Web Appendix for

Ordering the Extraction of Polluting Nonrenewable Resources

Appendix A: *Maximizing Cumulative Extraction of Coal along a Hotelling Path implies that the Stock of Pollution is Always at the Ceiling*

For $X^0 \in (X_2^H, X_1^H)$, we have a common scarcity rent $\lambda(X^0) \in (\bar{p}_1, \bar{p}_2)$.Consider the Hotelling path $d(\lambda^0(X^0)e^{\rho t})$, written in reduced form as $d(t)$, over the time interval $[0, \Delta_{12})$ during which $p(t) = \lambda^0(X^0)e^{\rho t}$ increases from $\lambda^0(X^0)$ to $\bar{p}_2$. Then $\Delta_{12} = \dfrac{\ln \bar{p}_2 - \ln \lambda^0(X^0)}{\rho}$. Let the extraction sequence be given by $\{x_1, x_2\}$ such that $x_1 + x_2 = d(t)$. We show that among paths starting from $Z^0 = \bar{Z}$ and satisfying the constraint $Z(t) \le \bar{Z}$, $Z(\Delta_{12}) = \bar{Z}$, maximizing the extraction of coal given a stock of gas implies that the ceiling constraint is always binding. That is, we must have $Z(t) = \bar{Z}$ over the entire interval $[0, \Delta_{12}]$. Define $\hat{x}_1(t)$ and $\hat{x}_2(t)$ to be the extraction rates of gas and coal when the pollution stock is at the ceiling as defined in equation (13). The maximization problem can be written as:

$$\underset{x_2}{Maximize} \int_0^{\Delta_{12}} x_2(t)dt$$

subject to

$$\dot{Z}(t) = \theta_1(d(t) - x_2(t)) + \theta_2 x_2(t) - \alpha Z(t),$$

$$Z^0 = \bar{Z}, \bar{Z} - Z(t) \ge 0, t \in [0, \Delta_{12}], Z(\Delta_{12}) = \bar{Z},$$

$$d(t) - x_2(t) \ge 0, \quad x_2(t) \ge 0.$$

The Lagrangian can be written as

$$L = x_2 + \pi[\theta_1(d - x_2) + \theta_2 x_2 - \alpha Z] + \nu[\overline{Z} - Z] + \overline{\gamma}_2[d - x_2] + \underline{\gamma}_2 x_2 .$$

The first order conditions are

(A1) $\dfrac{\partial L}{\partial x_2} = 0 \Leftrightarrow 1 + \pi[\theta_2 - \theta_1] = \overline{\gamma}_2 - \underline{\gamma}_2$,

(A2) $\dot{\pi} = -\dfrac{\partial L}{\partial Z} \Leftrightarrow \dot{\pi} = \alpha\pi + \nu$,

(A3) $\nu \geq 0,\ \ \nu[\overline{Z} - Z] = 0$,

(A4) $\overline{\gamma}_2 \geq 0,\ \ \overline{\gamma}_2(d - x_2) = 0,$ and

(A5) $\underline{\gamma}_2 \geq 0,\ \ \underline{\gamma}_2 x_2 = 0$.

The shadow price of pollution π must be non-positive and continuous. First we show that $\hat{x}_1(t)$ and $\hat{x}_2(t), t \in (0, \Delta_{12})$ satisfy the above first order conditions (A1-A5). Since $\hat{x}_2(t) \in (0, d(t))$, both $\overline{\gamma}_2(t), \underline{\gamma}_2(t)$ equal zero. Then (A1) implies $\pi = \dfrac{-1}{\theta_2 - \theta_1} < 0$.

