

“The Nature of Credit Constraints and Human Capital”

Web Appendix

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This document is a companion (online) appendix to our paper “*The Nature of Credit Constraints and Human Capital*.” The first three sections describe some important details of our analysis. The fourth section shows that the main results of the paper are fairly robust to introducing a number of important extensions to the model.

I. Student Loans in the U.S.

This section describes the structure and enforcement of GSL programs and private student lending in the U.S.

A. *GSL Programs in the U.S.*

The largest program is the Stafford Loan program, which awarded nearly \$50 billion to students in the 2003-04 academic year.¹ The Parent Loans for Undergraduate Students (PLUS) awarded \$7 billion to parents of undergraduate students during the same period. On a much smaller scale, the Perkins Loan program disbursed \$1.6 billion to a small fraction of students from very low-income families. See The College Board (2006) for details about financial aid disbursements and their trends over time.

Table A1 reports loan limits (based on the dependency status and class of the student) for Stafford and Perkins programs for the period 1993-2007. Dependent students could borrow up to \$23,000 from the Stafford Loan Program over the course of their undergraduate careers. Independent students could borrow roughly twice that amount, although most traditional undergraduates do not fall into this category. Qualified undergraduates from low-income families could receive as much as \$20,000 in Perkins loans, depending on their need and post-secondary institution. However, amounts offered through this program have typically been less than mandated limits. Since 1993-94, the PLUS loan program no longer has a fixed maximum borrowing limit; however, parents still cannot borrow more than the total cost of college net of other financial aid. Student borrowers can defer loan re-payments until six (Stafford) to nine (Perkins) months after leaving school. Repayment of PLUS loans typically begins within 60 days of loan disbursement.

In real terms, cumulative Stafford loan limits for undergraduates were nearly identical in 2002-03 to what they were twenty years earlier. (NLSY79 and NLSY97 respondents made their college attendance decisions around these two periods.) While the government nominally increased loan

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¹The Stafford program offers both subsidized and unsubsidized loans, with the latter available to all students and the former only to students demonstrating financial need. The government waives the interest on subsidized loans while students are enrolled; it does not do so for unsubsidized loans. Prior to the introduction of unsubsidized Stafford Loans in the early 1990s, Supplemental Loans to Students (SLS) were an alternative source of unsubsidized federal loans for independent students.

TABLE 1—BORROWING LIMITS FOR STAFFORD AND PERKINS STUDENT LOAN PROGRAMS (1993-2007)

	Stafford Loans		Perkins Loans
	Dependent Students	Independent Students*	
Eligibility Requirements	Subsidized: Financial Need Unsubsidized: All Students		Financial Need
Undergraduate Limits:			
First Year	\$2,625	\$6,625	\$4,000
Second Year	\$3,500	\$7,500	\$4,000
Third-Fifth Years	\$4,000	\$8,000	\$4,000
Cum. Total	\$23,000	\$46,000	\$20,000
Graduate Limits:			
Annual		\$18,500	\$6,000
Cum. Total**		\$138,500	\$40,000

Notes:

* Students whose parents do not qualify for PLUS loans can borrow up to independent student limits from Stafford program.

** Cumulative graduate loan limits include loans from undergraduate loans.

limits (especially for upper-year college students) in 1986-87 and 1993-94, inflation has otherwise eroded these limits away.²

GSL loans are more strictly enforced relative to typical unsecured private loans. Except in very special circumstances, these loans cannot be expunged through bankruptcy. If a suitable repayment plan is not agreed upon with the lender once a borrower enters default, the default status is reported to credit bureaus and collection costs (up to 25 percent of the balance due) may be added to the amount outstanding. (Formally, a borrower is considered to be in default once a payment is 270 days late.) Up to 15 percent of the borrower's wages can also be garnished. Moreover, federal tax refunds can be seized and applied toward any outstanding balance. Other sanctions include a possible hold on college transcripts, ineligibility for further federal student loans, and ineligibility for future deferments or forbearances.

B. Private Lending

The design of private lending programs is broadly consistent with the problem of lending under limited repayment incentives. Private lenders directly link credit to educational investment expenditures and indirectly to projected earnings. All private student loan programs require evidence of post-secondary school enrollment, offering students credit far in excess of what is otherwise offered in the form of more traditional uncollateralized loans. While many private student lending programs are loosely structured like federal GSL programs, they vary substantially in their terms and eligibility requirements. Some private lenders clearly advertise that they consider the school attended, course of study, and college grades in determining loan packages.

²From 1982-83 to 2002-03, Stafford borrowing limits for undergraduates declined by 44 percent for first-year students and 25 percent for second-year students, while they increased by about 20 percent for college students enrolled in years three through five. For most of this period, loan limits for independent undergraduates remained about twice the amounts available to dependent students. Stafford loan limits for graduate students declined by about 35 percent in real terms from 1986-87 to 2006-07, roughly the time NLSY97 respondents would have began attending graduate school.

Until the ‘Bankruptcy Abuse Prevention and Consumer Protection Act of 2005’, individuals could discharge private student loans through bankruptcy. Thus, enforcement of private student loans was regulated by U.S. bankruptcy code. Borrowers filing for bankruptcy under Chapter 7 must pay a court and filing fees of up to a few thousand dollars and surrender any non-collateralized assets (above an exemption) in exchange for discharging all debts; however, most school-leavers considering bankruptcy have few if any assets. Furthermore, bankruptcy shows up on an individual’s credit report for ten years, limiting future access to credit. Bankruptcy may spill over into other domains as well (e.g. banks, mortgage companies, landlords, and employers often request credit reports from potential customers or employees). Finally, U.S. bankruptcy requires “good faith” attempts to meet debt obligations, which may make it difficult for former students to expunge their debts if current income levels are high. After reviewing the punishments associated with Chapter 7 bankruptcy, Livshits, MacGee, and Tertilt (2007) argue that they are well-approximated by a temporary period of both wage garnishments and exclusion from credit markets.

II. NLSY79 and NLSY97 Data

We now discuss some of the data used in the paper.

The NLSY79 is a random survey of American youth ages 14-21 at the beginning of 1979, while the NLSY97 samples youth ages 12-16 at the beginning of 1997.³ Since the oldest respondents in the NLSY97 recently turned age 24 in the 2004 wave of data, we analyze college attendance as of age 21 in both samples.

Individuals are considered to have attended college if they *attended* at least 13 years of school by the age of 21. For the 1979 cohort, we use average family income when youth are ages 16-17, excluding those not living with their parents at these ages. In the NLSY97 data, we use household income and net wealth reported in 1997 (corresponding to ages 13-17), dropping individuals not living with their parents that year. Family income includes government transfers (e.g. welfare and unemployment insurance), but it does not subtract taxes. Net wealth is the value of all assets (e.g. home and other real estate, vehicles, checking and savings, and other financial assets) less loans and credit card debt. We use AFQT as a measure of cognitive ability. It is a composite score from four subtests of the Armed Services Vocational Aptitude Battery (ASVAB) used by the U.S. military: arithmetic reasoning, word knowledge, paragraph comprehension, and numerical operations. These tests are taken by respondents in both the NLSY79 and NLSY97 during their teenage years as part of the survey process. Since AFQT percentile scores increase with age in the NLSY79, we determine an individual’s quartile based on year of birth. (AFQT percentile scores in the NLSY97 have already been adjusted to account for age differences.)

III. Proofs and Other Aspects of the Two-Period Model

In the text, we assume preferences with a constant IES; however, in this appendix, we allow for general preferences satisfying $u'(c) > 0$ and $u''(c) < 0$. The IES is defined as $\eta(c) \equiv \frac{-cu''(c)}{u'(c)}$.

The set of constrained individuals: For each ability level a , the various forms of credit constraints define a threshold wealth level below which the agent is constrained (and above which he is not). We now characterize those thresholds.

