Online Appendix to "Cultural Change as Learning: The Evolution of Female Labor Force Participation over a Century" by Raquel Fernández

For Online Publication Only

A Data

Earnings

For earnings data prior to 1940, I rely on numbers provided in Goldin (1990) who uses a variety of sources (Economic Report of the president (1986), Current Population Reports, P-60 series, and the U.S. Census among others) to calculate earnings for men and women.[60] As there is no data for earnings in 1880 and 1910, these points are constructed using a cubic approximation with the data from 1890 -1930 (inclusive).

To construct the earnings sample from 1940 onwards I used the 1% IPUMS samples of the U.S. Census for yearly earnings (incwage) to calculate the median earnings of white 25-44 years old men and women who were working full time (35 or more hours a week) and year round (40 or more weeks a year) and were in non-farm occupations and not in group quarters.[61] As is commonly done, observations that report weekly earnings less than a cutoff are excluded.[62] Prior to 1980, individuals report earnings from the previous year, weeks

[60] See Goldin (1990) pages 64-65 and 129 for greater detail about the earnings construction for various years. I use the data for white men and women.

[61] The sample is limited to full-time year-round workers because hourly wages are not reported. The sample could have been restricted to include only married men and women, but I chose not to do this in order to be consistent with the data from the earlier time period.

[62] The latter is calculated as half the nominal minimum wage times 35 hours a week and nominal weekly wages are calculated by dividing total wage and salary income last year by weeks worked last year. See, for example, Katz and Autor (1999). This procedure is somewhat more problematic for the decades 1940-1960, when the federal minimum wage did not apply to all workers (prior to the 1961 amendment, it only affected those involved in interstate commerce). Nonetheless, I use the same cutoff rule as in Goldin and Margo (1992) as a way to eliminate unreasonably low wages. Note that by calculating median earnings, I do not have to concern myself with top-coding in the Census.

worked last year, and hours worked last week. From 1980 onwards, individuals are asked to report the "usual hours worked in a week last year." Hence for these years we include only people who answered 35 or more hours to that question and we drop the restriction on hours worked last week. In 1960 and 1970, the weeks and hours worked information was reported in intervals. We take the midpoint of each interval for those years.

Sample weights (PERWT) were used as required in 1940, 1990, 2000. In 1950 sample line weights were used since earnings and weeks worked are sample line questions. The 1960-1980 samples are designed to be nationally representative without weights.

For the education categories, college is defined as having at least one year of schooling after high school. High School is defined as having at most a high school degree or no more than 13 years of education.

Figure 11 shows the evolution of female and male median earnings over the 120 year period 1880-2000 (with earnings expressed in 1967 dollars). In order to compare data sets, the figure plots both the numbers obtained from the calculations above as of 1940 (they are shown in (red) dots) as well as Goldin's numbers (which continue to 1980 and are shown in (blue) x's). The only significant difference is with male earnings in 1950 which are higher for Goldin.[63]

Married Women's LFP: US and International

For the LFP numbers I used the 1% IPUMS samples for 1880, 1900-1920, 1940-1950, 1980-2000, and the 0.5% sample in 1930 and the 1970 1% Form 2 metro sample. Since the individual census data is missing for this 1890, I use the midpoint between 1880 and 1900. I restricted the sample to married white women (with spouse present), between the ages of 25-44, born in the US, in non-agricultural occupations and living in non-farm, non-institutional quarters.

For the education categories, college is defined as having at least one year of schooling after high school. High School is defined as having at most a high school degree or no more

[63] Goldin's 1950 number is from the Current Population Reports, series P-60 number 41 (January 1962). It is for all men over 14 which may explain the discrepancy since our census figure leaves out men older than 44 who would, on average, have higher earnings.

than 13 years of education.

The data for Great Britain (except 2010) is from Table 1 in Costa (2000) for married women between the ages either from 15-64 or 16-59, depending on the year. She obtains this data from various sources (see her footnote in Table 1). For 2010, the data is from the Office of National Statistics website (`www.statistics.gov.uk`).

