Web Appendix A Additional Proofs

Proof of Lemma A1: If m_A is uninformative, then $\nu_1 = \nu_0 = 0.5$. This requires $N_x = 1 - N_z$, which is only possible if $\theta_A \leq 0.5$. Because m_A is uninformative, m_B must be uninformative: $a_1^B = a_0^B$. For biased B to be willing to randomize, his reputation cost must be zero, which implies that biased A randomizes such that $\frac{N_z}{N_x} = \frac{1-N_y}{1-N_w}$. Also, biased B needs to randomize so that biased A's reputation cost is zero: $\Pr(m_A = 1|m_B = 1) = \Pr(m_A = 1|m_B = 0)$. Recall that biased A's agenda-pushing benefit and reputation cost are both filtered by a factor $N_w - (1 - N_y)$, thus biased B can randomize such that $N_w = 1 - N_y$, which is only possible if $\theta_B \leq 0.5$. One uninformative equilibrium is for biased A to choose $x = z = \frac{1-2\theta_A}{2(1-\theta_A)}$, and for biased B to choose $y = w = \frac{1-2\theta_B}{2(1-\theta_B)}$. In this equilibrium, $N_x = N_z = 0.5, N_y = N_w = 0.5$.

Proof of Corollary 5: Suppose that $x > 0, y > 0$ in equilibrium. Recall that biased A, B's indifference conditions are given in equation (A5) and (A6): $\xi(x, y) = 0$ and $\psi(x, y; \beta) = 0$. Let ψ_3 be the partial derivative of ψ with respect to β. Differentiate with respect to β, then we have $\xi_1 x' + \xi_2 y' = 0$ and $\psi_1 x' + \psi_2 y' + \psi_3 = 0$. The changes of x, y with respect to β are respectively:

$$\frac{dx}{d\beta} = \frac{\psi_3 \xi_2}{\xi_1 \psi_2 - \xi_2 \psi_1} > 0; \ \frac{dy}{d\beta} = -\frac{\psi_3 \xi_1}{\xi_1 \psi_2 - \xi_2 \psi_1} > 0.$$

Similarly it can be shown that both x, y increases in α.

Proof of Corollary 7: In Part (1), for given p_A and α, Proposition 6 shows that biased A always reports $m_A = 1$ if $\alpha \leq \alpha^d$. Because the cutoff α^d increases in θ_A, and $\alpha^d = p_A - 0.5$ at

$\theta_A = 0$, $x^d = 0$ if $\alpha \leq p_A - 0.5$ for all θ_A. For any $\alpha > p_A - 0.5$, biased A reports $m_A = 0$ with a positive probability for some θ_A. Moreover, x^d approaches zero if θ_A is arbitrarily close to zero, $x^d = 0$ if $\theta_A = 0$, and $\lim_{\theta_A \to 0} \frac{x^d}{\theta_A} = \frac{2\alpha}{2p_A - 1} - 1$.

If $x^d > 0$, differentiate the LHS of biased A's indifference condition (A7) with respect to x^d:

$$\text{(WA1)}\quad \frac{(2p_A - 1)(1 - x^d)}{(2 - N_x^d)^2} + \alpha\theta_A(1 - x^d)\left[\frac{1}{(N_x^d)^2} + \frac{p_A^2(1 - p_A)}{(1 - p_A N_x^d)^2} + \frac{p_A(1 - p_A)^2}{(1 - (1 - p_A)N_x^d)^2}\right],$$

which is clearly positive. Next, differentiate the LHS of (A7) with respect to θ_A, we have:

$$\text{(WA2)}\quad \frac{(2p_A - 1)(1 - x^d)}{(2 - N_x^d)^2} - \frac{\alpha x^d}{(N_x^d)^2} + \alpha p_A(1 - p_A)\left[\frac{1 - p_A x^d}{(1 - p_A N_x^d)^2} + \frac{1 - (1 - p_A)x^d}{(1 - (1 - p_A)N_x^d)^2}\right],$$

which is strictly negative at θ_A sufficiently close to 0. Moreover, expression (WA2) itself strictly increases in θ_A. Next, there exists a cutoff value $\overline{\theta}_A$ such that $x^d = 0$ if $\theta_A \geq \overline{\theta}_A$. The cutoff $\overline{\theta}_A$ is implicitly defined by $g(p_A, \overline{\theta}_A, \alpha) = 0$, where

$$g(p_A, \theta_A, \alpha) \equiv \frac{2p_A - 1}{2 - \theta_A} - \frac{\alpha(1 - \theta_A)(1 - 2p_A(1 - p_A)\theta_A)}{(1 - p_A\theta_A)(1 - (1 - p_A)\theta_A)}.$$

At $\theta_A = \overline{\theta}_A$ and $x^d = 0$, (WA2) is positive. Because of the monotonicity of (WA2) with respect to θ_A, the sign of (WA2) changes only once from negative to positive. Since the LHS of (A7) strictly increases in x^d, by the implicitly function theorem, x^d first increases in θ_A, then decreases, and becomes zero when $\theta_A \geq \overline{\theta}_A$.