Substituting in (A2) and using the fact that π is a constant yields

$$\nu = \frac{\alpha}{\theta_2 - \theta_1} > 0, t \in (0, \Delta_{12}).$$

We show that if $Z < \overline{Z}$ over any interval $(t_1, t_2) \subseteq [0, \Delta_{12}]$ such that along its boundary, $Z(t_1) = \overline{Z} = Z(t_2)$, it leads to a contradiction. From (A2) and (A3), we have

(A6) $\pi(t) = \pi(t_1)e^{\alpha(t - t_1)},\ \ t \in (t_1, t_2)$

There are five possible cases:

(i). $\exists t_0$ and $t_3 : 0 \le t_0 < t_1$ and $t_2 < t_3 \le \Delta_{12}$ such that $Z(t) = \overline{Z}$ for $t \in [t_0, t_1] \cup [t_2, t_3]$.

Then by definition, $x_2(t) = \hat{x}_2(t)$ hence $\pi(t) = -\dfrac{1}{\theta_2 - \theta_1}$ for $t \in [t_0, t_1] \cup [t_2, t_3]$. By (A6), we have $lim_{t \uparrow t_2} \pi(t) = -\dfrac{1}{\theta_2 - \theta_1} e^{\alpha(t_2 - t_1)} < \dfrac{-1}{\theta_2 - \theta_1}$, which implies a discontinuity in the path of π at $t = t_2 < \Delta_{12}$, a contradiction since it must be continuous.

(ii). $t_1 = 0$ and $\exists t_3 : t_2 < t_3 \le \Delta_{12}$ with $Z(t) = \overline{Z}, t \in [t_2, t_3]$. We then have $\pi(t_2) = -\dfrac{1}{\theta_2 - \theta_1} = \pi^0 e^{\alpha t_2}$ where $\pi(0)$ is written as π^0. Thus for $t \in (0, t_2)$,

$1 + \pi(t)[\theta_2 - \theta_1] = 1 + \pi(t_2) e^{-\alpha(t_2 - t)}(\theta_2 - \theta_1) = 1 - e^{-\alpha(t_2 - t)} > 0$. From (A1), (A4) and (A5), $\overline{\gamma}_2(t) > 0$ hence $x_2(t) = d(t) > \overline{x}_2$ since $Z^0 = \overline{Z}$. This implies $Z(t) > \overline{Z}, t \in (0, t_2)$ a contradiction.

(iii). $\exists t_0 : 0 \le t_0 < t_1$ with $Z(t) = \overline{Z}, t \in [t_0, t_1], t_2 = \Delta_{12}$. By (A6), $\pi(t) = \dfrac{-e^{\alpha(t - t_1)}}{\theta_2 - \theta_1}$ so $1 + \pi(t)[\theta_2 - \theta_1] < 0$ hence $\underline{\gamma}_2(t) > 0$ so that by (A5), $x_2(t) > 0$. Since $Z(t_1) = \overline{Z}$ and $x_1(t) = d(t) < \overline{x}_1, t \in (t_1, t_2)$, we have $lim_{t \uparrow \Delta_{12}} Z(t) < \overline{Z}$ which violates the terminal condition $Z(\Delta_{12}) = \overline{Z}$.

(iv). $t_1 = 0, t_2 = \Delta_{12}$. This yields $\pi(t) = \pi^0 e^{\alpha t}, t \in (0, \Delta_{12})$. Suppose $\pi^0 > \dfrac{-1}{\theta_2 - \theta_1}$. Then $1 + \pi^0[\theta_2 - \theta_1] > 0$. Since π is continuous (or by (A6)) there exists $\varepsilon > 0$ such that $1 + \pi(\varepsilon)[\theta_2 - \theta_1] > 0$. By (A1), $\overline{\gamma}_2(t) > 0$ and we get the same contradiction as in part (ii). Now consider $\pi^0 \le \dfrac{-1}{\theta_2 - \theta_1}$. Then $1 + \pi(t)[\theta_2 - \theta_1] < 0$ so that $\underline{\gamma}_2(t) > 0$ and $x_2(t) = 0, t \in (0, \Delta_{12})$. But $x_2(t)$ can be positive over some subinterval of $(0, \Delta_{12})$ and since we maximize its integral over $[0, \Delta_{12}]$, it must be. Hence a contradiction.

