

Web Appendix for:
Monetary-Fiscal Policy Interactions and Indeterminacy
in Post-War U.S. Data

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Approximate model

Detrend

The technology process A_t induces a common trend in output Y_t , consumption C_t , real wage $\frac{\tilde{w}_t(i)}{P_t}$, government purchases G_t , government debt B_t/P_t , tax revenues T_t , and transfers S_t . In addition, introducing non-zero steady state inflation also creates a trend in nominal prices. Since we will solve the model through a local approximation of its dynamics around a steady state, we first detrend the variables as

$$\tilde{Y}_t \equiv \frac{Y_t}{A_t}, \quad \tilde{C}_t \equiv \frac{C_t}{A_t}, \quad \text{and} \quad \frac{\tilde{w}_t(i)}{P_t} \equiv \frac{w_t(i)}{P_t A_t}.$$

Note that the fiscal variables, $b_t = B_t/P_t Y_t$, $g_t = G_t/Y_t$, $\tau_t = T_t/Y_t$, and $s_t = S_t/Y_t$ are already stationary. We then rewrite the equilibrium conditions in terms of the detrended variables, compute the non-stochastic steady state, and then take a first-order approximation around the steady state.

First order approximation

We define the log deviations of a variable X_t from its steady state \bar{X} as $\hat{X}_t = \ln X_t - \ln \bar{X}$, except for four fiscal variables: $\hat{b}_t = b_t - \bar{b}$, $\hat{g}_t = g_t - \bar{g}$, $\hat{\tau}_t = \tau_t - \bar{\tau}$, and $\hat{s}_t = s_t - \bar{s}$. We denote the mean growth rate of the technology shock $a_t \equiv A_t/A_{t-1}$ by a . The approximated model equations are then given by

$$\begin{aligned} \hat{C}_t &= \frac{a}{a+\eta} E_t \hat{C}_{t+1} + \frac{\eta}{a+\eta} \hat{C}_{t-1} - \frac{a-\eta}{a+\eta} \left(\hat{R}_t - E_t \hat{\pi}_{t+1} \right) + \frac{a}{a+\eta} E_t \hat{a}_{t+1} - \frac{\eta}{a+\eta} \hat{a}_t + \frac{a-\eta}{a+\eta} \hat{d}_t \\ \hat{\pi}_t &= \frac{\beta}{1+\gamma\beta} E_t \hat{\pi}_{t+1} + \frac{\gamma}{1+\gamma\beta} \hat{\pi}_{t-1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\varphi\theta)(1+\gamma\beta)} \left[\left(\varphi + \frac{a}{a-\eta} \right) \left(\hat{Y}_t - \hat{Y}_t^* \right) - \frac{\eta}{a-\eta} \left(\hat{Y}_{t-1} - \hat{Y}_{t-1}^* \right) \right] + \hat{u}_t \\ \hat{R}_t &= \rho_R \hat{R}_{t-1} + (1-\rho_R) \left[\phi_\pi (\hat{\pi}_t - \hat{\pi}_t^*) + \phi_Y \left(\hat{Y}_t - \hat{Y}_t^* \right) \right] + \varepsilon_{R,t} \\ \hat{\tau}_t &= \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau) \left[\psi_b \left(\hat{b}_{t-1} - \hat{b}_{t-1}^* \right) + \psi_Y \left(\hat{Y}_t - \hat{Y}_t^* \right) + \psi_g \hat{g}_t \right] + \varepsilon_{\tau,t} \\ \hat{b}_t &= \beta^{-1} \hat{b}_{t-1} + \beta^{-1} \bar{b} \left(\hat{R}_{t-1} - \hat{\pi}_t - \hat{Y}_t + \hat{Y}_{t-1} - \hat{a}_t \right) + \hat{g}_t - \hat{\tau}_t + \hat{s}_t \\ \hat{Y}_t^* &= \frac{\eta}{\varphi(a-\eta) + a} \hat{Y}_{t-1}^* + \frac{a}{[\varphi(a-\eta) + a](1-g)} \hat{g}_t - \frac{\eta}{[\varphi(a-\eta) + a](1-g)} \hat{g}_{t-1} - \frac{\eta}{\varphi(a-\eta) + a} \hat{a}_t. \\ \hat{Y}_t &= \hat{C}_t + \frac{1}{1-g} \hat{g}_t \\ \hat{g}_t &= \rho_g \hat{g}_{t-1} + \varepsilon_{g,t} \\ \hat{d}_t &= \rho_d \hat{d}_{t-1} + \varepsilon_{d,t} \\ \hat{a}_t &= \rho_a \hat{a}_{t-1} + \varepsilon_{a,t} \\ \hat{u}_t &= \rho_u \hat{u}_{t-1} + \varepsilon_{u,t} \\ \hat{s}_t &= \rho_s \hat{s}_{t-1} + \varepsilon_{s,t} \\ \hat{\pi}_t^* &= \rho_\pi \hat{\pi}_{t-1}^* + \varepsilon_{\pi,t} \\ \hat{b}_t^* &= \rho_b \hat{b}_{t-1}^* + \varepsilon_{b,t} \end{aligned}$$

where we have defined two scaled shocks

$$\begin{aligned}\hat{d}_t &= (1 - \rho_\delta) \hat{\delta}_t \\ \hat{u}_t &= -\frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha(1 + \varphi\theta)(1 + \gamma\beta)} \frac{1}{\theta - 1} \hat{\theta}_t.\end{aligned}$$

Solution and estimation method

For details of the solution method under determinacy, see Sims (2002). For details of the solution method under indeterminacy, see Lubik and Schorfheide (2004). We follow the method of Lubik and Schorfheide (2004), except for one modification that we describe below.

