Online Appendix for "Bargaining with Arrival of New Traders" by William Fuchs and Andrzej Skrzypacz

Proof of Claims in Section III.C. $\Pi(v_1)$ can be re-written as:

$$\Pi(v_1) = \gamma v_1 + (1-\gamma)\left(\int_0^{v_1} x f(x)\, d(x) + (1 - F(v_1))\, v_1\right)$$

Hence,

$$\begin{aligned}\Pi'(v_1) &= \gamma + (1-\gamma)(v_1 f(v_1) + (1 - F(v_1)) - f(v_1) v_1) \\ &= \gamma + (1-\gamma)(1 - F(v_1)) \\ &= 1 - F(v_1) + F(v_1)\gamma\end{aligned}$$

Therefore:

$$\frac{\partial \Pi'(v_1)}{\partial \gamma} = F(v_1) > 0$$

Therefore, the larger γ the larger $\Pi'(v)\ \forall v$ and from Proposition 2 this implies that delay is decreasing in the number of different buyer classes. (*ii*) and (*iii*) follow from noting that $\Pi(v_1)$ is decreasing in n since the second term of $\Pi(v_1)$ is smaller than v_1 and using equations (8) and (9) which respectively characterize the seller's value and prices. ■

Proof of Lemma 1 (Section IV). For $k > V^*$, $p_A(k)$ is a solution to the F.O.C.:

$$p - \frac{(F(k) - F(p))}{f(p)} = V^*$$

Now, the LHS is decreasing in k.[1] We claim that it is increasing in p if the marginal revenue is downward sloping. The derivative of the LHS with respect to p is:

$$1 - \frac{-f^2(p) - (F(k) - F(p)) f'(p)}{f^2(p)} = 2 + \frac{(F(k) - F(p)) f'(p)}{f^2(p)}$$

which if $f'(p) > 0$ is positive for all k and if $f'(p)$ is < 0 it is the smallest for $k = 1$, but then this expression is positive by assumption.

Hence the LHS of the F.O.C. is increasing in p for all k and decreasing in k, which implies that $p_A(k)$ is strictly increasing.

For $k \leq V^*$ the seller cannot get more than V^*, which he can guarantee by offering $p_A(k) = V^*$ and trading with probability 0. ■

[1] Hence, if $p_A(k)$ is strictly increasing, the problem (19) is supermodular in k and p, guaranteeing that the F.O.C. is sufficient.