Exogenous Constraints: The threshold $w_{\min}^X(a)$ is defined by $d^U(a, w_{\min}^X(a)) = \bar{d}^X$, and therefore it is increasing in a . Consumption smoothing implies that $w_{\min}^X(a) \geq h^U(a) - \bar{d}^X$ (the minimum wealth needed to finance $h^U(a)$ given maximum borrowing) and that $w_{\min}^X(a)$ is

³See Belley and Lochner (2007) for details on their sample and variables, which we employ here.

steeper than $h^U(a)$ as a function of a . To see this, implicit differentiation leads to $\frac{dw_{\min}^X(a)}{da} = \frac{\partial d^U(a, w_{\min}^X)}{\partial a} / \frac{\partial d^U(a, w_{\min}^X)}{\partial w} > \frac{\partial d^U(a, w_{\min}^X)}{\partial a} > \frac{dh^U(a)}{da} > 0$.

GSL Programs: The threshold $w_{\min}^G(a) \equiv \max\{w_{\min}^X(a), \tilde{w}_{\min}(a)\}$, where $\tilde{w}_{\min}(a)$ is defined by $h^U(a) = d^U(a, \tilde{w}_{\min}(a))$. It is increasing in a because $d^U(\cdot, w)$ is steeper than $h^U(\cdot)$. To see that $w_{\min}^X(a)$ is steeper than $\tilde{w}_{\min}(a)$, use implicit differentiation to obtain $\frac{d\tilde{w}_{\min}(a)}{da} = \frac{dw_{\min}^X(a)}{da} + \frac{\partial h^U}{\partial a} / \frac{\partial d^U}{\partial w} < \frac{dw_{\min}^X(a)}{da}$.

GSL Programs plus Private Lenders: The threshold $w_{\min}^{G+L}(a)$ is defined by $d^U(a, w_{\min}^{G+L}(a)) = \kappa af(h^U(a)) + \min\{h^U(a), \bar{d}^G\}$. An instructive special case is when $\bar{d}^G = 0$ and only private lending is available in the economy. In this case, the threshold $w_{\min}^L(a)$ is defined by $d^U(a, w_{\min}^L(a)) = \kappa af(h^U(a))$, which increases at a slower rate in a than does $w_{\min}^X(a)$. Indeed, $w_{\min}^L(a)$ may even be decreasing in a if κ is large enough. Both of these facts can be seen from $\frac{dw_{\min}^L}{da} = \frac{dw_{\min}^X}{da} + \kappa \left(f(h^U) + R \frac{\partial h^U}{\partial a} \right) / \frac{\partial d^U}{\partial w} < \frac{dw_{\min}^X}{da}$ because $\frac{\partial d^U}{\partial w} < 0$. In the general case when both private and GSL credit is available, direct inspection reveals that $w_{\min}^{G+L}(a) < \min\{w_{\min}^G(a), w_{\min}^L(a)\}$. As with $w_{\min}^L(a)$, the threshold $w_{\min}^{G+L}(a)$ can be decreasing in a and may even be negative.

Proof of Lemma 1. Implicit differentiation of the expression defining $h^U(a)$ yields $\frac{dh^U(a)}{da} = -\frac{f'[h^U(a)]}{af''[h^U(a)]} > 0$. Using expression Euler condition for consumption, define

$$F \equiv u' [w + d - h^U(a)] - \beta Ru' [af[h^U(a)] - Rd] = 0.$$

From the implicit function theorem $\frac{\partial d^U(a, w)}{\partial a} = -\frac{\partial F}{\partial a} / \frac{\partial F}{\partial d}$, then

$$\frac{\partial d^U(a, w)}{\partial a} = \frac{\partial h^U(a)}{\partial a} + \beta R \frac{u'' [af[h^U(a)] - Rd] f[h^U(a)]}{u'' [w + d - h^U(a)] + \beta R^2 u'' [af[h^U(a)] - Rd]} > \frac{\partial h^U(a)}{\partial a} > 0,$$

where we have used $af'[h^U(a)] = R$. Similarly,

$$\begin{aligned} \frac{\partial d^U(a, w)}{\partial w} &= -\frac{u'' [w + d - h^U(a)]}{u'' [w + d - h^U(a)] + \beta R^2 u'' [af[h^U(a)] - Rd]} \\ &= -\frac{1}{1 + \beta R^2 \frac{u'' [af[h^U(a)] - Rd]}{u'' [w + d - h^U(a)]}}. \end{aligned}$$

Since the denominator is greater than one, the argument is complete. ■

Proof of Proposition 1. From the FOC define

$$F \equiv -u' (w + \bar{d}^X - h) + \beta af'[h] u' [af(h) - R\bar{d}^X] = 0.$$

The second order condition implies $\partial F / \partial h < 0$, which, combined with implicit differentiation, implies that $\text{sign} \left\{ \frac{\partial h}{\partial w} \right\} = \text{sign} \left\{ \frac{\partial F}{\partial w} \right\}$ and $\text{sign} \left\{ \frac{\partial h}{\partial a} \right\} = \text{sign} \left\{ \frac{\partial F}{\partial a} \right\}$. First, we have $\frac{\partial h}{\partial w} > 0$ since

$\frac{\partial F}{\partial w} = -u''(w + \bar{d}^X - h) > 0$. Second,

$$\begin{aligned} \frac{\partial F}{\partial a} &= \beta f'[h] u' [af(h) - R\bar{d}^X] \left\{ 1 + af(h) \frac{u'' [af(h) - R\bar{d}^X]}{u' [af(h) - R\bar{d}^X]} \right\} \\ &< \beta f'[h] u' [af(h) - R\bar{d}^X] \left\{ 1 - 1/\eta [af(h) - R\bar{d}^X] \right\}, \end{aligned}$$

where the first results from direct derivation, and the second from $u' > 0$, $u'' < 0$, $f' > 0$, $\bar{d}^X \geq 0$, and the definition of IES $\equiv \eta(\cdot)$. If $\eta(c) \leq 1 \forall c > 0$, the right-hand-side (RHS) of the last line is non-positive and $\frac{\partial F}{\partial a} < 0$. ■

Proof of Proposition 2. Using the FOC for the exogenous constraint model,

$$\hat{a}(w) \equiv \sup \left\{ \hat{a} : u'(w) \geq \beta \hat{a} f'[\bar{d}^G] u' [\hat{a} f(\bar{d}^G) - R\bar{d}^G] \right\},$$

which in principle could be $+\infty$. If $u(c) = c^{1-\sigma}/(1-\sigma)$, then $\hat{a}(w)$ is finite and given by $\hat{a} : w (\beta f'[\bar{d}^G])^{\frac{1}{\sigma}} = (\hat{a})^{\frac{\sigma-1}{\sigma}} f(\bar{d}^G) - R\bar{d}^G (\hat{a})^{\frac{1}{\sigma}}$. If $\sigma > 1$ (IES < 1), the RHS is strictly increasing and unbounded, so $\hat{a}(w)$ is finite. The rest is direct upon examination of optimality conditions under the three different cases. ■

Proof of Lemma 2. Part (i) is from direct inspection based on the thresholds as derived above. For part (ii), use the FOC for a constrained person with $a > \bar{a}$ (i.e. $d_g = \bar{d}^G$, $d_p = \kappa af(h)$ and $h > \bar{d}^G$) to define

$$\begin{aligned} F(h, \bar{d}^G, \kappa) &\equiv (\kappa af'(h) - 1) u' [w + \bar{d}^G + \kappa af[h] - h] \\ &\quad + \beta af'(h) (1 - \kappa R) u' [af(h) (1 - \kappa R) - R\bar{d}^G]. \end{aligned}$$

For constrained agents, with $a > \bar{a}$, we have that $u'(c_0) > \beta Ru'(c_1)$ and $af'(h) < R$. It is straightforward to verify that $\frac{\partial F}{\partial \bar{d}^G} > 0$, and $\frac{\partial F}{\partial \kappa} > 0$, and therefore, implicit differentiation implies the stated results. ■

