Web Appendix for "Reference-Dependent Consumption Plans" by Botond Kőszegi and Matthew Rabin

Appendix A: Modeling Rational Reference-Dependent Behavior

Throughout the paper, we have used the PPE solution concept to analyze a decisionmaker's behavior. But as we emphasized, this solution concept assumes the decisionmaker arrives at utility-maximizing credible beliefs. Since it is unlikely that a person would hold correct beliefs about all future contingencies from the moment of birth, and adjusting beliefs carries utility in our model, the question arises whether and in what situations she would actually arrive at PPE beliefs. In this appendix, we modify the model in the text to allow for unrestricted initial beliefs when the decisionmaker forms the first focused plans, and study the implications of rationality given our assumptions about preferences with minimal ancillary assumptions. In doing so we investigate the circumstances that would justify the application of PPE and various other solution concepts we and others have proposed. This in turn lets us address a closely related question: what solution concept may be appropriate for a researcher to employ as a function of what she is willing to assume about the decisionmaker's reference-dependent utility and information from before the period of analysis.

We make a single change relative to the model in Section 1. We assume that the decisionmaker starts off in period 0 with beliefs F_{-1} inherited from before she started focusing on the choice problem at hand. While we still assume that the beliefs F_0 through F_T must be rational, we do not impose any restrictions on initial beliefs, F_{-1}, so that we do not make any assumptions about the reasonableness of a person's initial unfocused beliefs. For given initial beliefs F_{-1}, we define our key solution concept, an optimal consistent plan, as follows:

Definition 2. Define the sets $\{D_t^*\}_{t=1}^T$ in the following backward-recursive way. A plan $d_t \in D_t$ is in D_t^* if, given the expectations generated by d_t, in any contingency, (i) it prescribes a continuation plan in D_{t+1}^* that maximizes the expectation of U^t; and (ii) it prescribes an action in period t that maximizes the expectation of U^t, assuming that future plans are made according to (i). A plan

$d_0 \in D_0$ is an *optimal consistent plan (OCP)* if it satisfies the above given the expectations F_{-1}.

The construction of the OCP is very similar to the recursive structure discussed in the text for PPE, except for the choice of the initial credible plan. As with PPE, to solve for OCP the decisionmaker has to think backwards. First, the decisionmaker must figure out, for each possible choice set D_T, which plans in D_T are credible. Any plan $d_T \in D_T$ induces beliefs F_{T-1} regarding consumption outcomes. To be consistent with rationality, d_T must then satisfy the constraint that given the expectations generated by d_T, self T will be willing to follow d_T. Knowing the set of credible plans D_T^* for each D_T, self $T-1$ can evaluate any action and any realization of uncertainty by making the best credible continuation plan for each such contingency. Then, we can define the set of credible plans D_{T-1}^* for any D_{T-1} analogously to the above, and continue similarly to period 1. In period 0, the person chooses the action and credible continuation plan that maximizes her expected utility given F_{-1}.

We think of OCP as the appropriate general solution concept determining behavior in our setting.[28] But it requires a tremendous amount of information about the decisionmaker and her choice problem, including her information from the moment of first focus to full execution, her preferences, and the initial beliefs. Practical applications of the model, however, are often likely to require predictions using more limited information about the person's choice and characterization of the problem. In the rest of this appendix, we develop various alternative solution concepts that permit predictions with less information, and identify circumstances when they would follow from assuming OCP given the broader (possibly unmodeled or unknown) situation.

