Appendix C to "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle".

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Consumption and housing choice with borrowing constraints: when do constraints binds?

In this appendix, I sketch the partial equilibrium problem of an infinitely-lived household that maximizes expected intertemporal utility. The household derives utility from the non-durable consumption c and from housing h. The household faces a budget constraint and a borrowing constraint tied to a fraction of the value of the durable asset. Income is produced according to a production function that can potentially take housing as input. The productivity variable is random and follows an $AR(1)$ process.

The main result of this appendix is that, when the model is parameterized with an amount of uncertainty that is sufficient to replicate the volatility which is observed in macroeconomic time series, such uncertainty is "too small" to generate a substantial amount of buffer-stock behavior in the model (loosely meant as borrowing less than the credit limit), provided that the borrowing constraint is tight enough (m, the loan-to-value ratio, is not too high), that relative risk aversion is not too large, that the gap between the interest rate and the discount rate is not too small.[1]

The model setup

I consider the partial equilibrium problem of optimal consumption and savings behavior of an agent who maximizes the discounted sum of future utility subject to an asset accumulation constraint and to a borrowing constraint tied to asset holdings up to a fraction m, which I call the loan-to-value ratio. I assume zero depreciation for the durable asset.[2] I assume that the interest rate is exogenous and lower than the time preference rate (otherwise, asset accumulation would converge to infinity, as shown by Gary Chamberlain and Charles Wilson, 2000).

My interest is in understanding which conditions are needed for such a model to generate instances in which the borrowing constraint does not hold with equality.

I consider the problem of a representative agent that maximizes expected discounted utility from consumption of both a nondurable good c_t and a durable asset h_t. The lifetime utility function is of the form:

$$U = E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t h_t^j \right)^{1-\rho} - 1}{1-\rho} \right) \text{ when } \rho > 0, \ \rho \neq 1$$

$$U = E_0 \left(\sum_{t=0}^{\infty} \beta^t \left(\log c_t + j \log h_t \right) \right) \text{ when } \rho = 1$$

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[1] Of course, the presence of uncertainty implies that in a model of the kind presented below there is precautionary saving, meant as the extra-increase in average total wealth (housing wealth less total outstanding debt, that is $h - b$) due to income uncertainty. What I am mainly interested in, however, is whether the model parameters generate fluctuations in the *ratio* between borrowing and housing (b/h).

[2] Allowing for positive depreciation does not affect the main results.

In the paper, I assume that relative risk aversion is 1, so that log utility and separability arise. The budget constraint is:

$$c_t = y_t + b_t - Rb_{t-1} - (h_t - h_{t-1})$$

where income is $y_t = A_t h_{t-1}^{\nu}$ and the borrowing constraint is:

$$b_t \leq m h_t.$$

One can interpret (apart from minor differences) this formulation as a simplified version of the impatient agents' problem in the paper when the following conditions hold: (1) prices are constant; (2) the interest rate is constant and higher than the discount rate; (3) the asset price is constant.

The stochastic process for A_t obeys the following:

$$\log(A_t) = 0.75 \log(A_{t-1}) + e_t, \ e_t \sim N\left(0, \sigma_e^2\right)$$

Fluctuations in A are therefore the only source of randomness in the model.[3] Altogether there are three state variables: (h_{t-1}, b_{t-1}, A_t).

The model is calibrated at quarterly frequencies. For expositional reasons, I keep the model formulation similar to the problem of the constrained households in the paper, by setting the following baseline parameters: $\nu = 0$, $j = 0.1$, $m = 0.55$, $R = 1.01$, $\beta = 0.95$. This way, the problem boils down to the problem of choice between consumption and a durable good in the presence of stochastic income, and the durable asset only provides utility services, without affecting total income produced.[4]

I experiment with several values for the σ_e, the conditional variance of income. In the estimated general equilibrium model of the paper, the unconditional standard deviation of quarterly detrended output is about 2 percent, which is roughly the value found in the data. A value of $\sigma_e = 0.013$ (which generates an unconditional standard deviation for A of $\sigma_A = 0.0197$) is what is needed to roughly replicate aggregate volatility.

