

# CONTRACT FORM, WAGE FLEXIBILITY AND EMPLOYMENT - APPENDIX

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ABSTRACT. In this appendix we provide the details of a model based on two uncontroversial hypotheses - firm productivity is expensive to measure and employment entails relationship-specific investments. These assumptions imply that firms would optimally choose fixed-wage contracts, and complement these with bonus pay when it is not too expensive to measure employee performance. We also derive a number of empirical implications of the model.

## 1. A MODEL OF EMPLOYMENT AND THE WAGE CONTRACT

In this appendix we outline the details for the model outlined in the text. The role of reliance/relationship specific investment is illustrated with a simple two period model. In period 1 the worker considers offers from two identical firms in a perfectly competitive labor market, indexed by  $J = \{A, B\}$ , and may also choose to enter unemployment/outside activity. Should the worker accept an offer from either firm she will pay a cost  $k^1$  in moving to the firm location, acquiring skills to do the job, and other investment activities. Should she decide in the second period to change firms or leave unemployment she would have to pay a mobility or investment cost  $k^2 > k^1$ . This assumption models specific investment - it is economically advantageous to contract in advance with a single firm.

However, all production and wage payments take place in period 2, and depending upon the realization of nature, it may be efficient to change jobs or enter unemployment at that time. Let us suppose that the productivity of firm  $i$  in period 2 is given by  $\theta_i + u$ , where  $\theta_i$  is an i.i.d. firm specific shock with distribution  $f(\theta)$ , and  $u$  is an economy wide shock with distribution  $g(u)$ . It is assumed that firm productivity is private information *ex post*, and hence wages cannot be conditioned upon firm productivity. Let  $\omega = \{\theta_A, \theta_B, u\}$  denote the *ex post* productivity state, and for notational convenience let  $h(\omega) = f(\theta_A) f(\theta_B) g(u)$  denote the joint density.

Let us further suppose that firm  $A$  can offer bonus pay that is a function of a costly measure  $s \in \{0, 1\}$ , that is 1 with probability  $P(\theta_A + u)$  at the end of period, where  $P$  is increasing with  $\theta_A + u$ . An important feature of

bonus-pay systems is that they are in general *enforceable*. Firms can escape paying wages by dismissing a worker, but under U.S. law firms cannot escape by firing a worker just before the end of the period to avoid paying the bonus. This requirement is governed by the so called doctrine of “good faith and fair dealing”.<sup>1</sup> This is important in the current context because it implies that a firm is obliged to pay the bonus even if the cost is greater than worker productivity. In the previous case we did not have to worry about firm profits in period 2 because they could always dismiss a worker.

Whether or not a firm can offer bonus pay depends upon the cost of implementing a measurement system. In order to evaluate the impact of contract form per se, it is assumed that monitoring costs for firm *A* are such that, in equilibrium, the worker is indifferent between the bonus-pay contract offered by firm *A* and the fixed-wage contract offered by firm *B*, and both firms make zero profits on these contracts. More precisely, firm *B* offers a fixed-wage contract  $w_B$  and firm *A* offers a bonus-pay contract  $\{w_A, b_A\}$ , where  $b_A$  is paid to the worker if and only if  $s = 1$ . It is assumed that after the state of the world is revealed, but before production begins, the firms can layoff workers. Since information is private, then the wage cannot be made conditional upon firm productivity, as in Hall and Lazear (1984). However, the current employer can renegotiate the wage upwards in the face of a credible outside option from the other firm.

## 2. NO CONTRACT EQUILIBRIUM

Consider first what happens when the worker does not accept a contract in period 1. In that case there is no need to pay for monitoring costs, and the two firms compete with wage offers. In order to get the worker to leave unemployment, the wage  $w^0$  must be at least:

$$w^0 = \bar{w} + k^2,$$

where  $\bar{w}$  is what the worker gets when unemployed (e.g. unemployment benefits), and  $k^2$  is the cost of accepting work in the second period. This captures one element of unemployment that has been discussed in the literature for the recent recession, namely that workers are more reluctant to move to find new jobs because of a decline in the value of their houses, which corresponds to a higher  $k^2$  than usual.<sup>2</sup>

Given that productivity is revealed, there is no benefit to bonus pay. The firms will match, but not exceed, each others offers. Let  $w^m(\omega) = \min(\theta_A, \theta_B) + u$  be the result of competition between firms A and B. The worker will accept the offer only if it is better than staying out of the labor

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<sup>1</sup>See Autor et al. (2004) and MacLeod and Nakavachara (2007) for a discussion.