To motivate our exercise, suppose that we (as researchers) "look in" at a person in period t, knowing her utility function and choice problem for all periods t and on—including any probabilistic beliefs she has about the choice problem in the morning of period t. What are the predictions we can

[28] Our model is a single-person but multi-self special case of a *dynamic psychological game* as introduced by Pierpaolo Battigalli and Martin Dufwenberg (2009) for multi-player games, and our definition of OCP is theoretically very similar to the solutions concepts they propose. Although we have not fully explored whether our ideas can be coded in their very rich framework, it seems that there are two important differences between our model and theirs. First, whereas we assume that a person can always affect her future selves' preferences merely by changing plans (so long as those plans are credible), in Battigalli and Dufwenberg's model this is only possible through taking different actions. This difference is natural since a person is likely to be able to choose her favorite plan for her *own* behavior but not for *others'* behavior. Second, as discussed above, we do not impose any restrictions on the reasonableness of a person's initial beliefs.

make as a function of possible ancillary assumptions about her prior decisionmaking environment and preferences?

The least restrictive predictions we might make are the following:

Definition 3. A plan $d_t \in D_t$ is *reference-dependent rational* (RDR) if there are beliefs F_{t-1} such that d_t satisfies Conditions (i) and (ii) of Definition 2 given the beliefs F_{t-1}.

The set of RDR plans is the set of plans that we would deem possible if we knew the decision-maker's situation and preferences from the time she wakes up on date t, but knew nothing about her initial beliefs, her earlier choice sets and behaviors, and so on. Redefining period t as period 0, this is just the union of all OCP's. Note that if period t is the time at which the first relevant decision is implemented, RDR is the set of behaviors consistent with our model when researchers are fully agnostic about prior preferences and any features of the environment that would not typically be included in the specification of a classical model. Indeed, once assumptions are made about the structures of $\gamma_{t',\tau}$ for all $t \leq t' \leq \tau \leq T$, the shape of μ, and the consumption dimensions, RDR is the set of predictions consistent with our model using solely the person's consumption utility and decision problem starting in period t—which is exactly the information used in conventional analysis.

We illustrate RDR and later concepts using the most important application in Kőszegi and Rabin (2006): shoe purchases. Suppose that in period t, a consumer might face a wide range of prices for shoes. Although RDR does not place restrictions on what the person had expected coming into period t, we know she will buy for very low prices and not buy for very high prices. More generally, we can say that the consumer uses a cutoff strategy, buying below a reservation price and not buying above it. But the reservation price can be anywhere within a range. If the consumer had expected not to buy, for example, she will experience paying as a loss and getting the shoes as merely a gain, so her reservation price will be low; if she had expected to buy, but at a low price, her reservation price would be moderate; and if she had expected to buy the shoes at a high price, her reservation price will be high.

If we are willing to make assumptions about the person's beliefs in period $t-1$ regarding the impending decision problem—in the case of shoe purchases, for instance, we would want to specify the person's previous beliefs about the probabilistic price distribution she would face—a stronger solution concept is appropriate:

Definition 4. A plan $d_t \in D_t$ is a *personal equilibrium* (PE) if $d_t \in D_t^*$.

We and others have previously used the term "personal equilibrium" as a solution concept defining reasonable planned behavior when people have beliefs-based preferences, and we use the same term here in a new context because our new definition shares the essential characteristics of the earlier ones. Namely, we assume that the person not only maximizes her utility given her beliefs, but also that her beliefs must turn out to be probabilistically correct given her own behavior and exogenous happenings. More precisely, the set of personal equilibria is the set of credible plans in the sense that if the person believed at the end of period $t-1$ that she would carry through the plan, she would indeed want to carry it through.[29] Despite the term "equilibrium", this amounts only to a rational-consistency requirement: the person can only make plans she believes she will follow through.

The restrictions PE places on possible prior beliefs relative to RDR can be illustrated using our example of shoe purchases above. As we have argued, the person always buys the shoes for very low prices. PE requires that she realize this and incorporate it into her expectations, which in turn means that when the probability of a low price is substantial, PE requires that she experience a loss from not getting the shoes, ruling out relatively low reservation prices.

While any PE plan is credible, different PE plans may generate different ex-ante expected utilities starting in period t. Our third, most restrictive solution concept, which is the one we have used in the text for decision problems starting in period 1, is that the person chooses the (typically unique) PE yielding the highest utility:

Definition 5. A plan $d_t \in D_t$ is a *preferred personal equilibrium* (PPE) if it is a PE, and it maximizes expected utility starting in period t among PE plans.