Proof Proposition 3. Part (1): If $a > \bar{a}$, the FOC is given by

$$F = u'(c_0) [\kappa af'(h) - 1] + \beta u'(c_1) af'(h) (1 - \kappa R) = 0,$$

where $c_0 = w + \kappa af(h) + \bar{d}^G - h$ and $c_1 = af(h) (1 - \kappa R) - R\bar{d}^G$. Moreover, notice that $\frac{\partial c_0}{\partial h} = \kappa af'(h) - 1 < 0$, and $\frac{\partial c_1}{\partial h} = af'(h) (1 - \kappa R) > 0$. To prove (i) notice that if the agent is constrained, then $u'(c_0) > \beta Ru'(c_1)$. Therefore, $F = 0$ implies $[1 - \kappa af'(h)] < \frac{af'(h)}{R} (1 - \kappa R) \implies af'(h) > R$, i.e. there is under-investment. To prove (ii), notice that $\text{sign}\left\{\frac{dh}{dw}\right\} = \text{sign}\left\{\frac{dF}{dw}\right\}$ and that $dF/dw = u''(c_0) [\kappa af'(h) - 1] > 0$. To prove (iii), first define for any $a \geq \bar{a}$ and $h \geq \bar{d}^G$ the fraction of labor earnings needed to pay back the maximum debt from the GSL: $\varrho(a, h) \equiv \frac{R\bar{d}^G}{af(h)} = \varrho(a) \frac{f(\bar{d}^G)}{f(h)}$, where $\varrho(a)$ is defined in the text. Next, compute

the derivative, re-group, simplify, and use the definition of $\eta(\cdot)$ and $\varrho(a, h)$

$$(1) \quad \frac{\partial F}{\partial a} = u'(c_0) \kappa f'(h) + (1 - \kappa a f'(h)) [-u''(c_0)] \kappa f(h)$$

$$(2) \quad + (1 - \kappa R) \beta f'(h) u'(c_1) \left[1 - \frac{1}{\eta(c_1)} \frac{1}{1 - \varrho(a, h)} \right].$$

Since the agent is constrained, we have that $u'(c_0) \geq \beta R u'(c_1)$. Using this inequality in the first term and ignoring the second term because it is always positive, obtain

$$\frac{\partial F}{\partial a} \geq \beta u'(c_1) f'(h) \left(R\kappa + (1 - \kappa R) \left[1 - \frac{1}{\eta(c_1)} \frac{1}{1 - \varrho(a, h)} \right] \right).$$

The RHS is positive when $\eta(c_1) > \frac{1 - \kappa R}{1 - \varrho(a, h)}$, which is stated as sufficient condition (A). Next, impose sufficient condition (B) on $\frac{dF}{da}$ in equation (1). Since $\beta R \leq 1$, we have $c_0 < c_1$ and $u'(c_0) \geq u'(c_1)$. Take $u'(c_1) f'(h) > 0$ as a common factor, and in the second term use the FOC implied equality $u'(c_1) = \frac{(1 - \kappa a f'(h))}{\beta(1 - \kappa R) a f'(h)} u'(c_0)$. Also, divide and multiply by c_0 and simplify to obtain:

$$\begin{aligned} \frac{\partial F}{\partial a} &= u'(c_1) f'(h) \left(\kappa \frac{u'(c_0)}{u'(c_1)} + \frac{\kappa (1 - \kappa a f'(h)) f(h)}{f'(h)} \frac{-c_0 u''(c_0)}{c_0 \frac{(1 - \kappa a f'(h)) u'(c_0)}{\beta(1 - \kappa R) a f'(h)}} \right) \\ &+ u'(c_1) f'(h) \beta (1 - \kappa R) \left(1 - \frac{1}{\eta(c_1) [1 - \varrho(a, h)]} \right), \\ &\geq u'(c_1) f'(h) \left\{ \kappa + \frac{\kappa \beta}{\eta(c_0)} \frac{1}{1 - \varrho(a, h)} + \beta (1 - \kappa R) \left[1 - \frac{1}{\eta(c_1) [1 - \varrho(a, h)]} \right] \right\}, \end{aligned}$$

where the second line uses the definition of $\varrho(a, h)$, the fact that $c_1 \geq c_0$ and that $u'(c_0) \geq u'(c_1)$. Finally, for $\eta(c)$ constant (as assumed in the text) or increasing,

$$\frac{\partial F}{\partial a} \geq u'(c_1) f'(h) \left\{ \kappa + \frac{\kappa \beta}{\eta(c_0)} \frac{1}{1 - \varrho(a, h)} + \beta (1 - \kappa R) \left[1 - \frac{1}{\eta(c_0) [1 - \varrho(a, h)]} \right] \right\}.$$

This inequality holds whenever the term inside brackets is positive, which holds if $\eta(c_0) \geq \frac{1}{1 - \varrho(a, h)} \frac{1 - \kappa(R+1)}{1 + \kappa(\beta^{-1} - R)}$. Since $\varrho(a, h) \leq \varrho(a)$, the sufficiency of condition (B) follows.

Part (2): The FOC in this case is given by

$$F \equiv u'(c_0) \kappa a f'(h) + \beta u'(c_1) [a f'(h) (1 - \kappa R) - R] = 0,$$

where $c_0 = w + \kappa a f(h)$ and $c_1 = a f(h) (1 - \kappa R) - R h$. To prove (i) notice that if the agent is constrained, then $u'(c_0) > \beta R u'(c_1)$ and $F = 0$ implies that $\kappa a f'(h) < \frac{R - a f'(h)(1 - \kappa R)}{R}$. Rearranging, we get $a f'(h) < R$, or equivalently $h > h^U(a)$ because of the strict concavity of $f(\cdot)$. To prove (ii), compute $\frac{dF}{dw} = \kappa a f'(h) u''(c_0) < 0$. The result follows from implicit differentiation ($\frac{\partial h^{G+L}(a, w)}{\partial w} = -\frac{\partial F / \partial w}{\partial F / \partial h}$) and the second order condition ($\partial F / \partial h < 0$). Similarly, for (iii)

$\text{sign} \left\{ \frac{\partial h^{G+L}(a, w)}{\partial a} \right\} = \text{sign} \left\{ \frac{\partial F}{\partial a} \right\}$. First, compute the derivative

$$\begin{aligned} \frac{\partial F}{\partial a} &= u'(c_0) \kappa f'(h) + \kappa a f'(h) u''(c_0) \frac{\partial c_0}{\partial a} \\ &\quad + \beta u'(c_1) [f'(h) (1 - \kappa R)] + \beta [a f'(h) (1 - \kappa R) - R] u''(c_1) \frac{\partial c_1}{\partial a}. \end{aligned}$$

Notice that only the second term in this expression can be negative. Take $\frac{1}{a}$ as a common factor and then add and subtract $R\beta u'(c_1)$ to get:

$$\begin{aligned} \frac{\partial F}{\partial a} &= \frac{1}{a} \left\{ u'(c_0) \kappa a f'(h) + \beta u'(c_1) [a f'(h) (1 - \kappa R) - R] \right\} \\ &\quad + \frac{1}{a} \left\{ R\beta u'(c_1) + \kappa a f'(h) u''(c_0) \frac{\partial c_0}{\partial a} a + \beta [a f'(h) (1 - \kappa R) - R] u''(c_1) \frac{\partial c_1}{\partial a} a \right\}. \end{aligned}$$

The FOC implies that the first line equals zero. Take $R\beta u'(c_1)$ as common factor and multiply and divide the second term by $u'(c_0)$:

$$\frac{\partial F}{\partial a} = \frac{R\beta u'(c_1)}{a} \left\{ 1 + \frac{1}{R} \left(\frac{u'(c_0) \kappa a f'(h)}{\beta u'(c_1)} \right) \frac{u''(c_0)}{u'(c_0)} \frac{\partial c_0}{\partial a} a + \frac{1}{R} [a f'(h) (1 - \kappa R) - R] \frac{u''(c_1)}{u'(c_1)} \frac{\partial c_1}{\partial a} a \right\}.$$

