{F^*t}}^2 + \frac{2-\nu}{2} \sigma_{p_{Ft}}^2 \right] + o(\|a^3\|) \end{aligned}$$

### C.3 Derivations of Price Dispersion Terms in Loss Functions

In the PCP case, we can write

$$(C46) \quad A_t N_t = A_t \int_0^1 N_t(f) df = Y_t \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df = Y_t V_t,$$

where  $V_t \equiv \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df$ . Taking logs, we can write:

$$(C47) \quad a_t + n_t = y_t + v_t.$$

We have  $v_t \equiv \ln \left( \int_0^1 e^{-\xi \hat{p}_{Ht}(f)} df \right)$ ,

where we define

$$(C48) \quad \hat{p}_{Ht}(f) \equiv p_{Ht}(f) - p_{Ht}.$$

Following Gali (2008), we note

$$(C49) \quad e^{(1-\xi)\hat{p}_{Ht}(f)} = 1 + (1-\xi)\hat{p}_{Ht}(f) + \frac{(1-\xi)^2}{2} \hat{p}_{Ht}(f)^2 + o(\|a^3\|).$$

By the definition of the price index  $P_{Ht}$ , we have  $\int_0^1 e^{(1-\xi)\hat{p}_{Ht}(f)} df = 1$ . Hence, from (C49),

$$(C50) \quad \int_0^1 \hat{p}_{Ht}(f) df = \frac{\xi-1}{2} \int_0^1 \hat{p}_{Ht}(f)^2 df + o(\|a^3\|).$$

We also have

$$(C51) \quad e^{-\xi\hat{p}_{Ht}(f)} = 1 - \xi\hat{p}_{Ht}(f) + \frac{\xi^2}{2} \hat{p}_{Ht}(f)^2 + o(\|a^3\|)$$

It follows, using (C50):

$$(C52)$$

$$\int_0^1 e^{-\xi\hat{p}_{Ht}(f)} df = 1 - \xi \int_0^1 \hat{p}_{Ht}(f) df + \frac{\xi^2}{2} \int_0^1 \hat{p}_{Ht}(f)^2 df + o(\|a^3\|) = 1 + \frac{\xi}{2} \int_0^1 \hat{p}_{Ht}(f)^2 df + o(\|a^3\|)$$

Note the following relationship:

$$(C53) \quad \int_0^1 \hat{p}_{Ht}(f)^2 df = \int_0^1 (p_{Ht}(f) - E_f(p_{Ht}(f)))^2 df + o(\|a^3\|) = \text{var}(p_{Ht}) + o(\|a^3\|).$$

Using our notation for variances,  $\sigma_{p_{Ht}}^2 \equiv \text{var}(p_{Ht})$ , and taking the log of (C52) we arrive at

$$(C54) \quad v_t = \frac{\xi}{2} \sigma_{p_{Ht}}^2 + o(\|a^3\|).$$

Substituting this into equation (C47), and recalling that  $\bar{y}_t = \bar{n}_t + a_t$ , we arrive at equation (C30). The derivation of (C31) for the Foreign country proceeds identically.

For the LCP model, we will make use of the following second-order approximation to the equation  $Y_t = C_{Ht} + C_{Ht}^*$ :

$$(C55) \quad y_t = \frac{\nu}{2} c_{Ht} + \left(\frac{2-\nu}{2}\right) c_{Ht}^* + \frac{1}{2} \left(\frac{\nu}{2}\right) \left(\frac{2-\nu}{2}\right) (c_{Ht}^2 + 2c_{Ht}c_{Ht}^* + c_{Ht}^{*2}) + o(\|a^3\|).$$

In the LCP model, we can write:

$$(C56)$$

$$A_t N_t = A_t \int_0^1 N_t(f) df = C_{Ht} \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df + C_{Ht}^* \int_0^1 \left(\frac{P_{Ht}^*(f)}{P_{Ht}^*}\right)^{-\xi} df = C_{Ht} V_{Ht} + C_{Ht}^* V_{Ht}^*,$$

where the definitions of  $V_{Ht}$  and  $V_{Ht}^*$  are analogous to that of  $V_t$  in the PCP model.