A model of this kind is fairly standard in partial equilibrium analyses of consumer behavior in presence of uncertainty, see e.g. Christopher Carroll (2000) or Sydney Ludvigson (1999). Ludvigson, for instance, assumes no capital, but ties borrowing to aggregate income and assumes that m is time-varying. Unlike traditional analyses, however, the model assumes that the investment good is both a productive asset and collateral for loan. To experiment for variable (shadow) price of the asset, I also tried versions of the above model with quadratic adjustment costs: most of the results presented below resulted to be robust to this change.

The question I want to address is: under which conditions does the model generate borrowing constraints which do not bind?

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[3] In the model of the paper, the quarterly theoretical autocorrelation of output is 0.65. In the (band-pass filtered) data, the autocorrelation of GDP is 0.86. Here, I set the autoregressive component of the only driving process for output at the average between these two values.

[4] I consider several values for the standard deviation of the innovation process. I approximate this process by using a five state Markov chain following the procedures described in George Tauchen (1986). I discretize the state space for the two states, housing and debt, using a 40×40 grid for the two variables with a uniform range that takes values from 20 percent less to 20 percent more than their steady state, non-stochastic values. In the simulations below, the bounds on h are very rarely binding. Such bounds h are also in accordance with the set-up of the paper in which the supply curve for housing is not flat.

Does the collateral constraint always bind?

A solution for the above problem can be summarized by a consumption rule $c_t = c(A_t, h_{t-1}, b_{t-1})$, an asset accumulation rule $h_t = h(A_t, h_{t-1}, b_{t-1})$ and a borrowing rule $b_t = b(A_t, h_{t-1}, b_{t-1})$. After such a solution is found, I use these decision rules to generate time profiles for the model variables.

Figure A.1 presents the results for the baseline case, showing a simulation of income y_t, consumption c_t and borrowing b_t over housing for 500 periods. Here, the borrowing constraint is binding 100 percent of the times. As a consequence, consumption closely tracks income, in good as in bad times.[5] On average, consumption is lower than income, since the individual ends up with a positive amount of debt to roll over on which interest is paid.

In Figure A.2, I consider a different example in which relative risk aversion is raised to $\rho = 5$ and the standard deviation of the innovation in productivity is $\sigma_e = 0.05$.[6] Here, buffer stock behavior emerges, and liquidity constraints bind less often. After a sufficiently long run of income shocks, the debt/asset ratio falls below the maximum loan-to-value (bottom panel). Although consumption continues to track income closely, the decision rules highlight the role played by precautionary behavior: liquidity constrained periods, in particular, are 76.8 percent of the total.

In general, there are four parameters that affect how often borrowing constraints bind. (1) The degree of risk aversion; (2) the volatility of the underlying income process; (3) the loan-to-value ratio; (4) the gap between the interest rate and the discount rate. How does each of these factors contribute?

In Figure A.3, I keep $\beta = 0.95$ and $m = 0.55$, as in the baseline case, and calculate the fraction of liquidity constrained periods as a function of risk aversion and income variability.[7] Not surprisingly, the borrowing constraint binds less frequently as risk aversion rises, and the effect is stronger when risk aversion is large.

For log utility, in fact, (relative risk aversion of $\rho = 1$), the borrowing constraint binds 100 percent of the cases if $\sigma_e \leq 0.06$: such a number would correspond to an unconditional standard deviation of aggregate income about four times larger than needed to replicate macroeconomic volatility. It takes very high risk aversion coupled with very high volatility to have precautionary behavior.

In Figure A.4, I consider how the frequency of borrowing constrained periods depends on m. To begin, in the baseline calibration for β, ρ and σ_e, the borrowing constraint holds 100 percent regardless of the value of m. If income volatility σ_e is raised from its baseline value of 1.3 percent to 5 percent, borrowing constraints are less likely to bind the higher m is. When risk aversion and income variability are high, the effect is stronger the higher is β.[8]

I therefore conclude that for a wide range of parameter configurations the assumption that the borrowing

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[5] Consumption does not track income exactly because the household can use both consumption and housing to smooth utility. For instance, in response to, say, a positive income shock, the household, *ceteris paribus*, increases both consumption and housing holdings, so that consumption rises less than one for one with income. The possibility to borrow more following the increase in h further increases consumption and housing demand, but the total effect is that consumption is slightly smoother than income.

[6] I keep the discount factor unchanged at $\beta = 0.95$. Increasing the discount factor reduces the impatience motive and increases the need to accumulate assets, but at the same time changes the non-stochastic steady-state.