Because of the FOC, the expression inside parentheses in the second term equals $[R - a f'(h) (1 - \kappa R)]$. After dividing and multiplying the last two terms by c_0 and c_1 , respectively, and using the definition of the IES, $\eta(c)$, and re-grouping:

$$\frac{\partial F}{\partial a} = \frac{R\beta u'(c_1)}{a} \left\{ 1 + \left[1 - \frac{a f'(h)}{R} (1 - \kappa R) \right] \left[\frac{1}{\eta(c_1)} \left(\frac{\partial c_1}{\partial a} \frac{a}{c_1} \right) - \frac{1}{\eta(c_0)} \left(\frac{\partial c_0}{\partial a} \frac{a}{c_0} \right) \right] \right\}.$$

The term $1 - \frac{a f'(h)}{R} (1 - \kappa R) > 0$, since there is over-investment, i.e. $a f'(h) < R$. Therefore, $\frac{\partial F}{\partial a}$ can only be negative if $\frac{1}{\eta(c_1)} \left(\frac{\partial c_1}{\partial a} \frac{a}{c_1} \right) - \frac{1}{\eta(c_0)} \left(\frac{\partial c_0}{\partial a} \frac{a}{c_0} \right)$. However, notice that since $\frac{\partial c_1}{\partial a} \frac{a}{c_1} = \frac{a f'(h) (1 - \kappa R)}{a f'(h) (1 - \kappa R) - R h} > 1$ and $\frac{\partial c_0}{\partial a} \frac{a}{c_0} = \frac{\kappa a f'(h)}{w + \kappa a f'(h)} < 1$, this possibility is ruled out if $\eta(c_1) \leq \eta(c_0)$, which clearly holds if $\eta(\cdot)$ is constant. ■

Proof Corollary 1. We first show that our normalized model ($p = \tau = 1$) is isomorphic to any model with arbitrary fixed (τ, p) . Define $e \equiv \tau h$, reflecting total investment expenditures measured in units of the consumption good, and re-write the tie-to-investment constraint as $d \leq e$. Earnings can be written as $y = p a f(e/\tau)$. Defining $\tilde{f}(e) \equiv p f(e/\tau)$, the function $\tilde{f}(\cdot)$ inherits all of the relevant properties of $f(\cdot)$ (e.g. positive, increasing, concave) assumed in the text. Therefore, for any pair (p, τ) , the maximization problem in terms of h and $f(\cdot)$ can be equivalently mapped into our normalized maximization problem using e and $\tilde{f}(\cdot)$.

Now, consider changes in p holding τ constant: the partial derivatives of h^X , h^G and h^{G+L} with respect to p are the same as the respective derivatives with respect to a (up to a multiplicative positive constant), so the proofs of Propositions 1-3 directly apply to the relationship between p and h or e . Finally, we discuss simultaneous increases in p and τ with p/τ increasing. Consider a new pair (p', τ') so that $\tau' > \tau$, and $p' = p x (\tau'/\tau) > p$, where $x \geq 1$. Assuming that

$f(h) = h^\alpha$ with $\alpha \in (0, 1)$, the new earnings function can be written as

$$y = p'a \left(\frac{e}{\tau'}\right)^\alpha = px \left(\frac{\tau'}{\tau}\right) a \left(\frac{e}{\tau}\right)^\alpha = p\hat{a} \left(\frac{e}{\tau}\right)^\alpha,$$

where $\hat{a} \equiv a \cdot \left[x \left(\frac{\tau'}{\tau}\right)^{1-\alpha}\right] > a$, since the term inside brackets is greater than one by assumption. Setting the problems in terms of e (expenditures on human capital investment), Propositions 1-3 establish the direction of change for constrained investment expenditures in the respective models. ■

IV. Extensions

Here, we consider three important extensions of the two-period model used in the paper and show that our main analytical results are robust to these generalizations.

- 1) **Endogenous parental transfers:** In the paper, we take individual/student wealth w as fixed and study the effects of changes in ability; we also consider the implications of changing w . In section I of this appendix, we allow for the endogenous determination of an individual's wealth $w \geq 0$ from parental bequests. We consider the effects of ability on investment holding parental wealth constant, as well as the effects of changing parental wealth.
- 2) **Ability affecting the cost to invest.** The paper implicitly assumes that the marginal cost of investment is the same for everyone. In Section II of this appendix, we allow this marginal cost to depend on individual ability.
- 3) **Generalized earnings function.** In Section III of this appendix, we generalize the earnings function to have non-unitary elasticity of substitution between ability a and human capital h . Here, we generalize our results about the ability – investment relationship, providing sufficient conditions in terms of the consumption intertemporal elasticity of substitution (IES) and the elasticity of substitution between ability and investment in human capital production.

For these extensions, we briefly discuss the alternative models and focus on whether they alter our main analytical results in the paper. Overall, we show that our main results are quite robust. While at a theoretical level, it is possible to define investment cost functions or human capital production functions that significantly alter certain results in our paper, these cases are extreme and empirically implausible. The implicit assumptions in our paper are common and largely consistent with available evidence. We conclude that there is no evidence that warrants the drastic departures from our assumptions required to overturn our main theoretical conclusions.

A. Endogenous Parental Transfers

We now show that our main analytical results in the paper extend to an intergenerational context where parents endogenously decide how much wealth to transfer to their children. In the paper, we present a number of comparative statics results on the relationship between investment and ability. Those results are *conditional on w* , i.e. they hold the young individual's wealth w constant. In this section, we obtain equivalent results that hold the individual's family wealth w_p constant and let w be determined endogenously. We also discuss the relationship between w_p and investment.

We consider three standard models of intergenerational transfers: “altruistic”, “warm-glow” and “paternalistic” parents. They differ in their assumptions about parental preferences. In the

first model, parents care about the *utility* of their child. In the second, parents directly care about the *amount they transfer* to their child. In the third model parents care about the amount of *human capital investment* of their child. For transparency and simplicity, we consider each of these three models in a two-generation model like that of Section II of the paper. Here, the parent is old and only lives during the first period. The child is young, lives two periods, invests in human capital in the first and earns income in the second.

We briefly summarize our results now and provide some detailed derivations in the rest of this section.

- 1) Under “altruistic” preferences, all of our qualitative results in Section II of the paper hold with the sole re-interpretation of initial wealth as parental wealth.
- 2) Under “warm-glow” preferences, all of our results in Section II of the paper hold interpreting wealth as either parental transfers or parental wealth.
- 3) Under “paternalistic” preferences (with a few additional conditions), all of our Section II of the paper results hold for exogenous constraints and endogenous constraints under limited commitment. They also hold for the GSL under a slight modification of the tied-to-investment constraint.

ALTRUISTIC PREFERENCES

Consider the standard altruistic model in which the utility of the parent, U^p , depends on his own consumption, c_p , and the utility of his child, U^c . The utility of the child depends on his current consumption, c_0 , and consumption in the next period, c_1 . Preferences are $U^p = u(c_p) + \theta U^c$ and $U^c = u(c_0) + \beta u(c_1)$, which follow standard one-sided altruistic models. The notation is the same as in our paper except for the new parameter $\theta \in [0, 1]$ defining parental altruism. The parent and the child observe the child’s ability, a , which impacts his future earnings as in the paper. Parents allocate their wealth, $w_p > 0$, between their own consumption and transfers to their children, $b \geq 0$. Given transfers, the child chooses consumption in both periods, human capital investment, and borrowing so as to maximize his own utility. For expositional purposes, we assume that the child has no initial wealth of his own (i.e. $w = b$); although, this assumption can easily be relaxed.

This problem can clearly be formulated as a Pareto optimal allocation problem (with weights 1 and θ on the parent and child, respectively), where the planner chooses c_p , c_0 , c_1 , d , and h . We now consider allocations under different forms of constraints.

