A Theory of Brain Drain and Public Funding for Higher Education in the U.S.

Felicia Ionescu Linnea Polgreen*

1 Technical Appendix

This appendix provides proofs for all the results in our paper. We present and number them in the order they appear in the paper, by each section.

II Enrollment Under Heterogeneity in Public Funding

Proposition 1.1. *There is a threshold $\overline{a}$ such that all individuals with ability $a > \overline{a}$ will enroll in college, and all individuals with $a < \overline{a}$ will not go to college.*

Proof. Wlog we restrict ourselves to the case where no one goes to college, $\mu_0 = 0$; there is an incentive for the most gifted to enroll, i.e. $g(0) - c > 1$. Let $G_1(a) = g(0)a - c$, so $G_1(0) < 1 < G_1(1)$. Then there exists $\overline{a}_1$ such that $G_1(\overline{a}_1) = 1$ and $\mu_1 = \int_{\overline{a}_1}^1 f(z)dz = 1 - \overline{a}_1 > \mu_0$. Let $G_2(a) = g(\mu_1)a - c$, so $G_2(0) < 1 < G_2(1)$. Then there exists $\overline{a}_2$ such that $G_2(\overline{a}_2) = 1$ and $\mu_2 = \int_{\overline{a}_2}^1 f(z)dz = 1 - \overline{a}_2 > \mu_1$. By induction, there exists $\overline{a}_n$ such that $G_n(\overline{a}_n) = 1$ and $\mu_n = \int_{\overline{a}_n}^1 f(z)dz = 1 - \overline{a}_n$ with $\{\mu_n\}$ an increasing, bounded sequence on $[0, 1]$ and $\{\overline{a}_n\}$ a decreasing, bounded sequence on $[0, 1]$. Then there exist μ and $\overline{a}$ such that $\lim \mu_n = \mu$ and $\lim \overline{a}_n = \overline{a}$ with $g(\overline{a})\overline{a} - c = 1$ and $\mu = 1 - \overline{a}$. □

*Department of Economics, Colgate University, 13 Oak Drive, Hamilton, NY, 13346, email: fionescu@mail.colgate.edu; College of Pharmacy, University of Iowa, 115 S. Grand Avenue, Iowa City, Iowa 52242 email: linnea-polgreen@uiowa.edu.

Proposition 1.2. *Given that state 1 subsidizes education, regardless of the subsidy type, a higher fraction, μ_1 will choose to go to college in state 1, $\mu_1 = \mu(\overline{a}_1) > \mu_2 = \mu(\overline{a}_2)$. The fraction, μ_1 increases with the subsidy.*

Proof. By Proposition 1.1, there exist $\overline{a}_1$ and $\overline{a}_2$ such that all agents of type $a > \overline{a}_1$ go to college in state 1, and all agents of type $a > \overline{a}_2$ go to college in state 2. If state 1 gives a subsidy regardless of ability level, $G(a) = g(\mu(a))a - c$ is the same as before. If state 1 chooses to condition the subsidy on ability level, the function is given by $G(a) = g(\mu(a))a - c(a)$, which is continuous and strictly increasing in a given $c'(a) < 0$, and $g(\mu) > g'(\mu)a$. The two fractions that go to college are given by $\mu_1 = \mu(\overline{a}_1) = \int_{\overline{a}_1}^{1} f(z)dz$ and $\mu_2 = \mu(\overline{a}_2) = \int_{\overline{a}_2}^{1} f(z)dz$. Since $c_1 < c$, it results that $g(\mu_2)\overline{a}_2 - c_1 > g(\mu_2)\overline{a}_2 - c = 1 = g(\mu_1)\overline{a}_1 - c_1$. This implies $\overline{a}_1 < \overline{a}_2$, and the result follows. □

III Brain drain

A The general problem

Proposition 1.3. *Given $\mu_1 = \mu(\overline{a})$, the fraction who enrolls in college in state 1, the fraction of agents who stay is $\lambda(\mu_1) \in [\frac{\mu_1}{2}, \mu_1]$ or $\lambda(\mu_1) = 0$.*

Proof. Suppose, by contradiction, that there is incomplete brain drain. Then $\lambda(\mu_1) < \mu_1 - \lambda(\mu_1)$ and since g is increasing it follows that $g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1) < 0$. Then for any a, $g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a < 0$. Since $t_1 > 0$, there is no agent who chooses to stay in state 1, i.e. $\lambda(\mu_1) = 0$. Contradiction. There is either complete or no brain drain, that is $\lambda(\mu_1) = 0$ or $\lambda(\mu_1) \in [\frac{\mu_1}{2}, \mu_1]$. □

Theorem 1.4. *Assume there is no brain drain. There is a level $\hat{a}$, such that for any $a > \hat{a}$, agents will choose to stay state 1, and for any $a < \hat{a}$, they choose to leave. For $\hat{a}$, the agent is indifferent if $\frac{w_s^1(\hat{a}) - t_1}{w_s^2(\hat{a})} = 1$. The fraction of agents who leave post graduation is given by $1 - \lambda(\overline{a}, \hat{a}) = \int_{\hat{a}}^{\overline{a}} f(z)dz$. (In the case of uniform distribution, $1 - \lambda(\hat{a}, \overline{a}) = (\hat{a} - \overline{a})$, and the fraction who stays by $\lambda(\hat{a}, \overline{a}) = (1 - \hat{a})$ with $\hat{a} \in (0, 1]$.)*

