

Online Appendix to ‘Authority versus Persuasion’

Eric Van den Steen*

January 7, 2009

Section 1 determines the equilibria and, based on that, Section 2 gives the proofs of the Propositions.

1 Equilibria

Persuasion and Beliefs Before starting the backwards induction, let me determine the players’ beliefs when the principal does collect new information. Remember that ν_P was the principal’s belief in X while ν_A was the agent’s belief in Y . Let the updated beliefs be denoted $\nu_i^- < \nu_i^+$ when the signal respectively contradicted or confirmed i ’s belief. (Note that while $.5 < \nu_i < \nu_i^+$, ν_i^- can be smaller or larger than $.5$.) Bayesian updating implies that

$$\nu_i^+ = \frac{p\nu_i}{p\nu_i + (1-p)(1-\nu_i)}$$

and

$$\nu_i^- = \frac{(1-p)\nu_i}{(1-p)\nu_i + p(1-\nu_i)}$$

Furthermore, the fact that $\nu_P > p$ implies that $\nu_P^- > .5$ so that the principal always believes that X is more likely to succeed.

The Decision and Effort of the Agent I now start the backwards induction. The fact that the project payoff is additively separable in the agent’s effort and decision allows me to treat them independently. For the agent’s effort, let $\mu_A(d_2)$ be the agent’s belief in the action that the principal chose for D_2 . In choosing effort, the agent then solves

$$\max_e \gamma_A \beta e \mu_A(d_2) - \beta \frac{e^2}{2}$$

So that the optimal effort $\hat{e} = \gamma_A \mu_A(d_2)$. Note that this will depend on the principal’s decision (through $\mu_A(d_2)$).

*Harvard Business School (evandensteen@hbs.edu)

The agent's choice of D_1 trades off the benefit from following his own belief against the cost of disobedience. Let $\tilde{\nu}_A$ be the agent's belief that Y is most likely to succeed. Taking into account that P always prefers X and that the agent does as the principal wants when indifferent, the agent will choose X (i.e., obey) iff

$$\alpha\gamma_A(1 - \tilde{\nu}_A) \geq \alpha\gamma_A\tilde{\nu}_A - c_d$$

or

$$c_d \geq \alpha\gamma_A(2\tilde{\nu}_A - 1)$$

or

$$\tilde{\nu}_A \leq \frac{1}{2} + \frac{c_d}{2\alpha\gamma_A}$$

Note that this is completely independent from the principal's decision on D_2 . One implication is that the agent always complies when $\tilde{\nu}_A \leq \frac{1}{2}$ which can happen when $\nu_A^- \leq \frac{1}{2}$ (which is the case when $p \geq \nu_A$).

The Decision of the Principal Since the principal chooses his decision D_2 simultaneously with the agent's decision and with the agent's choice of effort, she takes the level of effort as given. It is then a dominant strategy for the principal to choose X (given that $.5 < \nu_P^- < \nu_P < \nu_P^+$ and that her expected payoff from D_2 equals $\beta\gamma_P\mu_P(d_2)e$ where $\mu_P(d_2)$ is the principal's belief in the action she chooses for D_2).

It then follows that the subgame starting in period 2 is uniquely determined by the parameters. In particular, the principal always chooses $D_2 = X$, the agent always chooses $\hat{e} = \gamma_A(1 - \tilde{\nu}_A)$, and the agent chooses $D_1 = X$ iff $\tilde{\nu}_A \leq \frac{1}{2} + \frac{c_d}{2\alpha\gamma_A}$ and $D_1 = Y$ otherwise.