For France the data for 1921-1975 is from Table 1 in Reboud (1985) for married women, in non-agricultural sectors, between the ages of 15-69. For 1978-1981, the data is from Table 2 in Reboud (1985), for married women, ages 15-64. For 1990-2008, the data is from the National Institute for Statistics and Economic Studies website (`www.insee.fr`), for married women, ages 15-64.

Elasticities

I take the elasticity estimates from Blau and Kahn (2006) who use the March CPS 1989-1991 and 1999-2001 to estimate married women's own-wage and husband's-wage elasticities along the extensive margin. They impute wages for non-working wives using a sample of women who worked less than 20 weeks per year, controlling for age, education, race and region, and a metropolitan area indicator (page 42). They run a Probit on work (positive hours) including log hourly wages (own and husband's), non-wage income, along with the variables used to impute wages, both including and excluding education. The sample is restricted to married women 25-54 years old (with spouses in the same age range). I use the results obtained from the basic probit specification, which does not control for education, as this way the elasticity measure does not control for a measure of permanent income. This is preferable since an elasticity with respect to some measure of lifetime earnings is more appropriate for the model. I also chose the specification without children as a control variable as it is endogenous. Using the elasticities estimated from a specification with education controls does not affect the results as the elasticities are very similar (0.28 and -0.12 for 2000 and -.15 in 1990). For the year 2000, Blau and Kahn estimate an own-wage elasticity of 0.30 and the cross-elasticity (husband's wage) of -0.13. The cross elasticity in 1990 is -0.14.

Poll Data

I use the Gallup Poll data for years 1938, 1945 and 1970. From 1972 to 1998, I use data from the General Social Survey (GSS). The exact question varied somewhat over time. It was "Do you approve or disapprove of a married woman holding a job in business or industry if her husband is capable to support her?" in 1945; and "Do you approve or disapprove of a married woman earning money in business or industry if she has a husband capable of supporting her?" for the remaining years. The possible answers included "Yes", "No" or "No Opinion" / "Don't know", depending on the year.

B Calibration of the learning model

The Model with No Learning

Note that the wage elasticity ε (own, f, or cross, h) is given by:

$$\varepsilon_k = g\left(l^*\right) \frac{\partial l^*}{\partial w_k} \frac{w_k}{L} \tag{14}$$

$k = f, h$. Taking the ratio of the two elasticities and manipulating the expression yields a closed-form expression for γ, from which one can obtain a parameter value by using the earnings and elasticity numbers in 2000, i.e.,

$$\gamma = \frac{\log\left(1 - \frac{w_f}{w_h}\frac{\varepsilon_h}{\varepsilon_f}\right)}{\log\left(1 + \frac{w_f}{w_h}\right)} = 0.503 \tag{15}$$

Next one can use one of the elasticity expressions and the requirement that $G\left(l^*; \sigma_l\right) = L$ in 2000 to solve for β and σ_l. Note that since G is a normal distribution, one can write:

$$l^* = \sigma_l \Phi^{-1}\left(L\right)$$

where Φ^{-1} is the inverse of a standard normal distribution $N\left(0,1\right)$. After some manipulation, one obtains:

$$\sigma_l = \frac{A}{\exp\left(\frac{\Phi^{-1}(L)^2}{2}\right)} = 2.29 \tag{16}$$

where $A = \frac{w_f\left(w_f+w_h\right)^{-\gamma}}{\sqrt{2\pi}\varepsilon_f L}$. One can then solve for β directly from the definition of l^*, yielding $\beta = 0.321$.

