IV. Online Mathematical Appendix

Proof of the condition for $s^* < s^{**}$. From (6)-(7) we first easily check that, if $s^* = +\infty$, then $s^{**} = +\infty$. Next, when both $s^*$and s^{**} are finite, $s^* < s^{**}$ if and only if

$$(y_B - 2v_L)\left[v_H + \lambda v_L + (1-\lambda)\bar{v} - y_B\right] >$$
$$(y_B - v_H - v_L)\left[2(\lambda v_L + (1-\lambda)v_H) - y_B\right] \iff$$

Denoting $\Delta \equiv v_H - v_L$, this becomes

$$(y_B - 2v_L)\left[2v_L + \Delta + (1-\lambda)\Delta/2 - y_B\right] >$$
$$(y_B - 2v_L - \Delta)\left[2v_L + 2(1-\lambda)\Delta - y_B\right] \iff$$
$$(y_B - 2v_L)\left[\Delta + (1-\lambda)\Delta/2 - 2(1-\lambda)\Delta\right] >$$
$$-\Delta\left[2v_L + 2(1-\lambda)\Delta\right] \iff$$
$$(y_B - 2v_L)\left[-3/2 - \lambda/2 + 2\lambda\right] + 2(1-\lambda)\Delta > 0$$

or, finally, $2v_H + v_L > (3/2)y_B$.

Proof of Proposition 1. The result for $s < s^*$ was shown in the text. The others follow from Lemmas 1 and 2 below.

LEMMA 1: *For $s > s^*$, HL pairs must split.*

(a) One cannot have both HL and LL agreeing since this requires $s \le \min\left\{s^*, s^{**}\right\}$.

(b) One also cannot have HL agreeing and LL splitting. Otherwise, let $(\theta_H^*, \theta_L^* = 1 - \theta_H^*)$ be the shares agreed to in an HL pair and $(\theta', \theta")$, with $\theta' + \theta" > 1$ the incompatible shares demanded in an LL pair. If neither of θ' nor $\theta"$ equals θ_H^*, by deviating to θ_H^* the L in an HL pair can achieve a gain of $s(1-\lambda)(v_H - v_L) > 0$. Therefore, it must be that $\theta_H^* \in \left\{\theta', \theta"\right\}$, say $\theta' = \theta_H^*$. But the other partner can then deviate from $\theta"$to $1 - \theta' = \theta_L^*$, i.e. concede: he will remain identified as an L, but now achieve $(1+s)\theta_L^* y_B \ge (1+\lambda s)v_L + \lambda s\bar{v} > (1+s)\,v_L$, where the first inequality must hold in order for the L partner in an HL pair to agree. The deviation is thus profitable, so once again LL pairs cannot be sustained.

It follows from the Lemma that, for $s^* < s \le s^{**}$, at most the LL matches can be sustained; and indeed, we showed in the text that in this region the shares $(1/2, 1/2)$ allow these pairs to reach agreement.

LEMMA 2: *For $s > s^{**}$, LL pairs must split.*

(a) Once again HL and LL cannot both agree, as this requires $s \le \min\left\{s^*, s^{**}\right\}$.

(b) We also cannot have LL agreeing and HL splitting. Otherwise, let (θ_H^*, θ_L^*) with $\theta_H^* + \theta_L^* > 1$ be the incompatible shares demanded by H and L respectively in an unbalanced pair and $(\theta', 1-\theta')$ the shares agreed to in an LL pair, with $\theta' \ge 1/2$. Consider now a deviation by the partner who was getting $1-\theta'$, to some $\theta''' > \theta'$ and $\theta''' \notin \left\{\theta_H^*, \theta_L^*\right\}$, and distinguish the following cases.

(i) If $\theta' \neq \theta_H^*$ the non-deviating partner, who is still asking for the equilibrium share θ', remains unambiguously identified as L (by the first of our refinements), and the deviating partner as an H (by the second refinement, or by D1), thus achieving $(1+\lambda s)v_L + s(1-\lambda)v_H > (1+s)y_B/2$, since $s > s^{**}$. A fortiori, this is better than his equilibrium utility $(1+s)(1-\theta')y_B$.

(ii) If $\theta' = \theta_H^*$, this implies $\theta_H^* \ge 1/2$. The LL partner receiving $1-\theta_H^*$ in equilibrium (say, Player 1) can profitably deviate to $\theta''' > \theta_H^*$ (clearly $\theta" = 1 - \theta_H^* > 0$, otherwise LL pairs are unsustainable), with $\theta''' \neq \theta_L^*$. Indeed, by our first refinement Player 2 is then presumed to have played according to equilibrium (which stipulates θ_H^* for both H types and one side in LL pairs), while the fact that Player 1 broke the match identifies him, by $D1$, as an H type. Since $s > s^{**}$ this is again a profitable deviation.

It follows from the two Lemmas that, for $s > s^{**}$, no matches can be sustained, even through asymmetric equilibria.