To determine now the equilibria, it is useful to distinguish different cases along two dimensions. The first is the strength of persuasion. The key here is whether $\nu_A^- \leq .5$ or not. In particular, if $\nu_A^- > .5$ then the agent always chooses Y absent authority ('weak persuasion'), but if $\nu_A^- \leq .5$ then the agent does choose X , even absent authority, when the signal confirms the principal's belief ('strong persuasion'). The second dimension is the strength of authority. There are 4 cases that matter here:

1. $c_d \geq \alpha\gamma_A(2\nu_A^+ - 1)$ so that the agent always obeys ('strong authority'),
2. $\alpha\gamma_A(2\nu_A^+ - 1) > c_d \geq \alpha\gamma_A(2\nu_A - 1)$ so that the agent obeys unless the principal collected information and that confirmed the agent's view (' ν_A authority'),
3. $\alpha\gamma_A(2\nu_A - 1) > c_d \geq \alpha\gamma_A(2\nu_A^- - 1)$ so that the agent obeys only when the principal collected information and that confirmed the principal's view (' ν_A^- authority').
4. $\alpha\gamma_A(2\nu_A^- - 1) > c_d$ so that the agent never obeys ('no authority').

This gives a total of 8 cases. However, strong persuasion and no obedience requires both $\nu_A^- > \frac{1}{2} + \frac{c_d}{2\alpha\gamma_A}$ and $\nu_A^- \leq .5$, which is impossible. This leaves 7 cases to be considered.

I consider now the equilibrium case by case. For the further analysis, it is useful to define:

$$\begin{aligned}\theta &= \beta\gamma_A\gamma_P \{ [p\nu_P + (1-p)(1-\nu_P)]\nu_P^+(1-\nu_A^-) + [p(1-\nu_P) + (1-p)\nu_P]\nu_P^-(1-\nu_A^+) \} \\ &= \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \left\{ \frac{p^2}{(1-p)\nu_A + p(1-\nu_A)} + \frac{(1-p)^2}{p\nu_A + (1-p)(1-\nu_A)} \right\}\end{aligned}$$

$$\begin{aligned}\Delta_p &= \theta - \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \\ &= \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \left\{ \frac{(2p-1)^2\nu_A^2}{((1-p)\nu_A + p(1-\nu_A))(p\nu_A + (1-p)(1-\nu_A))} \right\}\end{aligned}$$

$$\Delta_a = \alpha\gamma_P(2\nu_P - 1)$$

and

$$\delta_a = \alpha\gamma_P(\nu_P + p - 1)$$

Note that since $.5 < p < \nu_P$, $0 < \delta_a < \Delta_a$.

The choice for persuasion and/or authority now completely depends on the principal's expected utility. I will use U^P , U_a^P , U_p^P , and U_{ap}^P for the principal's utility (excluding the costs of persuasion or authority) when, respectively, she uses neither authority nor persuasion, she uses only authority, she uses only persuasion, and she uses both authority and persuasion. I now study case by case.

Weak persuasion ($\nu_A^- > .5$), strong authority Remember that in this case, the agent always obeys upon authority but never chooses X absent authority.

If the principal does not use either authority or persuasion, the agent chooses Y and the principal's expected utility becomes:

$$U^P = \alpha\gamma_P(1-\nu_P) + \beta\gamma_A\gamma_P\nu_P(1-\nu_A)$$

If she uses 'authority only', her expected utility becomes:

$$U_a^P = \alpha\gamma_P\nu_P + \beta\gamma_A\gamma_P\nu_P(1-\nu_A)$$

It follows that her gain from authority (excluding c_a) equals

$$U_a^P - U^P = \alpha\gamma_P(2\nu_P - 1) = \Delta_a$$

If she uses 'persuasion only', her expected utility becomes

$$\begin{aligned}U_p^P &= [p\nu_P + (1-p)(1-\nu_P)] \{ \alpha\gamma_P(1-\nu_P^+) + \beta\gamma_A\gamma_P\nu_P^+(1-\nu_A^-) \} \\ &\quad + [p(1-\nu_P) + (1-p)\nu_P] \{ \alpha\gamma_P(1-\nu_P^-) + \beta\gamma_A\gamma_P\nu_P^-(1-\nu_A^+) \} \\ &= \alpha\gamma_P(1-\nu_P) + \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \left\{ \frac{p^2}{(1-p)\nu_A + p(1-\nu_A)} + \frac{(1-p)^2}{p\nu_A + (1-p)(1-\nu_A)} \right\} \\ &= \alpha\gamma_P(1-\nu_P) + \theta\end{aligned}$$

