

On the Timing and Pricing of Dividends: Web Appendix

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I. Cumulative returns

In Figure 1, we show the cumulative returns of investing \$1 in January 1996 until October 2009 for four strategies: (i) dividend strategy 1 (R_1), dividend strategy 2 (R_2), (iii) the S&P500, and (iv) 30-day T-bills (R_F).

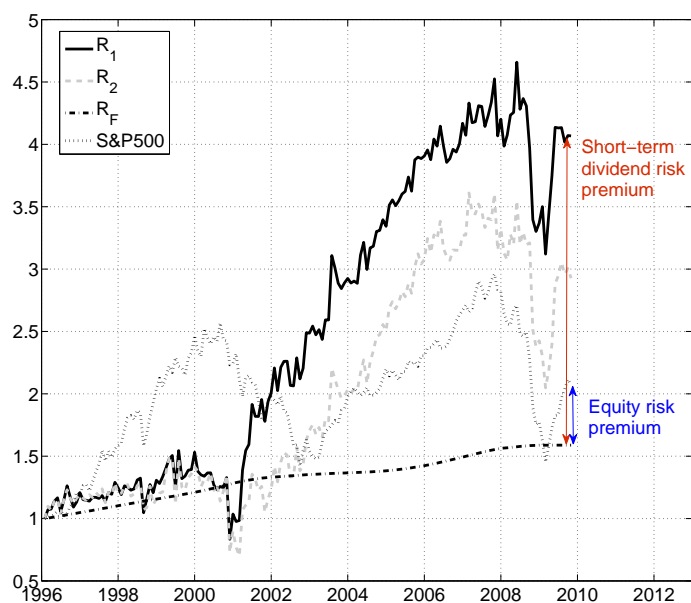


Figure 1. Cumulative returns of dividend strategies, the S&P500, and 30-day T-bills.

The figure shows the cumulative returns of investing \$1 in January 1996 until October 2009 for four strategies: (i) dividend strategy 1 (R_1), dividend strategy 2 (R_2), (iii) the S&P500, and (iv) 30-day T-bills (R_F).

The difference between the cumulative S&P500 return and T-bill returns is

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the realized cumulative excess return during this period, which can be used to estimate the equity risk premium; the difference between the first dividend strategy and T-bill returns is our preferred measure of the short-term dividend risk premium (cumulative).

II. Details dividend returns

The two trading strategies can be implemented for different maturities T . The specific maturities we follow for trading strategy 1 vary between 1.9 years and 1.3 years. To be precise, for trading strategy 1, we go long in the 1.874 year dividend claim on January 31st 1996, collect the dividend during February and sell the claim on February 29th 1996 to compute the return. The claim then has a remaining maturity of 1.797 years. We buy back the claim (or alternatively, we never sold it), go long in the 1.797 year claim, collect the dividend, and sell it on March 29th 1996. We follow this strategy until July 31st 1996 at which time the remaining maturity is 1.381 years. On this date a new 1.881 year contract is available so we restart the investment cycle at this time. We continue this procedure until October of 2009, which is the end of our sample.

For trading strategy 2, we follow the same maturities, apart from the fact that we go long in the 1.874 year dividend claim and short in the 0.874 dividend claim on January 31st 1996. On July 31st 1996 the remaining maturities are 1.381 years and 0.381 years at which point we restart the investment cycle in the 1.881 year contract and the 0.881 year contract available at that time.

III. CAPM and Fama-French Regressions

In Table 1 we repeat the regressions of Table 4, but instead of using excess returns on the S&P500 index, we now use excess returns on the aggregate market (mktrf). In Table 2, we repeat the regressions of Table 5, but instead of using firms in the S&P500 index only, we now use the standard Fama and French factors, labeled mktrf, hml and smb.

