A Appendix

Steady-State Distribution of Agent Types:

The steady-state distribution of agent types is given by the following system of equations. The economic intuition for these equations is discussed in connection with Equation (5) in the paper.

$$
\begin{aligned}
0 &= \dot{\mu}_{lo}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \\
0 &= \dot{\mu}_{hn}(t) &= -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t) \\
0 &= \dot{\mu}_{ho}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t) \\
0 &= \dot{\mu}_{ln}(t) &= 2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t).
\end{aligned} \tag{A.1}
$$

Two of the equations in (A.1) are redundant, so that, together with (3)–(4), (A.1) forms a well-posed system. The system can be reduced to the quadratic equation

$$
0 = 2\lambda\mu_{hn}^2 + \left(2\lambda\left(\frac{\Theta}{\bar{\theta}} - \frac{\lambda_u}{\lambda_d + \lambda_u}\right) + \lambda_u + \lambda_d\right)\mu_{hn} - \lambda_u\left(1 - \frac{\Theta}{\bar{\theta}}\right). \tag{A.2}
$$

We use below the following result, which follows from calculations in Duffie, Gârleanu, and Pedersen (forthcoming).

Lemma 3 *If $\bar{\theta} \geq \Theta$, the system of equations (3), (4), and (A.1) has a unique solution in $[0,1]^4$. The steady-state fraction of sellers μ_{lo} increases with λ_d and decreases with λ_u, while the fraction of buyers μ_{hn} decreases with λ_d and increases with λ_u. Both μ_{lo} and μ_{hn} decrease with the meeting intensity λ. Furthermore, μ_{lo} increases, while μ_{hn} decreases with $\bar{\theta}$.*

Proof of Proposition 1:

The value function coefficients are given by

$$
\begin{aligned}
0 &= rv_{lo} - \lambda_u(v_{ho} - v_{lo}) - 2\lambda\mu_{hn}(p - v_{lo} + v_{ln}) + \delta \\
0 &= rv_{ln} - \lambda_u(v_{hn} - v_{ln}) \\
0 &= rv_{ho} - \lambda_d(v_{lo} - v_{ho}) \\
0 &= rv_{hn} - \lambda_d(v_{ln} - v_{hn}) - 2\lambda\mu_{ho}(v_{ho} - v_{hn} - p) \\
p &= (v_{lo} - v_{ln})(1-q) + (v_{ho} - v_{hn})q.
\end{aligned}
\tag{A.3}
$$

The first equation means that an agent of type *lo* has a zero change in steady-state utility. The change in his utility is due to opportunity cost $-rv_{lo}$, expected change in intrinsic-type $\lambda_u(v_{ho} - v_{lo})$, trade $2\lambda\mu_{hn}(p - v_{lo} + v_{ln})$, and holding cost $-\delta$. The next three equations are similar. Direct solution of this system yields

$$
p = \frac{\delta}{r}\frac{r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q)}{r + \lambda_d + 2\lambda\mu_{lo}(1-q) + \lambda_u + 2\lambda\mu_{hn}q}\,. \tag{A.4}
$$

Given the dependence of $P(X_t)$ on X_t, it is immediate that

$$
\mathrm{var}_t(P(X_{t+\tau}) - P(X_t)) = \frac{\sigma_X^2}{r^2}\tau
$$

for constant τ. If τ is randomly distributed with constant arrival intensity $\lambda\mu_{hn}$,

$$\begin{aligned}
\text{var}_t(P(X_{t+\tau}) - P(X_t)) &= \frac{1}{r^2}\text{var}_t(X_{t+\tau} - X_t) \\
&= \frac{1}{r^2}\left[E_t(\text{var}_t(X_{t+\tau} - X_t|\tau)) + \text{var}_t(E_t(X_{t+\tau} - X_t|\tau))\right] \\
&= \frac{1}{r^2}[\sigma_X^2 E_t(\tau)] \;=\; \frac{\sigma_X^2}{r^2\lambda\mu_{hn}},
\end{aligned}$$

and it is clear that, when the VaR constraint (2) binds, the equilibrium holding θ is given by (9) or (10), depending on the nature of risk management.

$\square$

Proof of Proposition 2: The equilibrium with the two kinds of risk management is given by $f_i(\bar{\theta}) = \frac{r\bar{\sigma}}{\sigma_X}$, where $f_0(\bar{\theta}) = \bar{\theta}\sqrt{\tau}$ and $f_1(\bar{\theta}) = \frac{\bar{\theta}}{\sqrt{2\lambda\mu_{hn}(\bar{\theta})}}$. Clearly, $f_0 = f_1$ when $\tau = \frac{1}{2\lambda\mu_{hn}}$, so that the two equilibria are identical.