(v). Suppose $Z(t) < \overline{Z}$ except at a finite number of discrete points $0 \leq t_1 \leq t_2 \leq .. \leq t_n < \Delta_{12}$ such that $Z(t_i) = \overline{Z}, i = 1,2,..,n$. At time t_n the problem becomes the same initial problem but restricted to the interval $[t_n, \Delta_{12}]$ The argument from case (iv) applies. Then proceed by backward induction.

Appendix B: *Characterization of Optimal Hotelling Paths Starting from the Ceiling in Zone II*

Consider point C in Fig. B1. It is chosen so that the stock of coal at C is higher than at A. Let $A_C A'_C$ be the translation of the AA' curve through C. One possible path from C is to extract both resources while keeping the stock of pollution at the ceiling, $Z(t) = \overline{Z}$. This is a Hotelling path in which the common scarcity rent λ^0 corresponds to the initial aggregate stock at C. This rent must equal the one starting from point D on the AA' curve, since the global aggregate stock is equal for both and resources are perfect substitutes and the paths are Hotelling. From C, extraction proceeds along the $A'_C A_C$ curve. At any point on this curve, extraction rates are exactly equal to the corresponding points on the AA' curve obtained by drawing a 45° line as shown for C. This program ends at A_C which has the same aggregate stock X_2^H as in point A. The price of energy at A_C is $\overline{p}_2$, although the residual coal stock is lower than X_2^H . In general, any path from A_C to the origin may now be followed provided the ceiling constraint is not violated and $x_1 + x_2 = d(\lambda^0 e^{\rho t})$.

[Fig. B1 here]

Since the vector of endowments at C is under the AA' curve, there exist alternative extraction sequences that will not violate the ceiling. For example, we may use only natural gas at first. Since the aggregate endowment at C is strictly lower than X_1^H (C lies left of the 45° line through B'), scarcity rent will be higher at C and the extraction rate of natural gas $x_1(t) = d(\lambda^0(X^0))$ lower than $\bar{x}_1$. This path may cross the AA' curve and reach a point such as *E*. As the price of the resource increases, extraction of gas decreases, and the stock of pollution also decreases. At *E* the stock of pollution is lower than the ceiling and thus smaller than on the path $A'_C A_C$. Since the aggregate resource stock is higher at *E* relative to *A*, the common shadow price is lower. Coal can be used at rates higher than $\bar{x}_2$ beginning from *E* to go back to the AA' curve at point *F*. The stock of pollution will rise from *E* towards point *F*. The lengths of these two periods can be so chosen that the pollution stock at *F* is exactly $\bar{Z}$. The remaining path may follow the curve AA' until *A* followed by coal use until exhaustion.

Yet another alternative may be to use gas until point *G* on the AA' curve where $Z(t) < \bar{Z}$, then use coal until some location *H* where $Z(t) = \bar{Z}$. From there extraction can follow the translation of the AA'curve through *H*. Alternative sequences are possible including single or joint use of the two resources such that $\bar{Z}$ is not exceeded. Once the vector of stocks achieves the boundary *AB* of zone I, the proportion of each resource that can be used in response to the common scarcity rent is no longer restrained. For instance, from location A_C, only coal can be used until exhaustion, and the ceiling will not be violated. This is not possible for initial coal stocks larger than X_2^H such as from point *C*.

An important feature of extraction from any location *C* in zone II is that the residual vector of stocks must stay either on or above the $A'_C A_C$ curve for some initial period. Paths such as *CJK* are not allowed since they imply extraction of the polluting resource at rates higher than $\bar{x}_2$ and violation of the ceiling constraint.