Now we can express the gain from persuasion as

$$\begin{aligned} U_p^P - U^P &= \beta\gamma_A\gamma_P\nu_P(1 - \nu_A) \left\{ \frac{p^2}{(1-p)\nu_A + p(1-\nu_A)} + \frac{(1-p)^2}{p\nu_A + (1-p)(1-\nu_A)} - 1 \right\} \\ &= \Delta_p \end{aligned}$$

Furthermore, when choosing between purely authority and purely persuasion, the gain from persuasion is

$$U_p^P - U_a^P = (U_p^P - U^P) - (U_a^P - U^P) = \Delta_p - \Delta_a$$

If the principal uses both authority and persuasion, her expected utility becomes

$$\begin{aligned} U_{ap}^P &= \alpha\gamma_P\nu_P + [p\nu_P + (1-p)(1-\nu_P)] \{ \beta\gamma_A\gamma_P\nu_P^+(1-\nu_A^-) \} + [p(1-\nu_P) + (1-p)\nu_P] \{ \beta\gamma_A\gamma_P\nu_P^-(1-\nu_A^+) \} \\ &= \alpha\gamma_P\nu_P + \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \left\{ p \frac{p}{(1-p)\nu_A + p(1-\nu_A)} + (1-p) \frac{(1-p)}{p\nu_A + (1-p)(1-\nu_A)} \right\} \\ &= \alpha\gamma_P\nu_P + \theta \end{aligned}$$

The gain from persuasion is then

$$\begin{aligned} U_{ap}^P - U_a^P &= \alpha\gamma_P\nu_P + \theta - \alpha\gamma_P\nu_P - \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \\ &= \Delta_p \end{aligned}$$

so that

$$U_{ap}^P - U_a^P - U_p^P + U^P = \Delta_p - \Delta_p = 0$$

Since the criteria for persuasion and authority are completely independent, the equilibria for this case are thus:

- The principal will use persuasion whenever $c_p \leq \Delta_p$.
- The principal will use authority whenever $c_a < \Delta_a$.

Strong persuasion ($\nu_A^- \leq .5 < \nu_A$), **strong authority**. The outcomes without persuasion are obviously identical to the ones above:

$$\begin{aligned} U^P &= \alpha\gamma_P(1-\nu_P) + \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \\ U_a^P &= \alpha\gamma_P\nu_P + \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \\ U_a^P - U^P &= \alpha\gamma_P(2\nu_P - 1) = \Delta_a \end{aligned}$$

The outcome with both persuasion and authority is also identical to the one above:

$$\begin{aligned} U_{ap}^P &= \alpha\gamma_P\nu_P + [p\nu_P + (1-p)(1-\nu_P)] \{ \beta\nu_P^+\gamma_A\gamma_P(1-\nu_A^-) \} \\ &\quad + [p(1-\nu_P) + (1-p)\nu_P] \{ \beta\nu_P^-\gamma_A\gamma_P(1-\nu_A^+) \} \\ &= \alpha\gamma_P\nu_P + \theta \\ U_{ap}^P - U_a^P &= \Delta_p \end{aligned}$$

If she uses ‘persuasion only’, the principal’s expected utility is

$$\begin{aligned}
 U_p^P &= [p\nu_P + (1-p)(1-\nu_P)] \{ \alpha\gamma_P\nu_P^+ + \beta\gamma_A\gamma_P\nu_P^+(1-\nu_A^-) \} \\
 &\quad + [p(1-\nu_P) + (1-p)\nu_P] \{ \alpha\gamma_P(1-\nu_P^-) + \beta\gamma_A\gamma_P\nu_P^-(1-\nu_A^+) \} \\
 &= [p\nu_P + (1-p)(1-\nu_P)] \{ \alpha\gamma_P(1-\nu_P^+) + \beta\gamma_A\gamma_P\nu_P^+(1-\nu_A^-) \} \\
 &\quad + [p\nu_P + (1-p)(1-\nu_P)] \alpha\gamma_P(2\nu_P^+ - 1) \\
 &\quad + [p(1-\nu_P) + (1-p)\nu_P] \{ \alpha\gamma_P(1-\nu_P^-) + \beta\gamma_A\gamma_P\nu_P^-(1-\nu_A^+) \} \\
 &= \alpha\gamma_P p + \theta
 \end{aligned}$$

