**Appendix for "Who Bears Aggregate Fluctuations and How?"**
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## 1. Estimating the sensitivity of group consumption growth to aggregate consumption growth

In Table 1 of our paper, we sort households into groups based on CEX consumption in period t-1 and consider growth rates in group consumption from period t-1 to t. Since consumption is measured with error, we are partly sorting on period t-1 measurement error and this generates a mechanically negative bias in growth rates for the top groups (those with high t-1 consumption levels) and a mechanically positive bias in growth rates for the bottom groups. Estimating regressions in logs implies that these mechanical biases in our mean growth rates will not lead to inconsistent estimates of the sensitivity of group consumption growth to aggregate consumption growth. For simplicity, we lay out the argument assuming just one household in a given group, but the conclusions apply to the case with many households in each group.

Suppose the true model relating group consumption growth and aggregate consumption growth rates is

$$1+g_t^{true}=\beta_0+\beta_1\left(1+g_t^{aggr}\right)+\varepsilon_t \tag{1}$$

and the relation between true group consumption and observed group consumption is

$$c_t^{obs} = c_t^{true}u_t, \qquad u_t \text{ independent of } c_t^{true},\ E\left(u_t\right)=1 \tag{2}$$

$$c_{t-1}^{obs} = c_{t-1}^{true}e_{t-1}, \qquad e_{t-1} \text{ independent of } c_{t-1}^{true},\ E\left(e_{t-1}\right)=1 \tag{3}$$

where $u_t$ and $e_{t-1}$ are measurement errors assumed independent of each other and of true group consumption (at both $t-1$ and $t$) and aggregate consumption. This measurement error structure implies that in growth rates,

$$1+g_t^{obs}=\frac{u_t}{e_{t-1}}\left(1+g_t^{true}\right) \tag{4}$$

Substitute (1) into (4) to get

$$1+g_t^{obs}=\frac{u_t}{e_{t-1}}\left[\beta_0+\beta_1\left(1+g_t^{aggr}\right)+\varepsilon_t\right]=\frac{u_t}{e_{t-1}}\beta_0+\frac{u_t}{e_{t-1}}\beta_1\left(1+g_t^{aggr}\right)+\frac{u_t}{e_{t-1}}\varepsilon_t \tag{5}$$

An OLS regression of $1+g_t^{obs}$ for a given group on $1+g_t^{aggr}$ will lead to an estimator of the group's $\beta_1$ which has probability limit $\beta_1 \text{plim}\frac{1}{T}\Sigma\frac{u_t}{e_{t-1}}$. For top groups,

$$p\lim\frac{1}{T}\Sigma\frac{u_t}{e_{t-1}}=E\left(\frac{u_t}{e_{t-1}}|c_{t-1}^{obs}>\text{cutoff}_{t-1}^{top}\right). \tag{6}$$

We expect this to be substantially below one, since $c_{t-1}^{obs}>\text{cutoff}_{t-1}^{top}$ can be achieved either by having large true consumption $c_{t-1}^{true}$ or large measurement error $e_{t-1}$, implying that households in the top groups will tend to have above average values of $e_{t-1}$. Conversely, we expect $E\left(\frac{u_t}{e_{t-1}}|c_{t-1}^{obs}\leq\text{cutoff}_{t-1}^{top}\right)$ to be above one.[1]

[1] A qualifier is needed for the statement about top groups since there is an additional effect which tends to increase the mean of $\frac{u_t}{e_{t-1}}$. This extra effect is that the unconditional distribution of $\frac{u_t}{e_{t-1}}$ does not have mean one. If $u_t$ and $e_{t-1}$ are each unconditionally log-normally distributed with equal means and variances, then $\ln E\left(\frac{u_t}{e_{t-1}}\right)=E\left(\ln\frac{u_t}{e_{t-1}}\right)+\frac{1}{2}V\left(\ln\frac{u_t}{e_{t-1}}\right)=V\left(\ln u_t\right)$ so $E\left(\frac{u_t}{e_{t-1}}\right)=\exp\left(V\left(\ln u_t\right)\right)>1$.

Consistent estimation of each group's $\beta_1$ can be achieved by using log growth rates. Suppose that the true group growth rates and aggregate growth rates are small. Then (1) can be approximated by

$$1 + \ln\left(1 + g_t^{true}\right) \simeq \beta_0 + \beta_1 \left(1 + \ln\left(1 + g_t^{aggr}\right)\right) + \varepsilon_t. \tag{7}$$

It follows that

$$\begin{aligned} \ln(1 + g_t^{obs}) &= \ln\left(1 + g_t^{true}\right) + \ln\left(\frac{u_t}{e_{t-1}}\right) \\ &\simeq (\beta_0 + \beta_1 - 1) + \beta_1 \ln\left(1 + g_t^{aggr}\right) + \varepsilon_t + \ln\left(\frac{u_t}{e_{t-1}}\right). \end{aligned} \tag{8}$$

Estimating (7) by OLS leads to a consistent estimate of $\beta_1$ since the measurement error term is additive and assumed independent of aggregate growth. For this reason, Panel A, B, C and D of Table 1 (and Panel C and D of Table 2 and 3) all use log growth rates.

