STRIKE THREE: DISCRIMINATION, INCENTIVES AND EVALUATION

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Web Appendix

Appendix– A Model of Bias-Induced Changes in Pitcher Strategies

Consider the following simple representation of the interaction between the pitcher and hitter. Denote the horizontal distance from the center of the plate $\tilde{y}$. Assume for simplicity that the pitcher can control the width of pitches (i.e., the horizontal dimension), but not their height. Further suppose that the batter is left-handed, and that the pitcher never aims left of center, i.e., $\mu = 0$. This simplifying assumption is little more than a normalization, although a realistic one, as pitchers are usually cautious to avoid hitting the batter.

The game unravels as follows.

1. The pitcher moves first. He can select his aim, $\mu \geq 0$, but not the final pitch location, $\tilde{y}$, which is random. On average, the pitcher's aim is correct, i.e., $E(\tilde{y}) = \mu$.

2. The batter moves next. A batter must decide whether to swing or not soon after a pitch is thrown, but before it reaches its final location $\tilde{y}$. To capture this timing, the batter's swing decision is made immediately after observing μ.[1]

3. If the batter does not swing, two outcomes are possible. For a given value of μ, with probability $s(\mu)$, the pitch is called a strike, and confers the batter a payoff S. With

[1]This strict timing assumption is not crucial. Instead, it is a simplified way of modeling that the batter makes his swing decision under imperfect information. For example, the batter could instead observe a noisy signal of $\tilde{y}$ without changing the results.

probability $1 - s(\mu)$, the pitch is called a ball, with payoff $B > S$. We assume $s' < 0$, $s'' < 0$, i.e., that pitches aimed closer to the plate are more likely to be called strikes, and at an increasing rate.

4. If the batter swings, two additional outcomes are possible. With probability $h(\mu)$, the batter gets a hit, and enjoys a payoff H. With probability $1 - h(\mu)$, the batter does not get a hit, with payoff $N < H$.[2] Similar to the assumptions for s, we assume $h' < 0$, $h'' < 0$.

The Batter's Problem:

To determine whether he swings at a pitch with expected location μ, the batter compares his expected payoff from swinging,

$$\pi(swing|\mu) = h(\mu)H + [1 - h(\mu)]N \tag{A1}$$

with that from not swinging,

$$\pi(no\ swing|\mu) = s(\mu)S + [1 - s(\mu)]B. \tag{A2}$$

Lemma 1. *Assume* $\pi(swing \mid \mu = 0) > \pi(no\ swing) \mid \mu = 0)$, *so that a batter always prefers to swing at a pitch aimed down the center of the plate. Then there exists a unique cutoff M whereby if: i)* $\mu < \hat{\mu}$, *the batter strictly prefers to swing, ii)* $\mu > \hat{\mu}$, *the batter strictly prefers to not swing, and iii)* $\mu = \hat{\mu}$, *the batter is indifferent between swinging and not.*

Proof. $\partial(\pi(swing|\mu)) / \partial\mu < 0$, *which follows from the assumptions that: i) called strikes are assumed to be more likely when thrown closer to the plate,* $s' < 0$, *ii) the batter's expected*

[2] Here, N captures the average payoff of swinging and missing, (S), and hitting into an out.

payoff from called balls is higher than that from called strikes, B > S. By similar logic, ∂(π(no swing|μ)) / ∂μ > 0. The convexity assumptions s′′, h′′< 0 then guarantee a single crossing for (π (swing) |μ) and (π (no swing) |μ), which we denote $\hat{\mu}$.

The intuition for Lemma 1 is straightforward. Batters will not attempt to hit pitches that have very little chance of being called a strike should they not swing, i.e., for sufficiently low values of μ. Moreover, the cutoff for swinging $\hat{\mu}$ is a function of the payoffs *S, B, H*, and *N* that correspond to the possible outcomes of the plate appearance. Generally, these payoffs will depend on game conditions, such as the score, the count, runners on base, or other factors that determine the payoffs to each outcome. For example, with runners on second and third base but no outs, the benefit of a hit (*H*) is substantial, where the cost of hitting into an out (*N*) is relatively small. In this situation, the batter will be less selective at the plate, which increases the swinging cutoff $\hat{\mu}$. We do not model differences in these payoffs across plate appearances, although the present set-up easily allows for this extension.

