

APPENDIX

Demographics and the politics of capital taxation in a life-cycle economy

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December 19, 2008

A Generalizing the analytical model

This appendix considers a more general case for the model in Section 3 of the paper.

Here I denote periods $t = 1, 2$. The population is divided into two types of agents, i , *old* and *young*, with sizes μ^{old} and $1 - \mu^{old}$ respectively. Agents own productive inputs in the form of capital k_t^i and one unit of labour, which earn an interest rate R_t and a wage rate w_t respectively. The income from the two inputs is taxed at rates τ_t^k and τ_t^l respectively. The individual after-tax income can be divided between current consumption, c_t^i , and capital next period, k_{t+1}^i . Firms produce output in each period with a Cobb-Douglas production function of labour and capital, with a capital share $\alpha \in (0, 1)$. The government uses the tax revenues to fund an amount of government spending equivalent to a share g of output. Assume $1 - g \geq \alpha$. Markets for inputs and output are competitive.

In the first period, the endowments of capital across types, k_1^i , as well as the tax rates, τ_1^k and τ_1^l , are given. Individuals choose investment and consumption to maximise their utility. In period 1, next-period tax rates will be chosen by an individual of a designated group i in order to maximise her own utility. An agent's utility depends only on her lifetime consumption, and is represented by $\log c_t + \beta \log c_{t+1}$, with $\beta > 0$.

Define the after-tax current income for an individual in group i at time t as $y_t^i \equiv (1 - \tau_t^k)R_t k_t^i + (1 - \tau_t^l)w_t$. Similarly, for the aggregate economy the after-tax income is $y_t \equiv (1 - \tau_t^k)R_t k_t + (1 - \tau_t^l)w_t$. In equilibrium, factor markets clear so $k_t = \sum_i \mu^i k_t^i$, the government constraint holds so $g = \alpha \tau_t^k + (1 - \alpha)\tau_t^l$, and factor prices equal their marginal products so $w_t = (1 - \alpha)k_t^\alpha$ and $R_t = \alpha k_t^{\alpha-1}$. Then it follows that these measures of disposable income can be written as

$$y_t^i = k_t^\alpha \left[\alpha(1 - \tau_t^k) \left(\frac{k_t^i}{k_t} - 1 \right) + 1 - g \right] \quad (\text{A1})$$

and

$$y_t = k_t^\alpha (1 - g) \quad (\text{A2})$$

Consider first the equilibrium determination of capital at $t = 2$ for an already given tax rate on capital τ_2^k , and initial incomes y_{t-1} and y_{t-1}^i . The individual decision problem

at $t - 1$ consists of the choice of investment that maximises utility for given disposable income, factor prices and taxes, subject to the budget constraints $c_{t-1}^i = y_{t-1}^i - k_t^i$ and $c_t^i = (1 - \tau_t^l)w_t + (1 - \tau_t^k)R_t k_t^i$. The standard Euler equation $c_t^i = \beta(1 - \tau_t^k)R_t c_{t-1}^i$ must be satisfied. Individual investment thus is

$$k_t^i = (\beta/(1 + \beta))y_{t-1}^i - (1/(1 + \beta))((1 - \tau_t^l)/(1 - \tau_t^k))(w_t/R_t), \quad t = 2$$

. Using the conditions for market clearing, factor prices, and the government constraint,

$$k_t^i = \frac{\beta}{1 + \beta}y_{t-1}^i - \frac{1}{1 + \beta} \left(\frac{1}{\alpha} \frac{1 - g}{1 - \tau_t^k} - 1 \right) k_t \quad (\text{A3})$$

Aggregating investment over the total population, a relation between the capital tax rate and aggregate investment emerges:

$$k_t = y_{t-1} INV(\tau_t^k), \quad t = 2 \quad (\text{A4})$$

with the investment-output ratio a negative function of the tax rate

$$INV(\tau_t^k) = \frac{\beta}{1 + \beta} \left[1 + \frac{1}{1 + \beta} \left(\frac{1}{\alpha} \frac{1 - g}{1 - \tau_t^k} - 1 \right) \right]^{-1}$$