IV. Dividend strips in the external habit formation model

We first summarize some of the key equations of the John Y. Campbell and John H. Cochrane (1999) habit formation model. The stochastic discount factor is given by:

$$(1) \quad M_{t+1} = \delta G^{-\gamma} e^{-\gamma(s_{t+1} - s_t + v_{t+1})},$$

where G represents consumption growth, γ is the curvature parameter, v_{t+1} is unexpected consumption growth, and s_t is the log consumption surplus ratio whose dynamics are given by:

$$(2) \quad s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)v_{t+1},$$

Dep. Var.	$R_{1,t+1} - R_{f,t}$		$R_{2,t+1} - R_{f,t}$	
c	0.0073 (0.0052)	0.0069 (0.0055)	0.0069 (0.0062)	0.0063 (0.0068)
mktrf	0.4721 (0.1704)	0.5006 (0.1635)	0.4847 (0.2015)	0.5274 (0.1797)
AR(1)	- -	-0.2889 (0.1083)	- -	-0.3287 (0.0827)
R^2	0.0877	0.1709	0.0604	0.1682

Table 1—Monthly returns on the two trading strategies and the market portfolio.

The table presents OLS regressions of the returns on trading strategies 1 and 2 (dependent variables) on the market portfolio. Newey-West standard errors in parentheses. When an AR(1) term is included, the intercept is adjusted by one minus the AR(1) coefficient, such that the intercept is comparable to the regressions without AR(1) term.

Dep. Var.	$R_{1,t+1} - R_{f,t}$	$R_{2,t+1} - R_{f,t}$
c	0.0065 (0.0046)	0.0053 (0.0055)
mktrf	0.4880 (0.1519)	0.5712 (0.1742)
hml	0.1393 (0.2154)	0.3744 (0.2622)
smb	0.0751 (0.1493)	-0.0279 (0.1796)
R^2	0.0915	0.0811

Table 2—Monthly Returns on the Two Trading Strategies and the Three Factor Model.

The table presents OLS regressions of the returns on trading strategies 1 and 2 (dependent variables) on the Fama French three factor model. Newey-West standard errors in parentheses.

where $\lambda(s_t)$ is the sensitivity function which is chosen such that the risk free rate is constant, see Campbell and Cochrane (1999) for further details. Dividend growth in the model is given by:

$$(3) \quad \Delta d_{t+1} = g + w_{t+1}.$$

We solve the model using the solution method described in Jessica A. Wachter (2005). Let $D_t^{(n)}$ denote the price of a dividend at time t that is paid n periods in the future. Let D_{t+1} denote the realized dividend in period $t + 1$. The price of the first dividend strip is simply given by:

$$(4) \quad D_t^{(1)} = E_t(M_{t+1}D_{t+1}) = D_t E_t\left(M_{t+1} \frac{D_{t+1}}{D_t}\right).$$

The following recursion then allows us to compute the remaining dividend strips:

$$(5) \quad D_t^{(n)} = E_t(M_{t+1}D_{t+1}^{(n-1)}).$$

The return on the n^{th} dividend strip is given by:

$$(6) \quad R_{n,t+1} = \frac{D_{t+1}^{(n-1)}}{D_t^{(n)}}.$$

V. Dividend strips in the long-run risks model

The technology processes are given by:

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \varepsilon_{x,t+1}, \\ \Delta c_{t+1} &= \mu_c + x_t + \varepsilon_{c,t+1}, \\ \Delta d_{t+1} &= \mu_d + \phi x_t + \varepsilon_{d,t+1}, \\ \sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma (\sigma_t^2 - \mu_\sigma) + \varepsilon_{\sigma,t+1}, \end{aligned}$$

and we define $\varepsilon_{t+1} \equiv (\varepsilon_{c,t+1}, \varepsilon_{x,t+1}, \varepsilon_{\sigma,t+1}, \varepsilon_{d,t+1})'$. We assume:

$$\varepsilon_{t+1} \mid \mathcal{F}_t \sim N(0, \Sigma_t),$$

where:

$$\Sigma_t = \Sigma_0 + \Sigma_1 \sigma_t^2.$$

For the return on total wealth, we have:

