Appendices to
"The Role of Trading Frictions in Real Asset Markets"

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A Model

In this section, I lay out a model of a decentralized market with two-sided search (Mortensen and Wright, 2002) to theoretically investigate the effects of market thickness on asset allocations and prices. As in the recent literature on search in financial markets, the model adapts the framework introduced by Diamond's (1982) seminal paper.

I model frictions of reallocating assets explicitly. In particular, each agent contacts another agent randomly, and this is costly for two reasons: 1) there is an explicit search cost c that an agent pays in order to actively meet another (random) active agent; and 2) there is a time cost in that all agents discount future values by the discount rate $r > 0$.

A.1 Assumptions

Time is continuous and the horizon infinite. There are a total mass $S' > 0$ of assets (i.e., the thickness of the asset market), and a mass $A = S' + B'$ of agents, with $B' > S'$. All agents are risk-neutral and discount the future at the positive rate $r > 0$.

Agents are differentiated by the exogenous productivity parameter $z \geq 0$. The exogenous productivity z is distributed in the population according to the cumulative distribution function $F(z)$ and follows an independent stochastic process: Each agent receives a new draw from $F(z)$ at the instantaneous rate λ

Each agent can own either zero or one asset.[1] An agent z who owns an asset chooses the endogenous utilization h of the asset to maximize the instantaneous payoff given by the difference between revenue zh and costs $\frac{h^2}{2}$ from operating the asset:

$$\pi(z) = \max_h zh - \frac{h^2}{2}.$$

Hence, the optimal capacity utilization is equal to productivity (i.e., $h^* = z$), and $\pi(z) = \frac{1}{2}z^2$ are the instantaneous profits.[2]

Agents can trade assets, and an agent who wants to trade an asset pays a search cost c. An agent who wants to trade makes contacts with other traders pairwise independently at Poisson arrival times with intensity $\gamma > 0$. Given a contact, because of the random-matching assumption, the probability that a buyer (seller) makes contact with a seller (buyer) is S (B), where S and B are the stocks of active sellers and active buyers. In other words, the mass of active sellers S (active buyers B) is the subset of the mass of potential sellers S' (potential buyers B') that has paid the search cost c. Thus, conditional on making contact, all traders are "equally likely" to be contacted. On aggregate, contacts between sellers and buyers occur continually at a total (almost sure) rate of γBS.

[1] Hence, I do not consider quantity decisions, like Duffie, Gârleanu and Pedersen (2005 and 2007), Miao (2006), Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2007 and 2008). It will be clear that the intuition applies more generally. See Lagos and Rocheteau (2009) for a model that considers quantity decisions.

[2] Under more general forms of complementarity between productivity z and capacity utilization h, the optimal capacity utilization is still an increasing function of productivity.

Once an active buyer and an active seller meet, they negotiate a price to trade. I assume that an active buyer and an active seller negotiate a price according to generalized Nash bargaining, where $\theta \in (0, 1)$ denotes the bargaining power of the buyer.

A.2 Solution

The potential seller of an aircraft with productivity z can put it up for sale or keep operating it. In the former case, he pays the search cost c and meets potential trading partners at rate γB. In the latter case, he enjoys the flow profit $\pi(z)$. Similarly, a potential buyer can pay the search cost c and meet active sellers at rate γS, or can wait to search later when his profitability changes. We, thus, have four categories of agents: active and non-active buyers and sellers. I denote by $V_B(z)$ the value function of an active buyer, and by $W_B(z)$ the value function of a non-active buyer. Similarly, $V_S(z)$ and $W_S(z)$ are the value functions of an active and non-active seller, respectively.

Intuitively, a potential buyer prefers to be an active buyer when his productivity z is sufficiently high, and a potential seller prefers to be an active seller when her productivity is sufficiently low. Moreover, an active buyer that has just bought an aircraft does not immediately become (i.e., before z changes) an active seller since he would have been better off not buying the aircraft and not paying the search cost. Thus, when an active buyer and active seller meet and trade, they become a non-active seller and a non-active buyer, respectively.