UNRESTRICTED ALLOCATIONS

The intergenerational problem with exogenous constraints is

$$(3) \quad \begin{aligned} & \max_{\{c_p, c_0, c_1, d, h\}} \{u(c_p) + \theta [u(c_0) + \beta u(c_1)]\}, \text{ subject to} \\ & w_p + d = c_p + c_0 + h \\ & c_1 = af(h) - Rd. \end{aligned}$$

It is straightforward to show that unconstrained optimal investment is the same as in the paper (i.e. it solves $af'(h^U(a)) = R$). It is trivial to show that Lemma 1 holds (where d now reflects total family borrowing/savings).

EXOGENOUS CREDIT CONSTRAINTS

Now assume that the additional exogenous borrowing constraint is imposed: $d \leq \bar{d}^X$.

If the constraint does not bind, investment is given by $h^U(a)$. However, if the borrowing constraint binds, the first order conditions become

$$\begin{aligned} u' \left(w_p + \bar{d}^X - c_0 - h \right) &= \theta u' (c_0) \\ \beta a f' (h) u' \left(a f (h) - R \bar{d}^X \right) &= u' (c_0). \end{aligned}$$

To characterize $h^X(a, w_p)$ in this case, apply implicit differentiation and Cramer's rule:

$$\frac{\partial h^X(a, w_p)}{\partial a} = \frac{-[u''(c_p) + \theta u''(c_0)] \partial F_{\text{PAPER}}^X}{|A| \partial a},$$

where F_{PAPER}^X is the expression in the Appendix C of the paper from which we derive our proposition for the exogenous constraints model. The matrix

$$A = \begin{bmatrix} u''(c_p) + \theta u''(c_0) & u''(c_p) \\ -u''(c_0) & \beta a \left\{ f''(h) u'[c_1] + a [f'(h)]^2 u''[c_1] \right\} \end{bmatrix}$$

has a positive determinant $|A| > 0$. Since $-[u''(c_p) + \theta u''(c_0)] > 0$,

$$\text{sign} \left\{ \frac{\partial h}{\partial a} \right\} = \text{sign} \left\{ \frac{\partial F_{\text{PAPER}}^X}{\partial a} \right\}.$$

The main lesson is quite simple. Adding altruistic parents and intergenerational transfers does not affect the sign of the relationship between ability and investment in the young person's human capital. However, the magnitudes may change because the level of ability of the young individual will also impact the consumption of his parents.

For constrained families, one can similarly show that investment in the child's human capital will be strictly increasing in parental wealth, w_p . Thus, an extension of Proposition 1 of the paper holds referring to family wealth w_p rather than the individual's wealth w .

GSL

Now consider the case in which credit is available from GSL programs. Given the results in the previous section, it is easy to see that our characterization of the ability-investment relationship with altruistic parents is the same as in the paper. The GSL removes the conflict between net income maximization and consumption smoothing. The key assumption here is that lending is tied to the child's investment and cannot be used to finance the consumption of either the parent or the child. All qualitative results go through where wealth refers to w_p .

PRIVATE LENDING WITH LIMITED COMMITMENT

Consider now our benchmark $G + L$ model in the paper, and as in the paper, start with the case when $\bar{d}^G = \rho(a) = 0$ and $\kappa > 0$. This is a special case of private lending only. In this environment, the family problem is subject to the endogenous debt constraint $d \leq \kappa a f(h)$.

Obviously, if the borrowing constraint does not bind, optimal investment is $h^L(a, w_p) = h^U(a)$. Now, assume that it binds. The first order conditions reduce to

$$\begin{aligned} u'(w + \kappa a f(h) - c_0 - h) &= \theta u'(c_0), \\ \beta(1 - \kappa R) a f'(h) u'[(1 - \kappa R) a f(h)] &= u'(c_0) [1 - \kappa a f'(h)]. \end{aligned}$$

Applying implicit differentiation and Cramer's rule,
(4)

$$\frac{\partial h^L(a, w_p)}{\partial a} = \frac{-[u''(c_p) + \theta u''(c_0)] \left(\frac{\partial \mathbf{F}_{\text{PAPER}}^L}{\partial a} \right) + [-u''(c_0) [\kappa a f'(h) - 1] \{u''[c_p] \kappa f(h)\}]}{|A|}$$

where now

$$(5) \quad A = \begin{bmatrix} u''(c_p) + \theta u''(c_0) & -u''[c_p] [\kappa a f'(h) - 1] \\ u''(c_0) [\kappa a f'(h) - 1] & \frac{\partial \mathbf{F}_{\text{PAPER}}^L}{\partial h} \end{bmatrix}$$

and $\mathbf{F}_{\text{PAPER}}^L$ is the expression in Appendix C of the paper that defines the first order conditions for the $G + L$ model when $\bar{d}^G = \varrho(a) = 0$. It can be directly verified that $|A| > 0$. In the numerator, the terms $-u''(c_0) [\kappa a f'(h) - 1] \{u''[c_p] \kappa f(h)\}$ and $[-(u''(c_p) + \theta u''(c_0))]$ are both positive. Therefore, a sufficient condition for $\frac{\partial h^L(a, w_p)}{\partial a} > 0$ is that the analogous expression in the model of our paper is positive (i.e. $\frac{\partial \mathbf{F}_{\text{PAPER}}^L}{\partial a} > 0$). It is straightforward to show that investment is strictly increasing in parental wealth, w_p , for constrained families.

BASILINE MODEL: PUBLIC AND PRIVATE LENDING WITH LIMITED COMMITMENT

From the GSL and private lending results just presented, it is evident that the analytical results for the baseline model in the paper can be extended to a model with endogenous parental transfers where both, private and public lending operate along the lines assumed in the paper. Define, as in the paper, the ability threshold \bar{a} as the maximum ability for which the GSL credit suffices by itself to finance the unconstrained level of investment, i.e. $\bar{a} f'(\bar{d}^G) = R$. Parallel arguments to those above indicate that Proposition 3 of the paper hold for w_p instead of w . In particular, over-investment for $a < \bar{a}$ can arise as poor parents and children seek to expand their access to private credit as a means to increase their consumption.

In sum, we have shown that our results regarding the implied relationship between investment and ability are robust to the introduction of altruistic parents that endogenously determine w . This is important since “altruistic parents” is the leading economic model for parental transfers and bequests. All qualitative results with respect to initial wealth in the paper hold in this framework with respect to parental wealth, w_p .

We now briefly consider two other models of parental transfers popular in the literature.

WARM-GLOW PREFERENCES

A common (and simpler) alternative assumption in the literature is that parents do not value the utility of children but instead assign value to the amount of resources they transfer to them. In this model, the utility of parents is given by $U^P = u(c_p) + v(b)$, where v is a strictly increasing and strictly concave function of bequests, b .

The problem of the parent is simply to maximize U^P , which leads to the bequest function $b^*(w_p)$. If Inada conditions for $u(\cdot)$ and $v(\cdot)$ hold, optimal bequests satisfy $u'(w_p - b) =$

$v'(b)$, and bequests $b^*(w_p)$ are strictly increasing in w_p and independent of ability.⁴ Since w_p determines w , the economic problems of the child (under all forms of constraints) are exactly the same as in the paper.

All results in our paper, without qualification, hold in this model, because bequests do not respond to ability a or to the lending opportunities faced by the child. Since bequests are increasing in parental wealth, all of our qualitative results regarding the youth's initial wealth hold for both the child's wealth/bequests as well as parental wealth, w_p .

PATERNALISTIC PREFERENCES

Another commonly used intergenerational model assumes that parents take pride in the schooling attainment of their children. Parental preferences will be given $U^p = u(c_p) + v(h)$, where v is an increasing and concave function. In this case, parental transfers depend positively on parental wealth/income w_p and the child's schooling expenditures h . Keane and Wolpin (2001) essentially estimate a model consistent with this assumption.