The gain from persuasion is then

$$\begin{aligned}
 U_p^P - U^P &= \alpha\gamma_P p + \theta - \alpha\gamma_P(1-\nu_P) - \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \\
 &= \delta_a + \Delta_p
 \end{aligned}$$

Furthermore, when choosing between purely persuasion and purely authority, the gain from persuasion is

$$\begin{aligned}
 U_p^P - U_a^P &= \alpha\gamma_P p + \theta - \alpha\gamma_P\nu_P - \beta\gamma_A\gamma_P\nu_P(1-\nu_A) \\
 &= \delta_a - \Delta_a + \Delta_p < \Delta_p
 \end{aligned}$$

The gain from authority when using persuasion is

$$U_{ap}^P - U_p^P = \alpha\gamma_P\nu_P + \theta - \alpha\gamma_P p - \theta = \Delta_a - \delta_a$$

It follows that

$$U_{ap}^P - U_a^P - U_p^P + U^P = \Delta_p - (\delta_a + \Delta_p) < 0$$

so that persuasion and authority are substitutes.

To find the equilibria, note that if $c_p \leq \Delta_p$ then there will always be persuasion. If $\Delta_p < c_p \leq \delta_a + \Delta_p$ then there will be ‘persuasion only’ absent authority. And if $\delta_a + \Delta_p < c_p$ then there will never be persuasion.

Furthermore, if $c_a < \Delta_a - \delta_a$ then there will always be authority. If $\Delta_a - \delta_a \leq c_a < \Delta_a$ then there will be ‘authority only’ when there is no persuasion. Finally, when $c_a \geq \Delta_a$ then there will be no authority at all.

The one issue remaining is what happens when authority and persuasion are both possible but exclusive. In that case, the principal uses persuasion iff

$$U_p^P - U_a^P = \delta_a - \Delta_a + \Delta_p \geq c_p - c_a$$

The equilibria are thus as follows:

- If $c_p \leq \Delta_p$, then there is always persuasion. Moreover, there is also authority iff $c_a < \Delta_a - \delta_a$.
- If $\Delta_p < c_p \leq \Delta_p + \delta_a$ then we have the following:
 - If $c_a < c_p - (\Delta_p - \Delta_a + \delta_a)$ then the principal uses authority.
 - If $c_a \geq c_p - (\Delta_p - \Delta_a + \delta_a)$ then the principal uses persuasion.
- If $c_p > \Delta_p + \delta_a$ then there is never persuasion. If $c_a < \Delta_a$ then there is authority.

Weak persuasion, ν_A -authority The payoffs for U^P , U_p^P , and U_a^P are unchanged from the case with weak persuasion and strong authority:

$$\begin{aligned} U^P &= \alpha\gamma_P(1 - \nu_P) + \beta\gamma_A\gamma_P\nu_P(1 - \nu_A) \\ U_p^P &= \alpha\gamma_P(1 - \nu_P) + \theta \\ U_a^P &= \alpha\gamma_P\nu_P + \beta\gamma_A\gamma_P\nu_P(1 - \nu_A) \\ U_p^P - U^P &= \Delta_p \\ U_a^P - U^P &= \alpha\gamma_P(2\nu_P - 1) = \Delta_a \end{aligned}$$

When using both authority and persuasion, the employee only obeys when the signal confirms the principal's belief, so that the principal's expected utility equals:

$$\begin{aligned} U_{ap}^P &= [p\nu_P + (1 - p)(1 - \nu_P)] \{ \alpha\gamma_P\nu_P^+ + \beta\gamma_A\gamma_P\nu_P^+(1 - \nu_A^-) \} \\ &\quad + [p(1 - \nu_P) + (1 - p)\nu_P] \{ \alpha\gamma_P(1 - \nu_P^-) + \beta\gamma_A\gamma_P\nu_P^-(1 - \nu_A^+) \} \\ &= \alpha\gamma_P p + \theta \end{aligned}$$

Finally,

$$\begin{aligned} U_{ap}^P - U_p^P &= \alpha\gamma_P(p + \nu_P - 1) = \delta_a \\ U_{ap}^P - U_a^P &= \alpha\gamma_P p + \theta - \alpha\gamma_P\nu_P - \beta\gamma_A\gamma_P\nu_P(1 - \nu_A) \\ &= \Delta_p - \Delta_a + \delta_a \end{aligned}$$

so that $U_{ap}^P - U_a^P - U_p^P + U^P = \Delta_p - \Delta_a + \delta_a - \Delta_p < 0$.