I now derive formally the value functions for active and inactive agents, and the transaction prices at which trade occurs. These value functions allow me to pin down the equilibrium conditions and to characterize the endogenous distribution of productivities and capacity utilizations of active firms and the endogenous distribution of transaction prices.

Numerical solutions shows that the key element affecting the first two moments of the productivity/capacity utilization and transaction price distributions is how potential sellers' and buyers' cutoff values—the values at which potential sellers and buyers are indifferent between being active or inactive—change with the thickness of the market—that is, with S'. In particular, the higher the sellers' cutoff value and the lower the buyers' cutoff value, the higher is the average and the lower is the variance of potential sellers' productivity. A higher average z, then, translates into a higher average transaction price, and a lower variance of valuation into a lower variance of transaction prices. In turn, sellers' and buyers' cutoff values are determined by the (endogenous) total number of meetings γBS.

As spelled out in more detail in Section II of the paper, the key economic force is that the trading technology exhibits increasing returns to scale. Hence, sellers' reservation value increases and buyers' reservation value decreases as the asset market gets thicker. In turn, as the mass of assets increases: 1) assets have a higher turnover; 2) the average productivity z and average capacity utilization h of firms increase; 3) the dispersion of firms' productivity and the dispersion of firms' capacity utilizations decrease; 4) assets have a higher average price; and 5) assets have a lower dispersion of transaction prices.

A.2.1 Value Functions

Consider an agent with productivity b and no asset. The agent can choose to pay the search cost c and search, or he can decide to stay inactive.

If the agent decides to be an active buyer, his value function $V_B(b)$ satisfies

$$\begin{aligned} rV_B(b) &= -c + \gamma S \int \max\{W_S(b) - p(b,s) - V_B(b), 0\} dG_S(s) \\ &\quad + \lambda \int (\max\{V_B(z), W_B(z)\} - V_B(b)) dF(z) \end{aligned} \tag{A1}$$

where $G_S(s)$ is the endogenous equilibrium distribution of active sellers (which is derived below). Equation (A1) has the usual interpretation of an asset-pricing equation. An active buyer with productivity b pays the search cost c. At any date, at most, one of two possible events might happen to him: 1) At rate γS, he meets an active seller. If he trades, he becomes an inactive seller and, thus, obtains a capital gain equal to $W_S(b) - p(b,s) - V_B(b)$. If he doesn't trade, he has no capital gain. 2) At rate λ, he receives a new productivity draw. After learning his new productivity, he decides whether to remain an active buyer (in which case he has a capital gain/loss equal to $V_B(z) - V_B(b)$) or to become an inactive buyer (in which case he has a capital gain/loss equal to $W_B(z) - V_B(b)$).

Similarly, the value function $V_S(s)$ of an active seller with productivity s satisfies the following Bellman equation:

$$\begin{aligned} rV_S(s) &= -c + \pi(s) + \gamma B \int \max\{p(b,s) + W_B(s) - V_S(s), 0\} dG_B(b) \\ &\quad + \lambda \int (\max\{V_S(z), W_S(z)\} - V_S(s)) dF(z). \end{aligned} \tag{A2}$$

where $G_B(b)$ is the endogenous equilibrium distribution of active buyers (which, again, is derived below). The interpretation of equation (A2) is now straightforward. An active seller receives an instantaneous payoff flow equal to the difference between her profitability $\pi(s)$ and the search cost c. At rate γB, she meets an active buyer. If she trades, she obtains a capital gain equal to $p(b,s) + W_B(s) - V_S(s)$. If she does not trade, she has no capital gain. At rate λ, she receives a new productivity draw. After learning her new productivity, she decides whether to remain an active seller (in which case she has a capital gain/loss equal to $V_S(z) - V_S(s)$) or to become an inactive seller (in which case she has a capital gain/loss equal to $W_S(z) - V_S(s)$).

The value functions W_B and $W_S(s)$ of an inactive buyer and of an inactive seller with productivity s satisfy:

$$rW_B = \lambda \int (\max\{V_B(z), W_B\} - W_B) dF(z) \tag{A3}$$

$$rW_S(s) = \pi(s) + \lambda \int (\max\{V_S(z), W_S(z)\} - W_S(s)) dF(z). \tag{A4}$$

Equations (A3) and (A4) say that the flow value of an inactive trader is equal to the instantaneous profits (0 for a buyer, $\pi(s)$ for a seller) plus the expected capital gain/loss. Thus, the value of an inactive buyer is independent of his profitability.

