Reference-Dependent Preferences and Labor Supply: The Case of New York City Taxi Drivers

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Appendix - Derivation of the Likelihood Function

First note that if driver i stops after trip t, he did not stop after the first $t-1$ trips. The probability of observing driver i stop after the t^{th} trip conditional on a particular value of T_{ij} is

$$
\begin{aligned}
Q_{ijt}|T_{ij} &= (1-(P^c_{ijt}|T_{ij}))\cdot\prod_{k=1}^{t-1}P^c_{ijk}|T_{ij} \\
&= (1-\Phi[X_{ijt}\beta+\delta I[T_{ij}>Y_{ijt}]])\cdot\prod_{k=1}^{t-1}\Phi[X_{ijk}\beta+\delta I[T_{ij}>Y_{ijk}]] \qquad \text{(A.1)}
\end{aligned}
$$

In order to derive the unconditional shift probability, I use the distribution of T_{ij} to "integrate out" the random component in the reference level (μ_{ij} in equation (8) as follows. For a driver who stops after the t^{th} trip, there are t+1 possible intervals for the reference level of income to fall relative to accumulated income after each trip during the shift. The reference level of income may be

- less than Y_{ij1},
- in one of the t-1 intervals $Y_{ij(k-1)}<T_{ij}\leq Y_{ijk}$, or
- above Y_{ijt}.

Suppose driver i on shift j stops after trip t_{ij}. Using the information on accumulated income after each trip on a shift, the unconditional shift probability associated with driver i on shift j is

$$
\begin{aligned}
Q_{ij} &= (Q_{ijt_{ij}}|T_{ij}\leq Y_{ij1})\cdot Pr(T_{ij}\leq Y_{ij1}) \\
&+ \sum_{h=2}^{t_{ij}}\left[(Q_{ijt_{ij}}|(Y_{ij(h-1)}<T_{ij}<Y_{ijh}))\cdot Pr(Y_{ij(h-1)}\leq T_{ij}<Y_{ijh})\right]
\end{aligned}
$$

$$(A.2) \qquad + \quad (Q_{ijt_{ij}}|T_{ij} > Y_{ijt}) \cdot Pr(T_{ij} > Y_{ijt_{ij}}).$$

The conditional shift probabilities in this expression follow from equation A.1:

- The probability of observing a driver stop after trip t_{ij} conditional on the reference income level being less than income after the first trip is

$$(A.3) \qquad Q_{ijt_{ij}}|(T_{ij} \le Y_{ij1}) = (1 - \Phi[X_{ijt_{ij}}\beta]) \cdot \prod_{k=1}^{t_{ij}-1} \Phi[X_{ijk}\beta].$$

- The probability of observing a driver stop after trip t_{ij} conditional on the reference income level being in the one of the $t_{ij} - 1$ possible intervals $Y_{ij(h-1)}$ to Y_{ijh} is

$$(A.4) \quad Q_{ijt_{ij}}|(Y_{ij(h-1)} < T_{ij} \le Y_{ijh}) = (1 - \Phi[X_{ijt_{ij}}\beta]) \cdot \prod_{k=1}^{h-1} \Phi[X_{ijk}\beta + \delta] \cdot \prod_{k=h}^{t_{ij}-1} \Phi[X_{ijk}\beta].$$

- The probability of observing a driver stop after trip t conditional on the reference income level being greater than income after trip t is

$$(A.5) \qquad Q_{ijt_{ij}}|(T_{ij} > Y_{ijt}) = (1 - \Phi[X_{ijt_{ij}}\beta + \delta]) \cdot \prod_{k=1}^{t_{ij}-1} \Phi[X_{ijk}\beta + \delta].$$

It remains to write the probabilities of the reference income falling in each of the $t + 1$ intervals. These probabilities follow from the definition of T_{ij} in equation (8):

- The probability that the reference income level is no greater than income after the first trip is

$$(A.6) \qquad Pr(T_{ij} \le Y_{ij1}) = \Phi[(Y_{ij1} - \theta_i)/\sigma].$$

- The probability that the reference income level lies in one of the $t - 1$ possible intervals $Y_{ij(k-1)}$ to Y_{ijk} is

$$\begin{aligned} Pr(Y_{ij(k-1)} < T_{ij} \le Y_{ijk}) &= Pr(T_{ij} \le Y_{ijk}) - Pr(T_{ij} \le Y_{ij(k-1)}) \\ (A.7) \qquad &= \Phi[(Y_{ijk} - \theta_i)/\sigma] - \Phi[(Y_{ij(k-1)} - \theta_i)/\sigma]. \end{aligned}$$

- The probability that the reference income level is greater than the income after trip t is

$$Pr(T_{ij} > Y_{ijt}) = 1 - \Phi[(Y_{ijt} - \theta_i)/\sigma]. \tag{A.8}$$

The probabilities defined in equations A.3-A.8 specify the components of the unconditional probability Q_{ij}, defined in equation A.2, for driver i observed to end shift j after trip t_{ij}. Assuming each shift for a driver is an independent observation, the likelihood function appropriate to this model is defined as

$$L = \prod_{i=1}^{n} \prod_{j=1}^{m_i} Q_{ij}, \tag{A.9}$$

where n denotes the number of drivers in the sample and m_i is the number of shifts for driver i in the sample.