The gain from using (only) persuasion rather than (only) authority is

$$\begin{aligned} U_p^P - U_a^P &= U_p^P - U^P - (U_a^P - U^P) \\ &= \Delta_p - \Delta_a \end{aligned}$$

The equilibrium is then as follows:

- If $c_p \leq \Delta_p - \Delta_a + \delta_a$ then always persuasion. Moreover, the principal will also use authority if $c_a < \delta_a$.
- If $\Delta_p - \Delta_a + \delta_a < c_p \leq \Delta_p$ then she will use persuasion if $c_p \leq c_a + \Delta_p - \Delta_a$ and authority otherwise.
- If $\Delta_p < c_p$ then she will never use persuasion but she will use 'authority only' if $c_a < \Delta_a$.

Strong persuasion, ν_A -authority Under persuasion, only ν_A^- would obey when exerting authority, but ν_A^- would comply anyways since persuasion is strong, so that $U_{ap}^P = U_p^P$. It follows that authority and persuasion are completely exclusive. When using only authority or only persuasion, this case is identical to the case with strong authority (and strong persuasion).

The equilibrium is then as follows:

- If $c_p \leq \Delta_p + \delta_a$ and

- if $c_a < c_p - (\Delta_p - \Delta_a + \delta_a)$ then authority.
- If $c_a \geq c_p - (\Delta_p - \Delta_a + \delta_a)$ then persuasion.

- If $c_p > \Delta_p + \delta_a$ then there is never persuasion. If $c_a < \Delta_a$ then there is authority.

It also follows that $U_{ap}^P - U_a^P - U_p^P + U^P = -\Delta_a < 0$.

Weak persuasion, ν_A^- -authority In this case, authority when used alone is never obeyed, so that $U_a^P = U^P$ and authority alone cannot be optimal. On the other hand, the payoffs for U^P and U_p^P are the same as in the case with weak persuasion but strong authority:

$$\begin{aligned} U^P &= \alpha\gamma_P(1 - \nu_P) + \beta\gamma_A\gamma_P\nu_P(1 - \nu_A) \\ U_p^P &= \alpha\gamma_P(1 - \nu_P) + \theta \\ U_p^P - U^P &= \Delta_p \end{aligned}$$

The principal's utility when using both persuasion and authority equals

$$\begin{aligned} U_{ap}^P &= [p\nu_P + (1 - p)(1 - \nu_P)] \{ \alpha\gamma_P\nu_P^+ + \beta\gamma_A\gamma_P\nu_P^+(1 - \nu_A^-) \} \\ &\quad + [p(1 - \nu_P) + (1 - p)\nu_P] \{ \alpha\gamma_P(1 - \nu_P^-) + \beta\gamma_A\gamma_P\nu_P^-(1 - \nu_A^+) \} \\ &= \alpha\gamma_P p + \theta \end{aligned}$$

so that

$$\begin{aligned} U_{ap}^P - U^P &= \Delta_p + \delta_a \\ U_{ap}^P - U_p^P &= \alpha\gamma_P p - \alpha\gamma_P(1 - \nu_P) = \delta_a \end{aligned}$$

and thus

$$U_{ap}^P - U_p^P - U_a^P + U^P = \delta_a > 0$$

So the equilibrium is then as follows:

- If $c_p \leq \Delta_p$ then the principal always uses persuasion. Moreover, the principal will also use authority if $c_a < \delta_a$.
- If $c_p > \Delta_p$, then the principal will use both authority and persuasion if $c_p + c_a < \Delta_p + \delta_a$. Otherwise the principal uses neither of the two.