When an active buyer b and an active seller s meet, if they trade, the negotiated price

$$p(b,s) = \theta(V_S(s) - W_B) + (1-\theta)(W_S(b) - V_B(b)) \tag{A5}$$

is the solution to the following symmetric-information bargaining problem:[3]

$$\begin{aligned} &\max_p [W_S(b) - p - V_B(b)]^\theta [p + W_B - V_S(s)]^{1-\theta} \\ \text{subject to:} \quad &W_S(b) - p \geq V_B(b) \text{ and } p + W_B \geq V_S(s). \end{aligned}$$

Using the equilibrium price (A5), the value function of an active buyer b becomes

$$\begin{aligned} rV_B(b) + c &= \gamma S\theta \int \max\{-V_S(s) + W_B + W_S(b) - V_B(b), 0\} dG_S(s) \\ &+ \lambda \int (\max\{V_B(z), W_B(z)\} - V_B(b)) dF(z). \end{aligned} \tag{A6}$$

Similarly, a value function of an active seller s is:

$$\begin{aligned} rV_S(s) + c &= \pi(s) + \gamma B(1-\theta) \int \max\{-V_S(s) + W_B + W_S(b) - V_B(b), 0\} dG_B(b) \\ &+ \lambda \int (\max\{V_S(z), W_S(z)\} - V_S(s)) dF(z). \end{aligned} \tag{A7}$$

Since $V_B(b)$ is increasing in b, there exists a reservation value R_B such that only buyers with productivity $b \geq R_B$ (and, hence, profits $\pi(b) \geq \pi(R_B)$) have positive gains from trade. R_B satisfies

$$V_B(R_B) = \frac{\lambda \int (\max\{V_B(z), W_B\}) dF(z)}{r+\lambda}.$$

Similarly, there exists reservation value R_S such that only sellers with productivity $s \leq R_S$ (profits $\pi(s) \leq \pi(R_S)$) have positive gains from trade. R_S satisfies

$$V_S(R_S) = \frac{\pi(R_S) + \lambda \int \max\{V_S(z), W_S(z)\} dF(z)}{r+\lambda}.$$

It is easy to show that for c sufficiently large, $R_B > R_S$.

When r is small, $-V_S(s) + V_B(s) + V_S(b) - V_B(b) \geq 0$ for all possible meetings of active buyers and sellers (Mortensen and Wright, 2002). Moreover, $\frac{\partial V_B(x)}{\partial \pi(x)}$ satisfies

$$(r + \lambda + \gamma S\theta) \frac{\partial V_B(x)}{\partial \pi(x)} = \frac{\gamma S\theta}{r+\lambda}.$$

[3]The characteristics of aircraft markets described in Section I support the assumption of symmetric information.

Similarly, $\frac{\partial V_S(x)}{\partial \pi(x)}$ satisfies

$$\frac{\partial V_S(x)}{\partial \pi(x)} = \frac{1}{r+\lambda+\gamma B(1-\theta)}.$$

Thus, both $V_B(x)$ and $V_S(x)$ are linear in $\pi(x)$, with slopes given by $\frac{\partial V_B(x)}{\partial \pi(x)}$ and $\frac{\partial V_S(x)}{\partial \pi(x)}$, respectively. Furthermore, we can use the conditions on the marginal traders $V_S(R_S)$ and $V_B(R_B)$ to find the intercepts. Thus, we can rewrite the value function of an active buyer as

$$V_B(b) = k_B(\pi(b) - \pi(R_B)) + \frac{\lambda \int \max\{V_B(z), W_B\} dF(z)}{r+\lambda} \tag{A8}$$

where $k_B = \frac{\gamma S\theta}{(r+\lambda)(r+\lambda+\gamma S\theta)}$. Similarly, the value function of an active seller is

$$V_S(s) = k_S(\pi(s) - \pi(R_S)) + \frac{\pi(R_S) + \lambda \int \max\{V_S(z), W_S(z)\} dF(z)}{r+\lambda} \tag{A9}$$