Appendix C: ***Determining Optimal Paths for Initial Endowments in Zones III, IV and V***

C1. ***Initial Endowments in Zone III***

On the line $A'B'$ (see Fig.C1) aggregate stocks of the two resources must sum to X_1^H. The common value of the initial scarcity rent λ^0 equals $\overline{p}_1$. Consider point D in zone III, with stocks $D=(D_1,D_2)$ and point $D'=(D_1',D_2)$ on line $A'B'$ with $D_1 > D_1'$. Then starting from D gas is consumed first at the maximal rate $\overline{x}_1$ over a time interval $\Delta_1 = \frac{D_1 - D_1'}{\overline{x}_1}$ until D' is reached. The price of energy at D' is $\overline{p}_1$. The initial value of the common scarcity rent is $\lambda^0 = \overline{p}_1 e^{-\rho\Delta_1}$. In this first period, $\lambda(t) = \overline{p}_1 e^{-\rho(\Delta_1 - t)}$. Since $p(t) = \overline{p}_1 = \lambda(t) - \mu(t)\theta_1$, we have

$$\mu(t) = \begin{cases} \frac{-\overline{p}_1[1-e^{-\rho(\Delta_1-t)}]}{\theta_1}, & t\in[0,\Delta_1) \\ 0 & t\in[\Delta_1,+\infty) \end{cases}.$$

All points on any line parallel to $A'B'$ must have the same scarcity rent as well as the same length of the first period when only gas is extracted. The further right the location of this line, the lower the scarcity rent, the longer is the period of gas extraction and higher in absolute terms the starting value of $\mu(t)$.

[Fig. C1 here]

C2. *Initial Endowments in Zone V*

Consider an initial endowment F (Fig. 2) detailed in Fig.C2, with endowments $(0,F_2)$ such that $F_2 > X_2^H$. Coal is used at the maximum rate $\overline{x}_2$ until the stock decreases to X_2^H. The energy price is constant at $\overline{p}_2$. The length of this phase is given by $\Delta_2 = \frac{F_2 - X_2^H}{\overline{x}_2}$. The initial scarcity rent of coal is $\lambda_2^0 = \overline{p}_2 e^{-\rho\Delta_2}$. The larger the value of F_2, the longer is the duration of this phase and smaller the scarcity rent of coal. The next phase is pure Hotelling of duration $\Delta_2^H = \frac{\ln c_r - \ln \overline{p}_2}{\rho}$. For a phase with joint use to occur at the beginning, initial resource endowments must be higher than at F.

[Fig. C2 here]

Suppose Δ_{12} is the duration of this first phase starting from *G*. Then the additional stocks required for the segment *G* to F are given by $\int_0^{\Delta_{12}} x_i(t)dt, i = 1,2$ where $x_i(t)$ are given by (13). The maximum length of this phase is $\frac{\ln \bar{p}_2 - \ln \bar{p}_1}{\rho}$ because the initial price of energy is $\bar{p}_1$ and the final price $\bar{p}_2$. Consider point *H* with a higher stock of coal. Then starting from *H*, the duration of the intermediate phase Δ_2 will be longer. Moreover, consider point *K* with the same stock of gas as in *G*. The duration of the phase from *K* to *H* is exactly the same as from *G* to *F*. The consumption of the two resources is exactly equal and the stock of pollution is at the ceiling. However, the resource prices and scarcity rents are not equal. The maximum length of this phase is $\frac{\ln \bar{p}_2 - \ln \bar{p}_1}{\rho}$ which corresponds to starting stocks at F' and G'. Thus the curve HH' is a vertical translation of FF' and of AA' which of course has no intermediate phase Δ_2.