In Part (2), for given θ_A and α, if $x^d > 0$, then biased A's indifference condition (A7) implicitly defines a function $f(x^d, p_A)$ such that x^d is the solution to $f(x^d, p_A) = 0$. From (A7),

$x^d > 0$ at p_A just above 0.5. Differentiate with respect to p_A:

$$\text{(WA3)} \qquad \frac{\partial f}{\partial p_A} = \frac{2}{2 - N_x^d} - \frac{(2p_A - 1)\alpha\theta_A(1-\theta_A)(1-x^d)(2-N_x^d)}{(1-p_A N_x^d)^2(1-(1-p_A)N_x^d)^2}.$$

Clearly, (WA3) is positive if p_A is sufficiently close to 0.5. Also, (WA3) is decreasing since the second derivative of f with respect to p_A is negative. Since (WA1) is positive, $\frac{dx^d}{dp_A} < 0$ if p_A is sufficiently close to 0.5. As p_A increases, there are two possibilities. First, if $\alpha \leq \frac{1}{2-\theta_A}$, then there exists a cutoff value $\hat{p}_A$ such that if $p_A \geq \hat{p}_A$, the LHS of IC (A3) is always larger than the RHS at $x^d = 0$. In this case, x^d first decreases in p_A and becomes zero if $p_A \geq \hat{p}_A$. Second, if $\alpha \geq \frac{1}{2-\theta_A}$, then $x^d > 0$ for all p_A. If p_A is sufficiently close to 1, and if $\alpha\theta_A \geq \frac{1}{2}$, we have $\frac{dx^d}{dp_A} > 0$. Thus if $\alpha \geq \max\left\{\frac{1}{2-\theta_A}, \frac{1}{2\theta_A}\right\}$, there exists a cutoff $\overline{p}_A$ such that x^d decreases in p_A for $p_A \in (\frac{1}{2}, \overline{p}_A]$, but increases in p_A otherwise.

Proof of Proposition 9: we first compare biased A's ex ante expected payoffs for a given θ_A before studying his channel choice.

Step 1: biased A's ex ante expected payoffs. Let $\mathbb{E}U_A^d, \mathbb{E}U_A^i$ be, respectively, biased A's expected equilibrium payoff from direct and indirect communication before observing s_A. Then:

$$\text{(WA4)} \qquad \mathbb{E}U_A^d = \Pr(\eta = 1 | m_A = 1) + \frac{1}{2}\alpha \sum_{\eta} \Pr(A = o | m_A = 1, \eta),$$

is biased A's ex ante expected payoff if he always reports $m_A = 1$. Note that if $x^d > 0$, biased A is indifferent between reporting $m_A = 1$ or $m_A = 0$ if $s_A = 0$; and thus his ex ante payoff is the same as if he always reports $m_A = 1$. Also, $\mathbb{E}U_A^d$ amounts to a weighted sum of C's posterior beliefs. The same sum of C's prior beliefs is simply $0.5+\alpha\theta_A$. Moreover, if $x^d>0$, we have:

$$\begin{aligned}
& \mathbb{E}U_A^d - (0.5+\alpha\theta_A) \\
= \; & \Pr(\eta=1|m_A=1) - [\Pr(\eta=1|m_A=1)\Pr(m_A=1) + \Pr(\eta=1|m_A=0)\Pr(m_A=0)] \\
+ \; & \frac{1}{2}\alpha\sum_{\eta}\Pr(A=o|m_A=1,\eta) - \alpha[\Pr(A=o|m_A=1)\Pr(m_A=1) + \Pr(A=o|m_A=0)\Pr(m_A=0)] \\
= \; & [\Pr(\eta=1|m_A=1) - \Pr(\eta=1|m_A=0)]\Pr(m_A=0) \\
+ \; & \frac{1}{2}\alpha\left[\pi_{A(1,1)} + \pi_{A(1,0)} - \pi_{A(1,1)}\Pr(m_A=1|\eta=1) - \pi_{A(1,0)}\Pr(m_A=1|\eta=0)\right] - \alpha\pi_{A(0,0)}\Pr(m_A=0) \\
= \; & \left[a_1^B - a_0^B - \alpha[\pi_{A(0,0)} - p_A\pi_{A(1,0)} - (1-p_A)\pi_{A(1,1)}]\right]\Pr(m_A=0) \\
= \; & 0.
\end{aligned}$$

The first equality holds due to the law of iterated expectations, and the last equality holds because it is simply biased A's indifference condition (A7). Thus biased A gets $0.5+\alpha\theta_A$ if $x^d>0$. If $x^d=0$, then (A7) does not hold, and thus $\mathbb{E}U_A^d > 0.5+\alpha\theta_A$. Similarly, biased A's ex ante expected payoff from indirect communication is:

(WA5) $$\mathbb{E}U_A^i = \Pr(\eta=1|m_B=1) + 0.5\alpha\sum_{\eta}\Pr(A=o|m_B=1,\eta).$$

Moreover, if $x>0$ in equilibrium, $\mathbb{E}U_A^i$ is equal to $0.5+\alpha\theta_A$; but if $x=0$, then $\mathbb{E}U_A^i > 0.5+\alpha\theta_A$. Therefore if $x^d>0, x>0$, biased A is ex ante indifferent between these channels.

Step 2: Compare biased A's ex ante expected payoffs for a given θ_A. Observe that cutoff α^i from Proposition 4 increases in θ_A and decreases in θ_B. For any given θ_A, since direct communication is equivalent to $\theta_B=1$, and $\alpha^d=\alpha^i$ at $\theta_B=1$, $\alpha^d<\alpha^i$. As shown in

Proposition 6, $x^d = 0$ if $\alpha \leq \alpha^d$ and $x^d > 0$ otherwise. Holding the intermediary's characteristics fixed, a similar cutoff $\alpha_2 \in (\alpha^d, \alpha^i]$ exists with indirect communication such that $x = 0$ if $\alpha \leq \alpha_2$ and $x > 0$ otherwise. A similar cutoff $\beta_2 \leq \beta^i$ also exists for the intermediary B, holding agent A's characteristics fixed.