Strong persuasion, ν_A^- -authority In this case, authority has no effect (since an agent with belief ν_A does not obey, while an agent with ν_A^- would obey but complies anyways even without authority), and is thus never optimal. It also follows that $U_{ap}^P = U_p^P$ and $U_a^P = U^P$ so that $U_{ap}^P - U_p^P - U_a^P + U^P = 0$. The utilities for U^P , U_p^P , and thus $U_p^P - U^P$ are the same as in the case of 'strong persuasion, strong authority'. It follows that the equilibrium is that the principal will use persuasion iff $c_p \leq \Delta_p + \delta_a$.

Weak persuasion, no authority With no authority, authority will never be used and $U_{ap}^P = U_p^P$ and $U_a^P = U^P$ so that $U_{ap}^P - U_p^P - U_a^P + U^P = 0$. The utilities for U^P , U_p^P , and thus $U_p^P - U^P$ are the same as in the case of ‘weak persuasion, strong authority’. It follows that the equilibrium is that the principal will use persuasion iff $c_p \leq \Delta_p$.

2 Proofs

Proof of Proposition 1: It suffices to show for each of the cases with $c_d < \alpha\gamma_A(2\nu_A - 1)$ that $U_{ap}^P - U_p^P - U_a^P + U^P \geq 0$ and for each of the cases with $c_d \geq \alpha\gamma_A(2\nu_A - 1)$ that $U_{ap}^P - U_p^P - U_a^P + U^P \leq 0$. These follow from the derivations of the equilibria. That concludes the proof. ■

Proof of Proposition ??: Note first that the boundaries between the different cases are independent of β . It thus suffices to show this case by case. Note also that Δ_p increases in β , while Δ_a and δ_a are independent of β . It follows that we can simply consider the effect of an increase in Δ_p . In what follows, I will assume that c_p changes along the horizontal axis and c_a changes along the vertical axis when describing regions in the parameter space.

The result is straightforward for the cases with weak persuasion and either strong authority or no authority and for the case with strong persuasion and ν_A^- -authority.

Consider next the case with strong persuasion and strong authority. Points that are originally in the $c_p \leq \Delta_p$ region are not affected by an increase in Δ_p . Points that are originally in the $c_p > \Delta_p + \delta_a$ either remain in that region and are not affected or become part of the middle region. But any such point with $c_a \geq \Delta_a$ then falls in the persuasion part. Any such point with $c_a < \Delta_a$ either remains ‘authority only’ or becomes ‘persuasion only’. It follows that the Proposition holds for these two regions. Consider finally points that are in the middle region. Any such point that satisfied $c_a \geq c_p - (\Delta_p - \Delta_a + \delta_a)$ and that remains in the middle region will still satisfy that condition. Any such point that satisfied $c_a < c_p - (\Delta_p - \Delta_a + \delta_a)$ and that remains in the middle region either remains ‘authority only’ or becomes ‘persuasion only’. The result thus also holds for such points. Consider finally any point from the middle region that goes to the left region. Any such point that satisfies $c_a \geq c_p - (\Delta_p - \Delta_a + \delta_a)$ will then satisfy $c_a \geq \Delta_a - \delta_a$ (since at the boundary between the left and middle region, $c_p = \Delta_p$) and thus remain in the ‘persuasion only’ part. Any such point that satisfies $c_a < c_p - (\Delta_p - \Delta_a + \delta_a)$ either becomes ‘persuasion only’ or persuasion and authority. It follows that the Proposition holds for such points. The proof for the cases with ν_A -authority are completely analogous. That proves the Proposition. ■

Proof of Proposition ??: Since Δ_p increases in γ_A while Δ_a and δ_a are independent of γ_A , the proof that this holds true on a case by case basis is analogous to the proof of Proposition ??.