This basic inability of the model absent learning to match the historical data is robust to a wide range of values for the elasticities (I explored with values ranging from twice to half of those in Blau and Kahn). It is also robust to alternative specifications of the share of consumption that a woman obtains from her husband's earnings. In particular, one can modify the model so that the wife obtains only a share $0 < \alpha \leq 1$ of her husband's earnings as joint consumption. The results obtained from recalibrating the model using values of α that vary from 0.1 to 1 is shown in Figure 12. As is clear from the figure, this modification does little to remedy the basic problem. Furthermore, introducing any sensible time variation in this share would not help matters as it would require women to have obtained a much larger share of husband's earnings in the past in order to explain the much lower participation rates then. Since women's earnings relative to men's are higher now than in the past, most reasonable bargaining models would predict the opposite, i.e., greater bargaining power and hence a higher share of male earnings than in the past.[64]

The failure of the model without learning is also robust to the exact choice of earnings series. For example, one might argue that, over time, the average hours worked by women has changed and this intensive margin is not incorporated into the model. In order to more fully account for this margin, rather than use the median earnings of full-time women, I constructed a series of the median annual earnings for all working women from 1940 to 2000. The sample consisted of 25-44 year old women who were born in the U.S., not living in group quarters, and working in a non-farm occupation. The adjustment to earnings was sizeable, ranging from 18% to 30% lower depending on the decade. This resulted in different parameter values ($\gamma = 0.49$, $\beta = .25$, $\sigma_l = 2.01$) but the predicted path of LFP generated

[64] Note that, in any case, to obtain the very low LFP numbers in 1880 would require women to fully share husband's earnings in that decade and to obtain a share of only 0.0001 of husband's earnings in the year 2000.

was similar to the one obtained with the original series and hence still wildly overpredicted LFP.

The Model with Learning

After noting that $\frac{\partial \bar{l}}{\partial w_k} = \frac{\partial \underline{l}}{\partial w_k}$, $k = f, h$ and using some algebra, one can show that the ratio of the elasticities in this model can be written as:

$$\frac{\varepsilon_{w_f}}{\varepsilon_{w_h}} = \frac{\frac{\partial \underline{l}}{\partial w_f}}{\frac{\partial \underline{l}}{\partial w_h}} \frac{w_f}{w_h}$$

Noting further that $\frac{\partial \underline{l}}{\partial w_k} = \frac{\partial l^*}{\partial w_k}$, this implies that by performing the same manipulations as in the previous subsection one obtains (15), and thus the same value of γ as in the earnings only model, i.e., $\gamma = 0.503$.

In order to calculate a daughter's conditional probability of working (as a function of her mother's work behavior), one needs to specify, in addition to how private signals are inherited, how mothers and daughters are correlated in their l_j types. As a benchmark, I assume that the correlation is zero, i.e., the l type is a random draw from the normal distribution $G(\cdot)$ that is *iid* across generations.[65] Signals, on the other hand, are perfectly inherited. Thus, given a signal s we can define l_s as the l type that is just indifferent between working and not at that signal value (i.e., $s^*_{l_s} = s$). Hence, the probability that a woman with signal s works is $G(l_s)$, i.e., it is the probability that her l type is smaller than l_s. Rearranging the expression for s^*_l in (7), we obtain

$$l_{st} = \frac{\underline{l}_t + \bar{l}_t \exp\left(\lambda_t - \left(\frac{\beta_H - \beta_L}{\sigma^2_\varepsilon}\right)(s - \bar{\beta})\right)}{1 + \exp\left(\lambda_t - \left(\frac{\beta_H - \beta_L}{\sigma^2_\varepsilon}\right)(s - \bar{\beta})\right)} \tag{17}$$

And, using Bayes rule and $\beta^* = \beta_L$, we can calculate the probability that a daughter works

[65] Thus, this model yields a postive correlation between a mother and her daughter's work "attitudes" ($E_{it}\beta + l_i$ and $E_{i',t+1}\beta + l_{i'}$ where i indexes the mother and i' the daughter).

