

# Fairness and Redistribution, a Comment

## Online Appendix

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*We provide a counterexample to Theorem 1 in Alesina and Angeletos (2005). The key to the counterexample is that in their proof they assume that the median voter is that with the median values of the shock, and in general she is not.*

*JEL: D31, E62, H2, P16.*

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AA define the government's choice of policy in p. 967: "The optimal policy maximizes the utility of the median voter." They then identify the median voter as the individual with the median values of the shocks: "Assuming that luck has zero mean and median, the median voter, denoted by  $i = m$ , is an agent with characteristics  $\delta_m = \text{median}(\delta_i)$  and  $\eta_m = 0$ ." In general, however, the median voter is not the individual with the median values of the shocks. Due to this wedge, one can produce a counterexample to Theorem 1.

AA define an equilibrium as a tax policy  $\tau^*$  such that when the anticipated policy is  $\tau_e = \tau^*$ , the ex-post optimal policy (the one preferred by the median voter) is  $\tau^*$ . Theorem 1 then says that an equilibrium always exists and corresponds to any fixed point of

$$f(\tau_e) = \arg \min_{\tau \in [0,1]} \left\{ \begin{array}{l} \frac{\delta_m}{2} (1 - \alpha) \tau^2 + \tau^2 (1 - \alpha \tau_e - (1 - \alpha) \tau)^2 \gamma \sigma_\delta^2 + \\ (1 - \tau)^2 \gamma \sigma_\eta^2 - \tau (1 - \alpha \tau_e - (1 - \alpha) \tau) (\bar{\delta} - \delta_m) \end{array} \right\}$$

where  $f(\tau_e)$  is the set of tax rates that maximize the utility of the individual with the median values of the shocks.

We now show that for some distributions and parameter values, 0 is an equilibrium, but it is not a fixed point of  $f$ : with an expected tax rate of  $\tau_e = 0$ , the median voter's preferred tax rate is 0, and if the government maximizes his utility, then it chooses  $\tau = 0$ , so that it is an equilibrium. In contrast, when the expected tax rate is 0, the utility of the individual with the median values of the shocks is maximized for  $f(0) = 1$ , showing that 0 is not a fixed point of  $f$ .

First, set  $\gamma = 0$ ,  $\alpha = \frac{442}{443}$  and  $\bar{\delta} = \frac{819}{400}$  and  $\delta_m = 2$ .<sup>1</sup> Note that if  $\tau_e = 0$ , we obtain that  $f$  minimizes

$$(1 - \alpha) \tau^2 - \tau (1 - (1 - \alpha) \tau) \frac{19}{400}$$

which implies  $f(0) = 1$ . Hence, what we will show to be an equilibrium,  $\tau_e = \tau = 0$ ,

<sup>1</sup>It is easy to build counterexamples to the theorem assuming  $\gamma > 0$ , but the calculations are simpler with  $\gamma = 0$ , and  $\gamma = 0$  is allowed by AA.

is not a fixed point of  $f$  (this is not a consequence of having chosen  $\gamma = 0$ ).

We now give distributions of  $\delta$  and  $\eta$  satisfying these restrictions, but for which the equilibrium tax rate is  $\tau_e = \tau = 0$ , but 0 is not a fixed point of  $f$ .

The distributions  $\delta_i$  and  $\eta_i$  are

$$p_\delta(x) = \begin{cases} \frac{19}{40} & x = \frac{19}{10} \\ \frac{2}{40} & x = 2 \\ \frac{19}{40} & x = \frac{22}{10} \end{cases} \quad \text{and} \quad p_\eta(x) = \begin{cases} \frac{1}{10} & x = \frac{61}{400} \\ \frac{8}{10} & x = 0 \\ \frac{1}{10} & x = -\frac{61}{400} \end{cases}.$$

We assume also that  $\delta_i$  and  $\eta_i$  are independent. In this case, the mean and median of  $\eta$  are 0. Also, the median of  $\delta$  is  $\delta_m = 2$  and its mean is  $\frac{819}{400}$ .

Since  $\gamma = 0$ , and  $2\bar{\delta} > \max \delta_i$ , preferences are single peaked and the optimal tax rate  $\tau^*$  for a person  $(\delta_i, \eta_i)$  is given by  $dU_i/d\tau = 0$ , or

$$(1) \quad \frac{dU_i}{d\tau} = (\bar{\delta} - \delta_i)(1 - \alpha\tau_e - \tau(1 - \alpha)) - \tau\bar{\delta}(1 - \alpha) - \eta_i \Rightarrow \tau^* = \frac{(\bar{\delta} - \delta_i)(1 - \alpha\tau_e) - \eta_i}{(1 - \alpha)(2\bar{\delta} - \delta_i)}$$

or 0 if  $\tau^* < 0$  or 1 if  $\tau^* > 1$ . From equation (1), and the border conditions  $1 \geq \tau \geq 0$  we get that the optimal tax rates for each combination of shocks, if  $\tau_e = 0$ , is given by

$\eta \setminus \delta$	$\frac{19}{10}$	2	$\frac{22}{10}$	Pr
$\frac{61}{400}$	0	0	0	$\frac{1}{10}$
0	1	1	0	$\frac{8}{10}$
$-\frac{61}{400}$	1	1	0	$\frac{1}{10}$
Pr	$\frac{19}{40}$	$\frac{2}{40}$	$\frac{19}{40}$	

A tax rate of 0 accumulates the votes of  $\frac{211}{400}$ % of the population, and therefore “the” median voter is any voter whose preferred tax rate is 0 (and not the individual with shocks  $(\delta, \eta) = (2, 0)$ , whose preferred tax rate is 1, as claimed in AA).

One possible solution is to assume that shocks in AA are symmetric, so as to ensure that the median voter is the individual with the median values of the shocks. It is possible then to show that, although preferences are not single peaked, this individual’s preferred tax rate is a Condorcet winner (see Di Tella, Dubra and MacCulloch (2010) and Di Tella and Dubra, 2011). In this case, however, the model can no longer capture a Meltzer-Richard motive for redistribution.