However, γ_A does affect the boundaries between the different authority cases, so that I now have to show that the Proposition also holds when going from one case to another. An increase in γ_A reduces authority. Consider first the case with weak persuasion. With strong authority, the principal uses ‘persuasion only’ when $c_p \leq \Delta_p$ and $c_a \geq \Delta_a$. With ν_A -authority, she will never use persuasion when $c_p > \Delta_p$. Moreover, when $c_p \leq \Delta_p$, she will use ‘persuasion only’ at least when $c_a \geq \Delta_a$ and she will never use ‘persuasion only’ when $c_a < \delta_a$. With ν_A^- -authority, she will never use ‘persuasion only’ when $c_p > \Delta_p$. Moreover, when $c_p \leq \Delta_p$, she will use ‘persuasion only’ iff $c_a \geq \delta_a$. Finally, with no authority she will use only persuasion iff $c_p \leq \Delta_p$. This proves the result for all transitions between cases with weak persuasion. The argument for strong persuasion is analogous. That proves the Proposition. ■

Proof of Proposition ??: Note first that the boundaries between the different cases are independent of ν_P . It thus suffices to show this case by case.

The statement for $c_d < \alpha\gamma_A(2\nu_A - 1)$ follows directly from the fact that Δ_p , Δ_a , and δ_a all increase in ν_P .

The statement for $c_d \geq \gamma_A(2\nu_A - 1)$ in the case with weak persuasion and strong authority also follows directly from the fact that Δ_p increases in ν_P . Consider next the case with strong persuasion and strong authority. Points that are originally in the $c_p \leq \Delta_p$ region remain there and thus keep persuasion. Points that are originally in the $c_p > \Delta_p + \delta_a$ can only increase in persuasion. Consider finally points that are in the middle region. Any such point that goes to the left region increases in persuasion. Any such point that remains in the middle region and that satisfied $c_a \geq c_p - (\Delta_p - \Delta_a + \delta_a)$ will still satisfy that condition if

$$\frac{d(\Delta_p - \Delta_a + \delta_a)}{d\nu_P} = \beta\gamma_A\gamma_P(1-\nu_A) \left\{ \frac{(2p-1)^2\nu_A^2}{((1-p)\nu_A + p(1-\nu_A))(p\nu_A + (1-p)(1-\nu_A))} \right\} - \alpha\gamma_P \geq 0$$

or if

$$\frac{\beta}{\alpha} \geq \frac{((1-p)\nu_A + p(1-\nu_A))(p\nu_A + (1-p)(1-\nu_A))}{\gamma_A(1-\nu_A)(2p-1)^2\nu_A^2}$$

so that the result holds for that case if

$$\epsilon \geq \frac{((1-p)\nu_A + p(1-\nu_A))(p\nu_A + (1-p)(1-\nu_A))}{\gamma_A(1-\nu_A)(2p-1)^2\nu_A^2}$$

Given the similarity of their equilibria, the same condition also works for strong persuasion and ν_A -authority. So consider finally weak persuasion and ν_A -authority. The above condition guaranteed that $\Delta_p - \Delta_a + \delta_a$ increases, so that points originally in the $c_p \leq \Delta_p - \Delta_a + \delta_a$ remain there and thus keep persuasion. Points that are originally in the $c_p > \Delta_p$ can only increase in persuasion. Points that are originally in the $\Delta_p - \Delta_a + \delta_a < c_p \leq \Delta_p$ and move to the left region also can only increase in persuasion. All that remains is to consider points in that middle region that stay in that middle region. Any such point that satisfied

$c_a \geq c_p - (\Delta_p - \Delta_a)$ will still satisfy that condition if

$$\frac{d(\Delta_p - \Delta_a)}{d\nu_P} = \beta\gamma_A\gamma_P(1-\nu_A) \left\{ \frac{(2p-1)^2\nu_A^2}{((1-p)\nu_A + p(1-\nu_A))(p\nu_A + (1-p)(1-\nu_A))} \right\} - 2\alpha\gamma_P \geq 0$$

or if

$$\frac{\beta}{\alpha} \geq \frac{2((1-p)\nu_A + p(1-\nu_A))(p\nu_A + (1-p)(1-\nu_A))}{\gamma_A(1-\nu_A)(2p-1)^2\nu_A^2}$$

so that the result holds if

$$\epsilon \geq \frac{2((1-p)\nu_A + p(1-\nu_A))(p\nu_A + (1-p)(1-\nu_A))}{\gamma_A(1-\nu_A)(2p-1)^2\nu_A^2}$$

which also implies the earlier condition. This concludes the Proposition. ■