where $k_S = \frac{1}{r+\lambda+\gamma B(1-\theta)}$. Thus, the transaction price when seller s and buyer b meet is equal to:

$$\begin{aligned} p(b,s) &= \theta\left(k_S(\pi(s) - \pi(R_S)) + \frac{\pi(R_S)}{r+\lambda}\right) + (1-\theta)\left(\frac{\pi(b)}{r+\lambda} - k_B(\pi(b) - \pi(R_B))\right) + \\ &\quad \int_{R_S} \frac{\lambda \pi(z)}{r(r+\lambda)} dF(z) + \frac{\lambda}{r}\int^{R_S}\left(k_S(\pi(z) - \pi(R_S)) + \frac{\pi(R_S)}{r+\lambda}\right) dF(z) - \\ &\quad \int_{R_B} \frac{\lambda k_B(\pi(z) - \pi(R_B))}{r} dF(z). \end{aligned} \tag{A10}$$

A.2.2 Distributions of Buyers and Sellers

I now derive the endogenous equilibrium distribution of potential buyers B' and potential sellers S' and the endogenous distribution of active buyers B and active sellers S.

Let $g_{S'}(\cdot,t)$ and $g_{B'}(\cdot,t)$ be the distributions of potential sellers and potential buyers, respectively. Consider a small interval of time of length ϵ. Up to terms in $o(\epsilon)$, the distribution of potential sellers $g_{S'}(\cdot,t)$ evolves from time t to time $t+\epsilon$ according to:

$$g_{S'}(z,t+\epsilon) = \begin{cases} \gamma S\frac{B'}{S'}\epsilon g_{B'}(z,t) + \lambda\epsilon f(z,t) + (1-\lambda\epsilon) g_{S'}(z,t) & \text{for } R_B \le z \\ \lambda\epsilon f(z,t) + (1-\lambda\epsilon) g_{S'}(z,t) & \text{for } R_S \le z < R_B \\ \lambda\epsilon f(z,t) + (1-\lambda\epsilon - \gamma B\epsilon) g_{S'}(z,t) & \text{for } z < R_S. \end{cases}$$

Similarly, the distribution of potential buyers $g_B(\cdot,t)$ evolves over time according to:

$$g_{B'}(z,t+\epsilon) = \begin{cases} \lambda\epsilon f(z,t) + (1-\lambda\epsilon - \gamma S\epsilon) g_{B'}(z,t) & \text{for } R_B \le z \\ \lambda\epsilon f(z,t) + (1-\lambda\epsilon) g_{B'}(z,t) & \text{for } R_S \le z < R_B \\ \gamma B\frac{S'}{B'}\epsilon g_{S'}(z,t) + \lambda\epsilon f(z,t) + (1-\lambda\epsilon) g_{B'}(z,t) & \text{for } z < R_S. \end{cases}$$

Steady state requires that traders' flows for each interval of the distribution functions of potential buyers B' and potential sellers S' are equal to traders' flows out. Thus, rearranging and taking the limit for $\epsilon \to 0$, the endogenous distribution of potential sellers S' satisfies

$$g_{S'}(z) = \begin{cases} \frac{B'}{S'}\frac{\gamma S}{\lambda+\gamma S} f(z) + f(z) & \text{for } R_B \le z \\ f(z) & \text{for } R_S \le z < R_B \\ \frac{\lambda}{\lambda+\gamma B} f(z) & \text{for } z < R_S. \end{cases}$$

Similarly, the endogenous distribution of potential buyers satisfies

$$g_{B'}(z) = \begin{cases} \frac{\lambda}{\lambda+\gamma S} f(z) & \text{for } R_B \le z \\ f(z) & \text{for } R_S \le z < R_B \\ \frac{S'}{B'}\frac{\gamma B}{\lambda+\gamma B} f(z) + f(z) & \text{for } z < R_S. \end{cases}$$

Hence, the distribution of active sellers is

$$g_S(z) = \begin{cases} 0 & \text{for } R_S \le z \\ \frac{f(z)}{F(R_S)} & \text{for } z < R_S \end{cases} \tag{A11}$$

and of active buyers is

$$g_B(z) = \begin{cases} \frac{f(z)}{1-F(R_B)} & \text{for } R_B \le z \\ 0 & \text{for } z < R_B. \end{cases} \tag{A12}$$

A.3 Equilibrium

The equilibrium conditions determine the four endogenous variables (R_S, R_B, S, B).