Because it yields the highest expected utility beginning in period t among plans she knows she will carry through, it may appear that a person would always plan to play the PPE. Indeed, we

[29] Notice that although D_t^* is introduced as part of the definition of OCP (Definition 2) beginning in period 0, its definition requires only the person's decision problem starting in period t.

have invoked exactly this intuition in our previous research to motivate the use of PPE. But the model here says that this justification is not always appropriate because it ignores the possibility of prospective gain-loss utility: if one has to *change* one's plans—and incur possible disappointments in some dimensions of future consumption—to get things "right," choosing the PPE may be unattractive.

Yet PPE is the appropriate solution concept whenever two conditions are satisfied. First, it must be the case that the decisionmaker had known about the distribution of choice sets in a period t' when her γ's for outcomes starting in period t are zero (that is, $\gamma_{t',\tau} = 0$ for all $\tau \geq t$). This is likely to be the case if she had known about her decision problem very early, when she cared little about changing plans. In addition, while we do not formalize issues of focusing in this paper, it seems that updating unfocused beliefs (in period $t' = 0$) generates little or no immediate gain-loss utility. In either case, the person can update her plans in period t' at no cost, and so would like to form PPE plans. A second requirement is that having formed a PPE plan, the person does not want to switch to another PE plan before period t. This condition is less likely to be binding, because if the person does not like to play the other PE in period t, it will typically not be pleasant to switch to it.[30]

The framework and definitions above also allow us to clarify the circumstances under which each of the reduced-form concepts we have introduced and applied in previous papers (Kőszegi and Rabin 2006, 2007)—which use the same terms PE and PPE—might be appropriate. To link those single-decision, single-outcome models to the current one, suppose that all non-trivial consumption outcomes occur in period $t \geq 1$, and all uncertainty about this outcome is resolved in the same period. The person selects the lottery determining this outcome from the choice set D in a given period $t' \leq t$. Because consumption outcomes are realized and uncertainty is resolved in period t, the decisionmaker's utility collapses to a simple form. Let F be the lottery of outcomes the person receives in period t, and G the previous period's expectations regarding those outcomes.

[30] The condition is met, for example, for all of the types of situations we consider in Section 4 of this paper, even if the relevant decision is made in some period $t > 1$. When A3$'$ holds, the condition is also met in the single-decision setting of Proposition 9 below.

The decisionmaker's utility in period t is then

$$U(F|G) = \sum_{k=1}^{K} E_{F^k}\left[m^k(c^k) + N^k(c^k|G^k)\right].$$

The following proposition states how to check for PE and PPE in this situation.

Proposition 9. *Suppose all outcomes occur and all uncertainty is resolved in period t, and the outcome lottery is the result of a single choice from the choice set D. If the choice is made in period t, then (i) $F \in D$ is a PE distribution of outcomes if and only if $U(F|F) \geq U(F'|F)$ for all $F' \in D$; (ii) $F \in D$ is a PPE distribution of outcomes if and only if it is a PE, and there is no PE distribution $F'' \in D$ such that $U(F''|F'') > U(F|F)$; and (iii) if A3$'$ holds and there is a $t' < t$ such that $\gamma_{t',t} = 0$, then OCP induces a PPE starting in period t. If the choice from D is made in period $t' < t$ and $\gamma_{t',t} = 0$, $F \in D$ is an OCP distribution of outcomes if and only if $U(F|F) \geq U(F'|F')$ for all $F' \in D$.*

Our old notion of PE is a situation where the stochastic outcome of a person's utility-maximizing choice given expectations is identical to the expectations (Kőszegi and Rabin 2006, Definition 1). Proposition 9 says that if the person makes a single decision in period t with consumption also occurring in period t, our current notion of PE is equivalent to the old notion. In the same situation, our old notion of PPE—the PE that maximizes ex-ante expected utility (Kőszegi and Rabin 2006, Definition 2)—is equivalent to the current notion. Furthermore, if A3$'$ holds and there is a period in which the person can update her beliefs at no hedonic cost, then OCP indeed implies PPE because the two conditions discussed above are met.