A natural assumption in this model is that parents fully decide on the investment on their child. If so, human capital investments would be given by the condition $u'(w_p - h) = v'(h)$. In this case, human capital investments would be fully pinned down by the wealth of the family w_p and *completely unrelated to the child's ability*. Given that the child's ability is probably correlated to the parent's ability, then the unconditional correlation between ability and investment is positive. However, once we control for familial resources, as we do in the paper, then the correlation should be zero. This is clearly at odds with the evidence presented in our paper as well in a huge literature on schooling decisions. Moreover, the model would imply that investment should always be increasing in the parent's wealth, even for unconstrained individuals. This is also in stark contrast with the evidence. Therefore, even if the assumption of fully parental control of h is the natural one for pre-, primary and even perhaps secondary schooling, it is a bad assumption if we are thinking about college and beyond.

More interestingly, let us assume that investment in human capital is the result of a simultaneous-moves game between the parent and the child. The parent influences the human capital decisions of the child by making transfers and the child influences the transfer decision-making of the parents by investing in schooling, i.e. what the parent cares about. The problem of parents is to allocate their initial wealth w_p between their own consumption c_p and transfers $\tau \geq 0$ to their child. The problem of children is to allocate their resources (foregone earnings w_0^{fe} plus parental transfers τ) between their own consumption c_0 and investment h . Given the parent's wealth w_p and the child's ability a , the equilibrium in this model is described by a pair of functions $\tau(h; w_p, a)$ and $h(\tau; w_p, a)$ that indicate, respectively, the best response of transfers to the investment of the child and the best response of investments to the transfers of the parent.

We argue that our results continue to hold in this formulation under an empirically plausible restriction on $v(\cdot)$. To this end, let x denote the investments directly borne by the child. In terms of x , an equilibrium is a pair of functions $\{\tau(x), x(\tau)\}$ of best responses for parents and children, respectively. Parents make transfers $\tau = \tau(x)$, and children invest $x(\tau)$. Given $\tau(x)$, total human capital investments are $h = x + \tau(x)$. The period $t = 0$ budget constraint for the child is $c_0 + x \leq w_0^{fe} + d$, and his second period earnings are $y = af[x + \tau(x)] \equiv ag(x)$. The formulation of the game in terms of x is equivalent to the formulation in terms of h as long as $x > 0$, which is the case whenever some component of investment (e.g. the child's own time) is non-transferable.

⁴If $v(\cdot)$ does not satisfy Inada conditions and youth have some initial wealth of their own (say from foregone earnings), then bequests can be zero for poorer households and positive for richer households. In this case, $b^*(w_p)$ is still entirely pinned down by parental wealth w_p and does not depend on the child's ability, investments, future consumption or borrowing.

As long as the resulting function $g(\cdot)$ is increasing and concave (it must be so around the point of optimality), this problem is the same as that of our paper where we now replace total investment h with investment borne by the child x . Therefore, our comparative static results for the ability – investment relationship under exogenous borrowing constraints (EXC) and under private lenders with limited commitment (LC) apply for x . If the GSL is defined such that youth cannot borrow more than they invest themselves (subject to upper limit d_{max}), those results go through as well. Moreover, as long as h is increasing in x , our results on ability also hold for total investment, h . This will be the case as long as $\tau'(x) > -1$ (i.e., at the margin, parents do not reduce their children's consumption when they increase their investment).

In sum, we show that simple and reasonable conditions on the behavior of transfers ensure that key results in the paper can be extended to parental wealth w_p .

B. Cost of investment dependent on ability

In the paper, we assume that the (marginal) cost of investment is invariant to the ability of the individual. Here, we examine whether our main results regarding the ability – investment relationship are robust to relaxing this assumption. A **decreasing cost** can arise, because more able students receive more fellowships and other forms of aid. In addition, more able students may find studying more enjoyable or easier. In these cases, the full cost (pecuniary and non-pecuniary) of investing h is decreasing in ability a . By contrast, an **increasing cost** arises when time is an important input in human capital production and the value of time is increasing in ability a (i.e. foregone earnings).

We first explore the implications of allowing the cost of investment to depend on ability. Then, we discuss the empirically relevant shape of the investment cost function, and whether our conclusions about the role of ability under exogenous constraints in the paper are warranted.

IMPLIED BEHAVIOR OF INVESTMENT

Assume that an individual with ability a that invests h units in human capital bears a total cost equal to $g(a)h$. The marginal (and average) cost of each unit invested is $g(a) > 0$, which may be increasing, decreasing, or constant in a .

UNRESTRICTED ALLOCATIONS

The unrestricted investment amount $h^U(a)$ is defined by $g(a) = R^{-1}af'(h)$, and

$$(6) \quad h^U(a) = f'^{-1} \left[\frac{Rg(a)}{a} \right].$$

Unlike our baseline model, unconstrained investment can no longer be determined from the earnings function alone. It will also depend on the cost function $g(a)$, which may or may not be observable.

If we define $\varepsilon \equiv \frac{ag'(a)}{g(a)}$, the cost elasticity of investment, then it is natural to distinguish between a few different cases. If costs are very elastic to ability, $\varepsilon \geq 1$, then unconstrained investment is decreasing in ability, an empirically uninteresting case. Our baseline model implicitly assumes $\varepsilon = 0$. A modest positive elasticity, $0 < \varepsilon < 1$ yields a positive relationship between unconstrained investment and ability, but one which is weaker than our baseline model. A negative elasticity, $\varepsilon < 0$, yields a stronger ability – unconstrained investment relationship than our baseline case. Not surprisingly, the latter case will also tend to create a positive ability – investment relationship with exogenous constraints under more general conditions than our baseline model. In all other cases, we are more likely to observe a negative relationship between ability and investment than in our baseline model.

EXOGENOUS BORROWING CONSTRAINTS: $d \leq \bar{d}^X$.

When the exogenous constraint binds, individuals

$$\max_h \left\{ u \left[w + \bar{d}^X - g(a)h \right] + \beta u \left[af(h) - R\bar{d}^X \right] \right\}.$$

The first order condition is $F \equiv -u'(c_0)g(a) + \beta af'(h)u'(c_1) = 0$. Using implicit differentiation, we note that $\text{sign} \left\{ \frac{\partial h^X}{\partial a} \right\} = \text{sign} \left\{ \frac{\partial F}{\partial a} \right\}$. After simplifying:

$$(7) \quad \frac{\partial F}{\partial a} = \beta f'(h)u'(c_1) \left\{ 1 - \varepsilon + \frac{1}{\eta(c_0)} \left(\frac{\partial c_0}{\partial a} \frac{a}{c_0} \right) - \frac{1}{\eta(c_1)} \left(\frac{\partial c_1}{\partial a} \frac{a}{c_1} \right) \right\},$$

where $\eta(c_i) \equiv -u'(c_i) / [c_i u''(c_i)]$ is the IES and ε is the cost elasticity of investment as defined above.

In the baseline model, $\varepsilon = 0$ and $\frac{\partial c_0}{\partial a} = 0$. Therefore, $\frac{\partial F}{\partial a} = \beta f'(h)u'(c_1) \left\{ 1 - \frac{1}{\eta(c_1)} \left(\frac{\partial c_1}{\partial a} \frac{a}{c_1} \right) \right\}$. Since $\frac{\partial c_1}{\partial a} \frac{a}{c_1} = \frac{af(h)}{af(h) - R\bar{d}^X} \geq 1$, a sufficient condition for $\frac{\partial F}{\partial a} < 0$ is $\eta(c_1) < 1$, as stated in the paper.

For the general case, observe that $\frac{\partial c_0}{\partial a} \frac{a}{c_0} = \frac{-g'(a)h}{w + \bar{d}^X - g(a)h} a = -\varepsilon \left(\frac{g(a)h}{w + \bar{d}^X - g(a)h} \right)$. If $\varepsilon > 0$ (i.e. cost of investment is increasing in ability as with foregone earnings) then $\frac{\partial c_0}{\partial a} \frac{a}{c_0} < 0$ and the perverse ability – investment relationship of Proposition 1 holds more strongly than in our baseline model. When $IES < 1$ investments decrease at a faster rate with ability than in our baseline model; when $IES \geq 1$ and our baseline exogenous constraints model can imply a positive relationship, an extended model with $\varepsilon > 0$ can lead to a negative one.⁵ In general, the more the investment costs increase in ability, the higher the IES must be for the exogenous constraint model to predict a positive relationship.