Definition 1 *A steady-state equilibrium is a set of reservation values (R_S, R_B), and a stock of active buyers and sellers (B, S) satisfying the following conditions:*

1. *The reservation values (R_S, R_B) satisfy the following indifference conditions:*[4]

$$c = \gamma S\theta \int \left(k_S (\pi(R_S) - \pi(s)) - \frac{\pi(R_S) - \pi(R_B)}{r+\lambda} \right) dG_S(s) \tag{A13}$$

$$c = \gamma B (1-\theta) \int \left(\frac{\pi(b) - \pi(R_S)}{r+\lambda} - k_B (\pi(b) - \pi(R_B)) \right) dG_B(b) \tag{A14}$$

   where $G_S(s)$ and $G_B(b)$ are the cumulative distribution functions of active sellers and active buyers, respectively. $G_S(s)$ and $G_B(b)$ are derived from the probability density functions $g_S(s)$ and $g_B(b)$ defined in (A11) and (A12);

2. *Active buyers are all potential buyers with productivity above R_B, and active sellers are*

[4]The conditions are obtained by combining equations (A6), (A8) and (A3), and equations (A7), (A9) and (A4), respectively.

all potential sellers with productivity below R_S:

$$\begin{aligned} B &= (1 - G_{B'}(R_B)) B' = B' (1 - F(R_B)) \frac{\lambda}{\lambda + \gamma S} \\ S &= G_{S'}(R_S) S' = S' \frac{\lambda}{\lambda + \gamma B} F(R_S). \end{aligned}$$

A.4 A Numerical Illustration

Unfortunately, the equilibrium conditions do not admit an explicit solution of the endogenous variables (R_S, R_B, S, B) as a function of the exogenous parameters. Thus, in order to understand how market thickness S' affects the equilibrium distribution of productivity and prices, I fix values of the exogenous parameters and the exogenous distribution $F(z)$, and then solve the model numerically.[5] More precisely, the numerical solutions illustrate how moments of the distributions of productivities and prices change as the thickness of the asset market S' increases, while holding the ratio of potential sellers (and, thus, assets) S' and potential buyers B' constant.

Figure 1 illustrates several features of the equilibrium. The behavior of the endogenous variables (R_S, R_B) plotted in the first plot (first row, first column) is the key to understanding the effects of market thickness on asset allocations and asset prices. The plot shows that sellers' reservation value R_S increases and buyers' reservation value R_B decreases as the number of assets increases. This is intuitive: When the asset market is thin, trading frictions are high. Thus, sellers rationally choose to hold on to assets with a thin market for longer periods of time in case their productivity z rises in the future. As market thickness increases, frictions vanish. Hence, the reservation values R_S and R_B converge, and their common limit is given by the Walrasian benchmark R^* that solves $1 - F(R^*) = \frac{S'}{S'+B'}$.[6]

The second plot (first row, second column) shows that the turnover of assets $\frac{\gamma BS}{S'}$ increases as the mass of assets increases. This is due to two reasons: 1) Sellers' cutoff value is higher, so the probability that assets are put on the market for sale is higher; and 2) the meeting rate is higher, so conditional on being on the market, assets trade faster.

The plots in the second row document the effects of market thickness on capacity utilization. The third plot (second row, first column) shows how the average capacity utilization $E(h) = \int h(z) g_{S'}(z) dz$ varies with the mass of assets. Moreover, since each agent chooses the capacity utilization h to be equal to its productivity parameter z, the distribution $g_{S'}(z)$ reflects the efficiency of the allocation of assets. The plot clearly shows that average capacity utilization and average productivity increase as the asset market becomes thicker. This suggests that, on average, assets are more efficiently allocated when their market becomes thicker. The plot also shows that the average capacity utilization and productivity converge to the Walrasian benchmark given by $\mu + \sigma \frac{\phi\left(\frac{R^*-\mu}{\sigma}\right)}{1-\Phi\left(\frac{R^*-\mu}{\sigma}\right)}$, where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard

[5]The numerical values of the exogenous parameters are: $\theta = .5$; $c = 150$; $r = .05$; $\gamma = .2$; $\lambda = .2$; $B' = 1.5S'$ and $F(z)$ is the normal distribution with mean μ equal to 20 and standard deviation σ equal to 5.