To see this, recall from the proof of Proposition 4 that if $x > 0, y > 0$, $\xi(x, y) = 0, \psi(x, y) = 0$; and that if $\alpha \geq \alpha^i$, $x > 0$ in the unique agenda-pushing equilibrium. If $\beta \leq \beta^i$, then Proposition 4 shows that $x = 0, y = 0$ is the unique equilibrium if $\alpha < \alpha^i$, and thus $\alpha_2 = \alpha^i$ in this case. If $\beta > \beta^i$, then there exists a $y' > 0$ such that $\psi(0, y') = 0$, that is, $y^{BR}(0) = y'$, or $y^{BR}(x) \geq y'$ for all $x \geq 0$. If $\alpha < \alpha^i$, then α_2 is implicitly defined such that $\xi(0, y') = 0$ at $\alpha = \alpha_2$. If $\alpha > \alpha_2$, then $\xi(0, y') < 0$, which implies that $x^{BR}(y) > 0$ for all $y \geq y'$. Thus in any equilibrium, $x > 0$. If $\alpha \leq \alpha_2$, then $x = 0, y = y'$ is an equilibrium. Also, because biased A's best response is steeper than biased B's in any interior equilibrium, no other equilibrium exists. Finally, because $\xi(0, 1) < \xi(0, y') = 0$ at α_2 and $\xi(0, 1) = 0$ at α^d, $\alpha^d < \alpha_2$. Given Step 1, we can see that for a given θ_A and a given intermediary, if $\alpha \in [0, \alpha^d]$, $x^d = x = 0$. If $\alpha \in (\alpha^d, \alpha_2]$, then $x^d > 0, x = 0$, and $\mathbb{E}U_A^i(\theta_A) > \mathbb{E}U_A^d(\theta_A)$. Finally, if $\alpha > \alpha_2$, then $x^d > 0, x > 0$ and $\mathbb{E}U_A^i(\theta_A) = \mathbb{E}U_A^d(\theta_A)$.

Step 3: Biased A's ex ante channel choice. Recall that objective A is assumed to use direct communication with probability $\mu \in (0, 1)$ and biased A chooses direct communication with probability γ. From Bayes' rule, agent A's interim objectivity given his channel choice is:

$$\theta_A^d \equiv \Pr(A=o|\text{direct}) = \frac{\theta_A \mu}{\theta_A \mu + (1-\theta_A)\gamma}; \theta_A^i \equiv \Pr(A=o|\text{indirect}) = \frac{\theta_A(1-\mu)}{\theta_A(1-\mu)+(1-\theta_A)(1-\gamma)}.$$

Clearly, $\theta_A^d \geq \theta_A \geq \theta_A^i$ if $\mu \geq \gamma$ and vice versa. Biased A chooses a channel by comparing $\mathbb{E}U_A^d(\theta_A^d)$ with $\mathbb{E}U_A^i(\theta_A^i)$. From Step 1 and 2 above, the expected payoff of biased A if he uses direct communication is $0.5+\alpha\theta_A^d$ if $x^d>0$; if $x^d=0$, it becomes:

$$\text{(WA6)} \qquad \mathbb{E}U_A^d(\theta_A^d) = \frac{2p_A-1}{2-\theta_A^d} + (1-p_A) + \frac{\alpha\theta_A^d}{2}\left[\frac{p_A}{1-(1-p_A)\theta_A^d} + \frac{1-p_A}{1-p_A\theta_A^d}\right].$$

His expected payoff from indirect communication is $0.5+\alpha\theta_A^i$ if $x>0$. If $x=0$, it becomes:

$$\text{(WA7)} \quad \mathbb{E}U_A^i(\theta_A^i) = \frac{2p_A-1}{2-\theta_A^i N_y} + (1-p_A) + \frac{\alpha\theta_A^i}{2}\left[\frac{1-(1-p_A)N_y}{1-(1-p_A)\theta_A^i N_y} + \frac{1-p_A N_y}{1-p_A\theta_A^i N_y}\right].$$

First, it is never part of the equilibrium for biased A to use indirect communication exclusively. Suppose $\gamma=0$ is part of an equilibrium, then $\theta_A^d=1$ and $\theta_A^i<\theta_A$. Thus $\mathbb{E}U_A^d(1)>\mathbb{E}U_A^i(\theta_A^i)$ from (WA6) and (WA7), which is a contradiction.

Next, if $\gamma=1$, then $\theta_A^i=1$ and $\theta_A^d=\overline{\theta}_A^d \equiv \frac{\theta_A\mu}{\theta_A\mu+(1-\theta_A)} < \theta_A$. If $\alpha \in [\alpha^d, \alpha_2]$, by Step 2, $x^d>0, x=0$ at θ_A, and $\mathbb{E}U_A^d(\theta_A) < \mathbb{E}U_A^i(\theta_A)$. In this case, if $\gamma=1$, biased A's expected payoff is $0.5+\alpha\overline{\theta}_A^d$. Because α^d increases in θ_A^d, biased A starts randomizing at a smaller α if $\theta_A^d=\overline{\theta}_A^d$, that is, $\alpha^d(\overline{\theta}_A^d)<\alpha^d(\theta_A)$. At $\theta_A^i=1$, biased A's expected payoff if he uses indirect communication is simply $\mathbb{E}U_A^i(1)=\frac{2p_A-1}{2-N_y}+(1-p_A)+\alpha$. Clearly, $\mathbb{E}U_A^i(1)>0.5+\alpha\overline{\theta}_A^d$ for any $\overline{\theta}_A^d$. If α is sufficiently close to zero, however, $x^d=0, x=0$ at $\theta_A^d=\overline{\theta}_A^d$. Also, using

(WA6), we can see that $\mathbb{E}U_A^d(\overline{\theta}_A^d) > \mathbb{E}U_A^i(1)$ if $\overline{\theta}_A^d$ is sufficiently large. In this case, there exists an equilibrium in which $\gamma = 1$. Otherwise, because $\mathbb{E}U_A^d(1) > \mathbb{E}U_A^i(\theta_A^i)$ at $\gamma = 0$ and $\mathbb{E}U_A^d(\overline{\theta}_A^d) < \mathbb{E}U_A^i(1)$ at $\gamma = 1$, and both $\mathbb{E}U_A^d$, $\mathbb{E}U_A^i$ are continuous, there exists a mixed strategy equilibrium: $\gamma \in (0,1)$. Finally, if $\alpha \geq \alpha_2$, then from Step 2, $\mathbb{E}U_A^d(\theta_A) = \mathbb{E}U_A^i(\theta_A)$. Because biased A receives the same expected payoff in either channel, he is indifferent, and thus $\gamma = \mu$ is an equilibrium.