given that her mother worked as:

$$
\begin{aligned}
\Pr(DW_t|MW_{t-2}) &= \frac{\Pr(DW_t \text{ and } MW_{t-2})}{P(MW_{t-2})} \\
&= \frac{\int_{-\infty}^{\infty} \Pr(DW_t \text{ and } MW_{t-2}|s) f(s-\beta_L)ds}{L_{t-2}(\beta_L)} \qquad (18) \\
&= \frac{\int_{-\infty}^{\infty} G(l_{st})G(l_{s,t-2}) f(s-\beta_L)ds}{L_{t-2}(\beta_L)}
\end{aligned}
$$

where DW and MW stand for daughter works and mother worked, respectively. I use the predicted LFP from two periods earlier to calculate the probability that mothers worked (hence the $t-2$ in expressions such as $G(l_{s,t-2})$). Note that in (18), the probability that both mother and daughter worked, $\Pr(DW_t \text{ and } MW_{t-2}|s)$, is multiplied by $f(s-\beta_L)$ as this is the proportion of daughters (or mothers) who have a private signal s in any time period.

A similar calculation to the one above yields

$$
\Pr(DW_t|MNW_{t-2}) = \frac{\int_{-\infty}^{\infty} G(l_{st})(1-G(l_{s,t-2}))f(s-\beta_L)ds}{1-L_{t-2}(\beta_L)} \qquad (19)
$$

where MNW denotes a mother who did not work. The work risk ratio is thus given by

$$
R_t = \frac{\Pr(DW_t|MW_{t-2})}{\Pr(DW_t|MNW_{t-2})} \qquad (20)
$$

In order to estimate $\lambda_0, \sigma_\varepsilon, \sigma_\eta$,$\beta_H, \beta_L$, and σ_l I minimized the sum of the squared errors between the predicted and actual values of our calibration targets (see table 1). All statistics were weighted equally.

The simplex algorithm was used to search for an optimal set of parameters. Multiple starting values throughout the parameter space were tried (specifically over 2,000 different starting values with λ_0 ranging between [-10, -.01], σ_ε in [0.1, 5], σ_η in [0.01, 2], σ_l between [0.5, 4], β_L in [.01, 1], and β_H to be between [1, 10] units greater than β_L.

A period is 10 years. 500 different public shocks were generated for each period (these draws were held constant throughout the minimization process). For each shock, there is a corresponding public belief that subjects begin the next period with. For each belief, a different percentage of women will choose to work after they receive their private signals.

300 discrete types were assumed between $\underline{l}(w_h, w_f)$ and $\bar{l}(w_h, w_f)$ in each year to approximate the integral in equation 8. Then we average over the η shocks to determine the expected number of women working. We then back out the belief that would lead to exactly that many women working. This determines the path of beliefs.

The elasticities were calculated computationally by assuming either a 1% increase in female earnings or male earnings and calculating the corresponding changes in LFP predicted by the model in those histories in which the (original) predicted LFP was close to the true LFP value (specifically those histories in which the predicted LFP was within ± .05 of the true LFP that year). These elasticities were calculated individually for all histories meeting this criterion and were then averaged.

In order to approximate the integrals that are needed to compute $\Pr(DW_t|MW_{t-2})$ and $\Pr(DW_t|MNW_{t-2})$, 400 discrete signals from $\beta_L - 4\sigma_\varepsilon$ to $\beta_L + 4\sigma_\varepsilon$ were used.

Elasticity Paths

The path of elasticities predicted by the models is very different. See figure 13 below.

An Alternative Decomposition

It should be noted that there is not a unique way to decompose LFP in order to measure the quantitative importance of wages and beliefs. One could alternatively eliminate the $L\left(\overline{\lambda}\left(w_0\right), \overline{w}\right)$ curve and replace it with the LFP path that would result if beliefs followed the path obtained from the historical earnings series, $\overline{w}$, but wages were kept constant at their 1880 levels. This curve is shown in Figure 14 as $L\left(\overline{\lambda}\left(\overline{w}\right), w_0\right)$. The dynamic effect of wages is now given by the difference between $L\left(\overline{\lambda}\left(\overline{w}\right), w_0\right))$ and $L\left(\overline{\lambda}\left(w_0\right), w_0\right)$. These paths are obtained using the same constant 1880 earnings, but in the first trajectory beliefs evolve as they would with the historical earnings profile, whereas in the second beliefs follow the path they would have taken had wages not changed over time. The static effect of earnings is now measured as the difference between $L\left(\overline{\lambda}\left(\overline{w}\right), w_0\right)$ and $L\left(\overline{\lambda}\left(\overline{w}\right), \overline{w}\right)$, as beliefs evolve the same way for both series whereas earnings follow different paths.