[6]Given the numerical values assumed, the Walrasian limit R^* of R_S and R_B is equal to 21.26.

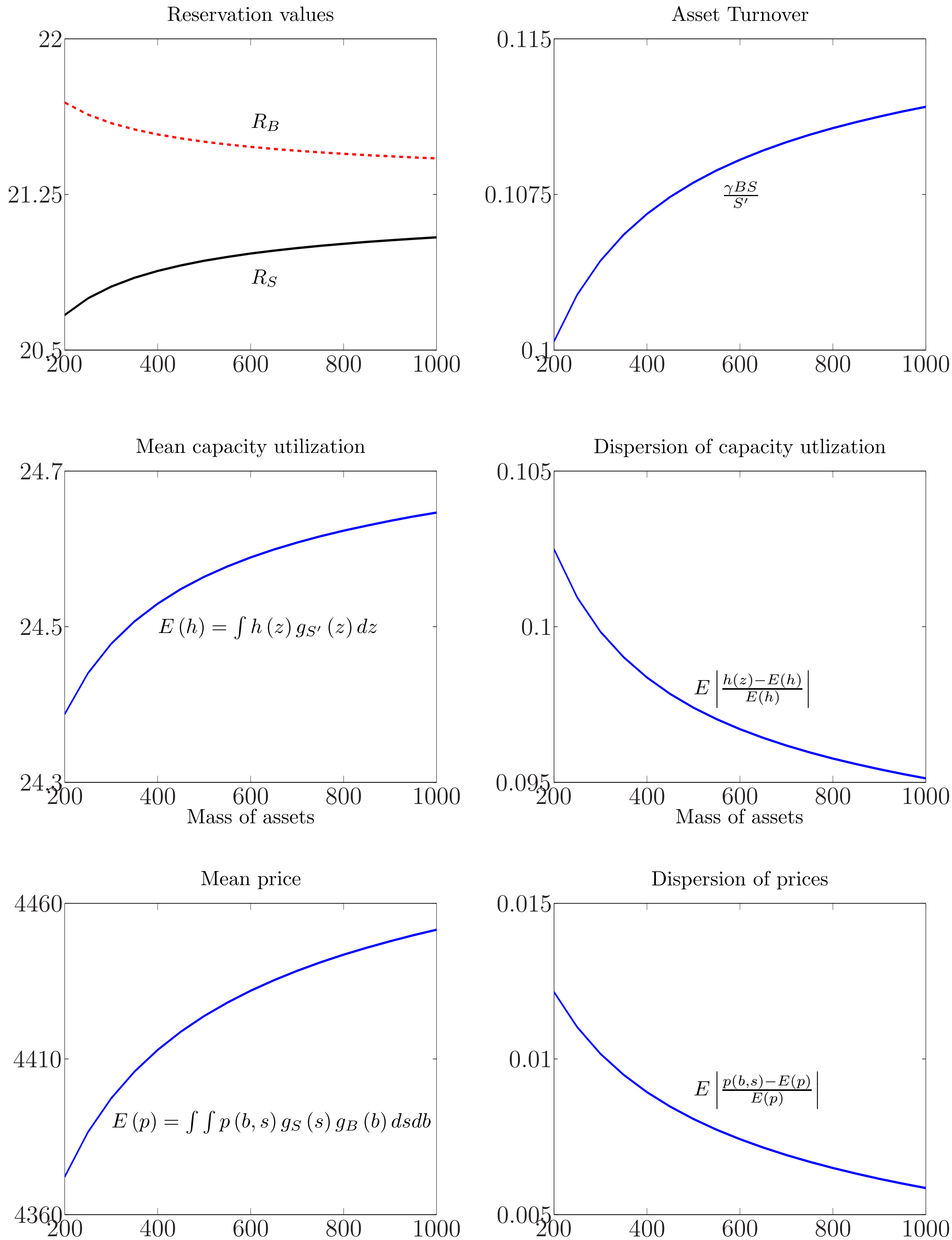

FIG. 1: Numerical solutions

normal p.d.f. and c.d.f., respectively.[7] The fourth plot (second row, second column) shows that the dispersion of capacity utilization $E\left|\frac{h(z)-E(h)}{E(h)}\right|$—which is identical to the dispersion of productivity $E\left|\frac{z-E(z)}{E(z)}\right|$—decreases as asset markets become thicker.

The plots in the third row document the effects of market thickness on asset prices. The fifth plot (third row, first column) of Figure 1 documents that the average asset price $E(p) = \int\int p(b,s)\, g_S(s)\, g_B(b)\, ds db$ increases when the asset market becomes thicker. Moreover, the price converges to the Walrasian price equal to $\frac{\pi(R^*)}{r}$.[8] The sixth plot (third row, second column) shows that the dispersion of transaction prices $E\left|\frac{p(b,s)-E(p)}{E(p)}\right|$decreases as the size of the asset market increases.[9]

B Calibration

The goal of this Appendix is to calibrate the model to assess the magnitude of trading frictions. To this end, I proceed in two steps: 1) I describe the additional assumptions that I need to impose given the available data; and 2) I calibrate a slightly modified version of the model to match moments of the data.

1. The model says that the profits that the asset generates are an increasing function of the productivity of a carrier, and utilization is an increasing function of the productivity of the carrier, as well. Thus, profits generated by an aircraft are an increasing function of utilization, and the data confirm this necessary step (see footnote 17).

   However, calculating the magnitude of trading frictions requires us to understand the exact way in which aircraft utilization translates into aircraft prices—i.e., we need to know the exact functional form that links utilization and profits. Important missing pieces of information, for example, are the revenues and the costs generated by one hour of utilization for each aircraft. This information is not readily available in the data.