During the period when resources are jointly extracted, their marginal cost must be equal, i.e., $\lambda_1(t) - \mu(t)\theta_1 = \lambda_2(t) - \mu(t)\theta_2$. Define the terminal time for this phase as Δ_{12}. Then $p(t) = \bar{p}_2 e^{-\rho(\Delta_{12}-t)}$. As in (13), we can write $x_1 = \frac{\theta_2[d(\bar{p}_2 e^{-\rho(\Delta_{12}-t)}) - \bar{x}_2]}{\theta_2 - \theta_1}$ and $x_2 = \frac{\theta_1[\bar{x}_1 - d(\bar{p}_2 e^{-\rho(\Delta_{12}-t)})]}{\theta_2 - \theta_1}$. Thus $\frac{dx_1}{dt} < 0, \lim_{t \uparrow \Delta_{12}} x_1(t) = 0$ and $\frac{dx_2}{dt} > 0, \lim_{t \uparrow \Delta_{12}} x_2(t) = \bar{x}_2$. Note that extraction depends upon the time variable $\Delta_{12} - t$ and not on calendar time. Equating the marginal costs of gas and coal at time Δ_{12} gives $\lambda_1(\Delta_{12}) - \mu(\Delta_{12})\theta_1 = \lambda_2(\Delta_{12}) - \mu(\Delta_{12})\theta_2$. Since the shadow prices and $-\mu$ all grow at the rate of discount, we have $\lambda_1^0 - \lambda_2^0 = -\mu^0(\theta_2 - \theta_1)$. Substituting the initial value of the scarcity rent of coal given by $\lambda_2^0 = \bar{p}_2 e^{-\rho(\Delta_{12}+\Delta_2)}$, we get

$\mu^0 = \mu(\Delta_{12})e^{-\rho\Delta_{12}} = \dfrac{-\overline{p}_2 e^{-\rho\Delta_{12}}(1-e^{-\rho\Delta_2})}{\theta_2}$. For points located on the AA' curve where $\Delta_2 = 0$, we have $\mu^0 = 0$ so that $\lambda_1^0 = \lambda_2^0$. Both resources are perfect substitutes and regulation is non-binding.

Finally we show that coal is used exclusively beyond Δ_{12}, i.e., the marginal cost of gas is higher than $\overline{p}_2$ in the interval $(\Delta_{12}, \Delta_{12} + \Delta_2)$. In this period, we have

$\lambda_1(t) - \mu(t)\theta_1 - \overline{p}_2 = \lambda_1(\Delta_{12})e^{\rho(t-\Delta_{12})} - \mu(t)\theta_1 - \overline{p}_2 =$

$[\lambda_2(\Delta_{12}) - \mu(\Delta_{12})(\theta_2 - \theta_1)]e^{\rho(t-\Delta_{12})} - \mu(\Delta_{12})e^{\rho(t-\Delta_{12})}\theta_1 - \overline{p}_2 =$

$[\lambda_2(\Delta_{12}) - \mu(\Delta_{12})\theta_2]e^{\rho(t-\Delta_{12})} - \overline{p}_2 = \overline{p}_2 e^{\rho(t-\Delta_{12})} - \overline{p}_2 = \overline{p}_2(e^{\rho(t-\Delta_{12})} - 1) > 0$. In the final interval $(\Delta_{12} + \Delta_2, \Delta_{12} + \Delta_2 + \Delta_2^H)$ regulation is no longer active hence $\mu(t) = 0$. The marginal cost of the resource is its scarcity rent. Since $\lambda_1(t) > \lambda_2(t)$, coal is cheaper than natural gas.

C3. *Initial Endowments in Zone IV*

Consider the vertical line through $\overline{X}_1^0$ (points such as A', F', H' in Fig. C2) where the resource price is $\overline{p}_1$ and the phase of joint use at the ceiling is of maximum duration and points with a higher stock of gas, such as $E = (E_1, F_2')$. The path must be dynamically consistent and we already know the extraction sequence from location F'. We show that only gas is consumed at its maximum rate $\overline{x}_1$ from E to F'. The duration of this phase is given by $\Delta_1 = \dfrac{E_1 - X_1^0}{\overline{x}_1}$. We only need to show that the marginal cost of coal is higher than that of gas which equals $\overline{p}_1$ in this period. The proof mimics the one above but on the interval $[0, \Delta_1)$. We have $\lambda_2(t) - \mu(t)\theta_2 - \overline{p}_1 = \lambda_2(\Delta_1)e^{\rho(t-\Delta_1)} - \mu(t)\theta_2 - \overline{p}_1 =$