We now proceed to prove Proposition 9. Note from (WA6) that $\mathbb{E}U_A^d$ strictly increases in θ_A^d (and decreases in γ). If β is sufficiently low, then $y = 0$ and $N_y = \theta_B$, and thus $\mathbb{E}U_A^i$ strictly increases in θ_A^i:

$$\frac{\partial}{\partial \theta_A^i}\mathbb{E}U_A^i = \frac{(2p_A - 1)N_y}{(2 - \theta_A^i N_y)^2} + \frac{\alpha}{2}\left[\frac{1-(1-p_A)N_y}{(1-(1-p_A)\theta_A^i N_y)^2} + \frac{1 - p_A N_y}{(1 - p_A\theta_A^i N_y)^2}\right] > 0.$$

Because $\mathbb{E}U_A^d(\theta_A) > \mathbb{E}U_A^i(\theta_A)$ at $\alpha = 0$ and $\mathbb{E}U_A^d(\theta_A) < \mathbb{E}U_A^i(\theta_A)$ at $\alpha = \alpha^d$, there exists a cutoff $\alpha^s < \alpha^d$ such that $\mathbb{E}U_A^d(\theta_A) = \mathbb{E}U_A^i(\theta_A)$ at $\alpha = \alpha^s$. Also, since $y = 0$, recall from above that $x = 0$ if $\alpha < \alpha^i$. Therefore $\mathbb{E}U_A^d(\theta_A) < \mathbb{E}U_A^i(\theta_A)$ if $\alpha \in (\alpha^s, \alpha^i]$.

Next, suppose that $\alpha \leq \alpha^s$. If $\theta_B < \overline{\theta}_A^d$, then $\mathbb{E}U_A^d(\overline{\theta}_A^d) > \mathbb{E}U_A^i(1)$ for α sufficiently small. Because $\mathbb{E}U_A^d$ decreases in γ, biased A uses direct channel exclusively in the unique equilibrium. If $\theta_B \geq \overline{\theta}_A^d$, then a mixed strategy equilibrium exists: $\gamma < 1$. Moreover, $\mathbb{E}U_A^d(\overline{\theta}_A^d) < \mathbb{E}U_A^i(1)$ at $\gamma = 1$; and $\mathbb{E}U_A^d(\theta_A^d) \geq \mathbb{E}U_A^i(\theta_A^d)$ at $\gamma = \mu$. Because $\mathbb{E}U_A^d$ decreases in γ and $\mathbb{E}U_A^i$ increases in γ, the equilibrium is unique: $\gamma \in [\mu, 1)$. If $\alpha \in [\alpha^s, \alpha^i]$ instead, similar arguments can show that $\gamma \in (0, \mu)$ in the unique mixed strategy equilibrium. If $\alpha \geq \alpha^i$, then $\gamma = \mu$ is the unique equilibrium because of the monotonicity of biased A's payoffs and biased A's indifference between the channels at θ_A.

Proof of Corollary 10: if β is sufficiently low, $y = 0$ and thus $N_y = \theta_B$. Also, from Proposition 9, we know that in this case, biased A uses direct communication with the same probability as objective A if $\alpha > \alpha^i$. If $\alpha \leq \alpha^i$, then $x = 0$ at θ_A. Differentiate $\mathbb{E}U_A^i$ with respect to N_y:

$$\frac{\partial}{\partial N_y}\mathbb{E}U_A^i = \frac{(2p_A - 1)\theta_A^i}{(2 - \theta_A^i N_y)^2} - \frac{\alpha}{2}\left[\frac{(1-p_A)(1-\theta_A^i)\theta_A^i}{(1-(1-p_A)\theta_A^i N_y)^2} + \frac{p_A(1-\theta_A^i)\theta_A^i}{(1-p_A\theta_A^i N_y)^2}\right].$$

Biased A's response to N_y depends on α. From Proposition 4, x increases in N_y if $x > 0$; and biased A is indifferent between $x = 0$ or $x > 0$ at $\alpha = \alpha^i$. If α is sufficiently close to α^i, then $\mathbb{E}U_A^i$ decreases in N_y (and thus θ_B), but still increases in θ_A^i (and thus γ). Therefore γ increases in θ_B in the mixed strategy equilibrium if α is smaller, but sufficiently close to α^i.

Proof of Proposition 11: since the proof is similar to that of Lemma 2 and Proposition 4, only a sketch is offered. Biased B's agenda-pushing strategy is to report $m_B = 0$ if $m_A = 0$, but to report $m_B = 1$ if $m_A = 1$ with probability w: $y = 1, w \in [0, 1)$. The decision maker's action and all posterior objectivity are as given in the proof of Lemma 2, but biased B's incentive constraints are different. Recall that $\nu_0 = \Pr(\eta = 1|m_A = 0)$ and $\nu_1 = \Pr(\eta = 1|m_A = 1)$. For biased B to report $m_A = 0$ and $m_A = 1$ truthfully, the following ICs must hold:

(WA8) $$a_1^B - a_0^B \geq \beta[\nu_0\pi_{B(1,1)} + (1-\nu_0)\pi_{B(1,0)} - \nu_0\pi_{B(0,1)} - (1-\nu_0)\pi_{B(0,0)}];$$

(WA9) $$a_1^B - a_0^B \leq \beta[\nu_1\pi_{B(1,1)} + (1-\nu_1)\pi_{B(1,0)} - \nu_1\pi_{B(0,1)} - (1-\nu_1)\pi_{B(0,0)}].$$