   Moreover, the separate identification of many parameters is problematic. Although the model is highly non-linear, so that all parameters affect all outcomes, intuitively revenues and costs parameters determine aircraft prices, and trading parameters determine turnover rates. More precisely, revenues/profits $\pi(h)$ and trading cost parameter c affect one single outcome—i.e., aircraft prices. Thus, there is one equation in two unknown parameters. Similarly, the parameters λ—the persistence of carriers' profitability—and γ—the rate of traders' pairwise meetings—determine the turnover rate.

   In addition, one additional important piece of information that determines turnover, utilizations and prices is the ratio of buyers to sellers $\frac{B'}{S'}$. The paper keeps it constant,

[7] The numerical value of the Walrasian limit is equal to 24.83.

[8] The numerical value of the Walrasian limit is equal to 4522.7.

[9] I am reporting the quantities $E\left|\frac{h(z)-E(h)}{E(h(z))}\right|$ and $E\left|\frac{p(b,s)-E(p)}{E(p)}\right|$ to measure the dispersion of capacity utilizations and prices, rather than the more common variances, because the empirical analysis is based on these quantities $E\left|\frac{h(z)-E(h)}{E(h)}\right|$ and $E\left|\frac{p(b,s)-E(p)}{E(p)}\right|$. The variances of capacity utilizations and prices display identical qualitative patterns—i.e., they decrease as the number of assets increases.

and the qualitative results do not depend on how much larger B' is than S'. The ratio $\frac{B'}{S'}$ determines the ratio of active buyers to active sellers $\frac{B}{S}$, and $\frac{B}{S}$ determines the aggregate number of transactions γBS and the individual trading rates. Potentially, I could recover the magnitude of the ratio $\frac{B'}{S'}$ if I knew how long it takes for a seller to sell an aircraft and for a buyer to buy an aircraft. Unfortunately, as described in Section III.B, the data offer little indication of the magnitudes of these delays and, hence, no indication of what the ratio $\frac{B'}{S'}$ is. Thus, the Poisson rate γ and the ratio $\frac{B'}{S'}$ cannot be separately identified.