<sup>2</sup>Though mobility costs are recognized as important, Kaplan and Shulhoder-Wold (2011) find that the effect, as measured by interstate mobility, seems to be small.

market, and hence the equilibrium wage is:

$$w^2(\omega|k^2) = \begin{cases} \bar{w}, & \text{if } w^m(\omega) \leq \bar{w}, \\ w^m(\omega), & \text{if not.} \end{cases}$$

The worker will leave unemployment if and only if at least one firm has a productivity higher than  $\bar{w}$ , in which case the worker must pay  $k^2$ . Let the  $I(\omega|k^2)$  be 1 if and only if the worker leaves unemployment, namely if and only if  $\max(\theta_A, \theta_B) + u - k^2 \geq \bar{w}$ . The utility of a worker who chooses not to accept a labor contract in period 1 is given by:

$$U^{NC}(k^2) = \int_{\omega} (w^2(\omega|k^2) - k^2 I(\omega|k^2)) h(\omega) d\omega.$$

Clearly utility is decreasing in  $k^2$ . The expected wage of an employed worker conditional upon the labor market demand  $u$  is given by:

$$w^{NC}(u, k^2) = \int_{\theta_A} \int_{\theta_B} (w^2(\theta_A, \theta_B, u|k^2) \times I(\theta_A, \theta_B, u|k^2)) f(\theta_A) f(\theta_B) d\theta_A d\theta_B.$$

In this case we have that observed market wages are *increasing* with  $k^2$  because workers need a higher wage to compensate them for the cost of entering the labor market. In addition, we have that observed wages are increasing with labor market demand  $u$ . These observations can be summarized in the following proposition.

**Proposition 1.** *Under no contract worker utility and employment falls with mobility costs  $k^2$ . Observed wages are positively correlated with aggregate demand shocks  $u$ .*

### 3. CONTRACT EQUILIBRIUM

In this section we work out the equilibrium fixed-wage and bonus-pay contracts. Both contract forms exist in equilibrium because of the variation in monitoring costs across firms (as discussed in MacLeod and Parent (1999)). To simplify comparison of the effect of contract alone we first compute the fixed-wage equilibrium contract. Next we work out the bonus-pay contract that maximizes firm profit and yields the same utility to the worker. We assume that the increase in firm profit is exactly offset by the cost of monitoring workers. In any market equilibrium we suppose that firms with costs below those of this marginal firm have strictly positive profits, a rent associated with a better monitoring technology. Firms with costs above this marginal firm will choose fixed-wage contracts or not enter at all.

**3.1. Fixed-wage Contract.** Consider now the zero profit fixed-wage contract. In period 1 firm  $B$  offers a fixed wage  $w^B$ , with the promise not to renegotiate the contract unless the worker obtains a better outside option. In order to evaluate the profit of the firm from a contract  $w^B$  we first define the set of states  $E^B(k^2, w^B)$  in which the worker remains matched to the

firm. This occurs whenever it is more efficient to work at the going wage than to be laid off, and leaving for firm A is not efficient:

$$(3.1) \quad E^B(k^2, w^B) = \{\omega | \theta_B + u \geq \max\{w^B, \theta_A + u - k^2\}\}.$$

When the worker stays with firm B the equilibrium wage is the best of the contract wage and the outside offer:

$$(3.2) \quad w_2^B(\omega, k^2, w^B) = \max\{w^B, \theta_A + u - k^2\}.$$

Given these we can define the profit of firm B when it makes a wage offer to the worker in period 1:

$$\Pi^B(w^B, k^2) = \int_{\omega \in E^B(k^2, w^B)} \theta_B + u - w_2^B(\omega, k^2, w^B) dh(\omega).$$

Observe that an increase in the contract wage  $w^B$  decreases the states at which there is trade, and also reduces firm revenue, hence profits fall with an increase in wages. We shall assume parameter values are such that it is profitable for firm B to offer a contract that the worker prefers to waiting until period 2. Let  $w^{B*}(k^2)$  be the unique solution to:

$$\Pi^B(w^{B*}(k^2), k^2) = 0.$$

It is straightforward to see that an increase in  $k^2$  decreases the worker's outside option in period 2 and hence leads to an *increase* in the first period wage.

The wrinkle here is that in period 2 the worker cannot observe  $\theta^B + u$ , thus, even if  $\theta^B + u < w^{B*}$ , the worker could never agree to a lower wage because the firm would always try to force the wage down regardless of  $\theta^B + u$ . Hence, as Hall and Lazear (1984) observe, asymmetric information leads to inefficient layoffs. However, the firm would always agree to match a credible outside offer. In contrast, models that rely upon the insurance motive would predict a *higher* utility when laid off! As Grossman and Hart (1981) show, one can modify the simple insurance model to obtain inefficient quits, but one has to use the same tactic as we do here, namely introduce asymmetric information.