In Definition 3 of Kőszegi and Rabin (2007), we introduced the solution concept *choice-acclimating personal equilibrium* (CPE) for situations in which a person commits to a lottery to be resolved in the future. A lottery is a CPE if it maximizes expected reference-dependent utility given that it determines both the reference lottery and the outcome lottery. Proposition 9 says that this is necessarily appropriate only if the relevant γ in the period of choice is zero. Similarly to PPE, our previous justification for CPE—that when choosing ahead of time the person commits to the lottery that maximizes expected utility at the time of resolution—ignored prospective gain-loss utility: it could be that changing plans to the option that maximizes expected utility in period t is painful, and hence the person will not do it.

Appendix B: An Elaboration of Monetary Preferences

This appendix characterizes monetary preferences in the setting of Section 3. Beyond saying that a person may asymmetrically attend to small gains and losses in wealth even when the background risk is large, our theory can identify exactly how she treats small risks as a function of her decisionmaking environment. In fact, in each of the cases we consider in this section, the person's attitude toward small gambles can be characterized by a "reduced-form" solution concept that depends only on the choice set D of small risks.

For our characterization, we first define two reduced-form utility functions for comparing a lottery the individual faces to a reference lottery. We will apply these utility functions to lotteries from D. Denote the mean of a lottery G by $\overline{G}$. For a function $v : \mathbb{R} \to \mathbb{R}$ satisfying assumptions $A0$ through $A4$, let

$$U(F|G) = \iint v(c-r)dG(r)dF(c), \text{ and} \tag{13}$$

$$V(F|G) = \int v(c-\overline{G})dF(c). \tag{14}$$

The functions U and V are different ways of evaluating the outcome lottery F relative to the reference lottery G. Under the utility function U, each outcome of F is evaluated relative to all possible outcomes under G, and the overall evaluation is an average of these gain-loss sensations. This is the utility function we postulated in our static models of reference-dependent utility (Kőszegi and Rabin 2006,2007). Under V, each outcome of F is compared only to the mean of the reference lottery.

Based on the above utility functions, we define reduced-form solution concepts for a decision-maker's choice from the choice set D.

Definition 6. $F \in D$ is a *risk-neutral choice* (RN) if for all $F' \in D$, $\overline{F} \geq \overline{F}'$. $F \in D$ is a *prospect-theory choice* (PT) if there is a U of the form 13 such that $U(F|0) \geq U(F'|0)$ for all $F' \in D$.

$F \in D$ is a *choice-acclimating personal equilibrium* (CPE) if there is a U of the form 13 such that $U(F|F) \geq U(F'|F')$ for all $F' \in D$. $F \in D$ is a *static preferred personal equilibrium* (SPPE) if there is a U of the form 13 such that $U(F|F) \geq U(F'|F)$ for all $F' \in D$, and for any $F'' \in D$ satisfying the same condition, $U(F|F) \geq U(F''|F'')$. $F \in D$ is a *Bell-Loomes-Sugden choice* (BLS) if there is a V of the form 14 such that $V(F|F) \geq V(F'|F')$ for all $F' \in D$. $F \in D$ is an *expected-value preferred personal equilibrium* (EVPPE) if there is a V of the form 14 such that $V(F|F) \geq V(F'|F)$ for all $F' \in D$, and for any $F'' \in D$ satisfying the same condition, $V(F|F) \geq V(F''|F'')$.

Any of the above concepts is *strict* if the weak inequalities in the definition are replaced with strict inequalities for all lotteries not equal to F.