2. With the previous caveats in mind, I proceed by fixing some parameters, and by imposing some assumptions on the functional form that links utilization and profits. Then, I use the model to infer the magnitudes of transaction costs by calibrating the key parameters to match some moments of the data.

   Specifically, I fix the bargaining power to $\theta = .5$ and the interest rate $r = .05$. It is well known that the bargaining power and the discount factor/interest rate are difficult parameters to calibrate in the data. In addition, I fix $\gamma = .1$, $\frac{B'}{S'} = 1.5$, and $c = 500,000$. Moreover, I assume that the profit function takes the following form:

$$\pi(h) = \alpha h,$$

   where α is a parameter to be calibrated. The above expression arises, for example, if the profit function is

$$\pi(z) = \max_h \alpha \left(z h^{1/2} - h \right)$$

   so that the optimal $h = \left(\frac{z}{2}\right)^2$ and $\pi(z) = \alpha \left(\frac{z}{2}\right)^2 = \alpha h$. I further assume that $F(z)$ is normal with mean μ and standard deviation σ to be calibrated.

   Further, I select the more popular aircraft type—the Boeing 737—to reduce the heterogeneity across aircraft types, and compute some moments from the data. More precisely, I take the average number of Boeing 737s during the period 1990-2002—equal to 2,700 units—and calculate the corresponding average flying hours, the standard deviations of flying hours, the average price from the "Blue Book" dataset, and the average turnover of Boeing 737s (I am not using any moment on the dispersion of transaction prices since the sample of transaction prices refer to earlier years.) Then, I take a ten-percent increase in the number of Boeing 737s, and apply the elasticities estimated in Tables 4 and 6 to the average flying hours and average price. Columns (1) and (2) in Panel A-Table B1 report the exact numeric value of the moments.

   Then, I then calibrate the parameters $(\alpha, \lambda, \mu, \sigma)$ so that the moments computed from the model are as close as possible to the moments in the data, as reported in Table B1. Panel B of Table B1 reports the implied parameters, and columns (3) and (4) of Panel A report the moments computed from the model at those parameters.

   Overall, the model matches the data moderately well. Moreover, the magnitudes of parameters seem reasonable. The parameter λ implies that the productivity of a carrier

Table B1: Moments and Parameters of the Calibration

Panel A: Moments	(1) Data	(2) Data	(3) Model	(4) Model
Number of Aircraft (Airtype)	2700	3000	2700	3000
Average Flying Hours $E(h)$	2724	2780	2751	2752
Standard Deviation of Flying Hours	239		239	
Average Price ($1,000)	32,950	33,930	33,433	33,445
Turnover	.11	.12	.114	.115

Panel B: Parameters	
γ	0.05
$\frac{B'}{S'}$	1.5
c	500,000
α	672.63
λ	0.217
μ	98.24
σ	7.17

Notes—This table contains details of the calibration of model parameters. Columns (1) and (2) in Panel A report the moments of the data that the model seeks to match. Columns (3) and (4) in Panel A report the corresponding moments computed from the model with the parameters reported in Panel B.

varies, on average, every five years $(\approx 1/\lambda)$. The parameter α means that one flying hour translates into approximately $672 of profits. More interestingly, the Poisson rate γ of the trading technology—fixed at .05—along with the endogenous mass of buyers implies that the individual rate of selling (buying) an aircraft is equal to $\gamma B \approx 3.98$ $(\gamma S \approx 3.87)$, so that it takes approximately $\frac{12}{\gamma B} \approx 3$ months to sell a Boeing 737 when the size of the market is 2,700 units. Since the annual search cost parameter c is fixed to $500,000 and it takes approximately three months to sell a Boeing 737, the total transaction costs are equal to approximately $\frac{c}{\gamma B} \approx \$125,000$ (in year-2000 values), or $\frac{\frac{c}{\gamma B}}{\alpha E(h)} \approx 6.7$ percent of the annual flow of profits from operating the aircraft.

As mentioned, these parameters and the implied transaction costs rely on several assumptions. Unfortunately, some of these assumptions are not directly testable with the available data. For these reasons, I view these parameters as suggestive. Moreover, the magnitudes that these parameters imply do not seem unreasonable.

References

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