$$\begin{aligned} R_{t+1}^c &= \frac{W_{t+1}}{W_t - C_t} \\ &= \frac{\exp(wc_{t+1})}{\exp(wc_t) - 1} \exp(\Delta c_{t+1}), \end{aligned}$$

and thus:

$$\begin{aligned}
r_{t+1}^c &= wc_{t+1} + \Delta c_{t+1} - \ln(\exp(wc_t) - 1) \\
&\simeq wc_{t+1} + \Delta c_{t+1} - \ln(\exp(E(wc_t)) - 1) - \frac{\exp(E(wc_t))}{\exp(E(wc_t)) - 1} (wc_t - E(wc_t)) \\
&= \kappa_0^c + \Delta c_{t+1} + wc_{t+1} - \kappa_1^c wc_t,
\end{aligned}$$

implying:

$$(7) \quad \kappa_0^c = -\ln(\exp(E(wc_t)) - 1) + \kappa_1^c E(wc_t),$$

$$(8) \quad \kappa_1^c = \frac{\exp(E(wc_t))}{\exp(E(wc_t)) - 1} > 1.$$

The stochastic discount takes the form:

$$m_{t+1} = c_0^m + c_1^m \Delta c_{t+1} + c_2^m (wc_{t+1} - \kappa_1^c wc_t),$$

where:

$$\begin{aligned}
wc_t &= A_0^c + A_1^c x_t + A_2^c \sigma_t^2, \\
c_0^m &= -\kappa_0^c - \frac{\gamma - 1}{1 - 1/\psi} (\ln \delta + \kappa_0^c), \\
c_1^m &= -\gamma, \\
c_2^m &= -\frac{\gamma - 1/\psi}{1 - 1/\psi}.
\end{aligned}$$

To compute $(\kappa_0^c, \kappa_1^c, A_0^c, A_1^c, A_2^c)$, we start from the Euler condition:

$$E_t(\exp(m_{t+1} + r_{t+1}^c)) = 1,$$

where $r_{t+1}^c = \ln(W_{t+1}/(W_t - C_t))$, which can be rewritten as:

$$E_t(m_{t+1}) + \frac{1}{2} V_t(m_{t+1}) + E_t(r_{t+1}^c) + \frac{1}{2} V_t(r_{t+1}^c) + Cov_t(m_{t+1}, r_{t+1}^c) = 0.$$

The five terms in this equation can be computed explicitly:

$$\begin{aligned}
E_t(m_{t+1}) &= E_t[c_0^m + c_1^m \Delta c_{t+1} + c_2^m (A_0^c + A_1^c x_{t+1} + A_2^c \sigma_{t+1}^2 - \kappa_1^c (A_0^c + A_1^c x_t + A_2^c \sigma_t^2))] \\
&= c_0^m + c_1^m \mu_c + c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma + c_2^m A_0^c (1 - \kappa_1^c) \\
&\quad + c_1^m x_t + c_2^m A_1^c (\rho_x - \kappa_1^c) x_t \\
&\quad + c_2^m A_2^c (\rho_\sigma - \kappa_1^c) \sigma_t^2
\end{aligned}$$

$$\begin{aligned}
m_{t+1} - E_t(m_{t+1}) &= c_1^m \varepsilon_{c,t+1} + c_2^m A_1^c \varepsilon_{x,t+1} + c_2^m A_2^c \varepsilon_{\sigma,t+1} \\
&\equiv \sigma'_m \varepsilon_{t+1}, \\
V_t(m_{t+1}) &= \sigma'_m \Sigma_t \sigma_m, \\
E_t(r_{t+1}^c) &= \kappa_0^c + \mu_c + A_2^c (1 - \rho_\sigma) \mu_\sigma + A_0^c - \kappa_1^c A_0^c + x_t + A_1^c (\rho_x - \kappa_1^c) x_t + A_2^c (\rho_\sigma - \kappa_1^c) \sigma_t^2, \\
r_{t+1}^c - E_t(r_{t+1}^c) &= \varepsilon_{c,t+1} + A_1^c \varepsilon_{x,t+1} + A_2^c \varepsilon_{\sigma,t+1}, \\
&\equiv \sigma'_{rc} \varepsilon_{t+1}, \\
V_t(r_{t+1}^c) &= \sigma'_{rc} \Sigma_t \sigma_{rc}, \\
Cov_t(m_{t+1}, r_{t+1}^c) &= \sigma'_m \Sigma_t \sigma_{rc}.
\end{aligned}$$

