Supplementary Material for "Designing Efficient Mechanisms for Dynamic Bilateral Trading Games," by Susan Athey and Ilya Segal

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Proof of Proposition 1: Recall that we consider IR constraints after any history of reports, including misrepresentations, since a player might choose to misrepresent for several periods and then (depending on the realizations of types) exit. However, for the simple model where types follow a first-order Markov process, after a buyer sees today's type, the fact that past reports were different than true types is irrelevant, since today's type provides all relevant information for predicting the future. Expected discounted payoffs in period t, given reports θ_t are

$$\theta_{i,t}\chi_i^*(\theta_t) + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\mathbb{E}_t\left[\tilde{\theta}_{i,\tau}\chi_i^*(\tilde{\theta}_\tau)\middle|\theta_{b,t}\right] + (1\{i=b\} - 1\{i=s\}) \cdot$$
$$\left(\varphi(\theta_t, \theta_{b,t-1}) - K + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\left(\mathbb{E}_{t-1}\left[\tilde{\theta}_{s,\tau}\chi_b^*(\tilde{\theta}_\tau)|\theta_{b,t-1}\right] - \mathbb{E}_t\left[\tilde{\theta}_{s,\tau}\chi_b^*(\tilde{\theta}_\tau)|\theta_{b,t}\right]\right)\right).$$

We simply compare these expressions to the outside option, where the seller keeps the object in the outside option. This condition combines the implied bounds on K for the worst-case scenario for the each player:

$$\begin{aligned}
& \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\mathbb{E}_t\left[\tilde{\theta}_{b,\tau}\chi_b^*(\tilde{\theta}_\tau)\middle|\underline{\theta}_b\right] + \\
& \underline{\varphi} + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\left(\mathbb{E}_t\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\bar{\theta}_b\right] - \mathbb{E}_{t-1}\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\underline{\theta}_b\right]\right) \\
\geq & \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\mathbb{E}_t\left[\tilde{\theta}_{s,\tau}\chi_b^*(\tilde{\theta}_\tau)\middle|\bar{\theta}_b\right] + \bar{\varphi} + \bar{\theta}_s \\
& + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\left(\mathbb{E}_t\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\underline{\theta}_b\right] - \mathbb{E}_{t-1}\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\bar{\theta}_b\right]\right).
\end{aligned}$$

The condition given in the proposition is sufficient for this. ■

Proof of Proposition 2: Payoffs to player i are

$$\theta_{i,t}\chi_i^*(\theta_t;z_t) + \sum_{\tau=t+1}^{\infty}\delta^{\tau-t}\mathbb{E}_{\tilde{\theta}}\left[\tilde{\theta}_{i,\tau}^*\chi_i^*(\tilde{\theta}_\tau^*;\tilde{z}_\tau)\middle|\,\sigma(z_t,\chi_b^*(\theta_t;z_t))\right]$$
$$+(1\{i=s\}-1\{i=b\})\cdot$$
$$\left(\begin{array}{c} -\varphi(\theta_t,z_{t-1})+K-\left(\Pr\left(\chi_b^*(\theta_{b,t},\tilde{\theta}_{s,t};z_t)=1|z_t\right)-\Pr\left(\chi_b^*(\tilde{\theta}_t;z_t)=1|z_t\right)\right)\cdot \\ \sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(\mathbb{E}_{\tilde{\theta}}\left[\tilde{\theta}_{s,\tau}^*\chi_s^*(\tilde{\theta}_\tau^*;\tilde{z}_\tau)|\sigma(z_t,1)\right]-\mathbb{E}_{\tilde{\theta}}\left[\tilde{\theta}_{s,\tau}^*\chi_s^*(\tilde{\theta}_\tau^*;\tilde{z}_\tau)|\sigma(z_t,0)\right]\right) \\ +\left(\Pr\left(\chi_b^*(\tilde{\theta}_{b,t},\theta_{s,t};z_t)=1|z_t\right)-\Pr\left(\chi_b^*(\tilde{\theta}_t;z_t)=1|z_t\right)\right)\cdot \\ \sum_{\tau=t+1}^{\infty}\delta^{\tau-t}\left(\mathbb{E}_{\tilde{\theta}}\left[\tilde{\theta}_{b,\tau}^*\chi_b^*(\tilde{\theta}_\tau^*;\tilde{z}_\tau)|\sigma(z_t,1)\right]-\mathbb{E}_{\tilde{\theta}}\left[\tilde{\theta}_{b,\tau}^*\chi_b^*(\tilde{\theta}_\tau^*;\tilde{z}_\tau)|\sigma(z_t,0)\right]\right) \end{array}\right).$$

The expression given in the proposition relies on finding a K such that this expression exceeds the outside options in the worst-case scenarios. ■