Turning to the determination in period $t - 1$ of the capital tax rate τ_t^k for $t = 2, 4$, assume now an individual belonging to group i chooses the tax rate on capital τ_t^k to maximise her own utility. This tax rate solves:

$$TAX \left(\tau_t^k, \frac{y_{t-1} INV(\tau_t^k)}{y_{t-1}^i} \right) = 0, \quad t = 2 \quad (\text{A5})$$

where the left hand term, proportional to the marginal utility, is

$$\begin{aligned} TAX \left(\tau_t^k, \frac{k_t}{y_{t-1}^i} \right) &\equiv 1 - \left(1 - \Lambda(\tau_t^k) \right) \frac{\beta}{1 + \beta} \left(\frac{k_t}{y_{t-1}^i} \right)^{-1} + \\ &\quad \left(\frac{1}{\alpha} \frac{1 - g}{1 - \tau_t^k} - 1 \right) \left(\frac{1}{1 + \beta} - \Lambda(\tau_t^k) \left(\frac{\alpha}{1 - \alpha} + \frac{1}{1 + \beta} \right) \right), \end{aligned}$$

which is clearly increasing in the second argument, and

$$\Lambda(\tau_t^k) \equiv \frac{\frac{1 - \alpha}{1 + \beta} \frac{1}{\alpha} \frac{1 - g}{1 - \tau_t^k}}{1 + \frac{1}{1 + \beta} \left(\frac{1}{\alpha} \frac{1 - g}{1 - \tau_t^k} - 1 \right)} < 1$$

One can use this condition to establish results analogous to Propositions 1 and 2 in the paper. Furthermore, one can also establish the general existence of equilibria where the partial marginal utility to taxing capital is increasing.

B The definition of equilibrium

This section gives details for the equilibrium defined in sections 5.1 and 5.2.

B.1 Equilibrium for given policy transition

An agent's individual type is characterised by her asset holdings a and age j . Age is important since it determines the agent's planning horizon and labour productivity. Individual asset holdings by any agent at time t have also been normalised by the level of technology at time $t - (J - 1)$. The aggregate state consists of the distribution of wealth defined earlier and the capital tax rate (A, τ_k) which will determine prices and the choice sets of the agents. Households decide on individual asset accumulation and labour supply to the market and these choices will thus depend on the individual state (j, a) and the aggregate state (A, τ_k) .

For a given transition for policies Ψ^τ , a recursive competitive equilibrium can be defined as a set of age-specific individual value and decision functions for asset holdings and leisure, $v_j(A, \tau_k, a; \Psi^\tau)$, $\psi_j^a(A, \tau_k, a; \Psi^\tau)$ and $l_j(A, \tau_k, a; \Psi^\tau)$, an aggregate law of motion for the distribution of wealth, $\Psi^A(A, \tau_k; \Psi^\tau)$, and an aggregate function for the supply of labour services, $H(A, \tau_k; \Psi^\tau)$, such that competitive firms and households behave optimally and all markets clear. More formally:

- (i) (*Optimisation*) $v_{J+1}(\dots) = 0$, and for $j = 1, \dots, J$ the decision rules $a' = \psi_j^a(A, \tau_k, a; \Psi^\tau)$ and $l = l_j(A, \tau_k, a; \Psi^\tau)$ solve:

$$\begin{aligned} v_j(A, \tau_k, a; \Psi^\tau) &= \max_{(a', l) \in \mathbb{R} \times [0, 1]} \{z_j(A, \tau_k, a, a', H, l) + \beta v_{j+1}(A', \tau'_k, a'; \Psi^\tau)\} \\ \text{s.t.} \quad &\tau'_k = \Psi^\tau(A, \tau_k) \\ &A' = \Psi^A(A, \tau_k; \Psi^\tau) \\ &H = H(A, \tau_k; \Psi^\tau) \end{aligned}$$

where, using the individual budget constraint,

$$\begin{aligned} z_j(A, \tau_k, a, a', H, l) \equiv & u((1 - \tau_l(A, H, \tau_k))\epsilon_j \lambda^{j-1} (1 - l)w(A, H) + \\ & \lambda^{j-1} b_j(A, H) + (1 + (1 - \tau_k)r(A, H))a - a', l) \end{aligned}$$

with

$$\begin{aligned} r(A, H) &= \alpha \left(H / \left(\sum_i \lambda^{1-i} A_i \mu_i \right) \right)^{1-\alpha} - \delta, \quad w(A, H) = (1 - \alpha) \left(\left(\sum_i \lambda^{1-i} A_i \mu_i \right) / H \right)^\alpha, \\ \tau_l(A, H, \tau_k) &= \frac{(G + T)F(\sum_i \lambda^{1-i} A_i \mu_i, H) - \tau_k r(A, H) \sum_i \lambda^{1-i} A_i \mu_i}{w(A, H)H}, \end{aligned}$$

and

$$b_j(A, H) = b_{s,j} \frac{F(\sum_i \lambda^{1-i} A_i \mu_i, H) \times T}{\sum_i \mu_i b_{s,i}}.$$

- (ii) (*Market clearing*) For $j = 1, \dots, J$,

$$\Psi_{j+1}^A(A, \tau_k; \Psi^\tau) = \psi_j^a(A, \tau_k, A_j; \Psi^\tau),$$

and $\Psi_1^A(A, \tau_k; \Psi^\tau) = 0$, and

$$H(A, \tau_k; \Psi^\tau) = \sum_j \mu_j \epsilon_j (1 - l_j(A, \tau_k, A_j; \Psi^\tau)).$$

Note that in the problem of part (i) the return function embeds the individual budget constraint, the standard formulas for equilibrium factor prices with arbitrage in asset returns (i.e., $r = R - \delta$), the government budget constraint, and the market clearing condition for capital. The term $\sum_j \lambda^{1-j} A_j \mu_j$ stands for aggregate capital, and the individual variable a describes the life-cycle pattern of asset holdings. Part (ii) contains conditions for market clearing through the aggregation of individual decision rules.

