Suppelemtary Appendix to Fehr and Goette (2006): "Do Workers work more when Wages are High? Evidence from a randomize field experiment."

Appendix A

In this appendix, we derive the quasi linear objective function in equation (3) of the paper from the underlying intertemporal maximization problem. The intertemporal optimization problem is

$$\max U = \sum \delta^t u(c_t, e_t; x_t) \text{ subject to } \sum (\hat{w}_t e_t + y_t - \hat{p}_t c_t)(1+r)^{-1} = 0$$

where u is strictly concave and twice differentiable in e and c, e is labor supply in period t, c is consumption in period t, x is a taste shift variable to allow for periods without work, $\hat{w}_t$ is the wage, $\hat{p}_t$ is the price of consumption goods, δ is the discount rate, and r is the interest rate. We assume that there are no liquidity constraints, and that the path of wages, prices, and the taste shifter are known, and that the interest rate is constant.

The first order conditions to this problem are

$$\delta^t u_c(c_t, e_t, x_t) = (1+r)^{-t} \lambda \hat{p}_t$$

$$-\delta^t u_e(c_t, e_t, x_t) = (1+r)^{-t} \lambda \hat{w}_t$$

where λ is the Langrange multiplier on the life-time budget constraint. Thus, it can be interpreted as the marginal utility of lifetime wealth. Define the discounted price as $p_t = (1+r)^{-t} \delta^{-t} \hat{p}_t$ and the discounted wage w_t analogously. The first order conditions then have the easily interpretable form

$$u_c(c_t, e_t, x_t) = \lambda p_t \tag{A1}$$

$$-u_e(c_t, e_t; x_t) = \lambda w_t \tag{A2}$$

Equation (A1) implies that, at every date t, the individual equates the marginal utility of consumption to the marginal utility of *lifetime* income λ times the discounted price of the consumption good. Similarly, when choosing how hard to work, the individual chooses effort such that the marginal disutility of effort is equal to the marginal utility of lifetime income times the discounted wage per unit of effort w_t. The model also allows for non-participation. If $-u_e(c_t, 0, x_t) < \lambda w_t$ it is optimal to choose $e = 0$.

It is possible to represent within-period preferences in terms of a static objective function. This is essentially a reformulation of the results in Browing, Deaton, and Irish (1985). Consider equation (A1) again. Since $u(.)$ is strictly concave, u_c is strictly decreasing in c. Thus, (A1) can be solved for c_t

$$c_t = u_c^{-1}(\lambda p_t, e_t, x_t) \tag{A3}$$

Substitute this into (A2) to obtain

$$-u_e(u_c^{-1}(\lambda p_t, e_t, x_t), e_t, x_t) = \lambda w_t \tag{A4}$$

Now consider the static one-period objective function

$$v(e_t) = \lambda w_t e_t - g(e_t, \lambda p_t, x_t) \tag{A5}$$

where λ is the lifetime marginal utility of income along the optimal path. Next we show that maximizing this static objective function is equivalent to solving the intertemporal maximization problem, that $g(\ )$ is convex and can be interpreted as the monetary equivalent of the disutility of effort. To see that this, define

$$g(e_t, \lambda p_t, x_t) = -\int_0^{e_t} u_e(u_c^{-1}(\lambda p_t, z, x_t), z, x_t)dz \tag{A6}$$

From the construction of $g(\ )$ in (A6), it is obvious that the first order condition (FOC) that results from the static one-period objective function is equivalent to the FOC (A4). To show that $g(\ )$ is convex in e, we need to show that the second derivative w.r.t. e is positive. We proceed in two steps: First, consider how the individual adjusts consumption to a small perturbation in labor supply along the optimal path, i.e., λ remains constant. Differentiation of (A1) yields:

$$\frac{dc_t}{de_t} = -\frac{u_{ce}}{u_{cc}}.$$

Now, take the second derivative of $g(\ )$ to obtain

$$g_{ee}(e_t, \lambda p_t, x_t) = -u_{ee} - u_{ce}\frac{dc_t}{de_t} = -u_{ee} + \frac{u_{ce}^2}{u_{cc}} = \frac{-1}{u_{cc}}\left(u_{cc}u_{ee} - u_{ce}^2\right)$$

To determine the signs of the terms, observe that the conditions for concavity of $u(\ )$ are $u_{cc} < 0$, $u_{ee} < 0$ and $u_{cc}u_{ee} - u_{ce}^2 > 0$. But this establishes the convexity of $g(\ )$, as claimed. Thus, in the canonical life-cycle model, a rational, forward looking individual behaves as if she maximized the one-period objective function (A5).