## 2. Estimating the fraction of aggregate risk borne by a group

In Table 1 Panel E we estimate of the fraction of risk borne by a given group. We assume the following true model

$$\left(\frac{\overline{c_t^{true}} - \overline{c_{t-1}^{true}}}{\overline{c_{t-1}^{aggr}}}\right) f_t = \delta_0 + \delta_1 \left(\frac{\overline{c_t^{aggr}} - \overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}\right) + v_t. \tag{9}$$

where bars indicate averages across households in a given group in a particular time period. $f_t$ is the fraction of the population that the group constitutes, and $\overline{c_t^{aggr}}$ is the average consumption in period $t$ for the full set of households. $\delta_1$ is the fraction of risk borne by the group. To implement this we regress $\frac{\overline{c_t^{obs}} - \overline{c_{t-1}^{obs}}}{\overline{c_{t-1}^{aggr}}} f_t$ on $\frac{\overline{c_t^{aggr}} - \overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}$ and argue here that this will lead to a consistent estimate of $\delta_1$ under reasonable assumptions.

Observe that under the measurement error structure laid out above,

$$\frac{\overline{c_t^{obs}} - \overline{c_{t-1}^{obs}}}{\overline{c_{t-1}^{aggr}}} = \frac{\overline{c_t^{true} u_t} - \overline{c_{t-1}^{true} e_{t-1}}}{\overline{c_{t-1}^{aggr}}} = \frac{\overline{c_t^{true}} - \overline{c_{t-1}^{true}}}{\overline{c_{t-1}^{aggr}}} + \left[\frac{\overline{c_t^{true} u_t} - \overline{c_t^{true}}}{\overline{c_{t-1}^{aggr}}} + \frac{\overline{c_{t-1}^{true}} - \overline{c_{t-1}^{true} e_{t-1}}}{\overline{c_{t-1}^{aggr}}}\right]. \tag{10}$$

Regressing $\frac{\overline{c_t^{obs}} - \overline{c_{t-1}^{obs}}}{\overline{c_{t-1}^{aggr}}} f_t$ on $\frac{\overline{c_t^{aggr}} - \overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}$ will lead to a consistent estimate of $\delta_1$ for a given group if $E\left(\left(\frac{\overline{c_t^{aggr}} - \overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}\right)\left(\frac{\overline{c_t^{true} u_t} - \overline{c_t^{true}}}{\overline{c_{t-1}^{aggr}}} + \frac{\overline{c_{t-1}^{true}} - \overline{c_{t-1}^{true} e_{t-1}}}{\overline{c_{t-1}^{aggr}}}\right)\right)$ for that group is zero.

We assume $E_h\left(c_{h,t-1}^{true} e_{h,t-1}\right) = E_h\left(c_{h,t-1}^{true}\right) E_h\left(e_{h,t-1}\right)$ and $E_h\left(c_{h,t}^{true} u_{h,t}\right) = E_h\left(c_{h,t}^{true}\right) E_h\left(u_{h,t}\right)$ and that the number of households in each cross-section is large so sample averages across households are close to their population means. This implies that for top groups (with a similar argument holding for bottom groups),

$$\begin{aligned} & E\left(\left(\frac{\overline{c_t^{aggr}} - \overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}\right)\left(\frac{\overline{c_t^{true} u_t} - \overline{c_t^{true}}}{\overline{c_{t-1}^{aggr}}}\right) | c_{t-1}^{obs} > \text{cutoff}_{t-1}^{top}\right) \\ = \; & E\left(\left(\frac{\overline{c_t^{aggr}} - \overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}\right)\left(\frac{\overline{c_t^{true}}}{\overline{c_{t-1}^{aggr}}}\right) | c_{t-1}^{obs} > \text{cutoff}_{t-1}^{top}\right) E\left(\overline{u_t} - 1 | c_{t-1}^{obs} > \text{cutoff}_{t-1}^{top}\right) \end{aligned} \tag{11}$$

and

$$E\left(\left(\frac{\overline{c_t^{aggr}}-\overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}\right)\left(\frac{\overline{c_{t-1}^{true}}-\overline{c_{t-1}^{true}e_{t-1}}}{\overline{c_{t-1}^{aggr}}}\right)|c_{t-1}^{obs}{>}\text{cutoff}_{t-1}^{top}\right)$$
$$= E\left(\left(\frac{\overline{c_t^{aggr}}-\overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}\right)\left(\frac{\overline{c_{t-1}^{true}}}{\overline{c_{t-1}^{aggr}}}\right)|c_{t-1}^{obs}{>}\text{cutoff}_{t-1}^{top}\right)E\left(1-\overline{e_{t-1}}|c_{t-1}^{obs}{>}\text{cutoff}_{t-1}^{top}\right) \quad (12)$$

In (11), $E\left(\overline{u_t}-1|c_{t-1}^{obs}{>}\text{cutoff}_{t-1}^{top}\right)=0$ under our maintained assumption that $u_t$ is independent of $c_{t-1}^{obs}$. In (12), $E\left(1-\overline{e_{t-1}}|c_{t-1}^{obs}{>}\text{cutoff}_{t-1}^{top}\right)$ is not zero, but $E\left(\left(\frac{\overline{c_t^{aggr}}-\overline{c_{t-1}^{aggr}}}{\overline{c_{t-1}^{aggr}}}\right)\left(\frac{\overline{c_{t-1}^{true}}}{\overline{c_{t-1}^{aggr}}}\right)|c_{t-1}^{obs}{>}\text{cutoff}_{t-1}^{top}\right)=0$ if growth in aggregate per household consumption is independent of the ratio of the group's average consumption in the last period to aggregate average consumption in the last period. In other words, $\delta_1$ is estimated consistently if $\frac{\overline{c_{t-1}^{true}}}{\overline{c_{t-1}^{aggr}}}$ does not have predictive power for aggregate growth from $t-1$ to $t$. We assume that this is approximately true.