Online Appendix for "A Numerical Investigation of the Potential for Negative Emissions Leakage"

By Niven Winchester and Sebastian Rausch*

This online appendix lays out the equilibrium conditions for the stylized model used in Section I of the paper "A numerical investigation of the potential for negative emissions leakage" and presents graphs summarizing additional sensitivity analyses obtained from the large-scale general equilibrium model used in Section II of the paper. We also provide the source code of the stylized numerical general equilibrium model used in Section I of the paper for download in a zip archive.

Our stylized general equilibrium model is a modified version of the regional interpretation of the model presented in Fullerton, Karney and Baylis (2011). Two regions ($r = \{East, West\}$) produce a good Y combining two inputs indexed by $f = \{K, C\}$ with decreasing marginal products in a constant returns to scale production functions. Let K denote the clean input, and C represents carbon inputs. The clean input can be considered to be a composite of labor and capital, and is assumed to be in fixed supply. We distinguish two cases: one in which the clean input is mobile and one in which it is immobile across regions. In response to a carbon tax, a firm can reduce its carbon per unit of output by additional use of abatement technology, that is by substituting from C into K. Y can be traded at no costs, and demand for Y in each region is derived from maximizing homothetic utility by choosing Y_{East} and Y_{West} subject to an income constraint, taking as given all market prices. We compare the long run equilibrium after imposing a tax on carbon inputs in the West, ignoring adjustments during the transition. We solve the model for alternative values of the elasticity of supply for carbon inputs, including a perfectly elastic case.

We now present the equilibrium conditions of the generalized FKB (2012) model that is used to derive the results from the first part of the paper. We employ numerical methods to solve for general equilibrium prices and quantities. More specifically, we formulate the model as a system of nonlinear inequalities and represent the economic equilibrium through two classes of conditions: zero profit and market clearance. The former class determines activity levels and the latter determines price levels. In equilibrium, each of these variables is linked to one inequality condition: an activity level to an exhaustion of product constraint and a commodity price to a market clearance condition. Following Mathiesen (1985) and Rutherford (1995), we formulate the model as a mixed complementarity problem.[1]

In equilibrium, activity levels for Y_r are determined by the following set of zero-

* Winchester: Joint Program on the Science and Policy of Global Change, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139 (e-mail: niven@mit.edu); Rausch: Department of Management, Technology and Economics, Zurichbergstrasse 18, 8032 Zurich, Switzerland (e-mail: srausch@ethz.ch). We are grateful to Don Fullerton, Samuel S. Kortum and session participants at the 2013 American Economic Association Annual Meeting for helpful suggestions. The Joint Program on the Science and Policy of Global Change is funded by the US Department of Energy and a consortium of government and industrial sponsors (for the complete list see http://globalchange.mit.edu/sponsors/all).

[1]A complementary-based approach has been shown to be convenient, robust, and efficient (Mathiesen, 1985; Rutherford, 1995). A characteristic of many economic models is that they can be cast as a complementary problem, i.e. given a function $F\colon \mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$, or, in short-hand notation, $F(z) \geq 0 \perp z \geq 0$. The complementarity format embodies weak inequalities and complementary slackness, relevant features for models that contain bounds on specific variables, e.g. activity levels which cannot a priori be assumed to operate at positive intensity. Numerically, we solve the model in GAMS using the PATH solver (Dirkse and Ferris, 1995).

profit conditions:

$$\Pi_r^Y = p_r^Y - \left(\sum_f \theta_{f,r}^Y w_{f,r}^{1-\sigma_r^Y}\right)^{1/(1-\sigma_r^Y)} \leq 0$$

$$\perp \quad Y_r \geq 0 \tag{1}$$

where p_r^Y, $\theta_{f,r}^Y$, $w_{K,r}$ denote output price indexes, the benchmark value share, and the price for the clean input, respectively. σ_r^Y represents the elasticity of substitution between K and C, i.e. a measure of the technological ease with which to abate carbon emissions, everything else equal. Let n_r denote the price for the carbon input "carbon emissions". The price gross of the carbon tax τ_r, $w_{C,r}$ is then given by:

$$w_{C,r} = n_r(1+\tau_r) \quad \perp \quad w_{C,r} \geq 0\,. \tag{2}$$

We consider the following case: $\tau_{West} > 0$ and $\tau_{East} = 0$.