[7] In the simulations below I include 5500 periods, and discard the first 500 observations.

[8] In general, the discount factor has little effect on the results: so long as $1/\beta$ stays above R, the borrowing constraint is binding in 100 percent of the cases. Keeping the other parameters of the baseline calibration unchanged, the borrowing constraint binds less than 100 percent only if β is greater than around 0.986.

constraint always holds is a very good approximation. In my view, there are two explanations for this result:

1. A representative agent model rules out the much larger idiosyncratic risk which instead is needed to replicate the microeconomic evidence on income volatility. For instance, Carroll and Wendy Dunn (1997) obtain buffer-stock effects because of the probability of unemployment that each agent faces.

2. Another potential explanation is related to studies that find a modest effect of income uncertainty on capital accumulation in the stochastic growth model (see Carroll (2000) for an example): there, little precautionary saving arises because the representative agent in the model has a large amount of capital. Here, the borrowers tend to overinvest in capital for two additional reasons: (1) durable assets reduce the need to hold a buffer stock of resources to shield consumption from income risk; in addition, (2) agents here "overinvest" in durable assets because they can loosen the borrowing constraint: once they do so, they end up with a very large amount of wealth, therefore the need to borrow less than the maximum possible amount becomes small.[9]

Caveats

The results here are obtained in a partial equilibrium context and without endogenizing the price of the durable asset. What happens, instead, if the price of the asset is endogenous? Here, I provide an intuitive answer to that question.

The key to understanding the behavior of the model when the housing supply curve is not flat is that, when the demand rises (in good times), the price of the collateral will go up: this will have two effects. The price effect works to reduce asset demand. The collateral effect drives asset demand up, leading to further relaxation of the borrowing constraint. If the second effect dominates, the collateral capacity for each unit of the asset pledged becomes procyclical, rising in good times, falling in bad times. This suggests that borrowing constraints might become "looser" in good times, thus offering potential for more buffer-stock behavior in good times, and for less in bad times. If so, borrowing constraints might be less likely to bind in good states of the world.

However, given the assumptions about the model parameters, this outcome seems unlikely in the baseline scenario. In the paper, for instance, asset price fluctuations are roughly of the same magnitude as the fluctuations in productivity. In the baseline case, whether borrowing constraints bind or not is insensitive to the value of m.[10] One can thus infer that even if asset prices were to change dramatically over the cycle, collateral constraint would always bind in the baseline case. This lends indirect support to the assumption that low uncertainty and small curvature of the utility function are sufficient to guarantee that the borrowing constraint is always binding over the relevant range.

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[9] A similar result is obtained by Antonia Díaz and Maria Luengo-Prado (2003): they find that the presence of collateralizable durable goods reduces the need for precautionary saving in an otherwise standard income fluctuation problem. In a similar vein, using a partial equilibrium model of consumption and mortgage choice under uncertainty, Erik Hurst and Frank Stafford (forthcoming) show how housing wealth can effectively be used as a hedge against adverse economic shocks.

[10] Quantitatively, an increase in m in the household problem leads to a reduction of the frequency of binding borrowing constraints only if high income volatility is coupled with a very high discount factor. Results are slightly different in the entrepreneurial problem instead: over a plausible range of parameters, an increase in m is likely to lead to an increase in buffer-stock behavior.

In experiments not reported here, I set $j = 0$ and allow for $\nu > 0$. With this modification, the problem becomes similar to that faced by the entrepreneur, since housing enter the production rather than the utility function. I find that, *ceteris paribus*, the entrepreneurial problem results in slightly larger (but not significantly different) buffer-stock behavior than the household problem: this happens because, when housing is needed to produce the consumption good (rather than to provide utility services), it can be used less effectively to smooth out income fluctuations, thus increasing the need not to use all the borrowing capacity during good times.[11]

References

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[4] **Díaz, Antonia and Luengo-Prado Maria J.** "Precautionary Savings and Wealth Distribution with Durable Goods". Working Paper, Universidad Carlos III de Madrid and Northeastern University, 2003.

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[6] **Ludvigson, Sydney.** "Consumption and Credit: A Model of Time-Varying Liquidity Constraints." *The Review of Economics and Statistics*, August 1999, *81*(3), pp. 434-47.