Unfortunately, no general analytical results for $\frac{\partial h^X}{\partial a}$ can be obtained when $\varepsilon < 0$. As such, we numerically explore the implied ability – investment relationship in this case by extending the *quantitative* model of the paper to allow for general costs of investment, $g(a) = (a/a_{\text{low}})^\varepsilon$ for a fixed value ε . We take all other parameter values from the baseline calibration. Here a_{low} is an arbitrary lower bound on which the marginal cost of investment is normalized to one. Our baseline model in the paper is $\varepsilon = 0$ because $g(a) = 1$ for all ability levels a .

Figure 1 shows the behavior of constrained investment for our baseline case $\varepsilon = 0$, two cases in which investment costs decrease with ability ($\varepsilon \in \{-0.5, -0.25\}$), and two cases in which investment costs increase with ability ($\varepsilon \in \{0.25, 0.5\}$).⁶ As already discussed, for values of $\varepsilon \leq 0$, we obtain a negative ability – investment relationship (when the constraint binds) given our IES of 0.5. More interestingly, we also obtain a strong negative relationship for $\varepsilon = -0.25$. Only when the cost elasticity of investment to ability falls to $\varepsilon = -0.5$ do we obtain a roughly flat (but still negative) ability – investment relationship. In sum, investment costs must be very strongly declining in ability to overcome the desire that constrained individuals have to smooth consumption.

Of course, such a strong negative elasticity of investment costs to ability also has significant effects on the relationship between ability and *unconstrained* investment levels. Indeed, $\varepsilon =$

⁵ If $\varepsilon \geq 1$, then $\frac{\partial h^X}{\partial a} < 0$ for any IES, while for $0 < \varepsilon < 1$ a sufficient condition for $\frac{\partial h^X}{\partial a} < 0$ is $\eta(c_1) \leq \frac{1}{1-\varepsilon}$.

⁶ For illustration purposes, we have extended the range of abilities considered so that a_{low} is half the estimated value for the lowest AFQT quartile (i.e. $a_{\text{low}} = 1/2 \times a_1$). The maximum is twice the estimated value for the highest quartile (i.e. $2 \times a_4$).

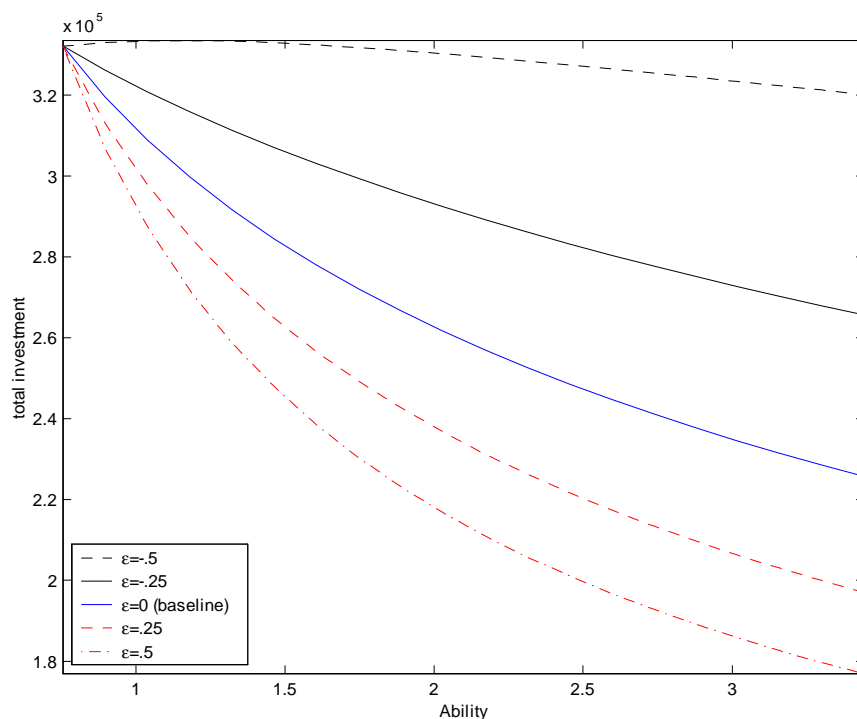


FIGURE 1. EXOGENOUS CONSTRAINTS MODEL: IMPLIED CONSTRAINED INVESTMENT-ABILITY RELATIONSHIP FOR THE MODEL EXTEND WITH $g(a) = \varepsilon_0 a^\varepsilon$ FOR $\varepsilon \in \{-.5, .25, 0, .25, .5\}$

−0.5 implies that *unconstrained* investment for the highest AFQT quartile should be 23 times the investment of the lowest AFQT quartile. Such ratio is unrealistically high. The observed ratio in the NLSY79 is much lower, only 12.4, and much closer to the implied ratio of 11.5 in our calibrated model.

DISCUSSION

The baseline model in the paper assumes investment costs are independent of ability. To the extent that foregone earnings are increasing in ability, this suggests investment costs should be increasing in ability. We show that this only strengthens our finding that an exogenous constraint model predicts a negative ability – investment relationship among constrained youth.⁷

Alternatively, if the marginal cost of ability is strongly decreasing in ability, either because smarter individuals find it easier to study or because they receive merit-based aid, it is theoretically possible to obtain a positive ability – investment relationship among constrained youth. Empirically, however, this seems extremely unlikely.

First, there is little evidence to suggest that the direct costs of investment are systematically decreasing in ability. If anything, merit aid is only likely to be important for the very high end of the ability distribution, while it is very low throughout the rest of the distribution. In the 1999-2000 academic year, fewer than 10% of all students at public post-secondary institutions

⁷For similar reasons, this would also tend to weaken any positive effects of ability on investment when individuals are constrained by private lenders as modeled in the paper.

received non-need based grants from the state or their institutions (Heller, 2003). Comparing this against our estimated 50 percentage point gaps in college attendance rates between the highest and lowest ability quartiles, it is clear that merit aid explains very little, if any, of the observed positive ability – schooling relationship. Although, merit aid is only relevant at the top end of the ability distribution, we observe a strong positive relationship between ability and schooling even at the bottom end. In sum, the data largely supports our assumption of a uniform net tuition cost of investment, at least for all except the very brightest.

Second, we have doubts on the quantitative importance of pure non-pecuniary factors (schooling in the utility function) because of their implications for measured returns to human capital investments. If all individuals are unconstrained (as what seems happened in the NLSY79), then measured marginal returns of human capital must be declining in ability. Indeed, when we take $\varepsilon = -0.5$ in our otherwise calibrated model, the returns to investments of the highest AFQT quartile are only 90% of the returns of the least able. Such implication seems at odds with the empirical evidence.

Assuming constant marginal cost of investment, our model is consistent with the fact that more able people acquire more skills per unit of time spent investing, and our calibration in the paper reflects this. If more able people enjoyed school much more than less able people, the estimated returns from earnings for more relative to less able individuals should not be enough to generate the differences in schooling we observe by ability. That is, there should be much larger gaps in schooling by ability than our model can account for through earnings differences alone if cost differences by ability were important.

Finally, we show that the elasticity of the cost with respect to ability would need to be very strong to generate a non-negative ability – investment relationship in the exogenous constraint model. In our opinion, such a strong negative elasticity is implausible, since it also implies a much stronger ability – investment relationship for unconstrained youth than we observe in the NLSY data.

C. Generalizing the human capital production function

To explore the importance of our assumptions about the human capital production function, we can generalize post-school earnings (in the two-period model) as follows: let $y = f(a, h)$, where $\frac{\partial f}{\partial h} > 0$, $\frac{\partial^2 f}{\partial h^2} < 0$, $\frac{\partial f}{\partial a} > 0$, and $\frac{\partial^2 f}{\partial h \partial a} > 0$. Otherwise, consider the same problem as in the paper.