The LHS of IC (WA8) and (WA9) is biased B's agenda-pushing benefit if he reports $m_B = 0$ instead of $m_B = 1$: $\Pr(\eta = 0|m_B = 0) - \Pr(\eta = 0|m_B = 1) = -a_0^B - (-a_1^B)$, which is $a_1^B - a_0^B$; and the RHS is the difference in his reputation cost. Similar to the proof of Lemma 2, if A's message is informative ($\nu_1 \neq \nu_0$), the RHS of IC (WA8) is always smaller than the RHS of IC (WA9) since

$$\beta(\nu_1 - \nu_0)[\pi_{B(1,0)} + \pi_{B(0,1)} - \pi_{B(0,0)} - \pi_{B(1,1)}] < 0.$$

Thus there are only two possibilities: $y = 1, w \in [0, 1)$ and $y \in [0, 1), w = 1$. Suppose $y \in [0, 1), w = 1$, we can show that if $\nu_1 > \nu_0$, then $a_1^B - a_0^B > 0$, but the RHS of IC (WA9) is negative, thus IC (WA9) cannot hold, a contradiction. If $\nu_1 < \nu_0$ and $y \in [0, 1), w = 1$, then biased A's reputation cost differs if he reports $m_A = 1$ instead of $m_A = 0$ given s_A, thus he must use either $x \in [0, 1), z = 1$ or $x = 1, z \in [0, 1)$. But in either case, $\nu_1 > \nu_0$, a contradiction. This establishes that if $\nu_1 \neq \nu_0$, biased B can only use his agenda-pushing strategy $y = 1, w \in [0, 1)$.

Next, biased A must use his agenda-pushing strategy $x \in [0, 1), z = 1$ if $y = 1, w \in [0, 1)$. Suppose that biased A is either fully truthful ($x = 1, z = 1$), of that he uses a strategy similar to biased B's: $x = 1, z \in [0, 1)$. In this case, biased A distorts $s_A = 1$ with some probability and biased B distorts $m_A = 1$ with some probability. Therefore when decision maker C hears $m_B = 1$, she knows that $s_A = 1$ and takes the highest action $a = p_A$. Also, given this strategy, $m_B = 1$ is a sign of objectivity for biased A. Consequently, biased A induces the highest action from C, and gets a higher expected reputational payoff from reporting $m_A = 1$. Therefore he would deviate and report $m_A = 1$, a contradiction. For the same reason, truth telling is impossible because the gain from reporting $m_A = 1$ given biased B's strategy is positive, but the reputational cost is zero. This proves that the only informative equilibrium is agenda-pushing.

An agenda-pushing equilibrium exists because both players' agenda-pushing benefit increases in their own truth-telling probabilities x and w. If IC (A3) and IC (WA9) don't hold because α, β are too low, then in equilibrium, $x = 0, w = 0$. If α, β are sufficiently high, then the LHS of IC (A3) is smaller than the RHS at $x = 0$, but strictly larger than that at $x = 1$, thus there exists a $x \in (0, 1)$ such that IC (A3) holds with equality. It defines biased A's best response function $x^{BR}(w)$. Similarly, $w^{BR}(x)$ exists. Because $x^{BR}(0) \in (0, 1), x^{BR}(1) \in (0, 1)$, $w^{BR}(0) \in (0, 1), w^{BR}(0) \in (0, 1)$, they intersect by the intermediate value theorem. Moreover, if IC (A3) holds with equality, it implicitly defines a function $\chi(x, w) = 0$. Differentiate $\xi(x, w)$ with respect to w, we have $\alpha\theta_A(1 - N_x)$ times:

$$\kappa_1^o \left[\frac{1 - 0.5N_x}{1 - (1 - 0.5N_x)N_w} - \frac{1 - p_A N_x}{1 - (1 - p_A N_x)N_w}\right] + \kappa_2^o \left[\frac{1 - 0.5N_x}{1 - (1 - 0.5N_x)N_w} - \frac{1 - (1 - p_A)N_x}{1 - (1 - (1 - p_A)N_x)N_w}\right];$$

$$\kappa_1^o \equiv \frac{p_A^2}{(1 - p_A N_x)[1 - (1 - p_A N_x)N_w]}, \kappa_2^o \equiv \frac{(1 - p_A)^2}{(1 - (1 - p_A)N_x))[1 - (1 - (1 - p_A)N_x)N_w]}.$$

Because biased A still distorts $s_A = 0$, the first term, which is associated with $\eta = 0$, is positive and dominates. This implies that an increase in w increases biased A's agenda-pushing benefit more than his reputation cost, thus x^{BR} decreases in w and $w^{BR}(x)$ decreases in x.

Proof of Proposition 12: let $y = y_n, w = y_n$, then biased A's incentive constraints and all the posterior objectivity are as given in Lemma 2. If α is sufficiently high, biased A is indifferent between reporting $m_A = 0$ and $m_A = 1$ if $s_A = 0$: IC (A3) holds with equality at x_n. Let $N_{xn} \equiv \theta_A + (1-\theta_A)x_n$. Differentiate this indifference condition with respect to y_n, we can show that the net effect of a change in y_n at $y_n = 1$ on biased A is $\alpha\theta_A(1 - N_{xn})$ times:

$$\begin{aligned} & \frac{p_A}{1-p_A N_{xn}}\left[\frac{1}{2-N_{xn}} - \frac{p_A}{1-p_A N_{xn}} + \frac{1}{N_{xn}}\left(\frac{2-N_x}{N_{xn}} - \frac{1-p_A N_{xn}}{p_A N_{xn}}\right)\right] \\ + \quad & \frac{1-p_A}{1-(1-p_A)N_{xn}}\left[\frac{1}{2-N_{xn}} - \frac{1-p_A}{1-(1-p_A)N_{xn}} + \frac{1}{N_{xn}}\left(\frac{2-N_{xn}}{N_{xn}} - \frac{1-(1-p_A)N_{xn}}{(1-p_A)N_{xn}}\right)\right]. \end{aligned}$$

Combine terms and rearrange, the above expression is positive if $N_{xn} < 2 - \sqrt{2}$, in which case x_n decreases in y_n; it is negative if $N_{xn} > 2 - \sqrt{2}$, in which case x_n increases in y_n.