[7] **Tauchen, George.** "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 1986, (20), pp. 177-81.

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[11] For instance, if $\sigma_e = 0.05$, $\beta = 0.95$, $\rho = 5$, $m = 0.55$, $j = 0$ and $\nu = 0.0919$, we have the same steady state housing holdings and debt as in the household problem (where $\nu = 0$ and $j = 0.1$). However, the frequency of binding borrowing constraints is 75.5 percent, as opposed to 76.8 percent in the household problem.

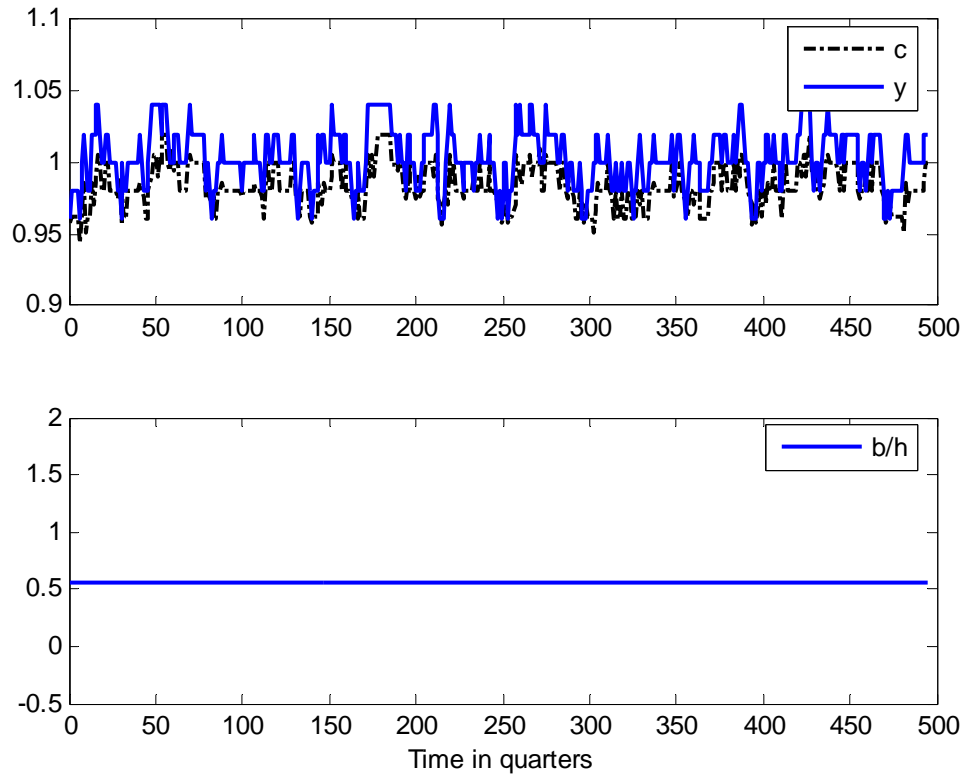# Figures

Figure A.1: Simulated Variables, Baseline Case

Figure A.2: Simulated Variables, High Volatility, High Discount Factor, And High Risk Aversion.
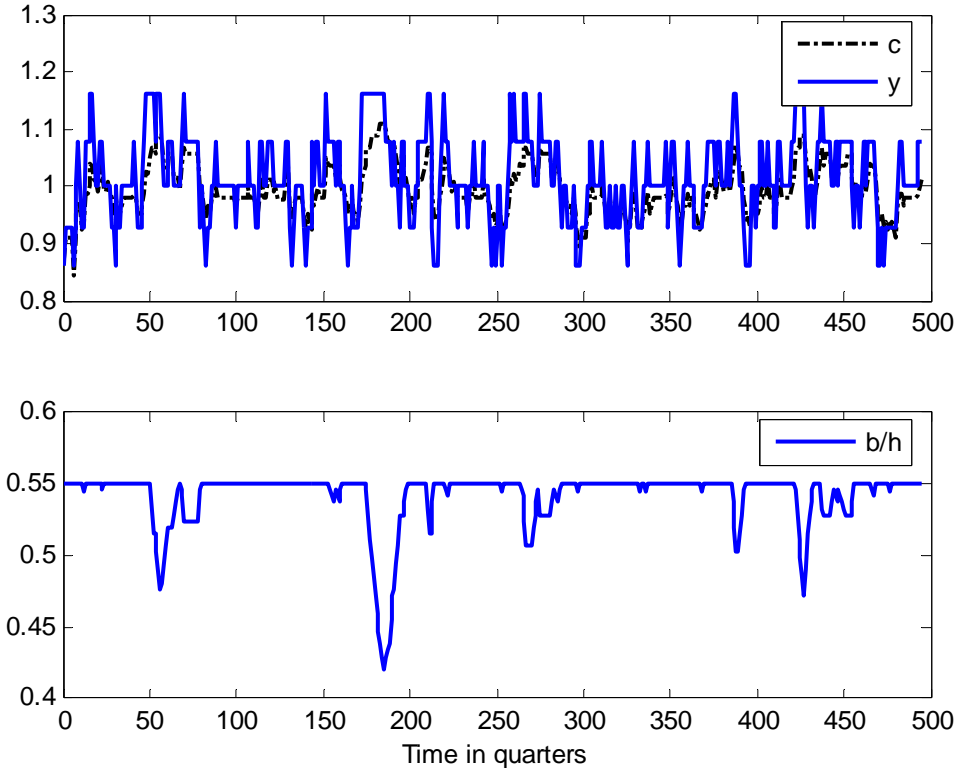
Figure A.3: Frequency Of Times The Borrowing Constraint Binds As A Function Of The Volatility For Different Degrees Of Risk Aversion.
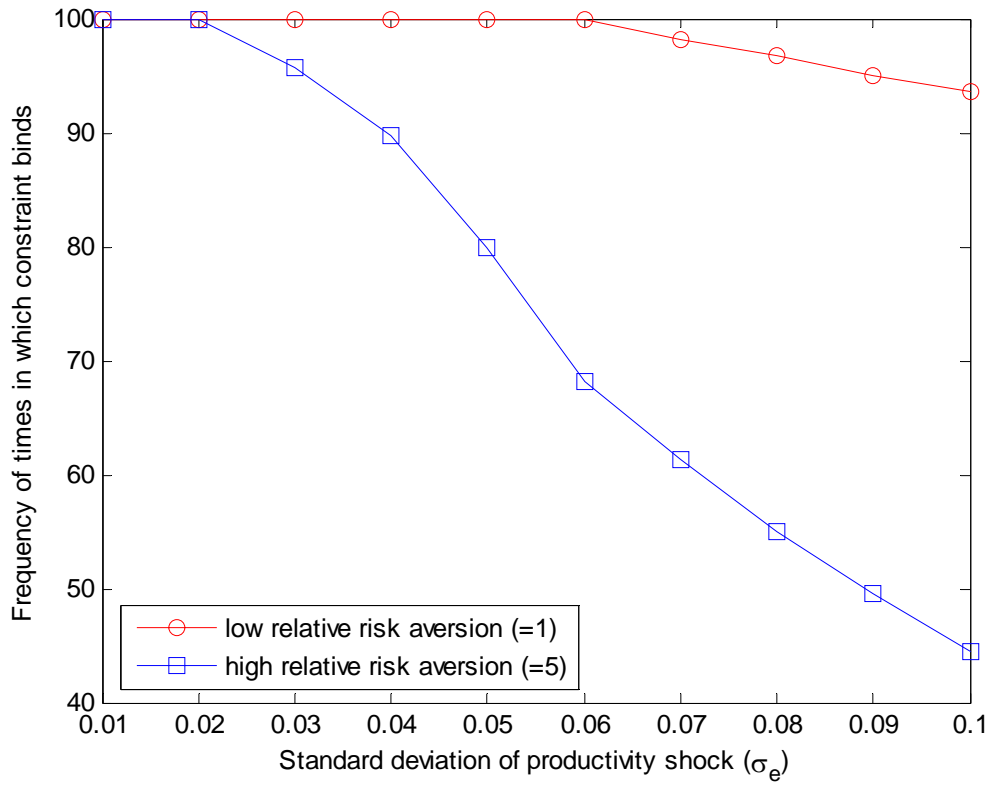
Figure A.4: Frequency Of Times The Borrowing Constraint Binds As A Function Of The Loan-To-Value m, For Different Values Of The Other Parameters.