This appendix presents algebraic details regarding the construction of the final versus intermediate goods trade decomposition. Looking at trade from the import side, we can rewrite the change in imports for each country as:

$$d_{IM_i} = \im_{m_i} d_{IM_i}^m + \im_{d_i} d_{IM_i}^d$$  \hspace{1cm} (1)

where $\im_{m_i}$ and $\im_{d_i}$ are total intermediate and final goods imports for country $i$, $m_{Ii}(t) = \sum_{j\neq i} \sum_s m_{ji}(s,t)$, and $d_{Ii}(t) = \sum_{j\neq i} d_{ji}(t)$. There are two important differences here between final and intermediate goods imports. First, final goods imports respond directly to changes in final demand ($\tilde{q}_i^d(t)$), while intermediate goods imports respond to changes in production ($\tilde{q}_i^m(t)$). Second, the import weights placed on each sector may differ for intermediate and final goods.

Aggregating across countries, the proportional change in world trade in final goods is

$$\hat{T}^d = \sum_i \left( \frac{\im_{d_i}^m}{T^d} \right) \hat{M}_i^d$$

and the proportional change in world trade in intermediate goods is
is \( \hat{T}^m = \sum_i \sum_t \left( \frac{im^m_i}{T^m_i} \right) \hat{M}^m_i \), with \( T^d = \sum_i im^d_i \) and \( T^m = \sum_i im^m_i \). These proportional changes can then be rewritten as:

\[
\hat{T}^d = \sum_t \left( \frac{T^d(t)}{\hat{T}^d(t)} \right) \hat{T}^d(t) \quad \text{with} \quad \hat{T}^d(t) = \sum_i \left( \frac{d_{Ii}(t)}{T^d(t)} \right) \hat{q}^d_{i}(t), \tag{3}
\]

\[
\hat{T}^m = \sum_t \left( \frac{T^m(t)}{\hat{T}^m(t)} \right) \hat{T}^m(t) \quad \text{with} \quad \hat{T}^m(t) = \sum_i \left( \frac{m_{Ii}(t)}{T^m(t)} \right) \hat{q}_i(t), \tag{4}
\]

where \( T^m(t) = \sum_i m_{Ii}(t) \) and \( T^d(t) = \sum_i d_{Ii}(t) \). Note that the index \( t \) here identifies the sector in which the goods are imported, for either final or intermediate use. For final goods, the sector in which goods are produced is identical to the sector in which goods are imported by definition. For intermediate goods, goods produced in sector \( s \) can be imported by another sector \( t \) as intermediates.