Appendix B

In this appendix, we provide a specific example that shows how non-separable time preferences can induce workers to increase the number of shifts but decrease the effort per shift in response to a wage increase. We consider a two-period model in which the workers one-period objective function is given by

$$v(e_t, e_{t-1}) = \lambda w e_t - g(e_t, e_{t-1}) .$$

We assume that if a worker does not work during a period she has a utility from leisure time of L_0 and that the effort cost function $g(\,)$ is given by

$$g(e_t, e_{t-1}) = e_t(1 + \alpha e_{t-1}) + 0.5 g e_t^2 .$$

If we ignore discounting and set $e_0 = 0$, total utility is given by

$$U = v(e_1, 0) + v(e_2, e_1) .$$

(a) If an individual works only one period, the first order condition for effort in this period is

$$\frac{\partial U}{\partial e_t} = \lambda w - 1 - g e_t = 0 \Leftrightarrow e_t^*(w) = \frac{\lambda w - 1}{g} .$$

Substituting this into the utility function, we get the overall utility of working one shift

$$U(\text{one shift}) = \frac{(\lambda w - 1)^2}{2g} + L_0 .$$

(b) If an individual works two shifts, the first order conditions for effort in the two periods are given by

$$\frac{\partial U}{\partial e_1} = \lambda w - 1 - g e_1 - \alpha e_2 = 0$$
$$\frac{\partial U}{\partial e_2} = \lambda w - 1 - g e_2 - \alpha e_1 = 0$$

The two first order conditions imply $e_1 = e_2$. Therefore,

$$e_1^{**}(w) = e_2^{**}(w) = \frac{\lambda w - 1}{g + \alpha}.$$

Substituting this into the objective function, we get

$$U(\text{two shifts}) = \frac{(\lambda w - 1)^2}{g + \alpha}$$

We can now examine the implications of this model for the number of shifts worked and effort exerted on a shift as a function of the wage *w*.

(i) Shifts: A rational individual works two shifts if $U(\text{two shifts}) > U(\text{one shift})$. This implies

$$\frac{(\lambda w - 1)^2}{g + \alpha} > \frac{(\lambda w - 1)^2}{2g} + L_0 \quad \text{(B1)}$$

Notice that, in this model, if $\alpha > g$, it is never optimal to work two shifts. The condition $\alpha > g$ has a straightforward interpretation: In this case, yesterday's effort raises today's marginal costs of effort by more than today's effort raises today's marginal costs of effort. Simplifying this inequality, we get

$$\lambda w - 1 > \sqrt{L_0 \frac{2g(g + \alpha)}{g - \alpha}}. \quad \text{(B2)}$$

Denote the wage that satisfies (B2) with equality by w'. As intuition suggests, the individual's participation is increasing in the wage: If *w* is large enough such that (B2) is satisfied, she will work two shifts.

(ii) Effort: To examine how effort responds to a change in wages, we choose two wage levels $w_L < w_H$ and set $w_L = w'$, i.e. the low wage is equal to the highest wage at which it is still optimal to work only one shift. If the wage is low, the individual works one shift, and effort is equal to

$$e_1^*(w_L) = \frac{\lambda w - 1}{g} = \frac{\sqrt{L_0 \frac{2g(g + \alpha)}{g - \alpha}}}{g} = \sqrt{L_0 \frac{2(g + \alpha)}{g(g - \alpha)}}$$

Effort on the high wage is equal to

$$e_1^{**}(w_H) = e_2^{**}(w_H) = \frac{\lambda w_H - 1}{g + \alpha}.$$

(iii) The response to a change from w_L *to* w_H: In this example, $e_1^*(w_L)$ exceeds $e_1^{**}(w_H) = e_2^{**}(w_H)$ if $w_H \in \left(w_L, \frac{g+\alpha}{g} w_L - \frac{\alpha}{\lambda g}\right]$. Thus, changing the wage from w_L to w_H may decrease effort per shift if the wage increase is not too high. Notice also that the effect crucially depends on L_0, the value of leisure. If $L_0 = 0$, the effect cannot occur, because the wage cancels from the participation condition (B1). Then the individual always works the same number of shifts, irrespective of the wage, and effort responds positively to the wage, *irrespective of the strength of the intertemporal spillover* α. By continuity, this also holds for some $L_0 > 0$. Thus, in our example intertemporal spillovers alone can produce the described response of shifts and effort to the wage only if the value of leisure is large enough.