This results in:

$$\begin{aligned}
&c_0^m + c_1^m \mu_c + c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma + A_2^c (1 - \rho_\sigma) \mu_\sigma + c_2^m A_0^c (1 - \kappa_1^c) + c_1^m x_t + c_2^m A_1^c (\rho_x - \kappa_1^c) x_t + \\
&c_2^m A_2^c (\rho_\sigma - \kappa_1^c) \sigma_t^2 + \frac{1}{2} \sigma'_m \Sigma_t \sigma_m + \kappa_0^c + \mu_c + x_t + A_0^c - \kappa_1^c A_0^c \\
&A_1^c (\rho_x - \kappa_1^c) x_t + A_2^c (\rho_\sigma - \kappa_1^c) \sigma_t^2 + \frac{1}{2} \sigma'_{rc} \Sigma_t \sigma_{rc} + \sigma'_m \Sigma_t \sigma_{rc} = 0.
\end{aligned}$$

By matching the coefficients on the constant, x_t , and σ_t^2 , we find the solutions for A_0^c , A_1^c , and A_2^c :

$$\begin{aligned}
0 &= c_0^m + c_1^m \mu_c + (1 + c_2^m) A_2^c (1 - \rho_\sigma) \mu_\sigma + c_2^m A_0^c (1 - \kappa_1^c) + \frac{1}{2} \sigma'_m \Sigma_0 \sigma_m \\
&\quad + \kappa_0^c + \mu_c + A_0^c + \frac{1}{2} \sigma'_{rc} \Sigma_0 \sigma_{rc} + \sigma'_m \Sigma_0 \sigma_{rc} + (1 - \kappa_1^c) A_0^c, \\
0 &= c_1^m + c_2^m A_1^c (\rho_x - \kappa_1^c) + 1 + A_1^c (\rho_x - \kappa_1^c), \\
0 &= c_2^m A_2^c (\rho_\sigma - \kappa_1^c) + \frac{1}{2} \sigma'_m \Sigma_1 \sigma_m + A_2^c (\rho_\sigma - \kappa_1^c) + \frac{1}{2} \sigma'_{rc} \Sigma_1 \sigma_{rc} + \sigma'_m \Sigma_1 \sigma_{rc}.
\end{aligned}$$

We solve this system numerically for (A_0^c, A_1^c, A_2^c) , where we impose:

$$E(wc_t) = A_0^c + A_1^c \mu_\sigma,$$

in (7) and (8).

The price of dividend strips can be computed recursively and are exponentially-affine in the state variables:

$$pd_t^n = A_0^{d(n)} + A_1^{d(n)} x_t + A_2^{d(n)} \sigma_t^2.$$

For a one-period strip, we have:

$$PD_t^1 = E_t(\exp(m_{t+1} + \Delta d_{t+1})),$$

where:

$$\begin{aligned} E_t(\Delta d_{t+1}) &= \mu_d + \phi x_t, \\ V_t(\Delta d_{t+1}) &= e_4' \Sigma_t e_4, \\ Cov_t(\Delta d_{t+1}, m_{t+1}) &= e_4' \Sigma_t \sigma_m, \end{aligned}$$

with e_4 denotes the fourth unit vector. We then have:

$$E_t(m_{t+1}) + \frac{1}{2} V_t(m_{t+1}) + E_t(\Delta d_{t+1}) + \frac{1}{2} V_t(\Delta d_{t+1}) + Cov_t(\Delta d_{t+1}, m_{t+1}) = A_0^{d(1)} + A_1^{d(1)} x_t + A_2^{d(1)} \sigma_t^2,$$