Taking a second-order log approximation to (C56), we have:

$$(C57) \quad a_t + n_t = \frac{\nu}{2} (c_{Ht} + v_{Ht}) + \left(\frac{2-\nu}{2}\right) (c_{Ht}^* + v_{Ht}^*) + \frac{1}{2} \left(\frac{\nu}{2}\right) \left(\frac{2-\nu}{2}\right) ((c_{Ht} + v_{Ht})^2 + 2(c_{Ht} + v_{Ht})(c_{Ht}^* + v_{Ht}^*) + (c_{Ht}^* + v_{Ht}^*)^2) + o(\|a^3\|)$$

We can follow the same steps as in the PCP model to conclude:

$$(C58) \quad v_{Ht} = \frac{\xi}{2} \sigma_{p_{Ht}}^2 + o(\|a^3\|)$$

$$(C59) \quad v_{Ht}^* = \frac{\xi}{2} \sigma_{p_{Ht}^*}^2 + o(\|a^3\|)$$

Substituting these expressions into (C57) and cancelling higher order terms, we find:

$$(C60) \quad a_t + n_t = \frac{\nu}{2}(c_{Ht} + \frac{\xi}{2}\sigma_{p_{Ht}}^2) + \left(\frac{2-\nu}{2}\right)(c_{Ht}^* + \frac{\xi}{2}\sigma_{p_{H^*t}}^2) + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c_{Ht}^2 + 2c_{Ht}c_{Ht}^* + c_{Ht}^{*2}) + o(\|a^3\|)$$

Then using equation (C55), we can write:

$$(C61) \quad a_t + n_t = y_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma_{p_{Ht}}^2 + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma_{p_{H^*t}}^2\right) + o(\|a^3\|).$$

Keeping in mind that  $\bar{y}_t = \bar{n}_t + a_t$ , we can write:

$$(C62) \quad \bar{n}_t = \tilde{y}_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma_{p_{Ht}}^2 + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma_{p_{H^*t}}^2\right) + o(\|a^3\|).$$

Following analogous steps for the Foreign country,

$$(C63) \quad \bar{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma_{p_{Ft}}^2 + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma_{p_{F^*t}}^2\right) + o(\|a^3\|).$$

Adding (C62) and (C63) gives us equation (C44).

Finally, to derive the loss functions for policymakers, we note that the loss function is the present expected discounted value of the period loss functions derived here (equation (C33) for the PCP model and (C45) for the LCP model.) That is, the

policymaker seeks to minimize  $-\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (u_{t+j} - u_{t+j}^{\max})$ .

Following Woodford (2003, chapter 6), we can see that, in the PCP model, if prices are adjusted according to the Calvo price mechanism given by equation (A39) for  $P_{Ht}$  that

$$(C64) \quad \sum_{j=0}^{\infty} \beta^j \sigma_{p_{Ht+j}}^2 = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{j=0}^{\infty} \beta^j \pi_{Ht+j}^2.$$

Analogous relationships hold for  $P_{Ft}^*$  in the PCP model, and for  $P_{Ht}$ ,  $P_{Ft}^*$ ,  $P_{Ft}$ , and  $P_{Ht}^*$  in the LCP model. We can then substitute this relationship into the present value loss

function,  $-\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (u_{t+j} - u_{t+j}^{\max})$ , to derive the loss functions of the two models presented in the text.

The text writes the loss function as:

$$(C65) \quad \Psi_t \propto \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\xi}{2\delta} \left( \frac{\nu}{2} (\pi_{Ht})^2 + \frac{2-\nu}{2} (\pi_{Ft})^2 + \frac{\nu}{2} (\pi_{Ft}^*)^2 + \frac{2-\nu}{2} (\pi_{Ht}^*)^2 \right)$$

We can use the fact that if  $\frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft} = \pi_t$ , then

$$\frac{\nu}{2}(\pi_{H_t})^2 + \frac{2-\nu}{2}(\pi_{F_t})^2 = \pi_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{F_t} - \pi_{H_t})^2.$$

This follows because for any  $a$ ,  $x$ , and  $y$ , if  $ax + (1-a)y = z$ , then  $ax^2 + (1-a)y^2 = z^2 + a(1-a)(x-y)^2$ .

Likewise,

$$\frac{\nu}{2}(\pi_{F_t}^*)^2 + \frac{2-\nu}{2}(\pi_{H_t}^*)^2 = (\pi_t^*)^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{H_t}^* - \pi_{F_t}^*)^2.$$

It follows that

$$\begin{aligned} & \frac{\nu}{2}(\pi_{H_t})^2 + \frac{2-\nu}{2}(\pi_{F_t})^2 + \frac{\nu}{2}(\pi_{F_t}^*)^2 + \frac{2-\nu}{2}(\pi_{H_t}^*)^2 \\ &= \pi_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{F_t} - \pi_{H_t})^2 + (\pi_t^*)^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{H_t}^* - \pi_{F_t}^*)^2 \\ \text{(C66)} \quad &= \pi_t^2 + (\pi_t^*)^2 + \left(\frac{\nu(2-\nu)}{2}\right)(s_t - s_{t-1}) \\ &= 2(\pi_t^R)^2 + 2(\pi_t^W)^2 + \left(\frac{\nu(2-\nu)}{2}\right)(s_t - s_{t-1}) \end{aligned}$$

The second equality follows because, under Calvo pricing, to a first order  $\pi_{F_t} - \pi_{H_t} = \pi_{F_t}^* - \pi_{H_t}^* = s_t - s_{t-1}$ . The third equality follows because

$$(\pi_t^R)^2 + (\pi_t^W)^2 = \frac{(\pi_t - \pi_t^*)^2 + (\pi_t - \pi_t^*)^2}{4} = \frac{\pi_t^2 + (\pi_t^*)^2}{2}.$$

Then substitute (C66) into (C65) to get the simplified loss function presented in the paper.