Risk neutrality simply means that the decisionmaker chooses the lottery with the highest mean. As we have emphasized in Section 3, previous theories of reference dependence imply that a person approaches risk neutrality for large amounts of background risk. A person accords to prospect theory if she maximizes the expectation of a reference-dependent utility function given a reference point of zero. A choice is a CPE if it maximizes the expectation of U given that it determines both the reference lottery and the outcome lottery. As mentioned in Section 1, this is the solution concept introduced in Kőszegi and Rabin (2007) for situations when the outcome of a lottery is resolved long after the decision. Similarly, a lottery in D is BLS if it maximizes the expectation of V given that it determines both the outcome lottery and the reference lottery. This solution concept is analogous to the disappointment-aversion models of Loomes and Sugden (1982) and Bell (1985). SPPE, first defined in Kőszegi and Rabin (2006) as PPE, is the person's favorite plan among plans she knows she will carry through if her reference point is determined by an expectation to carry through her plan and her utility function is U. Finally, EVPPE is the analogue of PPE when the person's utility function is V instead of U.

Because it gives rise to qualitatively different behavior, we consider two extreme types of background risk. One type is a uniform distribution on an interval $[x_l, x_h]$. The other is a bounded discrete distribution in which the distance between any two atoms is at least twice the absolute value of the largest possible realization of any lottery in D. While such background risk is somewhat artificial, it is of interest for two reasons. First, it captures in extreme form a situation where much of the risk a person faces comes in large increments—e.g. whether she gets a promotion—and the additional risk at hand is small relative to these increments. Second, because a decisionmaker in our model dislikes risks to consumption, she may endogenously choose consumption distributions that are discrete.

Our main interest is in determining how the decisionmaker evaluates small risks as the background risk to her consumption becomes arbitrarily large; that is, as $x_h - x_l \to \infty$ in the case of uniform background risk and the weight on the most probable atom approaches zero in the case of discrete background risk. In particular, we will identify to which of the solution concepts in Definition 6 behavior approaches as the background risk becomes large. But because of possible indifferences between options in D, we have to be careful (and unfortunately somewhat clumsy) in defining what this convergence means:

Definition 7. The concept X = RN, PT, CPE, SPPE, BLS, or EVPPE introduced in Definition 6 represents the decisionmaker's limiting behavior as the background risk becomes large if there is a utility function of the given form such that (i) if $F \in D$ satisfies the strict version of X, then F is an OCP choice for a sufficiently large background risk; and (ii) if $F' \in D$ does not satisfy the weak version of X, then F' is not an OCP choice for a sufficiently large background risk.

Table 1 summarizes the decisionmaker's limiting behavior as a function of when she learns about D, when she implements her choice, when the uncertainty from the chosen lottery is resolved, and the type of background risk. For the cases denoted with an asterisk (*), the result is true not only in the limit, but for *any* lottery of the given type.

As we emphasized in Section 3, uncertainty in the chosen small lottery that is resolved in period 1 generates immediate prospective gain-loss utility. This means that—although the specific concept summarizing behavior depends on further aspects of the timing of the decision problem—in each case the person attends asymmetrically to gains and losses coming from the small risk. The difference between discrete and uniform background distributions arises due to a difference in how the person evaluates news with the two types of background risk. With a discrete distribution, the additional small risk replaces each atom of the background risk with a small "local lottery," so when the person receives new about the small lottery, her prospective gain-loss utility is determined by

Learn	Implement	Resolved	Discrete	Uniform
0	0,1,2	2	RN	RN
0	0	1	CPE*	BLS
0	1	1	SPPE*	EVPPE
1	1	1	PT*	PT*
1	1	2	PT	RN
1,2	2	2	RN	RN

Table 1: Limiting Behavior for Large Background Risk
In the cases denoted by *, the result is true for any background risk of the given form.

a rank-dependent comparison between her old beliefs and new beliefs *regarding the small lottery.* With a uniform distribution, the small lottery does not change the distribution of outcomes over most of the support of period-2 consumption—the distribution remains uniform other than near the edges. As a result, when the person receives news, she only cares about the comparison of her new *mean* beliefs and old *mean* beliefs regarding the small risk. Hence, while the two types of background risk generate conceptually similar reduced-form solution concepts, these concepts are based on the utility function U in the case of a discrete distribution but on the utility function V in the case of a uniform distribution.

