Searching and Learning by Trial-and-Error
Web Appendix

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Proposition 6 *Consider two industries with mappings ψ_1 and ψ_2, and drift-variance pairs of (μ, σ^2) and $(\tau\mu, \tau\sigma^2)$,respectively, and that have identical status quo outcomes. In expectation, search in these industries is identical, with the exception that the expected size of experimental steps in Industry 2 is equal to $\frac{1}{\tau}$ that in Industry 1.*

Proof of Proposition 6: Suppose at time t the industries face the respective histories (denoting industry by a subscript):

$$\begin{aligned} h_1^t &= \{(0, \psi_1(0)), (p_1, \psi_1(p_1)), (p_2, \psi_1(p_2)), ..., (p_{t-1}, \psi_1(p_{t-1}))\}, \\ h_2^t &= \left\{(0, \psi_2(0)), \left(\frac{1}{\tau}p_1, \psi_2\left(\frac{1}{\tau}p_1\right)\right), \left(\frac{1}{\tau}p_2, \psi_2\left(\frac{1}{\tau}p_2\right)\right), ..., \left(\frac{1}{\tau}p_{t-1}, \psi_2\left(\frac{1}{\tau}p_{t-1}\right)\right)\right\}, \end{aligned}$$

with $\psi_1(p_t) = \psi_2\left(\frac{1}{\tau}p_t\right)$ for $t = 1, 2, ..., t$ and $p_0 = 0$. Straightforward algebra establishes that for each product, $p \in \mathbb{R}$, the distribution of possible outcomes in Industry 1 is equal to the distribution of outcomes in Industry 2 for product $\frac{1}{\tau}p$; that is $\psi_1(p) \overset{d}{=} \psi_2\left(\frac{1}{\tau}p\right)$. Facing the identical set of alternatives, the equilibrium choice across industries satisfies $p_{t,2}^* = \frac{1}{\tau}p_{t,1}^*$. As the industries share a common history at $t = 1$, the result follows by induction.

Corollary 5 *With drift uncertainty, the equilibrium strategy of entrepreneur* 1 *is:*
(i) Stable at $p_1^* = p_0^*$ if $\psi(p_0^*) \in [0, \alpha]$.
(ii) Experimental if $\psi(p_0^*) > \alpha$, where $E(\psi(p_1^*|h^1)) > \alpha$ and strictly increasing in $\psi(p_0^*)$.

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Proof of Corollary 5: Setting $\mu_1 = \mu - \nu$, $\mu_2 = \mu + \nu$ and denoting utility here with a tilde, the derivatives of expected utility become:

$$\begin{aligned}
\frac{dE\tilde{u}(z)}{dz} &= -\mu_1 [\psi(p_0^*) + \mu_1 (z - p_0^*)] - \mu_2 [\psi(p_0^*) + \mu_2 (z - p_0^*)] - \sigma^2, \\
&= -2\mu [\psi(p_0^*) + \mu (z - p_0^*)] - 2\nu^2 (z - p_0^*) - \sigma^2, \\
&= \frac{dEu(z)}{dz} - 2\nu^2 (z - p_0^*), \\
\frac{d^2 E\tilde{u}(z)}{dz^2} &= -\mu_1^2 - \mu_2^2 < -2\mu^2 < 0.
\end{aligned}$$

Part i follows from $\frac{dE\tilde{u}(z)}{dz}|_{z=p_0^*} = \frac{dEu(z)}{dz}|_{z=p_0^*}$. As $\frac{dEu(z)}{dz}|_{z=p_0^*} > 0$ implies $\frac{dE\tilde{u}(z)}{dz}|_{z=p_0^*} > 0$, part ii follows because $\frac{dE\tilde{u}(z)}{dz} < \frac{dEu(z)}{dz}$ for all $z > p_0^*$.

Corollary 6 *With drift uncertainty, the equilibrium strategy for the second entrepreneur is:*
(i) Stuck at $p_2^* = p_0^*$ if $\psi(p_1^*) > \hat{\gamma}_2$ where the cut-point $\hat{\gamma}_2$ is less extreme than in the baseline model; i.e., $\hat{\gamma}_2 < \gamma_2$, for γ_2 defined in Proposition 2.
(ii) Experimental if $\psi(p_1^*)$ is in a neighborhood of α.

Proof of Corollary 6: By stochastic dominance and $E\psi(p_1^*) < \psi(p_0^*)$, a realization $\psi(p_1^*) > \psi(p_0^*)$ induces more weight on μ_2 in posterior beliefs. Similarly, a realization $\psi(p_1^*) \approx \alpha < E\psi(p_1^*)$ puts more weight on μ_1. Both results then follow from straightforward algebra.

Proposition 7 *For a given known starting point, $(p_0^*, \psi(p_0^*))$, there is a δ' such that the monotonic-triangulating-stable dynamic holds if $\delta \in [0, \delta']$.*

Proof of Proposition 7: For the monotonic-triangulating-stable dynamic to not hold, the entrepreneur must in some period choose an experimental product over a known product that has a (weakly) better expected value. Let the deviation in period t be to z with p the nearest known product, and suppose $\psi(p) > 0$.

The current period cost of the deviation is variance and possibly expected value. The variance cost alone increases linearly in z if uncertainty is open-ended and at least quadratically if on a bridge. The benefit of the deviation accrues in subsequent periods. By the single period deviation principle, a realization of $\psi(z) > 0$ leaves the optimal strategy in effect: the deviation is thereafter ignored if $\psi(z) > \psi(p)$ and potentially stabilized at for $\psi(z) \in [0, \psi(p)]$. For realizations $\psi(z) < 0$ behavior is unclear, but is clearly inferior to obtaining the ideal outcome 0 in every subsequent period. The benefit from the deviation is, therefore, bounded by:

$$\varsigma\left(z|\psi\left(p\right),\delta\right)=\frac{\delta}{1-\delta}\left[\begin{array}{c}\int_{0}^{\psi(p)}\left(\psi\left(p\right)^{2}-x^{2}\right)\phi\left(x|0,z\sigma^{2}\right)dx\\+\left(1-\Phi\left(\psi\left(p\right)|0,z\sigma^{2}\right)\right)\psi\left(p\right)^{2}\end{array}\right],$$

where Φ and ϕ are the cdf and pdf of the normal distribution with mean 0 and variance $z\sigma^2$ (supposing uncertainty is open-ended). The benefit ς is continuous in z, and bounded by $\psi\left(p\right)^2$, which itself is bounded by $\psi\left(p_0^*\right)^2$. Thus, $\frac{d}{dz}\varsigma\left(z|\psi\left(p\right),\delta\right)$ is bounded for given starting point $\left(p_0^*,\psi\left(p_0^*\right)\right)$, and as $\varsigma\left(0|\psi\left(p\right),\delta\right)=0$, the deviation is not profitable for sufficiently small δ.

Proposition 8 *The monotonic-triangulating-stable dynamic holds for any weakly concave utility function.*

Proof of Proposition 8: An entrepreneur with weakly concave utility strictly prefers the expected value of a lottery with certainty to the lottery itself. The result then follows by comparing expected values of products in each phase.