## Appendix D

The model is closed with equations for monetary policy. This appendix solves the model algebraically when there are no cost push shocks and labor supply elasticity parameter,  $\phi$ , is set to zero. These solutions can be used to derive the impulse response functions in Figure 1 of the paper.

We also assume for simplicity that the foreign productivity shock is zero. (Since the model is symmetric, the solution for the response to foreign productivity shocks is straightforward.)

We assume the Home productivity shock follows the AR1 process given by:

$$a_t = \rho a_{t-1} + \varepsilon_t, \quad E_{t-1} \varepsilon_t = 0.$$

### D.1 PCP model

With no mark-up shocks, the Phillips curves, (B27) and (B28) simplify to:

$$(D1) \quad \pi_{Ht} = \delta \left[ \tilde{y}_t^R + \sigma \tilde{y}_t^W \right] + \beta E_t \pi_{Ht+1}$$

$$(D2) \quad \pi_{Ft}^* = \delta \left[ -\tilde{y}_t^R + \sigma \tilde{y}_t^W \right] + \beta E_t \pi_{Ft+1}^*$$

The optimal targeting rules can be written as:

$$(D3) \quad \tilde{y}_t - \tilde{y}_{t-1} + \xi \pi_{Ht} = 0$$

$$(D4) \quad \tilde{y}_t^* - \tilde{y}_{t-1}^* + \xi \pi_{Ft}^* = 0.$$

We will assume  $\tilde{y}_{-1} = \tilde{y}_{-1}^* = 0$ . It follows immediately from these equations that under the optimal policy,  $\tilde{y}_t = \tilde{y}_t^* = \pi_{Ht} = \pi_{Ft}^* = 0$ .

From equations (B18) and (B19) we have

$$(D5) \quad a_t = \frac{\sigma}{D} \bar{y}_t^R + \sigma \bar{y}_t^W$$

$$(D6) \quad a_t^* = -\left( \frac{\sigma}{D} \right) \bar{y}_t^R + \sigma \bar{y}_t^W$$

It follows that :

$$(D7) \quad \bar{y}_t = \frac{1+D}{2\sigma} a_t + \frac{1-D}{2\sigma} a_t^*$$

$$(D8) \quad \bar{y}_t^* = \frac{1+D}{2\sigma} a_t^* + \frac{1-D}{2\sigma} a_t$$

Since  $\tilde{y}_t = \tilde{y}_t^* = 0$ , these two equations solve for actual Home and Foreign output.

Since  $\tilde{s}_t = 2\tilde{y}_t^R$ , we have  $\tilde{s}_t = 0$ , or  $s_t = \bar{s}_t = a_t$ . Since  $q_t = 2(\nu-1)s_t$ , we have  $\tilde{q}_t = 0$ , which implies  $q_t = (\nu-1)a_t$ .

Then  $\pi_t = \frac{\nu}{2} \pi_{Ht} + \frac{2-\nu}{2} (e_t - e_{t-1} + \pi_{Ft}^*) = \frac{2-\nu}{2} (e_t - e_{t-1})$ . Assuming

$p_{H,-1} = p_{F,-1}^* = 0$ , we have  $p_{Ht} = p_{Ft}^* = 0$ , so  $s_t = e_t + p_{Ft}^* - p_{Ht} = e_t$ .

Therefore,  $\pi_t = \frac{2-\nu}{2} (a_t - a_{t-1}) = \frac{2-\nu}{2} ((\rho-1)a_{t-1} + \varepsilon_t)$ .

To calculate impulse responses for the exchange rate, we have, setting  $a_{t-1} = 0$ ,  $E_t e_{t+k} = E_t s_{t+k} = \rho^k \varepsilon_t$  for  $k > 0$ . For impulse responses for consumer price inflation, we have  $E_t \pi_{t+k} = \frac{2-\nu}{2} \rho^{k-1} (\rho-1) \varepsilon_t$  for  $k > 0$ .