Without borrowing constraints, optimal investment solves $\frac{\partial f}{\partial h} = R$ and optimal unconstrained investment, $h^U(a)$, is an increasing function of ability and is independent of w . As the following analysis shows, the elasticity of substitution between investment and ability in the production of human capital and the consumption intertemporal elasticity of substitution (IES) play key roles in determining the relationship between ability and investment for constrained borrowers. With preferences defined by a constant intertemporal elasticity of substitution (elasticity η) and a CES human capital production function (elasticity of substitution, ϕ), we obtain a negative ability investment relationship for youth constrained by exogenous borrowing constraints if $\eta < \phi$. In the case of endogenous constraints generated by limited commitment, we obtain a positive ability – investment relationship if $\eta > (1 - R\kappa)\phi$, so there are parameterizations that imply a negative ability – investment relationship under exogenous constraints but not under endogenous constraints. Under the GSL, investment behaves largely as discussed in the text. The discussion below provides a more detailed characterization of investment behavior assuming a general human capital production function.

When ability and investment are strong complements (i.e. $\phi < 1$), it may be possible to obtain a positive ability – investment relationship under exogenous constraints when the IES $\eta < 1$. The vast majority of empirical and theoretical studies on schooling, ability, and earnings assume a multiplicatively separable relationship between schooling and ability. This assumption fits our

data quite well, and we are not aware of any compelling evidence that suggests serious departures from this assumption. As such, the paper assumes multiplicative separability, which is equivalent to the more general case presented here with $\phi = 1$.

EXOGENOUS BORROWING CONSTRAINTS

Now, consider the exogenous borrowing constraint $d \leq \bar{d}^X$. Imposing the borrowing constraint, $d = \bar{d}^X$, constrained borrowers $\max_h u(w + \bar{d}^X - h) + \beta u(f(a, h) - R\bar{d}^X)$.

This yields FOC for investment:

$$-u'(w + \bar{d}^X - h) + \beta u'(f(a, h) - R\bar{d}^X) \frac{\partial f}{\partial h} = 0,$$

which implicitly defines optimal investment for constrained borrowers, $h^X(a, w)$, as a function of ability and initial assets. It is straightforward to show that investment is increasing in initial wealth, w .

Using implicit differentiation, one can show that for constrained persons

$$\begin{aligned} \text{sign} \left\{ \frac{dh^X}{da} \right\} &= \text{sign} \left\{ u''(c_2) \left(\frac{\partial f}{\partial h} \right) \left(\frac{\partial f}{\partial a} \right) + u'(c_2) \frac{\partial^2 f}{\partial h \partial a} \right\} \\ (8) \quad &= \text{sign} \left\{ \eta(c_2) - \left[\frac{\left(\frac{\partial f}{\partial h} \right) \left(\frac{\partial f}{\partial a} \right)}{f(a, h) \left(\frac{\partial^2 f}{\partial h \partial a} \right)} \right] \left(\frac{f(a, h)}{f(a, h) - R\bar{d}^X} \right) \right\}. \end{aligned}$$

where $\eta(c_2) = \frac{-u'(c_2)}{c_2 u''(c_2)}$ is the consumption intertemporal elasticity of substitution (at c_2).

Thus,

$$\frac{dh^X}{da} < 0 \quad \Leftrightarrow \quad \eta(c_2) < \left[\frac{\left(\frac{\partial f}{\partial h} \right) \left(\frac{\partial f}{\partial a} \right)}{f(a, h) \left(\frac{\partial^2 f}{\partial h \partial a} \right)} \right] \left(\frac{f(a, h)}{f(a, h) - R\bar{d}^X} \right).$$

For $\bar{d}^X \geq 0$, $f(a, h) \geq f(a, h) - R\bar{d}^X$ and

$$(9) \quad \frac{dh^X}{da} < 0 \quad \text{if} \quad \eta(c_2) < \frac{\left(\frac{\partial f}{\partial h} \right) \left(\frac{\partial f}{\partial a} \right)}{\left(\frac{\partial^2 f}{\partial h \partial a} \right) f(a, h)}.$$

If $f(a, h)$ is of CES form with elasticity of substitution ϕ , then

$$\frac{dh^X}{da} < 0 \quad \text{if} \quad \eta(c_2) < \phi.$$

The case in our paper is equivalent to this specification with $\phi = 1$.

GSL SYSTEM

Investment behaves as in the paper (where the new $h^X(a, w)$ replaces that of the text).

PRIVATE LENDERS WITH LIMITED COMMITMENT

Now, consider our private lending constraint: $d \leq \kappa f(a, h)$. Those constrained by this endogenous borrowing limit solve the following maximization problem:

$$\max_h \{u(w + \kappa f(a, h) - h) + \beta u[(1 - R\kappa)f(a, h)]\}.$$

This problem yields the FOC for investment:

$$F(h, a) \equiv u'(c_1) \left[\kappa \left(\frac{\partial f}{\partial h} \right) - 1 \right] + \beta u'(c_2)(1 - R\kappa) \left(\frac{\partial f}{\partial h} \right) = 0,$$

which implicitly defines optimal investment, $h^L(a, a)$, for constrained borrowers.

One can easily show that investment is increasing in initial assets, a . Regarding ability, one can show that $\text{sign}\left\{\frac{dh^L}{da}\right\} = \text{sign}\left\{\frac{\partial F}{\partial a}\right\}$. Note

$$\begin{aligned} \frac{\partial F}{\partial a} &= u''(c_1)\kappa \left(\frac{\partial f}{\partial a} \right) \left[\kappa \left(\frac{\partial f}{\partial h} \right) - 1 \right] + u'(c_1)\kappa \left(\frac{\partial^2 f}{\partial h \partial a} \right) \\ &+ \beta \left[u''(c_2)(1 - R\kappa)^2 \left(\frac{\partial f}{\partial h} \right) \left(\frac{\partial f}{\partial a} \right) + u'(c_2)(1 - R\kappa) \left(\frac{\partial^2 f}{\partial h \partial a} \right) \right]. \end{aligned}$$

Dividing this through by $-\beta(1 - R\kappa)c_2u''(c_2)\frac{\partial^2 f}{\partial h \partial a}$ and re-arranging terms:

$$\begin{aligned} &\text{sign} \left\{ \frac{dh^L}{da} \right\} \\ &= \text{sign} \left\{ \eta(c_2) - \left[\frac{\left(\frac{\partial f}{\partial h} \right) \left(\frac{\partial f}{\partial a} \right)}{f(a, h) \left(\frac{\partial^2 f}{\partial h \partial a} \right)} \right] + \frac{u''(c_1)\kappa \left(\frac{\partial f}{\partial a} \right) \left[1 - \kappa \left(\frac{\partial f}{\partial h} \right) \right]}{\beta(1 - R\kappa)c_2u''(c_2) \left(\frac{\partial^2 f}{\partial h \partial a} \right)} + \frac{-u'(c_1)\kappa}{\beta(1 - R\kappa)c_2u''(c_2)} \right\}, \end{aligned}$$

where we use the definition of $\eta(c_2)$ and $c_2 = (1 - R\kappa)f(a, h)$. Because $u'(c_1) > \beta R u'(c_2)$ when the borrowing constraint binds, the third term is positive and

$$\text{sign} \left\{ \frac{dh^L}{da} \right\} > 0 \quad \text{if} \quad \eta(c_2) > (1 - R\kappa) \left[\frac{\left(\frac{\partial f}{\partial h} \right) \left(\frac{\partial f}{\partial a} \right)}{f(a, h) \left(\frac{\partial^2 f}{\partial h \partial a} \right)} \right].$$

Clearly, there are parameterizations where $\frac{dh^L}{da} > 0$ but $\frac{dh^X}{da} < 0$, so this model can produce a positive relationship between investment and ability in cases where the exogenous constraint model does not.

If $f(a, h)$ is CES with elasticity of substitution ϕ ,

$$\text{sign} \left\{ \frac{dh^L}{da} \right\} > 0 \quad \text{if} \quad \eta(c_2) > (1 - R\kappa)\phi.$$

The case in the paper is equivalent to $\phi = 1$.

V. References

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