Proof. We know that less than half leave state 1, that is $\lambda(\overline{a}) > \frac{\mu_1}{2}$. Assume $\lambda(\mu_1) < \mu_1$, otherwise the conclusion is trivial. Then $\lambda(\mu_1) > \mu_1 - \lambda(\mu_1)$ and $g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1)) > 0$. Let a^* represent an agent who stays, i.e. $(g(\lambda(\mu)) - g(\mu - \lambda(\mu)))a^* > t_1$. For any $a > a^*$, $g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a > g(\lambda(\mu_1))a^* - g(\mu_1 - \lambda(\mu_1))a^* > t_1$, and a chooses to stay in state 1. Let a^{**} an agent who leaves, i.e. $(g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1)))a^{**} < t_1$. For any $a < a^{**}$, $g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a < g(\lambda(\mu_1))a^{**} - g(\mu_1 - \lambda(\mu_1))a^{**} < t_1$, and a chooses to leave state 1. Hence there is a $\hat{a} \in [\overline{a}, \frac{1-\overline{a}}{2}]$ such that for any $a > \hat{a}$, agents will choose to stay in state 1, and for any $a < \hat{a}$, they choose to leave. From the continuity of g, it follows that for $\hat{a}$, the agent is indifferent if $\frac{w_s^1(\hat{a}) - t_1}{w_s^2(\hat{a})} = 1$. □

B Heterogeneity in returns to scale in higher education

Proposition 1.5. *Consider the environment where no subsidy is provided. There is a threshold $\overline{a}_{DRS}$ such that all individuals with ability $a > \overline{a}_{DRS}$ will enroll in college and all individuals with ability $a < \overline{a}_{DRS}$ will not enroll in college.*

Proof. In the case $g(0) - c < 1$, the highest ability agent, $a = 1$, does not go to college. This implies that $\forall\, a$, if $g(0)a - c < 1$, no one goes to college, i.e. $\overline{a}_{DRS} = 1$.
In the case where $g(0) - c > 1$, let $G(a) = g(\mu(a))a - c$ with $G(0) = -c$ and $G(1) = g(0) - c$. Hence $G(0) < 1 < G(1)$, so by the Intermediate Value Theorem, there exists $\overline{a}_{DRS} \in (0, 1)$ such that $g(0)\overline{a}_{DRS} - c = 1$. In addition, since $\mu(a) = \int_a^1 f(z)dz$ is continuous and decreasing in a, and $g'(\mu) < 0$, $G(a)$ is strictly increasing in a, the threshold $\overline{a}_{DRS}$ is unique and the result follows. □

Proposition 1.6. *Regardless of the subsidy type, a higher fraction μ_{DRS}, will choose to go to school in state 1, $\mu_{DRS} = \mu(\overline{a}_1) > \mu_2 = \mu(\overline{a}_2)$. The fraction μ_{DRS}, increases with the subsidy.*

Proof. The result follows from Propositions 1.5 and 1.2. □

Proposition 1.7. *Given $\mu_1 = \mu(\overline{a}_1)$, the fraction who enroll in college in state 1, the fraction of agents who stay is $\lambda(\mu_1) \in [0, \frac{\mu_1}{2})$.*

Proof. Suppose, by contradiction, less than half of college graduates leave state 1, i.e. $\lambda(\mu_1) \geq \frac{\mu_1}{2}$. Then $\lambda(\mu_1) > \mu_1 - \lambda(\mu_1)$, and since g is decreasing, it follows that $g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1) < 0$. Then for any a, $g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a < t_1$. Hence agent a chooses to leave state 1. Contradiction. There is either complete or incomplete brain drain, that is $\lambda(\mu_1) \in [0, \frac{\mu_1}{2})$. □

Theorem 1.8. *There is a level $\hat{a}$, such that for any $a > \hat{a}$, agents will choose to stay in state 1, and for any $a < \hat{a}$, they choose to leave. For $\hat{a}$, the agent is indifferent as $\frac{w_s^1(\hat{a}) - t_1}{w_s^2(\hat{a})} = 1$. The fraction of agents who leave post graduation is given by $1 - \lambda(\overline{a}_1, \hat{a}) = \int_{\hat{a}}^{\overline{a}_1} f(z)dz$. (In the case of a uniform distribution, $1 - \lambda(\hat{a}, \overline{a}_1) = (\hat{a} - \overline{a}_1)$, and the fraction who stays by $\lambda(\hat{a}, \overline{a}_1) = (1 - \hat{a})$ with $\hat{a} \in (\frac{1-\overline{a}}{2}, 1])$.*

Proof. We know that more than half of college graduates leave state 1, that is $\lambda(\mu_1) < \frac{\mu_1}{2}$. Assume $\lambda(\mu_1) > 0$, otherwise the conclusion is trivial. Then $\lambda(\mu) < \mu - \lambda(\mu)$ and $g(\lambda(\mu)) - g(\mu - \lambda(\mu) > 0$. Let a^* represent an agent who stays, i.e. $(g(\lambda(\mu)) - g(\mu - \lambda(\mu)))a^* > t_1$. For any $a > a^*$, $g(\lambda(\mu))a - g(\mu - \lambda(\mu))a > g(\lambda(\mu))a^* - g(\mu - \lambda(\mu))a^* > t_1$, and a chooses to stay in state 1. Let a^{**} represent an agent who leaves, i.e. $(g(\lambda(\mu)) - g(\mu - \lambda(\mu)))a^{**} < t_1$. For any $a < a^{**}$, $g(\lambda(\mu))a - g(\mu - \lambda(\mu))a < g(\lambda(\mu))a^{**} - g(\mu - \lambda(\mu))a^{**} < t_1$, a chooses to leave state 1. Hence there is a $\hat{a} \in (0, 1]$ such that for any $a > \hat{a}$, agents will choose to stay in state 1 and for any $a < \hat{a}$, they choose to leave. From the continuity of g, it follows that for $\hat{a}$, the agent is indifferent if $\frac{w_s^H(\hat{a}) - t_1}{w_s^O(\hat{a})} = 1$. □