## D.2 LCP Model

Under LCP, we can write the Phillips curves as:

$$(D9) \quad \pi_t^R = \frac{\delta}{2} \tilde{q}_t + \beta E_t \pi_{t+1}^R$$

$$(D10) \quad \pi_t^W = \delta \sigma \tilde{y}_t^W + \beta E_t \pi_{t+1}^W.$$

The targeting rules are:

$$(D11) \quad \tilde{q}_t - \tilde{q}_{t-1} + 2\sigma \xi \pi_t^R = 0$$

$$(D12) \quad \tilde{y}_t^W - \tilde{y}_{t-1}^W + \xi \pi_t^W = 0.$$

Assuming at time period -1 all variables are at their efficient levels, these equations immediately imply that  $\pi_t^R = \pi_t^W = 0$ , which imply  $\pi_t = \pi_t^* = 0$ . Also,  $\tilde{y}_t^W = 0$  and  $\tilde{q}_t = 0$ .

Since  $\tilde{q}_t = 2(\nu-1)\tilde{y}_t^R + \nu(2-\nu)m_t = 0$ , we have  $m_t = \frac{-2(\nu-1)}{\nu(2-\nu)} \tilde{y}_t^R$ . Then, because

$$\tilde{s}_t = \frac{2\sigma}{D} \tilde{y}_t^R - \frac{(\nu-1)}{D} m_t, \text{ we find}$$

$$(D13) \quad \tilde{y}_t^R = \frac{\nu(2-\nu)}{2} \tilde{s}_t.$$

Note also that since  $\tilde{y}_t^W = 0$ , we have  $\tilde{y}_t = \tilde{y}_t^R$

The deviation of the relative price of imports from the efficient level is given by  $\tilde{s}_t = s_t - a_t$ . The evolution of  $s_t$  in turn is determined by the expectational difference equation:

$$(D14) \quad s_t - s_{t-1} = -\delta s_t + \beta E_t (s_{t+1} - s_t) + \delta a_t.$$

This equation has the solution

$$(D15) \quad s_t = \gamma_1 s_{t-1} + \gamma_2 a_t, \text{ where}$$

$$\gamma_1 = \frac{1 + \beta + \delta - ((1 + \beta + \delta)^2 - 4\beta)}{2\beta}, \quad \text{and} \quad \gamma_2 = \frac{\delta}{1 + \beta(1 - \gamma_1 - \rho) + \delta}.$$

However, recognizing that  $\delta \equiv (1 - \theta)(1 - \beta\theta) / \theta$ , we see that these solutions simplify to:

$$\gamma_1 = \theta \quad \text{and} \quad \gamma_2 = \frac{(1 - \theta)(1 - \beta\theta)}{1 - \beta\rho\theta}.$$

It follows that  $\tilde{s}_t = \theta \tilde{s}_{t-1} + (\gamma_2 - 1)a_t + \theta a_{t-1}$ .

To get impulse responses, with some algebra, we can show, setting  $a_{t-1} = 0$  and  $\tilde{s}_{t-1} = 0$ ,

$$(D16) \quad E_t \tilde{s}_{t+k} = \left[ \rho^k \left( \frac{(\gamma_2 - 1) + \theta}{\rho - \theta} \right) - \theta^{k+1} \frac{\gamma_2}{\rho - \theta} \right] \varepsilon_t.$$



It follows immediately from (D13) that

$$(D17) \quad E_t \tilde{y}_{t+k} = \frac{\nu(2-\nu)}{2} \left[ \rho^k \left( \frac{(\gamma_2 - 1) + \theta}{\rho - \theta} \right) - \theta^{k+1} \frac{\gamma_2}{\rho - \theta} \right] \varepsilon_t.$$

From the Phillips curve (B29), we have

$$(D18) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + \frac{D - (\nu - 1)}{2D} m_t \right] + \beta E_t \pi_{Ht+1}.$$

Substituting in the relationships  $m_t = \frac{-2(\nu - 1)}{\nu(2 - \nu)} \tilde{y}_t^R$  and (D13), we get:

$$(D19) \quad \pi_{Ht} = \frac{\delta(2-\nu)}{2} \tilde{s}_t + \beta E_t \pi_{Ht+1}.$$

The forward solution to this equation is given by:

$$(D20) \quad \pi_{Ht} = \frac{\delta(2-\nu)}{2} (\tilde{s}_t + \beta E_t \tilde{s}_{t+1} + \beta^2 E_t \tilde{s}_{t+2} + \dots).$$

Setting  $a_{t-1} = 0$  and  $\tilde{s}_{t-1} = 0$ , we can get impulse responses from this equation:

$$(D21) \quad E_t \pi_{Ht+k} = \frac{\delta(2-\nu)}{2} \left[ \frac{\rho^k}{1 - \beta\rho} \left( \frac{(\gamma_2 - 1) + \theta}{\rho - \theta} \right) - \frac{\theta^{k+1}}{1 - \beta\theta} \frac{\gamma_2}{\rho - \theta} \right] \varepsilon_t.$$