Mathematical Appendix

In order to solve for the market equilibrium, first set the inverse downstream world demand equal to the inverse downstream world supply. World downstream demand is $Q_d^D \equiv D_d + \widetilde{D}_d = (a - P_d)/b + (\widetilde{a} - P_d - \tau_d^D)/\widetilde{b}$, which can be written $P_d = \theta_1 - \widehat{b\widetilde{b}}Q_d^D - \widehat{b}\tau_d^D$, where $\widehat{x} = x/(x + \widetilde{x})$ and $\theta_1 = (a\widetilde{b} + \widetilde{a}b)/(b + \widetilde{b})$. World downstream supply is $Q_d^S \equiv S_d + \widetilde{S}_d = (P_d - c - P_u)/k + (P_d - \widetilde{c} - P_u - \tau_d^S)/\widetilde{k}$, which implies $P_d = \theta_2 + \widehat{k\widetilde{k}}Q_d^S + P_u + \widehat{k}\tau_d^S$, where $\theta_2 = (c\widetilde{k} + \widetilde{c}k)/(k + \widetilde{k})$. (When β and $\widetilde{\beta}$ are not restricted to be one, then $P_d = \theta_2 + \widehat{k\widetilde{k}}Q_d^S + (\widetilde{k}\beta/(k + \widetilde{k}) + \widehat{k\widetilde{\beta}})P_u + \widehat{k}\tau_d^S$.) In equilibrium, $Q_d = Q_d^D = Q_d^S$ and the downstream price clears both markets:

$$\theta_1 - \widehat{b\widetilde{b}}Q_d - \widehat{b}\tau_d^D = \theta_2 + \widehat{k\widetilde{k}}Q_d + P_u + \widehat{k}\tau_d^S. \tag{7}$$

Given β's of one, $Q_u^D = Q_d$. We can rearrange (7) to be the inverse upstream demand:

$$P_u = \theta_1 - \theta_2 - (\widehat{k\widetilde{k}} + \widehat{b\widetilde{b}})Q_u^D - \widehat{k}\tau_d^S - \widehat{b}\tau_d^D. \tag{8}$$

The upstream supply is $Q_u^S \equiv S_u + \widetilde{S}_u = (P_u - \varsigma)/\kappa + (P_u - \widetilde{\varsigma} - \tau_u^S)/\widetilde{\kappa}$, which can be written $P_u = \theta_3 + \widehat{\kappa\widetilde{\kappa}}Q_u^S + \widehat{\kappa}\tau_u^S$, where $\theta_3 = (\varsigma\widetilde{\kappa} + \widetilde{\varsigma}\kappa)/(\kappa + \widetilde{\kappa})$. In equilibrium, $Q^* = Q_d^* = Q_u^* = Q_u^D = Q_u^S$:

$$\begin{aligned} Q^* &= \theta_4 - \left(\widehat{k}\tau_d^S + \widehat{b}\tau_d^D + \widehat{\kappa}\tau_u^S\right)/\phi_1 \\ P_u^* &= \theta_5 - \phi_2\tau_d^D - \phi_3\tau_d^S + \phi_4\tau_u^S \\ P_d^* &= \theta_6 - \phi_6\tau_d^D + \phi_7\tau_d^S + \phi_8 P_u^*, \end{aligned} \tag{9}$$

where $\phi_1 = \widehat{k\widetilde{k}} + \widehat{b\widetilde{b}} + \widehat{\kappa\widetilde{\kappa}}$, $\phi_2 = \widehat{\kappa\widetilde{\kappa}}\widehat{b}/\phi_1$, $\phi_3 = \widehat{\kappa\widetilde{\kappa}}\widehat{k}/\phi_1$, $\phi_4 = \left(\widehat{k\widetilde{k}}\widehat{\kappa} + \widehat{b\widetilde{b}}\widehat{\kappa}\right)/\phi_1$, $\phi_5 = \widetilde{b}k\widetilde{k} + bk\widetilde{k} + b\widetilde{b}\widetilde{k} + b\widetilde{b}k$, $\phi_6 = bk\widetilde{k}/\phi_5$, $\phi_7 = b\widetilde{b}k/\phi_5$, $\phi_8 = (b\widetilde{b}\widetilde{k} + b\widetilde{b}k)/\phi_5$, $\theta_4 = (\theta_1 - \theta_2 - \theta_3)/\phi_1$, $\theta_5 = \widehat{\kappa\widetilde{\kappa}}\theta_4 + \theta_3$, and $\theta_6 = \left(a\widetilde{b}k\widetilde{k} + b\widetilde{b}c\widetilde{k} + \widetilde{a}bk\widetilde{k} + b\widetilde{b}\widetilde{c}k\right)/\phi_5$.