$[\lambda_1(\Delta_1) + \mu(\Delta_1)(\theta_2 - \theta_1)]e^{\rho(t-\Delta_1)} - \mu(\Delta_1)e^{\rho(t-\Delta_1)}\theta_2 - \overline{p}_1 =$

$[\lambda_1(\Delta_1) - \mu(\Delta_1)\theta_1]e^{\rho(t-\Delta_1)} - \overline{p}_1 = \overline{p}_1 e^{\rho(t-\Delta_1)} - \overline{p}_1 = \overline{p}_1(e^{\rho(t-\Delta_{12})} - 1) > 0$.

Figures for the Web Appendix

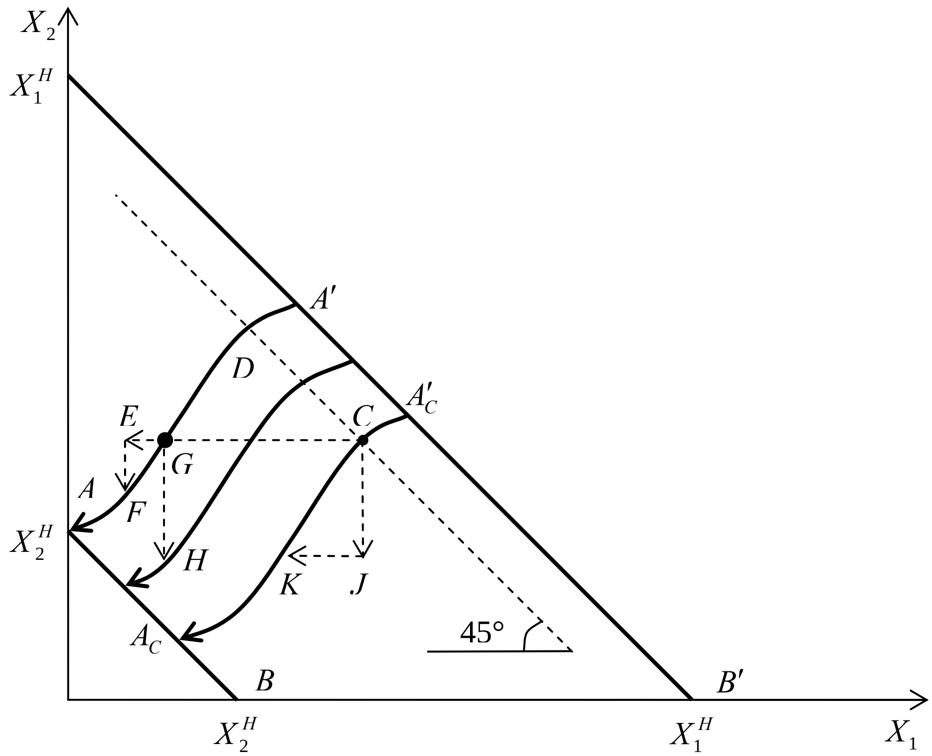

Fig.B1. Hotelling paths starting from Zone II

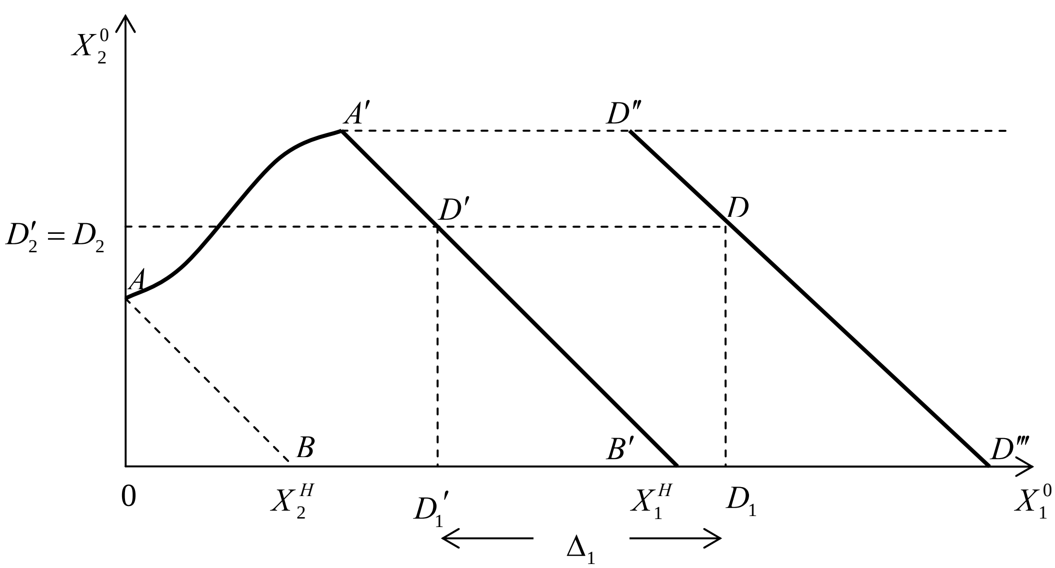

Fig.C1. Iso-scarcity rent loci in Zone III

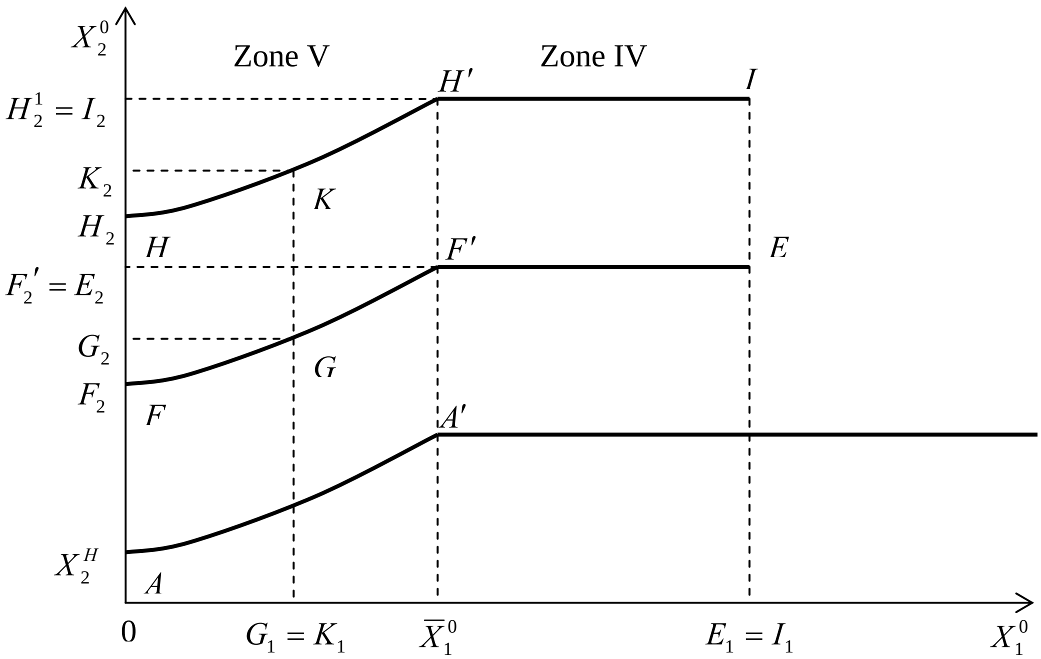

Fig.C2. Endowments in zones IV and V