leading to:

$$\begin{aligned} c_0^m + c_1^m \mu_c + c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma + c_2^m A_0^c (1 - \kappa_1^c) + c_1^m x_t + c_2^m A_1^c (\rho_x - \kappa_1^c) x_t + c_2^m A_2^c (\rho_\sigma - \kappa_1^c) \sigma_t^2 + \\ \frac{1}{2} \sigma_m' \Sigma_t \sigma_m + \mu_d + \phi x_t + \frac{1}{2} e_4' \Sigma_t e_4 + e_4' \Sigma_t \sigma_m = A_0^{d(n)} + A_1^{d(1)} x_t + A_2^{d(1)} \sigma_t^2, \end{aligned}$$

and thus:

$$\begin{aligned} A_0^{d(1)} &= c_0^m + c_1^m \mu_c + c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma + \frac{1}{2} \sigma_m' \Sigma_0 \sigma_m \\ &\quad + \mu_d + \frac{1}{2} e_4' \Sigma_0 e_4 + e_4' \Sigma_0 \sigma_m + c_2^m A_0^c (1 - \kappa_1^c), \\ A_1^{d(1)} &= c_1^m + c_2^m A_1^c (\rho_x - \kappa_1^c) + \phi, \\ A_2^{d(1)} &= c_2^m A_2^c (\rho_\sigma - \kappa_1^c) + \frac{1}{2} \sigma_m' \Sigma_1 \sigma_m + \frac{1}{2} e_4' \Sigma_1 e_4 + e_4' \Sigma_1 \sigma_m. \end{aligned}$$

The general recursion follows from:

$$\begin{aligned} PD_t^n &= E \left(M_{t+1} PD_{t+1}^{n-1} \frac{D_{t+1}}{D_t} \right) \\ &= E_t \left(\exp(m_{t+1} + \Delta d_{t+1} + pd_{t+1}^{n-1}) \right). \end{aligned}$$

We first compute the moments of $\Delta d_{t+1} + pd_{t+1}^{n-1}$:

$$\begin{aligned} E_t(\Delta d_{t+1} + pd_{t+1}^{n-1}) &= \mu_d + A_2^{d(n-1)} (1 - \rho_\sigma) \mu_\sigma + \phi x_t \\ &\quad + A_0^{d(n-1)} + A_1^{d(n-1)} \rho_x x_t + A_2^{d(n-1)} \rho_\sigma \sigma_t^2, \\ \Delta d_{t+1} + pd_{t+1}^{n-1} - E_t(\Delta d_{t+1} + pd_{t+1}^{n-1}) &= \varepsilon_{d,t+1} + A_1^{d(n-1)} \varepsilon_{x,t+1} + A_2^{d(n-1)} \varepsilon_{\sigma,t+1} \\ &\equiv \sigma_{pd}^n \varepsilon_{t+1}, \\ V_t(\Delta d_{t+1} + pd_{t+1}^{n-1}) &= \sigma_{pd}^n \Sigma_t \sigma_{pd}^n, \\ Cov_t(\sigma_{pd}^n \varepsilon_{t+1}, \sigma_m' \varepsilon_{t+1}) &= \sigma_{pd}^n \Sigma_t \sigma_m. \end{aligned}$$

This implies:

$$\begin{aligned}
& c_0^m + c_1^m \mu_c + c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma + c_2^m A_0^c (1 - \kappa_1^c) + \\
& c_1^m x_t + c_2^m A_1^c (\rho_x - \kappa_1^c) x_t + c_2^m A_2^c (\rho_\sigma - \kappa_1^c) \sigma_t^2 + \frac{1}{2} \sigma_m' \Sigma_t \sigma_m + \\
& \mu_d + A_2^{d(n-1)} (1 - \rho_\sigma) \mu_\sigma + \phi x_t + A_0^{d(n-1)} + A_1^{d(n-1)} \rho_x x_t + A_2^{d(n-1)} \rho_\sigma \sigma_t^2 + \frac{1}{2} \sigma_{pd}^{n'} \Sigma_t \sigma_{pd}^n + \sigma_{pd}^{n'} \Sigma_t \sigma_m \\
= & A_0^{d(n)} + A_1^{d(n)} x_t + A_2^{d(n)} \sigma_t^2,
\end{aligned}$$

