Appendix

Computation of the Equilibrium

In this appendix, I first explain the algorithm used to compute the benchmark and voucher equilibria, and then discuss specific aspects related to the treatment of household location and school choice. This Appendix refers to Models 2, 3 and 4, which include household idiosyncratic tastes. The algorithm for Model 1 is analogous to the one presented here, although with specific provisions to deal with the discreteness of household types. See Ferreyra (2002) for further details on the algorithm for Model 1.

The computation of the benchmark equilibrium for the Chicago metropolitan area takes between five and ten seconds in a 3Ghz processor, and the computation of the voucher equilibrium for vouchers ranging from $1,000 to $7,000 takes between one and seven hours depending on the voucher program and amount.

The Algorithm: Overview

Chart I depicts the algorithm that calculates an equilibrium. The computation of an equilibrium consists of two nested loops: an outer loop of *major iterations* for voting and adjustment of community compositions and school quality, and an inner loop of *minor iterations* for the choice of location and school type. The sequence of major iterations concludes when agents cannot gain any utility by moving or switching to a different type of school, and all endogenous variables have converged.

The algorithm, which is coded in ANSI C++, follows the steps explained below. Notice that steps 3 through 5 comprise the inner loop, in which property tax rates, community compositions, public school quality, spending per student, number of people in public schools, and property tax base are held constant and taken as given by households when making choices.

1. For the benchmark equilibrium, set up the community structure for the computational version of the model, and define initial prices for houses and non-residential property in all locations. For the voucher equilibrium, take the benchmark equilibrium as the starting point.
2. Households vote for tax rates and spending in public schools. In the first iteration while computing the benchmark equilibrium, households vote for property taxes in the district where their endowed houses are located, as though they all attended public schools. Otherwise, households vote for taxes in the districts where they would prefer to live. A district's new tax rate is the one chosen by the district's median voter. Given the new tax rates, districts' spending per student and public school quality are updated.

3. Households choose their optimal location and type of school given the election outcomes; when considering a location, households evaluate the public and private schools available there.
4. Once all households have made their choice, the algorithm computes the excess of demand for houses in each location and adjusts the price of houses in all neighborhoods proportionally to each neighborhood's excess of demand according to the following price adjustment rule: $P_1(d,h) = P_0(d,h) + cED_1\,(d,h)$, where the subscript *1* stands for the current iteration, and *0* for the previous one. *ED(d,h)* denotes the excess of demand for housing in district *d* and neighborhood *h*. The parameter *c* is the adjustment factor. It is set to 0.01 at the beginning of every round of price adjustments, and it is adjusted using a bisection rule as departures are detected from the convergence path on the way to the equilibrium.
5. Check whether there is a nonzero excess demand in some location. If there is, go back to (3), which will in turn lead to a new adjustment of prices in (4). The inner loop of location choice and price adjustments continues until supply equals demand in all locations. In each of these minor iterations, the value of a household's endowment is updated as the price of its endowed house changes during price adjustments. Households keep their endowment house throughout the inner loop.
6. Repeat steps (3), (4) and (5) until prices clear the housing market in each location.
7. Adjust community compositions, non-residential property tax base, number of public school households, and public school quality for each district.
8. If households have experienced utility gains in the current major iteration by moving and/or changing schools, start a new major iteration. The outer loop continues until households cannot gain any utility by moving or switching to a different school, and all endogenous variables have converged.

Further Details on the Algorithm

a. Benchmark Equilibrium

To simplify the explanation of the computational treatment of household location and school choice, some additional notation is in order. Each household *i* in the economy belongs to a non-idiosyncratic household type *j*, which represents a combination of income endowment y_n, house endowment valued at p_n , and religious type *k*. This non-idiosyncratic type exists with measure μ_j , and the total number of non-idiosyncratic types is $J = I \times H \times K$. In addition, location (*d*,*h*) has a measure of houses equal to μ_{dh} , so that district d has a total measure of houses equal to $\mu_d = \sum_h \mu_{dh}$. Let m denotes school type; m=1, 2, 3 represents public, private Catholic and private non-Catholic school respectively. Furthermore, the triplet (d,h,m) denotes the joint choice of location (d,h) and school type m.

When choosing (d,h,m), household i of non-idiosyncratic type j obtains utility $U_{idhm} = s^{\alpha}_{jdhm} c^{\beta}_{jdhm} k^{1-\beta-\alpha}_{dh} e^{\varepsilon_{idhm}}$. Since all households of non-idiosyncratic type j choosing (d,h,m) experience the same consumption and parental valuation of school quality, we can write this utility as $U_{idhm} = U^{*}_{jdhm} e^{\varepsilon_{idhm}}$. Define $\tilde{U} \equiv \log(U^{*})$. Hence, the share of non-idiosyncratic type j households who choose (d,h,m) equals $P\left(\tilde{U}_{jdhm} + \varepsilon_{idhm} > \tilde{U}_{jd'h'm'} + \varepsilon_{id'h'm'}\right)$ for all (d',h',m') not equal to (d,h,m). Since ε is distributed i.i.d. type I with scale parameter (1/b), this share equals $P_{jdhm} = \dfrac{\exp\left(\tilde{U}_{jdhm}/b\right)}{\sum_d \sum_h \sum_m \exp\left(\tilde{U}_{jdhm}/b\right)} = \dfrac{\left(U^{*}_{jdhm}\right)^{1/b}}{\sum_d \sum_h \sum_m \left(U^{*}_{jdhm}\right)^{1/b}}$. Thus, the total demand for any given location (d,h) is $\sum_j \sum_m P_{jdhm} \mu_j$, and the prices p_{dh} that equalize the demand for each location to the supply μ_{dh} clear the housing market.