Utility in each region is "produced" by combining locally-produced and imported varieties of Y. The level of utility is determined in equilibrium by the following conditions:

$$\Pi_r^U = p_r^U - \left(\sum_{r'} \theta_{r'}^U p^Y r'^{1-\sigma^C}\right)^{1/(1-\sigma^C)} \leq 0$$

$$\perp \quad U_r \geq 0 \tag{3}$$

where σ^C represents the elasticity of substitution between locally-produced and imported varieties, and θ_r^U denote the respective value shares.

Assuming that the revenue from the carbon tax is returned lump-sum, the income of the representative consumer in each region is given by:

$$M_r = w_{K,r}\overline{E}_r + n_r(1+\tau_r)\,\zeta_r \tag{4}$$

where $\overline{E}_r$ denotes the fixed supply of K and ζ_r is the supply of the carbon input, which is determined endogenously by targeting the price elasticity of fossil fuel supply, η. Note that this formulation assumes a constant elasticity supply function. If $\eta < \infty$, then this condition is given by:

$$\frac{\zeta_r - 1}{n_r - 1} = \eta \quad \perp \quad \zeta_r \geq 0\,. \tag{5}$$

If the supply of fossil fuels is perfectly elastic, i.e. $\eta = \infty$, then:

$$n_r = 1 \quad \perp \quad \zeta_r \geq 0\,. \tag{6}$$

Note that we calibrate the model such that initially all prices and quantities are equal to one. Hence, in Eq. (5) the left-hand side is simply the percentage change in the quantity of fossil fuel supplied over the percentage change in the price for fossil fuel, i.e. the price elasticity of supply.

Using Shephard's Lemma, differentiating the unit profit function with respect to input prices provides compensated demand coefficients, which appear subsequently in the market clearance conditions. The equilibrium price for clean inputs is determined by:

$$\overline{E}_r \geq \frac{\partial \Pi_r^Y}{\partial w_{K,r}} Y_r \quad \perp \quad w_{K,r} \geq 0\,. \tag{7}$$

The following condition determines the equilibrium price for carbon inputs:

$$\overline{E}_{C,r}\,\zeta_r \geq \frac{\partial \Pi_r^Y}{\partial n_r} Y_r \quad \perp \quad n_r \geq 0\,. \tag{8}$$

Locally non-satiated preferences imply that all income is exhausted, and hence the market for utility clears if:

$$U_r \geq \frac{M_r}{p_r^U} \quad \perp \quad p_r^U \geq 0\,. \tag{9}$$

In the equations above, we have represented the case where the clean input is immobile across regions. We also consider the case where K is mobile across regions, hence implying a uniform rental price for K, w_K. In this version of the model, Eq. (7) is replaced by:

$$\sum_r \overline{E}_r \geq \sum_r \frac{\partial \Pi_r^Y}{\partial w_K} Y_r \quad \perp \quad w_K \geq 0\,. \tag{10}$$

and $w_{K,r}$ in equations (1) and (4) is replaced by w_K.

REFERENCES

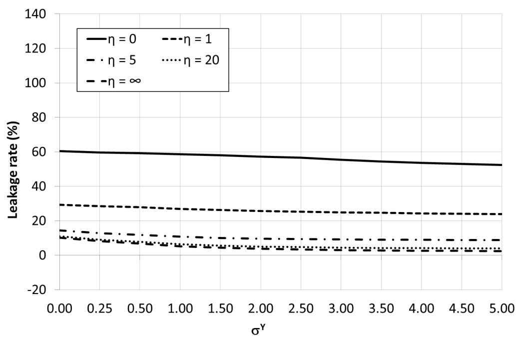
Dirkse, S. P., and M. C. Ferris. 1995. "The PATH Solver: a Non-monontone Stabilization Scheme for Mixed Complementarity Problems." *Optimization Methods and Software*, 5: 123–156.