To compute the worker's utility from the contract, we compute the wage for the two events we have defined above - the worker stays with firm B or leaves. If she stays ( $\omega \in E(w^{B*}, k^2)$ ), her wage is given by 3.2. If not, her wage *net* of mobility costs is given by:<sup>3</sup>

$$w_2^{B0}(\omega) = \max\{w^0, \theta_B + u\}.$$

Given this the workers utility under this contract wage is:

$$U^* \equiv U^{C*}(k^1, k^2) = \int_{\omega \in E(w^{B*}, k^2)} w_2^B(\omega, k^2, w^B) dh(\omega) + \int_{\omega \notin E(w^{B*}, k^2)} w_2^{B0}(\omega) dh(\omega) - k^1.$$

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<sup>3</sup>The wage setting process is modelled as an ascending auction, with firm B dropping out at wage  $\theta_B + u$ .

As long as  $k^2 > 0$  and there is strictly positive probability that employment is profitable in period 2, then there are  $k^1 > 0$  such that  $U^* > U^{NC}(k^2)$ . In such cases workers agree to a fixed-wage contract in advance, rather than having the wage set in the spot market in period 2. Under this contract whenever  $w^B > w^0$  then there are inefficiencies *ex post*. To formally close the model we should follow Hall and Lazear (1984) and make  $w^0$  or  $k^2$  uncertain and unobservable to the firm. This would ensure that a simple contract with severance pay is not efficient. This would complicate the model in well understood ways, without changing the basic incentive to contract *ex ante* due to the fact that  $k^1 < k^2$ . This modification is not necessary for our empirical implications.

**3.2. Bonus Pay.** In this section we show that if the firm acquires credible signals of worker productivity, they can be used to improve the efficiency of the fixed-wage contract. Specifically, suppose at a cost  $c$  the firm introduces a system to provide a verifiable measure of performance. It is well known that the creation of such a system can be difficult (Kerr (1975)). Moreover, we observe a great deal of heterogeneity in contract form across firms (MacLeod and Parent (1999)). Thus, we can suppose that  $c$  varies across firms to explain why bonus pay is not universal. For the current discussion, we derive the optimal contract at which a bonus-pay firm matches the utility a worker gets from a fixed-wage contract. Given that both contract forms are observed, it must be the case that in equilibrium bonus-pay firms are on the short side of the market. This will define a cutoff cost  $c^*$  such that all firms with monitoring costs below  $c^*$  offer bonus-pay contracts.

The bonus-pay contract is given by  $\{w^A, b^A\}$ , under which  $b^A$  is paid with probability  $P(\theta_A + u)$ . The states in which firm A keeps the worker under this contract are:

$$(3.3) \quad E^A(k^2, w^A, b^A) = \{\omega | \theta_A + u \geq \max\{w^A + b^A P(\theta_A + u), \theta_B + u - k^2\}\}.$$

Notice that the difference between equations 3.1 and 3.3 is the bonus term. If  $P(\cdot)$  is an increasing function, then the efficiency of the relationship can be increased with bonus pay. One can maintain expected compensation by moving some of the fixed-wage component to the bonus. This lowers the layoff threshold because for low  $\theta_A + u$  the expected bonus  $b^A P(\theta_A + u)$  is lower, and hence the firm can lower the productivity point at which it is efficient to layoff the worker. When the worker stays with firm A the equilibrium wage is the best of the contract wage and the outside offer:

$$(3.4) \quad w_2^A(\omega, k^2, w^A, b^A) = \max\{w^A + b^A P(\theta_A + u), \theta_B + u - k^2\}.$$

We do not explicitly model renegotiation costs, but observe that since the frequency of bonus pay rises with  $u$  then it is also the case that the outside option at another firm is also less likely to bind, which saves upon renegotiation costs. Given this, we can define the profit of firm A when it

makes a wage offer to the worker in period 1 as:

$$\Pi^A(w^A, b^A, k^2, c) = \int_{\omega \in E^B(w^B, k^2)} \theta^A + u - w_2^A(\omega, k^2, w^B) dh(\omega) - c,$$

where  $c$  is the monitoring cost.