implying for the coefficients:

$$\begin{aligned}
A_0^{d(n)} &= c_0^m + c_1^m \mu_c + c_2^m A_0^c (1 - \kappa_1^c) + c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma + \frac{1}{2} \sigma_m' \Sigma_0 \sigma_m + \mu_d \\
&\quad + A_2^{d(n-1)} (1 - \rho_\sigma) \mu_\sigma + A_0^{d(n-1)} + \frac{1}{2} \sigma_{pd}^{n'} \Sigma_0 \sigma_{pd}^n + \sigma_{pd}^{n'} \Sigma_0 \sigma_m, \\
A_1^{d(n)} &= c_1^m + c_2^m A_1^c (\rho_x - \kappa_1^c) + \phi + A_1^{d(n-1)} \rho_x, \\
A_2^{d(n)} &= c_2^m A_2^c (\rho_\sigma - \kappa_1^c) + \frac{1}{2} \sigma_m' \Sigma_1 \sigma_m + A_2^{d(n-1)} \rho_\sigma + \frac{1}{2} \sigma_{pd}^{n'} \Sigma_1 \sigma_{pd}^n + \sigma_{pd}^{n'} \Sigma_1 \sigma_m.
\end{aligned}$$

Finally, the one-period risk-free rate is given by:

$$\begin{aligned}
-r_t &= E_t(m_{t+1}) + \frac{1}{2} V_t(m_{t+1}) \\
&= c_0^m + c_1^m \mu_c + c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma + c_2^m A_0^c (1 - \kappa_1^c) \\
&\quad + c_1^m x_t + c_2^m A_1^c (\rho_x - \kappa_1^c) x_t + c_2^m A_2^c (\rho_\sigma - \kappa_1^c) \sigma_t^2 + \frac{1}{2} \sigma_m' \Sigma_t \sigma_m \\
&= -r_0 - r_x x_t - r_\sigma \sigma_t^2,
\end{aligned}$$

where

$$\begin{aligned}
r_0 &= -c_0^m - c_1^m \mu_c - c_2^m A_2^c (1 - \rho_\sigma) \mu_\sigma - c_2^m A_0^c (1 - \kappa_1^c) - \frac{1}{2} \sigma_m' \Sigma_0 \sigma_m, \\
r_x &= -c_1^m - c_2^m A_1^c (\rho_x - \kappa_1^c), \\
r_\sigma &= -c_2^m A_2^c (\rho_\sigma - \kappa_1^c) - \frac{1}{2} \sigma_m' \Sigma_1 \sigma_m.
\end{aligned}$$

In the model of Ravi Bansal and Amir Yaron (2004) it is assumed that:

$$\begin{aligned}
\Sigma_t &= \Sigma_0 + \Sigma_1 \sigma_t^2 \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_w^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \varphi_e^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varphi_d^2 \end{bmatrix} \sigma_t^2.
\end{aligned}$$

VI. Dividend strips in the rare disasters model

The setup of the Barro-Rietz rare disasters model as presented by Xavier Gabaix (2009) is as follows. Let there be a representative agent with utility given by:¹

$$(9) \quad E_0 \left[\sum_{t=0}^{\infty} e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} \right]$$

At each period consumption growth is given by:

$$(10) \quad \frac{C_{t+1}}{C_t} = e^g \times \begin{cases} 1 & \text{if there is no disaster at time } t+1 \\ B_{t+1} & \text{if there is a disaster at time } t+1 \end{cases}$$