Fullerton, Don, Dan Karney, and Kathy Baylis. 2011. "Negative Leakage." CESinfo Working Paper No. 3379.

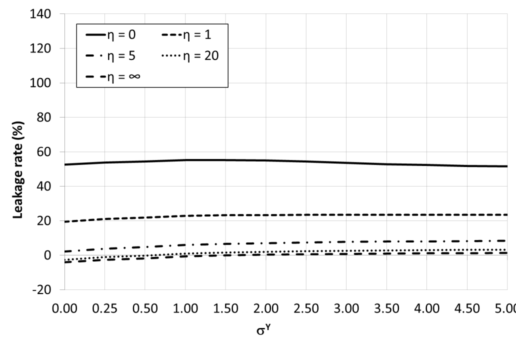
Fullerton, Don, Dan Karney, and Kathy Baylis. 2012. "Negative Leakage." Unpublished manuscript, University of Illinois at Urbana-Champaign.

Mathiesen, Lars. 1985. "Computation of Economic Equilibria by a Sequence of Linear Complementarity Problems." *Mathematical Programming Study*, 23: 144–162.

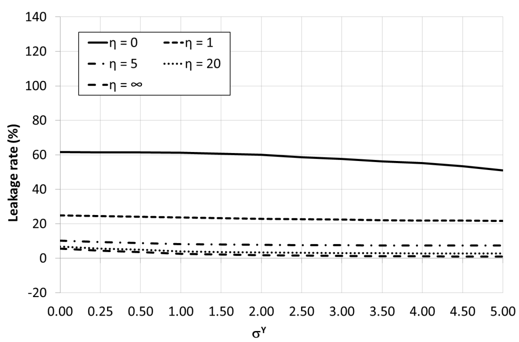
Rutherford, Thomas F. 1995. "Extension of GAMS for Complementarity Problems arising in Applied Economics." *Journal of Economic Dynamics and Control*, 19(8): 1299–1324.



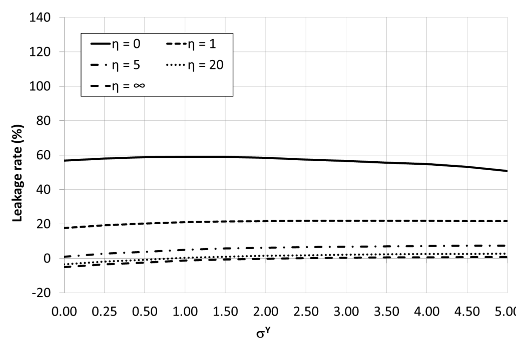
(a) Full factor mobility, base case trade elasticities.



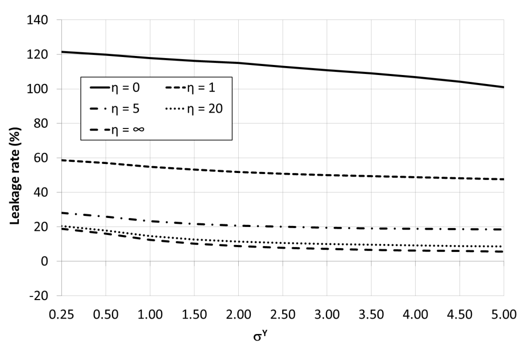
(b) No factor mobility, base case trade elasticities.



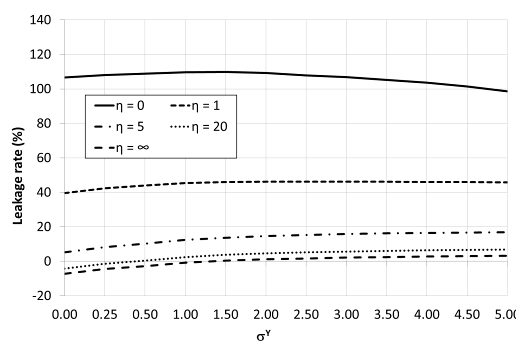
(c) Full factor mobility, low trade elasticities.



(d) No factor mobility, low trade elasticities.



(e) Full factor mobility, high trade elasticities.



(f) No factor mobility, high trade elasticities.

Figure 1. USREP model: Leakage for alternative trade elasticity values (%)