The marginal emissions in the foreign country for each policy will differ for each type of emissions. Emissions from upstream supply are $E_u^S = r_u^S S_u^* = r_u^S\left(P_u^* - \varsigma\right)/\kappa$, which are increasing in P_u^*. Hence, we can measure leakage from upstream supply as:

$$\begin{aligned} \partial E_u^S/\partial\tau_u^S &= r_u^S\phi_4/\kappa > 0 \\ \partial E_u^S/\partial\tau_d^S &= -r_u^S\phi_3/\kappa < 0 \\ \partial E_u^S/\partial\tau_d^D &= -r_u^S\phi_2/\kappa < 0. \end{aligned} \tag{10}$$

For downstream supply, the emissions are $E_d^S = r_d^S S_d^* = r_d^S(P_d^* - c - P_u^*)/k$, and depend on both P_d^* and P_u^*:

$$\begin{aligned} \partial E_d^S/\partial\tau_u^S &= -r_d^S\left(1 - \phi_8\right)\phi_4/k < 0 \\ \partial E_d^S/\partial\tau_d^S &= r_d^S\left[\phi_7 + \left(1 - \phi_8\right)\phi_3\right]/k > 0 \\ \partial E_d^S/\partial\tau_d^D &= -r_d^S\left[\phi_6 - \left(1 - \phi_8\right)\phi_2\right]/k \gtrless 0. \end{aligned}$$

Note that $\phi_8 < 1$, as we can rewrite $\phi_8 = x/(x + \upsilon)$, where x and υ are positive.

Emissions from downstream demand are $E_d^D = r_d^D D_d^* = r_d^D\left(a - P_d^*\right)/b$, which are decreasing in P_d^*:

$$\begin{aligned} \partial E_d^D/\partial\tau_u^S &= -r_u^S\phi_4\phi_8/b < 0 \\ \partial E_d^D/\partial\tau_d^S &= -r_u^S\left(\phi_7 - \phi_3\phi_8\right)/b \gtrless 0 \\ \partial E_d^D/\partial\tau_d^D &= r_u^S\left(\phi_6 + \phi_2\phi_8\right)/b > 0. \end{aligned} \tag{11}$$

We also measure the effectiveness of policies on domestic and global emissions. Note that downstream domestic firms emit $\widetilde{E}_d^S = \widetilde{r}_d^S \left(P_d - \widetilde{c} - P_u - \tau_d^S\right)/\widetilde{k}$. Thus, the ratio of the marginal leakage over the marginal reduction in domestic emissions is $r_d^S\widetilde{k}/\widetilde{r}_d^S k$ for policies that do not directly target foreign emissions. In general, the marginal effects of indirect policies on domestic agents are similar to those on foreign agents, just with domestic emissions rates and divisors. The direct effects are:

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\begin{aligned}
\partial\widetilde{E}_u^S/\partial\tau_u^S &= \widetilde{r}_u^S\left(\phi_4 - 1\right)/\widetilde{\kappa} \\
\partial\widetilde{E}_d^S/\partial\tau_d^S &= \widetilde{r}_d^S(\phi_7 + \left(1 - \phi_8\right)\phi_3 - 1)/\widetilde{k} \qquad (12) \\
\partial\widetilde{E}_d^D/\partial\tau_d^D &= \widetilde{r}_d^D(\phi_6 + \phi_2\phi_8 - 1)/\widetilde{b}.
\end{aligned}
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