The Welfare Costs of Imperfect Information

Another interesting exercise is to quantify the welfare costs of imperfect information. While this is not a policy-relevant calculation in that the government, for example, is not assumed to be better informed than any individual, it gives an idea of how costly mistaken beliefs were and how this cost evolved over time.

To quantify the losses from imperfect information we start by noting that if women were given the true value of β, only those individuals of type $l_j \in \left(\underline{l}_t, \bar{l}_t\right)$ who as a result of their private information did not work in time t would change their decisions (and thus their utility).[66] All women with $l_j \leq \underline{l}_t$ worked, and all those with $l_j \geq \bar{l}_t$ would choose not to work even if they knew the truth.[67]

Thus, to quantify the loss of welfare, we can calculate at each moment in time, for each $l_j \in \left(\underline{l}_t, \bar{l}_t\right)$ type, the amount of consumption, z_{jt}, that a woman would have to be given in order to make her as well off as she would have been had she worked:

$$\frac{(w_{ht} + z_{jt})^{1-\gamma}}{1-\gamma} = \frac{(w_{ht} + w_{ft})^{1-\gamma}}{1-\gamma} - \beta_L - l_j \tag{21}$$

To interpret equation (21), note that the right-hand side is the utility enjoyed by a woman of type l_j who works. Thus, the left-hand side solves for the consumption equivalent of that utility.

The proportion of a given l_j type who made the wrong decision is given by those whose private signals lay above s^*_{jt} (as expressed in equation (7)), i.e., a fraction $1 - F(s^*_{jt} - \beta_L; \sigma_\varepsilon)$. Integrating over the l_j types yields the aggregate welfare loss for these women at time t:

$$Z_t \equiv \int_{\underline{l}}^{\bar{l}} z_{jt}(1 - F(s^*_{jt} - \beta_L; \sigma_\epsilon))g(l_j)dl$$

In order to contrast this with the welfare enjoyed by working women, we need to translate

[66] The top (red) line in Figure 7 shows what the evolution of female LFP would have been with full information.

[67] Note that $\underline{l}_t, \bar{l}_t$ have time subscripts since their values depend on wages which are changing over time.

the utility of an l_j type who worked into consumption units. This is given by finding x_{jt} such that:

$$\frac{(w_{ht}+w_{ft}+x_{jt})^{1-\gamma}}{1-\gamma}=\frac{(w_{ht}+w_{ft})^{1-\gamma}}{1-\gamma}-\beta_L-l_j \tag{22}$$

The proportion of a given l_j women who made the correct decision is given by $F(s^*_{jt}-\beta_L;\sigma_\varepsilon)$ if $l_j \in (\underline{l}_t, \bar{l}_t)$ and equals one if $l_j \leq \underline{l}_{jt}$. Thus, the aggregate welfare of working women, expressed in consumption units, is given by:

$$W_t \equiv \int_{\underline{l}}^{\bar{l}} (w_{ht}+w_{ft}+x_{jt})F(s^*_{jt}-\beta_L;\sigma_\epsilon)g(l_j)dl + \int_{-\infty}^{\underline{l}} (w_{ht}+w_{ft}+x_{jt})g(l_j)dl \tag{23}$$

Lastly, the total welfare of non-working women expressed in units of consumption, is given by:

$$N_t \equiv (1-L_t)\, w_{ht}$$

Thus, the welfare lost as a result of imperfect information as a proportion of married women's welfare at time t, translated into consumption units, is given by $\widehat{c}_t$:

$$\widehat{c}_t \equiv \frac{Z_t}{W_t+N_t} \tag{24}$$

Note that $\widehat{c}_t$ gives the proportion of married women's average consumption lost as a result of imperfect information, when all utility is expressed in consumption units.