B.2 Equilibrium with endogenous policy transition

The type of political constitution assumed establishes that the next-period tax rate is determined in the current period by the preferences of some cohorts of agents. To derive these preferences over tax rates, agents think through all the current and future equilibrium consequences of alternative choices of τ'_k provided that future outcomes will be those associated with the equilibrium law of motion for policies Ψ^τ . Denote by $\tilde{v}_j(A, \tau_k, a, \tau'_k; \Psi^\tau)$ the utility function of τ'_k that represents these preferences for an agent of age $j = 1, \dots, J$. Formally, it is defined as

$$\tilde{v}_j(A, \tau_k, a, \tau'_k; \Psi^\tau) \equiv \max_{(a', l) \in R \times [0, 1]} \{z_j(A, \tau_k, a, a', H, l) + \beta v_{j+1}(A', \tau'_k, a'; \Psi^\tau)\}$$

subject to

$$\begin{aligned} A' &= \tilde{\Psi}^A(A, \tau_k, \tau'_k; \Psi^\tau) \\ H &= \tilde{H}(A, \tau_k, \tau'_k; \Psi^\tau) \end{aligned}$$

which yields decision rules $a' = \tilde{\psi}_j^a(A, \tau_k, a, \tau'_k; \Psi^\tau)$ and $l = \tilde{l}_j(A, \tau_k, a, \tau'_k; \Psi^\tau)$ such that

$$\tilde{\Psi}_{j+1}^A(A, \tau_k, \tau'_k; \Psi^\tau) = \tilde{\psi}_j^a(A, \tau_k, A_j, \tau'_k; \Psi^\tau),$$

for $j = 1, \dots, J$, $\tilde{\Psi}_1^A(A, \tau_k, \tau'_k; \Psi^\tau) = 0$, and

$$\tilde{H}(A, \tau_k, \tau'_k; \Psi^\tau) = \sum_j \mu_j \epsilon_j (1 - \tilde{l}_j(A, \tau_k, A_j, \tau'_k; \Psi^\tau)).$$

These preferences are defined over τ'_k 's that will in general differ from the value dictated by the equilibrium law of motion $\Psi^\tau(A, \tau_k)$. These deviations in the tax rate will have an effect on the labour supply and savings across cohorts of agents, and the functions $\tilde{H}(\dots, \tau'_k; \Psi^\tau)$ and $\tilde{\Psi}^A(\dots, \tau'_k; \Psi^\tau)$ describe the one-period aggregate equilibrium consequence of these alternative tax options. Since policies will be dictated by the equilibrium Ψ^τ from the next period on, the associated equilibrium value function $v_{j+1}(\dots; \Psi^\tau)$ evaluates the future impact of current tax choices.

The aggregation of preferences over tax rates just derived $\tilde{v}_j(\dots; \Psi^\tau)$ yields a political outcome. The particular aggregation rule $\Gamma(A, \tau_k; \Psi^\tau)$ depends on the prescriptions of the political constitution. The two regimes that I will explore imply that the outcome reflects the preferred policy of households associated with a particular age $m \in [1, J]$, using linear interpolation when m does not coincide with a cohort $j \in \{1, \dots, J\}$. In the first regime this decisive age is determined exogenously. If $m \in [j^* - 1, j^*]$ is the age of the designated

decisive age then

$$\Gamma(A, \tau_k; \Psi^\tau) = (j^* - m) \times \arg \max_{\tau'_k} \tilde{v}_{j^*-1}(A, \tau_k, A_{j^*-1}, \tau'_k; \Psi^\tau) + \\ (m - (j^* - 1)) \times \arg \max_{\tau'_k} \tilde{v}_{j^*}(A, \tau_k, A_{j^*}, \tau'_k; \Psi^\tau) \quad (\text{B1})$$