To characterize the posterior distribution of the structural parameters, a Markov Chain Monte Carlo simulation is used. We first find a mode of the posterior density numerically using *csminwel* by Christopher A. Sims. Then we use a random-walk Metropolis algorithm to draw a sample from the posterior distribution. The proposal density of the random-walk Metropolis algorithm is a Normal distribution whose mean is the previous successful draw and variance is the inverse of the negative Hessian at the posterior mode found before the simulation. The variance of the proposal density is scaled to achieve an acceptance rate of around 30%. For details of the random-walk Metropolis algorithm, see An and Schorfheide (2007). Marginal likelihoods are estimated using the modified harmonic mean estimator by Geweke (1999).

An identification problem in Lubik and Schorfheide (2004)

Lubik and Schorfheide (2004) use left singular vectors corresponding to zero singular values of a matrix singular value decomposition (SVD) to characterize the full set of indeterminacy solutions, or multiple equilibria, of a linear rational expectations (LRE) model. However, these left singular vectors are not identified because their singular values are degenerate. This appears to cause numerical instability in their solution method. For example, small changes in parameter values can easily lead to a large change in the likelihood of a LRE model under indeterminacy.

Because of this problem, $\Gamma_{0,\zeta}^*$ in Eq. (1) of the main text is not well identified. Since in our model the degree of indeterminacy is at most one, $\Gamma_{0,\zeta}^*$ is simply a vector. We identify $\Gamma_{0,\zeta}^*$ by normalizing its first entry to its norm. With this normalization, posterior density maximization and simulation of our model is stable and works well. The normalization would affect the posterior distribution of the entries of the matrix M in Eq. (1). However, those parameters in M do not have behavioral interpretations. What matters is the additional channel for the propagation of the fundamental shocks, $\Gamma_{0,\zeta}^* M$, whose posterior distribution is not affected by the normalization if the prior distribution for the entries of M is flat. Although our baseline prior for the entries of M is not completely flat, it is very diffuse and the effect of the normalization is not significant. We tried different specifications for the prior distribution for the entries of M , including a uniform prior distribution over $(-5, 5)$ and our results were robust to these variations. The same argument applies to those parameters related to the sunspot shock ζ_t .

Data

Definitions and sources

We use the following definitions for our data variables: per capita output = (personal consumption of nondurable+personal consumption of services+government consumption) / civilian noninstitutional population; annualized inflation = $400 \times \Delta \log(\text{GDP deflator})$; annualized interest rates = the quarterly average of daily effective federal funds rates; tax revenues = current tax receipts + contributions for government social insurance; government debt = market value of privately held gross federal debt; and government purchases = government consumption. Note that we use a single price level, GDP deflator, for all the model variables (e.g. output, government debt, tax revenues, and government purchases).

The effective federal funds rate and civilian noninstitutional population data were obtained from the FRED database of Federal Reserve Bank of St. Louis. The market value of privately held gross federal debt series was obtained from Federal Reserve Bank of Dallas. All the other data were taken from National Income and Product Accounts (NIPA) tables.

Measurement equations

The measurement equations of our model are then given by:

$$\begin{aligned} 100 \times \Delta \log(\text{Real per capita output}) &= \hat{Y}_t - \hat{Y}_{t-1} + \hat{a}_t + \bar{a}^* \\ \text{Annualized inflation} &= 4\hat{\pi}_t + 4\bar{\pi}^* \\ \text{Annualized interest rates} &= 4\hat{R}_t + 4(\bar{a}^* + \bar{\mu}^* + \bar{\pi}^*) \\ \frac{\text{Nominal tax revenue}}{\text{Nominal output}} &= \hat{\tau}_t + \bar{\tau}^* \\ \frac{\text{Nominal government debt}}{\text{Nominal output}} &= \hat{b}_t + \bar{b}^* \\ \frac{\text{Nominal government purchases}}{\text{Nominal output}} &= \hat{g}_t + \bar{g}^* \end{aligned}$$

where the following relationships hold

$$\begin{aligned} \beta &= \frac{1}{1 + \frac{\bar{\mu}^*}{100}} \\ a &= 1 + \frac{\bar{a}^*}{100} \\ \bar{\pi} &= 1 + \frac{\bar{\pi}^*}{100} \\ \bar{b} &= \frac{\bar{b}^*}{100} \\ \bar{\tau} &= \frac{\bar{\tau}^*}{100}. \end{aligned}$$

Sample means

	pre-Volcker (1960:1-1979:2)	post-Volcker (1982:4-2008:2)
\bar{a}^*	0.48	0.57
$\bar{\pi}^*$	4.38/4	2.57/4
\bar{R}^*	5.47/4	5.45/4
$\bar{\tau}^*$	25	24.33
\bar{b}^*	35.97	48.46
\bar{g}^*	24.38	21.43

References

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