Proof of Proposition 13: Consider the symmetric k-agent model where the decision maker $k+1$ chooses an action based on m_k. Let $\mathbb{E}U_i^k$ denote biased i's expected payoff, then he has two truth-telling ICs given $m_{i-1} = 0$ and $m_{i-1} = 1$ respectively ($s_1 = 0$ and $s_1 = 1$ respectively for biased agent 1):

$$\mathbb{E}U_i^k(m_i = 1|m_{i-1} = 0) \leq \mathbb{E}U_i^k(m_i = 0|m_{i-1} = 0);$$

$$\mathbb{E}U_i^k(m_i = 1|m_{i-1} = 1) \geq \mathbb{E}U_i^k(m_i = 0|m_{i-1} = 1).$$

Let biased i adopt an agenda-pushing strategy such that he reports $m_i = 1$ if $m_{i-1} = 1$ (if $s_1 = 1$ for biased 1), but $m_i = 0$ with probability x_i if $m_{i-1} = 0$ (if $s_1 = 0$ for biased 1). Arguments similar to those in the proof of Proposition 4 can show that each biased agent's net agenda-pushing benefit and his net reputation cost of lying are multiplied by a common factor $\Pr(m_k = 0|m_i = 0)$. Let $N_i = \theta + (1-\theta)x_i$, then biased i's agenda-pushing benefit (relative to his reputation cost) is simply:

$$\Pr(\eta = 1|m_k = 1) - \Pr(\eta = 1|m_k = 0) = \frac{2p_1 - 1}{2 - \prod_{i=1}^{k} N_i}.$$

Let biased l, $l \neq i$ report $m_{l-1} = 0$ truthfully with probability x_l, then biased i's net reputation cost is:

$$\begin{aligned} & \alpha[\Pr(i = o|m_k = 0) - p_A \Pr(i = o|m_k = 1, \eta = 0) - (1 - p_A)\Pr(i = o|m_k = 1, \eta = 1)] \\ = \ & \alpha\theta \left(\frac{1}{N_i} - \frac{p_A(1 - p_A \prod_l^k N_l)}{1 - p_A \prod_{i=1}^k N_i} - \frac{(1 - p_A)(1 - (1 - p_A) \prod_l^k N_l)}{1 - (1 - p_A) \prod_{i=1}^k N_i} \right). \end{aligned}$$

If α is sufficiently small, biased i always reports $m_i = 1$. If α is sufficiently high, there exists an interior agenda-pushing equilibrium such that for each biased i:

(WA10) $$\frac{2p_1 - 1}{2 - \prod_i^k N_i} = \alpha\theta \left(\frac{1}{N_i} - \frac{p_1(1 - p_1 \prod_l^k N_l)}{1 - p_1 \prod_{i=1}^k N_i} - \frac{(1 - p_1)(1 - (1 - p_1) \prod_l^k N_l)}{1 - (1 - p_1) \prod_{i=1}^k N_i} \right)$$

Observe from this indifference condition that $x_i = x_k$ for all i, is clearly an equilibrium. Moreover, no asymmetric agenda-pushing equilibrium exists.

Next, consider a $k+1$ agent model where the decision maker is $k+2$. If $x_k > 0, x_{k+1} > 0$, compare x_k with x_{k+1}, the truth-telling probability in the $k+1$ agent model. We can show that at $x_k = x_{k+1}$, the difference in the agenda-pushing benefit of any biased $i, i \leq k$, is:

$$\begin{aligned} & \Pr(\eta = 1|m_k = 1) - \Pr(\eta = 1|m_k = 0) - [\Pr(\eta = 1|m_{k+1} = 1) - \Pr(\eta = 1|m_{k+1} = 0)]] \\ = \ & \frac{2p_1 - 1}{2 - (N_k)^k} - \frac{2p_1 - 1}{2 - (N_{k+1})^{k+1}} = \frac{(2p_1 - 1)(N_k)^k}{(2 - (N_k)^k)(2 - (N_k)^{k+1})}. \end{aligned}$$

And at $x_k = x_{k+1}$, the difference in biased i's net reputation cost is:

$$\begin{aligned} & \alpha[\mathbb{E}_\eta \left[\Pr(i = o|m_{k+1} = 1, \eta) - \Pr(i = o|m_k = 1, \eta)\right]] \\ = \; & \alpha\theta(1 - N_k)^2 \left[\frac{p_1^2}{(1 - p_1(N_k)^k)(1 - p_1(N_k)^{k+1})} + \frac{(1 - p_1)^2}{(1 - (1 - p_1)(N_k)^k)(1 - (1 - p_1)(N_k)^{k+1})} \right]. \end{aligned}$$

Using biased i's indifference condition (WA10), we can show that at $x_k = x_{k+1}$, the difference in biased i's net agenda-pushing benefit is strictly smaller than the difference in his net reputation cost. This implies that at $x_k = x_{k+1}$, biased i in the $k + 1$ agent model prefers reporting $m_i = 1$ when $m_{i-1} = 0$. Thus x_{k+1} must decrease so that biased i is still indifferent between $m_i = 1$ and $m_i = 0$, implying $x_k > x_{k+1}$.

Web Appendix B Strategic Objective Agent and Channel Choice

As described in Section V.A, both objective and biased agents are strategic in this appendix. Agent A chooses a channel before observing s_A. He then sends a message to C if he has chosen direct communication; and otherwise to B who then sends a message to C. Objective A chooses the channel and message m_A to maximize his ex ante expected payoff $\mathbb{E}_\eta \left[-(a - \eta)^2 + \alpha^o \pi_A | m_A\right]$ if direct communication is chosen, and $\mathbb{E}_\eta \mathbb{E}_{m_B} \left[-(a - \eta)^2 + \alpha^o \pi_A | m_B\right]$ if indirect communication is chosen, where π_A is A's posterior objectivity. All other assumptions remain.