The second regime is democracy under a majority voting rule in pair-wise tax rate contests where the weight of each cohort is given by $\tilde{\mu}_j$ – its demographic size μ_j adjusted by its relative political power or influence I_j as defined above in section 3. It will be assumed that individual preferences over taxes are single-peaked so that the median voter theorem applies. In this case, the policy rule is as above in (B1) with the decisive age m determined in equilibrium as follows: First, for a given state, order age groups by decreasing preferred tax rate through the sequence $\{j_i(A, \tau_k; \Psi^\tau)\}_{i=1}^J$ with $j_i(A, \tau_k; \Psi^\tau) \in \{1, \dots, J\}$. Second calculate an upper bound for the age of the effective median voter as the value $j_{i^*}(A, \tau_k; \Psi^\tau)$ such that:

$$\sum_{i=1}^{i^*} \tilde{\mu}_{j_i(A, \tau_k; \Psi^\tau)} \geq .5, \quad \sum_{i=1}^{i^*-1} \tilde{\mu}_{j_i(A, \tau_k; \Psi^\tau)} \leq .5$$

Third, calculate the age of the decisive voter m as this upper bound minus the excess that it leaves over the 0.5 mark adjusted by the mass of voters in this cohort:¹

$$m(A, \tau_k; \Psi^\tau) = j_{i^*}(A, \tau_k; \Psi^\tau) - \left(\sum_{i=1}^{i^*} \tilde{\mu}_{j_i(A, \tau_k; \Psi^\tau)} - 0.5 \right) \frac{1}{\tilde{\mu}_{j_{i^*}(A, \tau_k; \Psi^\tau)}}.$$

One is now in a position to define an equilibrium as the policy transition consistent with the choices expressed through the political process. Formally, it is a fixed point Ψ^τ of the following mapping:

$$\Gamma(A, \tau_k; \Psi^\tau) = \Psi^\tau(A, \tau_k)$$

I will focus on steady states, which adds the requirement that the system be consistent with a constant τ_k and stationary distribution of wealth A :

$$A = \Psi^A(A, \tau_k; \Psi^\tau) \\ \tau_k = \Psi^\tau(A, \tau_k)$$

C Local dynamics

This appendix describes the (local) dynamic properties of the benchmark economy based on the coefficients of the laws of motion shown in Appendix B of the paper. This completes the discussion at the end of section 6 of the paper.

Consider first the rule that determines the tax rate in the next period. The positive entries in $\Psi^{\tau A}$ indicate that the level of assets currently held by most age groups has a positive effect on the voted capital tax rate for the next period. The reason must be found in

¹Note that in a life-cycle context where heterogeneity is two-dimensional (age and wealth) preferences over the tax rate on capital may be non-monotonic in age and the median voter might not coincide with the median-age effective voter, i.e., $j_i(A, \tau_k; \Psi^\tau) \neq i$. However this will not be the case in the settings studied in this paper.

the properties of Ψ^{AA} – to be discussed below – which imply that the present stock of assets influences positively tomorrow’s stock of capital which, in equilibrium, will lead to a lower rate of return. This lower return reduces the marginal cost of taxing capital income.

But not all of the elements of $\Psi^{\tau A}$ have this positive sign. The two negative entries correspond to the two politically decisive groups of ages 5 and 6. It is intuitive that the wealthier the voters who decide the policy outcome the weaker the support for capital taxation. Turning to the intertemporal link between tax rates, the positive $\Psi^{\tau\tau}$ indicates that the current tax rate has a positive influence on its value next period. This is explained by inspection of the vector $\Psi^{A\tau}$ which will imply that today’s tax raises aggregate investment and, consequently, will reduce the rate of return which makes future capital taxation less costly.

The previous arguments rest on properties of the law of motion for the wealth distribution, as characterized by Ψ^{AA} and $\Psi^{A\tau}$. The presence of positive large values along the lower diagonal of Ψ^{AA} is what implies that an increase in current assets holdings leads to higher asset positions in the next period. Note that most of the remaining elements are negative since everyone else’s wealth will undermine anyone’s rate of return. This is not true however of values in the fourth to sixth columns. These components account for the positive effect of the current and next-period voter’s wealth on an individual’s saving behavior. This follows because, as seen in regard to $\Psi^{\tau A}$, a richer voter means lower capital taxation to come. Turning to $\Psi^{A\tau}$, the positive entries for most age groups reveal that a higher current τ_k increases most individual’s net current resources since their main source of income is labour rather than capital. This causes savings to increase with the current τ_k . For the opposite reason, the last component is negative. This is not the only effect at work though. Because $\Psi^{\tau\tau}$ is positive, a higher current tax rate τ_k leads to a higher τ_k next period and an increase in after-tax earnings from work relative to the return to current savings. This explains the rest of the negative entries in $\Psi^{A\tau}$. Note that the unexpected increase in the component for assets between ages 10 and 11 is the result of this negative effect ceasing to have any effect after retirement.