The pricing kernel is then given by:

$$(11) \quad \frac{M_{t+1}}{M_t} = e^{-\delta} \times \begin{cases} 1 & \text{if there is no disaster at time } t+1 \\ B_{t+1}^{-\gamma} & \text{if there is a disaster at time } t+1 \end{cases}$$

where $\delta = \rho + g$. The dividend process for stock i takes the form:

$$(12) \quad \frac{D_{i,t+1}}{D_{it}} = e^{g_i D} (1 + \varepsilon_{i,t+1}^D) \times \begin{cases} 1 & \text{if there is no disaster at time } t+1 \\ F_{i,t+1} & \text{if there is a disaster at time } t+1 \end{cases}$$

where $\varepsilon_{i,t+1}^D > -1$ is an independent shock with mean 0 and variance σ_D^2 , and $F_{i,t+1} > 0$ is the recovery rate in case a disaster happens. The resilience of asset i is defined as:

$$H_{it} = p_t E_t^D \left[B_{t+1}^{-\gamma} F_{i,t+1} - 1 \right]$$

where the superscript D signifies conditioning on the disaster event. Define $\hat{H}_{it} = H_{it} - H_{i*}$, which follows a near-AR(1) process given by:

$$\hat{H}_{i,t+1} = \frac{1 + H_{i*}}{1 + H_{it}} e^{-\phi_H} \hat{H}_{it} + \varepsilon_{i,t+1}^H$$

where $\varepsilon_{i,t+1}^H$ has a conditional mean of 0 and a variance of σ_H^2 , and $\varepsilon_{i,t+1}^H$ and $\varepsilon_{i,t+1}^D$ are uncorrelated with the disaster event. Under the assumptions above, the stock price is given by:

$$P_{it} = \frac{D_{it}}{1 - e^{-\delta_i}} \left(1 + \frac{e^{-\delta_i - h_{i*}} \hat{H}_{it}}{1 - e^{\delta_i - \phi_H}} \right)$$

¹We thank Xavier Gabaix for providing us with this derivation.

where

$$\begin{aligned}\delta_i &= \delta - g_{iD} - h_{i*} \\ h_{i*} &= \ln H_{i*}\end{aligned}$$

Gabaix (2009) shows that the price at time t of a dividend paid in n periods is given by:

$$D_{it}^{(n)} = D_{it} e^{-\delta_i T} \left(1 + \frac{1 - e^{\phi_H n}}{\phi_H} \hat{H}_{it} \right)$$

and that the expected return on the strip, conditioning on no disaster is given by:

$$E_t [\ln R_{n,t+1}] = E_t \left[\ln \frac{D_{t+1}^{(n-1)}}{D_t^{(n)}} \right] \approx \delta - H_{it}$$

The expected return is the same across maturities, because strips of all maturities are exposed to the same risk in a disaster.

The volatility of the linearized return is given by:

$$\sigma_{n,t} = \sqrt{\sigma_D^2 + \left(\frac{1 - e^{-\phi_H n}}{\phi_H} \right)^2 \sigma_H^2}$$

which is increasing with maturity, due to the fact that higher duration cash flows are more exposed to discount rate shocks than short duration cash flows. Given that the expected return is constant across maturities and the volatility is increasing with maturity, the Sharpe ratio is decreasing with maturity.

VII. Dividend strips in the Lettau and Wachter model

In the model of Martin Lettau and Jessica A. Wachter (2007), the stochastic discount factor is assumed to be of the form:

$$(13) \quad M_{t+1} = \exp(-r_f - \frac{1}{2} x_t^2 + x_t \varepsilon_{d,t+1})$$

where x_t drives the price of risk and follows an AR(1) process:

$$(14) \quad x_{t+1} = (1 - \phi_x) \bar{x} + \phi_x x_t + \sigma_x \varepsilon_{t+1}$$

where ε_{t+1} is a 3x1 vector of shocks and σ_x is 1x3 vector. Dividend growth is predictable and given by:

$$(15) \quad \Delta d_{t+1} = g + z_t + \sigma_d \varepsilon_{t+1},$$

where

$$(16) \quad z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{t+1}.$$

Lettau and Wachter (2007) derive the prices of dividend strips in their model.

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