As argued in Section V.A, within a channel, reporting truthfully is optimal for objective A and B when their weights on reputation are sufficiently low or zero. Recall that μ is the probability that objective A chooses direct communication, and γ the probability that biased A chooses direct communication. Also, agent A's interim objectivity given his channel choice is θ_A^d and θ_A^i respectively. Using equation (4) in Section III.B, objective A's ex ante expected payoffs from direct communication and indirect communication are respectively:

$$\begin{aligned}
\mathbb{E}U_A^{od} &= -\frac{[1-(p_A^2+(1-p_A)^2)N_x^d]}{2(2-N_x^d)} + \frac{1}{2}\alpha^o \Pr(A=o|m_A=0) \\
&+ \frac{1}{2}\alpha^o \left[p_A \Pr(A=o|m_A=1,\eta=1) + (1-p_A)\Pr(A=o|m_A=1,\eta=0)\right]; \\
\mathbb{E}U_A^{oi} &= -\frac{[1-(p_A^2+(1-p_A)^2)N_x N_y]}{2(2-N_x N_y)} + \frac{1}{2}\alpha^o N_y \Pr(A=o|m_B=0) \\
&+ \frac{1}{2}\alpha^o \left[(1-(1-p_A)N_y)\pi_{A(1,1)} + (1-p_A N_y)\pi_{A(1,0)}\right].
\end{aligned}$$

Clearly, if $\alpha^o = 0$, objective A chooses direct communication if $N_x^d > N_x N_y$; otherwise he chooses indirect. Intuitively, if $N_x^d > N_x N_y$, C believes that she is more likely to receive message 0 truthfully with direct communication than with indirect communication. But if $\alpha^o > 0$, objective A's channel choice also depends on his expected posterior objectivity. In both cases, there exist two pooling equilibria: one in which both types of A choose direct communication ($\mu = \gamma = 1$); and another in which both choose indirect communication ($\mu = \gamma = 0$). Each equilibrium is supported by the out-of-equilibrium-path belief that A is biased if he is observed to have deviated. Furthermore, there does not exist any equilibrium in which objective A only uses one channel ($\mu = 0$ or 1), but biased A uses the other channel with any positive probability. Suppose so, biased A loses all reputation and his message has no effect on C, a contradiction.

This implies that if any other informative equilibrium exists, objective A must use both channels with positive probabilities in equilibrium.

Case I: Objective A has no reputational concerns: $\alpha^o = 0$. We claim that there does not exist any informative equilibrium in which both channels are used. To see this, note that objective A's indifference between channels requires that $\mathbb{E}U_A^{od} = \mathbb{E}U_A^{oi}$, or:

$$N_x^d = N_x N_y. \tag{WA11}$$

There are two cases to rule out regarding biased A's channel choice. First, no equilibrium exists in which biased A uses one channel exclusively. Suppose $\gamma = 0$, then if biased A deviates to direct communication, he is believed to be objective: $\theta_A^d = 1$. Thus his message is credible and he faces no reputation cost. Therefore biased A would deviate to direct communication, a contradiction. If $\gamma = 1$ instead, then $\theta_A^i = 1$, and biased A reports $m_A = 1$ if he uses indirect communication, which leads to a deviation payoff,

$$\frac{2p_A - 1}{2 - N_y} + 1 - p_A + \alpha,$$

by (WA7). If biased A plays a mixed strategy in direct communication, his ex ante expected payoff is $0.5 + \alpha\theta_A^d$, strictly smaller than his deviation payoff above. If $x^d = 0$ instead, then from (WA6), biased A gets:

$$\frac{2p_A - 1}{2 - \theta_A^d} + (1 - p_A) + \frac{\alpha\theta_A^d}{2}\left[\frac{p_A}{1 - (1 - p_A)\theta_A^d} + \frac{1 - p_A}{1 - p_A\theta_A^d}\right].$$

Since objective A is indifferent between channels and $N_x = 1$ in this putative equilibrium, $\theta^d_A = N_y$. Clearly, biased A's posterior objectivity is higher with indirect communication, and thus he would deviate to indirect communication in either case, a contradiction.

In the second case, biased A uses both channels with positive probabilities. Then we have:

$$\text{(WA12)}\qquad \frac{2p_A-1}{2-N^d_x}+(1-p_A)+\frac{\alpha\theta^d_A}{2}\left[\frac{1-p_A}{1-p_AN^d_x}+\frac{p_A}{1-(1-p_A)N^d_x}\right]$$
$$= \frac{2p_A-1}{2-N_xN_y}+(1-p_A)+\frac{\alpha\theta^i_A}{2}\left[\frac{1-p_AN_y}{1-p_AN_xN_y}+\frac{1-(1-p_A)N_y}{1-(1-p_A)N_xN_y}\right].$$

If $x > 0, x^d > 0$ in equilibrium, condition (WA12) becomes $0.5 + \alpha\theta^d_A = 0.5 + \alpha\theta^i_A$ by the law of iterated expectations, and thus $\theta^d_A = \theta^i_A$. By Corollary 8, $x^d > x$ and thus $N^d_x > N_xN_y$, contradicting condition (WA11). Next, if $x^d > 0, x = 0$, then the LHS of (WA12) becomes $0.5 + \alpha\theta^d_A$, while the RHS is greater than $0.5 + \alpha\theta^i_A$. This implies that $\theta^d_A > \theta^i_A$ and thus $N^d_x > \theta^i_AN_y$, contradicting (WA11) again. Next, if (WA11) holds, biased A must get the same expected posterior objectivity in both channels. If $x = 0, x^d = 0$, then (WA11) becomes $\theta^d_A = \theta^i_AN_y$. Substituting $\theta^d_A = \theta^i_AN_y$ into (WA12), we find that the LHS is strictly smaller than the RHS, a contradiction. Finally, if $x^d = 0, x > 0$, then the LHS of (WA12) is larger than $0.5+\alpha\theta^d_A$, and the RHS becomes $0.5 + \alpha\theta^i_A$, and therefore $\theta^d_A < \theta^i_A$. But since (WA11) holds, (WA12) cannot hold. This shows that no equilibrium exists in which both objective and biased A use both channels with positive probabilities.