As we have also discussed in Section 3, when the uncertainty in the small lottery is resolved in period 2, the resolution does not generate separate gain-loss utility, so the person integrates the lottery with the background risk and is typically risk-neutral to it. The only exception to this logic occurs when the person learns about D in period 1 and makes an immediate choice. In this case, her choice from D generates immediate news relative to her previous expectations of not having a choice, and hence induces prospective gain-loss utility. Nevertheless, when the background risk is uniform, she only cares about the mean of the chosen lottery, so despite prospective gain-loss utility in period 1 she is still risk neutral. But when the background risk is discrete, she compares the full distribution of the chosen lottery to her previous expectation of zero, so she behaves as postulated by prospect theory.

Appendix C: Proofs of Proposition 9 and Table 1

Proof of Proposition 9. All parts except for (iii) are obvious from the definition of OCP. To prove part (iii), it is sufficient to prove that if the person makes a PPE plan in period t', she does not switch away from this plan in a subsequent period; if this is the case, the person will clearly make a PPE plan. Suppose, then, that F is a PPE plan starting in period t. We prove by contradiction. Suppose that in period τ satisfying $t > \tau > t'$, the person strictly prefers to switch to plan $G \neq F$. Since she can only credibly switch to a PE plan, and there is no PE plan that she strictly prefers to F, to be preferred the switching itself must generate positive prospective gain-loss utility: $N(G|F) > 0$. Since the gains of G relative to F count less than the losses, this immediately implies that average consumption utility under G is strictly greater than under F: $E_G[m(c)] > E_F[m(c)]$. Next, we prove that for any dimension k, $N^k(G^k|F^k) \leq E_{G^k}[N^k(c^k|F^k)] - E_{F^k}[N^k(c^k|F^k)]$. Combined with the fact that consumption utility is higher under G, this means that in period t the person wants to switch away from a plan to choose F, so that F is not a PE, a contradiction.

To prove our claim, take any $p \in [0,1]$. We will show that

$$\mu(m^k(c_{G^k}(p)) - m^k(c_{F^k}(p))) \leq N^k(c_{G^k}(p)|F^k) - N^k(c_{F^k}(p)|F^k) \tag{15}$$

by considering two cases. First, if $c_{G^k}(p) \geq c_{F^k}(p)$, then

$$\mu(m^k(c_{G^k}(p)) - m^k(c_{F^k}(p))) = \eta(m^k(c_{G^k}(p)) - m^k(c_{F^k}(p))) \leq N^k(c_{G^k}(p)|F^k) - N^k(c_{F^k}(p)|F^k)$$

because the difference on the right-hand side is a mixture of gains and avoided losses rather than just a pure gain. Second, similarly, if $c_{G^k}(p) \leq c_{F^k}(p)$, then

$$\mu(m^k(c_{G^k}(p)) - m^k(c_{F^k}(p))) = -\eta\lambda(m^k(c_{G^k}(p)) - m^k(c_{F^k}(p))) \leq N^k(c_{G^k}(p)|F^k) - N^k(c_{F^k}(p)|F^k)$$

because the right-hand side is a mixture of eliminated gains and losses rather than a pure loss.

Finally, integrating Inequality 15 over p gives exactly the desired inequality. □

Proof of Table 1. We prove each case in turn; in the notation below, the three numbers refer to when the decision is learned, implemented, and resolved, respectively.

0,2,2; 1,2,2; 2,2,2: By Part 1 of Proposition 2 in Kőszegi and Rabin (2007), whenever the person implements the decision in period 2 her limiting behavior is risk neutral.

0,0,2: This is a direct implication of Part 1 of Proposition 6 in Kőszegi and Rabin (2007).