Our main interest is in how changes in the conditional strike function, $s(\mu)$, influence the batter's optimal behavior. Specifically, assume that the race/ethnicity match of the umpire and pitcher influences the probability that a pitch aimed at location μ will be called a strike. If the pitcher and umpire match (*M*), denote the conditional called strike probability $s_M(\mu)$. If they are different (*D*), the conditional strike probability becomes $s_D(\mu)$. To capture the idea that similar race or ethnicity helps the pitcher, we assume:

$$s_M(\mu) > s_D(\mu), \quad \forall \mu \qquad \text{(A3)}$$

In other words, the same pitch has a different probability of being called a strike, conditional on whether the umpire and pitcher have the same or different races or ethnicities.

Lemma 2. *When the pitcher and umpire share the same race/ethnicity, the batter swings at pitches further from the center of the plate. That is, the cutoff location under a match is strictly greater than the cutoff location otherwise, i.e.,* $\hat{\mu}_M > \hat{\mu}_D$.

Proof. *Denote* $\hat{\mu}_M$ *as the cutoff swinging location when* $s(\mu) = s_M(\mu)$ *and* $\hat{\mu}_D$ *as that when* $s(\mu) = s_D(\mu)$. *Suppose* $s(\mu) = s_M(\mu)$ *and* $\mu = \hat{\mu}_M$. *From equation (2), when* $s(\mu)$ *changes to* $s_D(\mu)$ *the expected payoff of not swinging declines by* $[s_M(\hat{\mu}_M) - s_D(\hat{\mu}_M)](S - B) > 0$, *while the payoff from swinging is unchanged. We can now use the proof for Lemma 1. Because* $\partial(\pi(swing \mid \mu)) / \partial\mu < 0$ *and* $\partial(\pi(no\ swing \mid \mu)) / \partial\mu > 0$, *the new cutoff* $\hat{\mu}_D$ *is strictly less than* $\hat{\mu}_M$.

Lemma 2 indicates that when the batter anticipates judgments that favor the pitcher, his optimal strategy changes. Expecting the umpire's bias to reduce his payoff from not swinging, the batter takes matters into his own hands by swinging at pitches that he would otherwise let pass. Empirically, this implies a distinct advantage to the pitcher, not only for pitches that are called, but also for pitches that are hit. We complete this exercise by extending consideration to the pitcher's optimal strategy.

The Pitcher's Problem:

The pitcher's choice variable is μ, the expected location of the pitch. His expected payoff is the inverse of the batter's. If the batter swings, then the pitcher's expected payoff is $-h(\mu)H-[1-h(\mu)]N$. If the batter does not swing, then his expected payoff is $-s(\mu)S-[1-s(\mu)]B$.

Lemma 3. *The pitcher's optimal pitch location is* $\hat{\mu}$*, so that the batter is indifferent between swinging and not.*

Proof. *The batter will swing at any pitch aimed at* $\mu < \hat{\mu}$*, but because* $\partial(\pi(swing \mid \mu)) / \partial\mu < 0$*, the pitcher is always strictly better off increasing μ given that the batter will swing. The batter will not swing at any pitch aimed at* $\mu > \hat{\mu}$*, but because* $\partial(\pi(no\ swing \mid \mu)) / \partial\mu > 0$*, the pitcher will always decrease μ given that the batter will not swing. It follows then that the optimal pitch location must be* $\hat{\mu}$*.*

The main prediction is that the umpire's bias influences not only called strikes and balls, but also pitches where the umpire's judgment plays no direct role. Lemma 3 shows that the umpire's judgment influences the choice of pitch location, which in turn influences the batter's incentive to swing at the ball. It follows that *conditional on swinging*, the batter is less likely to hit the ball when the umpire and pitcher share race or ethnicity. As indicated by the model, this is because pitches are, on average, more difficult to hit in these situations.