The sensitivity $\bar{\theta}'$ of the equilibrium position $\bar{\theta}$ to the ratio $\frac{\bar{\sigma}}{\sigma_X}$ is given by $f_i'\bar{\theta}' = r$. With simple risk management, it is clear that $f_0' > 0$, so that the equilibrium position $\bar{\theta}$ decreases if the volatility σ_X increases or the risk limit $\bar{\sigma}$ decreases. A decreasing $\bar{\theta}$ leads, in turn, to an increasing expected search time for sellers $(2\lambda\mu_{hn})^{-1}$ and a decreasing price, because $\partial\mu_{hn}/\partial\bar{\theta} > 0$ and $\partial\mu_{lo}/\partial\bar{\theta} < 0$, as stated by Lemma 3.

With liquidity-adjusted risk management, $f_1' > 0$ by the definition of a stable equilibrium, and, since $\partial\mu_{hn}/\partial\bar{\theta} > 0$, $f_1' < f_0'$. Hence, with liquidity-adjusted risk management, the effects of σ_X on the equilibrium quantities are larger in absolute value and of the same sign as with simple risk management. A stable equilibrium exists because $f_1 < \infty$ on (Θ, ∞), while $\lim_{x\to\Theta} f_1(x) = \lim_{x\to\infty} f_1(x) = \infty$, given that $\mu_{hn}(\Theta) = 0$ and $\lim_{x\to\infty}\mu_{hn}(x) > 0$.

Consider now the dependence on the meeting intensity λ. It holds that

$$\begin{aligned} 0 &= \frac{df_i}{d\lambda} \\ &= \frac{\partial f_i}{\partial \bar{\theta}}\frac{d\bar{\theta}}{d\lambda} + \frac{\partial f_i}{\partial \lambda} + \frac{\partial f_i}{\partial \mu_{hn}}\frac{d\mu_{hn}}{d\lambda} \\ &= \frac{\partial f_i}{\partial \bar{\theta}}\frac{d\bar{\theta}}{d\lambda} + \frac{\partial f_i}{\partial \lambda} + \frac{\partial f_i}{\partial \mu_{hn}}\left(\frac{\partial \mu_{hn}}{\partial \lambda} + \frac{\partial \mu_{hn}}{\partial \bar{\theta}}\frac{d\bar{\theta}}{d\lambda}\right), \end{aligned}$$

which can be solved for $\frac{d\bar{\theta}}{d\lambda}$.

With simple risk management, it follows that $\frac{d\bar{\theta}}{d\lambda} = 0$, as $\frac{\partial f_0}{\partial \lambda} = \frac{\partial f_0}{\partial \mu_{hn}} = 0$. With liquidity-adjusted risk management, $\frac{d\bar{\theta}}{d\lambda} > 0$, since $\frac{\partial f_1}{\partial \theta} + \frac{\partial f_1}{\partial \mu_{hn}}\frac{\partial \mu_{hn}}{\partial \theta} = \frac{df_1}{d\theta} > 0$ for a stable equilibrium. We also use the fact that $\frac{\partial f_1}{\partial \lambda} + \frac{\partial f_1}{\partial \mu_{hn}}\frac{\partial \mu_{hn}}{\partial \lambda} < 0$, which holds because $\frac{d(\lambda \mu_{hn})}{d\lambda} > 0$, as can be shown based on (A.2). Since $\frac{\partial f_1}{\partial \lambda} < 0$, the result on selling times also obtains.

The price effects follows from

$$\frac{dP}{d\lambda} = \frac{\partial P}{\partial \lambda} + \frac{\partial P}{\partial \bar{\theta}}\frac{d\bar{\theta}}{d\lambda}. \tag{A.5}$$

The first term gives the impact with simple risk management, while the second captures the additional impact introduced by adjusting risk management to liquidity. Since $\frac{\partial P}{\partial \bar{\theta}} > 0$ from Proposition 1 and (A.2) (a complete proof can be given along the lines of Duffie, Gârleanu, and Pedersen (forthcoming), the second term has the same sign as $\frac{d\bar{\theta}}{d\lambda}$. This sign was shown above to be positive, as is the sign of $\frac{\partial P}{\partial \lambda}$. The total effect is therefore larger with risk-adjusted risk management.

Similar reasoning establishes the results concerning the dependence on λ_d and λ_u.

□