To prove our statements for the rest of the cases, we solve for how the person feels about a period-1 change in beliefs. We start with discrete distributions. To state the following lemma, let Y be the highest absolute value of any realization of any lottery in D.

Lemma 2. *Suppose the background risk B is of the discrete type. Then, for any lotteries F, G distributed on $[-Y, Y]$ we have*

$$N(B+F|B+G) = N(F|G).$$

Proof of Lemma 2. For any atom x of B, let $B_{-}(x)$ be the probability of strictly smaller realizations of x, and $b(x)$ the weight on x. (Hence, $B_{-}(x) = B(x) - b(x)$). Since any realization of F has absolute value of at most Y, and the distance between any two atoms of B is at least $2Y$, for any atom x of B and any $y \in [-Y, Y]$ we have

$$(B+F)(x+y) = B_{-}(x) + b(x)F(y).$$

For any $p \in [0,1]$, let x_p be the atom of B that satisfies $B_{-}(x_p) \leq p < B(x_p)$. Using the above equality, we have

$$c_{B+F}(p) = c_F\left(\frac{p - B_{-}(x_p)}{b(x_p)}\right).$$

This implies the statement of the lemma.

We prove a similar lemma for the case of uniform background risk.

Lemma 3. *Suppose the background risk B is of the uniform type. Then*

(i) For any constant y, $N(B+y|B) = N(y|0)$.

(ii) For any lotteries F, G distributed on $[-Y, Y]$,

$$\lim_{x_h - x_l \to \infty} N(B+F|B+G) = \mu[\overline{F} - \overline{G}].$$

Proof of Lemma 3. Part (i) is obvious. To prove Part (ii), we start by noting that for any F distributed on $[-Y, Y]$ and any $c \in (x_l + Y, x_h - Y)$ we have

$$(B+F)(c) = \int \frac{c - (x_l + y)}{x_h - x_l} dF(y) = \frac{c - x_l - \overline{F}}{x_h - x_l} = B(c - \overline{F}).$$

Hence, for any p satisfying $p \in ((B+F)(x_l+Y), (B+F)(x_h-Y))$ and $p \in ((B+G)(x_l+Y), (B+G)(x_h-Y))$, we have $c_{B+F}(p) - c_{B+G}(p) = \overline{F} - \overline{G}$. Since $\lim_{x_h - x_l \to \infty}(B+F)(x_l+Y) = \lim_{x_h - x_l \to \infty}(B+G)(x_l+Y) = 0$, and $\lim_{x_h - x_l \to \infty}(B+F)(x_h-Y) = \lim_{x_h - x_l \to \infty}(B+G)(x_h-Y) = 1$, this implies the statement of the lemma.

Now we return to proving the rest of the cases in Table 1.

0,1,2: Since in the case 0,0,2 the decisionmaker becomes risk neutral in the limit, to prove that she will become risk neutral in this case it is sufficient to prove that if she had planned in period 0 to make a risk neutral choice, switching away from it in period 1 does not generate positive prospective gain-loss utility. This is obvious from Lemmas 2 and 3.

0,0,1; 0,1,1: Obvious from the definition of OCP and Lemma 2 and Lemma 3, Part (ii).

1,1,1: Obvious from the definition of OCP and Lemma 2 and Lemma 3, Part (i).

1,1,2: Suppose for a second that $\gamma_{1,2} = 0$, so that there is no prospective gain-loss utility in period 1. Then, by Part 1 of Proposition 6 in Kőszegi and Rabin (2007), the decisionmaker approaches risk-neutrality as the background risk becomes large. By Part (ii) of Lemma 3, with uniform background risk this does not change when prospective gain-loss utility is added in period 1. Now consider discrete background risk. By Lemma 2, for any lottery $F \in D$, $N(B+F|B) = N(F|0) = U(F|0)$ for some U satisfying Equation 13. Given that period-2 utility approaches risk neutrality, this implies that the person chooses according to prospect theory in period 1.

This completes the proof of all statements in the table. □