Case 2: Objective A also has reputational concerns: $\alpha^o > 0$. In this case, objective A no longer chooses channel solely based on C's belief of receiving message 0 in each channel, and it is possible for informative equilibria other than pooling ones to exist. For example, if α is sufficiently low, one possible equilibrium is for objective A to use both channels with positive probabilities ($\mu \in (0,1)$), and for biased A to only use direct communication in which he always reports $m_A = 1$ ($\gamma = 1$). In such an equilibrium, the following four conditions must hold:

$$\text{(WA13)}\ \frac{(p_A^2+(1-p_A)^2)N_x^d-1}{2(2-N_x^d)}+\frac{\alpha^o\theta_A^d}{2}\left[\frac{1}{N_x^d}+\frac{p_A^2}{1-(1-p_A)N_x^d}+\frac{(1-p_A)^2}{1-p_AN_x^d}\right]=$$
$$\frac{(p_A^2+(1-p_A)^2)N_xN_y-1}{2(2-N_xN_y)}+\frac{\alpha^o\theta_A^i}{2}\left[\frac{N_y}{N_x}+\frac{(1-(1-p_A)N_y)^2}{1-(1-p_A)N_xN_y}+\frac{(1-p_AN_y)^2}{1-p_AN_xN_y}\right];$$
$$\frac{2p_A-1}{2-N_x^d}\left[2(2p_A-1)-\frac{2p_A-1}{2-N_x^d}\right]\geq\alpha^o\theta_A^d\left[\frac{1}{N_x^d}-\frac{(1-p_A)^2}{1-p_AN_x^d}-\frac{p_A^2}{1-(1-p_A)N_x^d}\right];$$
$$\frac{2p_A-1}{2-N_x^d}+\frac{\alpha\theta_A^d}{2}\left[\frac{1-p_A}{1-p_AN_x^d}+\frac{p_A}{1-(1-p_A)N_x^d}\right]\geq\frac{2p_A-1}{2-N_xN_y}+\alpha;$$
$$\frac{2p_A-1}{2-N_x^d}>\alpha\theta_A^d\left[\frac{1}{N_x^d}-\frac{p_A(1-p_A)}{1-p_AN_x^d}-\frac{p_A(1-p_A)}{1-(1-p_A)N_x^d}\right].$$

(WA13) is objective A's indifference condition regarding the channel choice: $\mathbb{E}U_A^{od} = \mathbb{E}U_A^{oi}$. In this equilibrium, A is believed to be objective for sure if he uses indirect communication, and thus $\theta_A^i = 1$, $N_x = 1$. Because his expected posterior objectivity is higher with indirect communication, his indifference requires that the expected payoff is lower for the decision maker with indirect communication, or $N_x^d > N_xN_y$ in equilibrium. The second condition guarantees that objective A is willing to report truthfully if $s_A = 1$ in direct communication. Because $m_A = 1$ is more associated with the biased type, objective A is willing to report $m_A = 1$ if α^o is sufficiently low. The third condition guarantees that biased A does not want to deviate to indirect communication. If biased A deviates, he is believed to be objective and always reports $m_A = 1$.

But since he induces a higher action with direct communication, he is willing to forego the gain in reputation if α is sufficiently low. For the same reason, the fourth condition, which guarantees that biased A always reports $m_A = 1$ with direct communication, is satisfied for small α.

Observe that objective A always reports $m_A = 0$ if $s_A = 0$. Further, because A is considered objective for sure if he uses indirect communication, objective A's truth-telling IC is satisfied with indirect communication. Thus the above four conditions are sufficient for the equilibrium under consideration. To find parameter values for the equilibrium, note that N_y can be made arbitrarily close to 0 by choosing θ_B and β sufficiently small. It follows that $N_x^d = \theta_A^d > N_x N_y$ for all θ_A and μ. Hence the first part of the LHS of condition (WA13) is larger than the first part of the RHS. That is, C's expected payoff with direct communication is higher than that with indirect communication. Moreover, for the same N_y sufficiently close to 0, the second part of the LHS of condition (WA13), which is objective A's expected posterior objectivity, is strictly smaller than the second part of the RHS. Thus for any $\theta_A^d > 0$, there exists an α^o such that (WA13) is satisfied. Also, if θ_A^d is sufficiently small, the value α^o that makes (WA13) hold is close to 0. By choosing θ_A sufficiently close to 0, we can make θ_A^d arbitrarily small for any μ, and thus find a value of α^o that satisfies both condition (WA13) and the second condition above, which is objective A's truth-telling IC in direct communication. Because $N_x^d > N_x N_y$, the third and fourth condition are clearly satisfied if we choose α sufficiently small.

The above example shows that if objective A has reputational concerns, he may use both channels. Objective A faces a tradeoff between maximizing C's expected payoff and his own reputation, which is the key difference between the models where $\alpha^o = 0$ and $\alpha^o > 0$. It may also be possible to construct informative equilibria in which both types of A use both channels with positive probabilities. To construct such an equilibrium, six conditions need to hold. Condition (WA13) still needs to hold for objective A to be indifferent between channels, and (WA12) needs to hold for biased A to be indifferent, replacing the third condition above. Next, the second condition still needs to hold for objective A to report truthfully with direct communication, and a similar one is necessary to guarantee that he reports truthfully with indirect communication. The last two conditions are for biased A. Depending on his putative equilibrium strategy within each channel, two indifference conditions need to hold if he uses mixed strategies in both channels; two inequalities if he reports 1 in both channels; or a mix of the two possibilities.