R&D Investment, Exporting, and Productivity Dynamics

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Web Appendix

I. Computation of the Firm's Dynamic Problem

For a state vector $s_{it} = (\omega_{it}, z_{it}, k_i, , \Phi_t, e_{it-1}, d_{it-1})$ we evaluate each plant's conditional choice probabilities for exporting and R&D $P(e_{it}|s_{it})$ and $P(d_{it}|s_{it})$ by solving each plant's dynamic optimization problem utilizing equations 10, 11, 12, and 13 to calculate the value functions using the following algorithm:

1. Begin with an initial guess of the value function $V^0(s)$.

2. Calculate $EV^0 = \int_{z'}\int_{\omega'} V^0(z', \omega', e, d, k, \Phi)dF(\omega'|\omega, e, d)dF(z'|z)$, where $F(\omega'|\omega, e, d)$ is calculated using equation 8 and $F(z'|z)$ follows 9. Notice that EV^0 depends on e and d for two reasons: (1) both e and d affect future productivity, (2) entry into either activity involves a sunk cost.

3. Calculate V_t^{E0} and V_t^{D0} using equations 11 and 12. We can express them in analytical form depending on $EV^0(e, d)$. Using the shorthand $EV^0(1, 1)$ to stand for $EV^0(e = 1, d = 1)$, these can be written as:

$$\begin{aligned} V^{E0}(d_{-1}) \quad = \quad & P[\delta EV^0(1,1) - \delta EV^0(1,0) > d_{-1}\gamma^I + (1-d_{-1})\gamma^D] \cdot \\ & (\delta EV^0(1,1) - d_{-1}E(\gamma^I|\cdot) - (1-d_{-1})E(\gamma^D|\cdot)) + \\ & P[\delta EV^0(1,1) - \delta EV^0(1,0) \leq d_{-1}\gamma^I + (1-d_{-1})\gamma^D]\delta EV^0(1,0) \end{aligned}$$

and

$$\begin{aligned} V^{D0}(d_{-1}) \quad = \quad & P[\delta EV^0(0,1) - \delta EV^0(0,0) > d_{-1}\gamma^I + (1-d_{-1})\gamma^D] \cdot \\ & (\delta EV^0(0,1) - d_{-1}E(\gamma^I|\cdot) - (1-d_{-1})E(\gamma^D|\cdot)) + \\ & P[\delta EV^0(0,1) - \delta EV^0(0,0) \leq d_{-1}\gamma^I + (1-d_{-1})\gamma^D]\delta EV^0(0,0) \end{aligned}$$

4. Finally, we can get the value function $V^1(s)$ using equation 10 by:

$$\begin{aligned} V^1(s) \quad = \quad & \pi^D(\omega, k) + \\ & P[\pi^X(z,\omega,k,\Phi) + V^{E0}(d_{-1}) - V^{D0}(d_{-1}) > e_{-1}\gamma^F + (1-e_{-1})\gamma^S] \cdot \\ & (\pi^X(z,\omega,k,\Phi) + V^{E0}(d_{-1}) - e_{-1}E(\gamma^F|\cdot) - (1-e_{-1})E(\gamma^S|\cdot)) + \\ & P[\pi^X(z,\omega,k,\Phi) + V^{E0}(d_{-1}) - V^{D0}(d_{-1}) \leq e_{-1}\gamma^F + (1-e_{-1})\gamma^S]V^{D0}(d_{-1}) \end{aligned}$$

5. Iterate across steps 2-4 until $|V^{j+1} - V^j| < \epsilon$.

Since the state space for our problem is very large, we adopt John Rust's (1997) method to discretize the state space. We choose $N = 100$ low-discrepancy points for (ω, z). Denote the random grid points as $(\omega_1, z_1), ...(\omega_n, z_n), ...(\omega_N, z_N)$. The grid values for k are fixed with 8 categories. The firm's dynamic problem and value function $\hat{V}$ can be solved exactly on each grid point by the value function iteration method described in the previous section. For the data points that are not on the grid points, we can calculate EV using the discrete Markov operator given by:

$$\begin{aligned} EV &= \int_{z'}\int_{\omega'} V^0(z', \omega', e, d, k, \Phi) dF(\omega'|\omega, e, d) dF(z'|z) \\ &= \frac{1}{N}\sum_{n=1}^{N} \hat{V}(z_n, \omega_n, e, d, k, \Phi) p^N(z_n, \omega_n|z, \omega, e, d) \end{aligned}$$

where $p^N(z_n, \omega_n|z, \omega, e, d) = \frac{p(z_n|z)p(\omega_n|\omega,e,d)}{\sum_{n=1}^{N} p(z_n|z)p(\omega_n|\omega,e,d)}$. Then the calculations of V^E and V^D follow from steps 2-4 of the previous section.

II. Details of Bayesian MCMC Estimation

Define the set of dynamic parameters as: $\Theta = (\gamma^I, \gamma^D, \gamma^F, \gamma^S, \Phi^X, \rho_z, \sigma_\mu, \theta_0^e, \theta_0^d)$, where θ_0^e and θ_0^d are, respectively, the parameters for probit equations for the initial conditions of exporting and R&D. Using the model, the likelihood function defining the data set (D) as a function of the parameters $L(D|\Theta)$, and a set of prior distributions of Θ, the posterior distribution $P(\Theta|D)$ is well defined. We use MCMC techniques to calculate the moments of the posterior distribution. The details of our sampling algorithm follows the discussion and references in Das, Roberts, and Tybout (2007) closely. We adopt very diffuse priors for the parameters. The means of all fixed and sunk cost distributions are assumed to have priors that are N(0,1000). The prior for the export revenue intercept is N(0, 1000), for the autoregressive coefficient in the export demand shocks is U[-1,1], and log σ_μ is N(0,10) The means and standard deviations of the posterior distribution are reported in Table 5.