We can now compute the utility of the worker at firm A. The wage in period 2 if she leaves firm A is:

$$w_2^{A0}(\omega) = \max\{w^0, \theta_A + u\}.$$

Given this the workers utility under this contract wage is:

$$U^{BC}(w^A, b^A, k^1, k^2) = \int_{\omega \in E^A(w^A, b^A, k^2)} w_2^A(\omega, k^2, w^A, b^A) dh(\omega) + \int_{\omega \notin E^A(w^A, b^A, k^2)} w_2^{A0}(\omega) dh(\omega) - k^1.$$

Observe that since we have assumed the distribution of the idiosyncratic part to be the same for both firms, then  $U^{BC}(w^{B*}, k^1, k^2) = U^*$ . The bonus contract firm will choose the profit maximizing agreement that solves:

$$(3.5) \quad \max_{w^A, b^A} \Pi^A(w^A, b^A, k^2)$$

subject to:

$$\begin{aligned} U^{BC}(w^{B*}, k^1, k^2) &\geq U^*, \\ w^A &\geq 0. \end{aligned}$$

The second constraint may bind if it is optimal to fully load compensation upon bonus pay. If there are some layoffs at the optimal fixed-wage contract, and  $P' > 0$  then it is optimal for firm A to set  $b^A > 0$ . Let  $\{w^{A*}, b^{A*}\}$  be the optimal contract. If  $b^{A*} > 0$ , then it must be the case that there is a monitoring cost  $c^* > 0$  such that:

$$\Pi^A(w^{A*}, b^{A*}, k^2, c^*) = 0.$$

Firms with monitoring costs less than  $c^*$  will choose bonus-pay contracts, while firms with higher monitoring costs choose fixed pay contracts.

#### 4. IMPLICATIONS

Let us fix monitoring costs at  $c^*$  so that the worker is indifferent between firm A and firm B. We can then compare her employment experience in these two jobs. This also provides a rationale for estimating the effect of contract form on employment experience by running regressions with a bonus-pay indicator and worker fixed effects.

The model has two sets of implications. The first is to compare how firm A with firm B differ keeping the aggregate shock  $u$  fixed. First, observe that in order for the worker to be indifferent then the wage component  $w^{B*} > w^{A*}$ . We can define the point at which each firm lays off a worker conditional upon  $u$  as

$$\begin{aligned} \bar{\theta}^A(u) &= w^{A*} + b^{A*}P(\bar{\theta}^A(u) + u) - u, \\ \bar{\theta}^B(u) &= w^{B*} - u. \end{aligned}$$

The hypothesis that  $P' > 0$  implies that  $\bar{\theta}^B(u) > \bar{\theta}^A(u)$ , which in turn implies that employment is higher at firm A than at firm B. Since the worker at firm A is employed in more states, and in these states she is receiving an expected rent, this implies that her wage may be lower than at firm B. However, income in both firms is the same since we have no cost of effort in this model. If unemployment earnings are not included in the measure of compensation, the earnings in a bonus-pay job is higher than in a fixed-wage job. In summary we have:

**Proposition 2.** *Suppose a worker is indifferent between a fixed-wage and bonus-pay job then conditional upon labor demand we have:*

1. *Wages are slightly higher at the fixed-wage job.*
2. *Employment is higher at the bonus-pay job.*
3. *Total compensation from employment is higher at the bonus-pay job.*
4. *Wages are higher in the bonus-pay job in any period where a bonus is paid.*

We can also consider the implications of the model over the cycle and ask how variations in  $u$  affect compensation and employment. We suppose that  $u$  is lower when unemployment rate is higher, which is the labor demand variable we use in the empirical analysis. In that case we have the following implications. When the unemployment rate is higher, then since  $\bar{\theta}^B(u) > \bar{\theta}^A(u)$  we have that employment is lower for workers who initially held fixed-wage jobs. In addition, a bonus is less likely to be paid, hence wages for employed workers is lower in bonus-pay jobs than in fixed-wage jobs. Given that some fixed-wage jobs are the result of responding to outside options, which would be lower when the unemployment rate is higher, this implies a small negative effect of the unemployment rate on wages.

With regards to earnings, since a downturn causes more fixed-wage workers to lose their jobs, while the same bonus-pay workers keep their jobs at a lower wage, it follows that earnings fall more for employed bonus-pay workers than fixed-wage workers. These are summarized as follows

**Proposition 3.** *In response to a negative demand shock (higher unemployment rate) we have*

1. *Employment falls more at fixed-wage than bonus-pay jobs.*
2. *Wages fall more at bonus-pay than fixed-wage jobs.*
3. *Employment earnings fall more at bonus-pay than fixed-wage jobs.*

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