With regards to the voting equilibrium, households of non-idiosyncratic type j who choose (d,h,m) vote for their optimal property tax rate t_{jdhm} in district d's polls. The measure of such households is $P_{jdhm}\mu_j$. The median voter for this district is the household who votes for a tax rate equal to the median of the distribution of the selected tax rates.

b. Voucher equilibrium

Since households of non-idiosyncratic type j vary in their idiosyncratic preferences, they potentially make different benchmark equilibrium choices of location and school. Furthermore, their different location choices imply different amounts of capital gains or losses in the voucher equilibrium, which gives rise to different budget constraints. Hence, for computational reasons I re-define a non-idiosyncratic type as the set of households with the same income and house endowment, religious type, *and* benchmark equilibrium location choice. In other words, non-idiosyncratic type v is the set of households of the *original* non-idiosyncratic type j that choose benchmark equilibrium location $(\hat{d},\hat{h})$. In this re-definition, the number of non-idiosyncratic types equals $V = J \times H$. Hereafter, an asterisk denotes the benchmark equilibrium. Non-idiosyncratic type *v* has measure $\mu_v = \left(\sum_m P^*_{j\hat{d}\hat{h}m} \right) \mu_j$; of course, $\sum_d \sum_h \sum_m P^*_{jdhm} \mu_j = \mu_j$. During the computation of the voucher equilibrium, the budget constraint for type *v* equals $c + (1+t_d)p_{dh} + \max(T - \upsilon) = (1-t_y)y_n + p_n^* + \left(p_{\hat{d}\hat{h}} - p^*_{\hat{d}\hat{h}}\right)$, where υ is the voucher amount, $p^*_{\hat{d}\hat{h}}$ is the benchmark equilibrium price of the house chosen by the original idiosyncratic type *j* in location $(\hat{d}, \hat{h})$, $p_{\hat{d}\hat{h}}$ is the price for that house in the current iteration, and p_n^* is the proceeds from selling the endowed house in the benchmark equilibrium.

Denote by $V_{idhm} = V^*_{vdhm} e^{\varepsilon_{idhm}}$ the utility enjoyed by household *i* of non-idiosyncratic type *v* by choosing (*d*,*h*,*m*) under vouchers, and define $\tilde{V} = \log(V^*)$. Under vouchers, the share of households of non-idiosyncratic type *v* who choose (*d*, *h*, *m*) equals $P\left(\tilde{V}_{vdhm} + \varepsilon_{idhm} > \tilde{V}_{vd'h'm'} + \varepsilon_{id'h'm'}\right)$ for all (*d'*, *h'*, *m'*) not equal to (*d*, *h*, *m*). From the definition of the non-idiosyncratic type *v*, this share equals the share of households of the original non-idiosyncratic type *j* who choose (*d*, *h*, *m*) under vouchers conditional on having chosen $(\hat{d}, \hat{h})$ in the benchmark equilibrium, or

$$P\left(\tilde{V}_{jdhm} + \varepsilon_{idhm} > \tilde{V}_{jd'h'm'} + \varepsilon_{id'h'm'} \mid \max_m \left(\tilde{U}_{j\hat{d}\hat{h}m} + \varepsilon_{i\hat{d}\hat{h}m}\right) > \max_m \left(\tilde{U}_{j\bar{d}\bar{h}m} + \varepsilon_{i\bar{d}\bar{h}m}\right)\right) \text{ for all } (d, h, m)$$

different from (d', h', m') and all $(\hat{d}, \hat{h})$ different from $(\bar{d}, \bar{h})$.

Since this share does not have a closed form solution, I compute it by simulation as follows. I randomly draw *R* independent vectors of idiosyncratic tastes, each one of dimension $H \times 3$, such that all vector elements come from a type I extreme value distribution with scale parameter (1/*b*). I use the same *R* vectors for each non-idiosyncratic type *v*. For each non-idiosyncratic type *v*, I keep the N_v vectors whose elements are consistent with the type having chosen its benchmark equilibrium location. Choosing a sufficiently large *R* ensures that the simulated shares approximate well the benchmark equilibrium closed-form shares. In any given iteration, I compute the share of households of type *v* making choice (*d*,*h*,*m*) as $\hat{P}_{vdhm} = N_{vdhm} / N_v$, where the numerator is the number of vectors of idiosyncratic tastes that render (*d*,*h*,*m*) as the optimal choice for households of non-idiosyncratic type *v*. The computation of the demand for a given location, and of the voting equilibrium, is analogous to that in the benchmark equilibrium.

For the analysis of the outcomes of policy simulations, I proceed as follows. Since voucher effects differ across households of even the same non-idiosyncratic type, I keep track of the benchmark and voucher equilibrium choices for each household involved in the voucher simulation. In the case of

Chicago, the computation of the benchmark equilibrium manipulates J=750 non-idiosyncratic types, whereas the computation of the voucher equilibrium handles V=750x15=11,250 non-idiosyncratic types. These, in turn, become 750,000 households in order to simulate the choice shares as described above. By storing pre- and post- voucher information for each of these households, I am able to estimate the distribution of voucher effects for any desired group of households.

Chart I. Algorithm to Compute the Equilibrium