Figure 15 shows the evolution of $\widehat{c}_t$ over time. It starts out very high at some 39.29% of average consumption, decreases slowly until 1950, and then decreases dramatically to 0.19% by the year 2000. The very high numbers at the beginning are the consequence of the fact that the calibrated model implies that 63% rather than 2% of married women would have been working in 1880 had they possessed full information. A model in which the real cost of working evolved over time so that it was higher in 1880 than 2000 would imply smaller numbers as would a model in which the cost of working increased with the number of children.[68] Thus, these numbers should be taken as an upper bound to the costs of imperfect information.

[68]The total fertility rate of white women in 1880, for example, is estimated to have been 4.24 as compared to 2.05 in 2000 (Haines (2008)). On the other hand, fertility is an endogenous variable and women might have chosen to have fewer children had they faced a higher known opportunity cost.

Additional References for Online Appendix

References

[1] Goldin, Claudia and Robert A. Margo (1992),"The Great Compression: The Wage Structure in the United States at Mid-century," *The Quarterly Journal of Economics*, 107(1), 1-34.

[2] Haines, Michael, "Fertility and Mortality in the United States". EH.Net Encyclopedia, edited by Robert Whaples. March 19, 2008. URL http://eh.net/encyclopedia/article/haines.demography

[3] Katz, Lawrence and David Autor (1999), "Changes in the wage structure and earnings inequality," in O. Ashenfelter and D. Card (ed.), *Handbook of Labor Economics*, edition 1, vol. 3, chap. 26, 1463-1555, Elsevier.

C Appendix Figures:

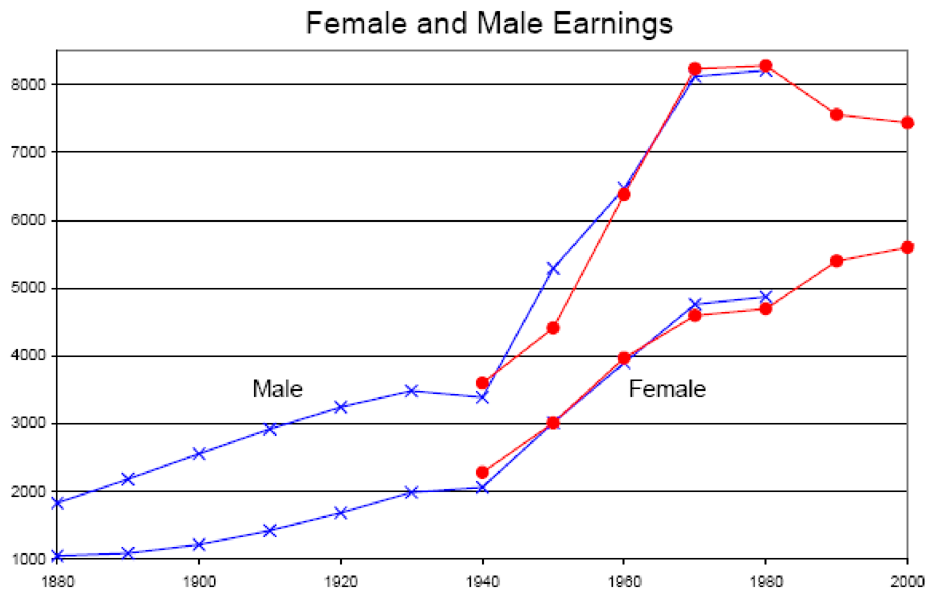

Figure 11: Crosses (blue) represent the yearly median earnings data from Goldin (1990), Table 5.1. Dots represent our calculations using U.S. Census data (red). They are the median earnings of white men and women between the ages of 25-44 in non-farm occupations and not living in group quarters. All earnings are expressed in 1967 $. See text for more detail.

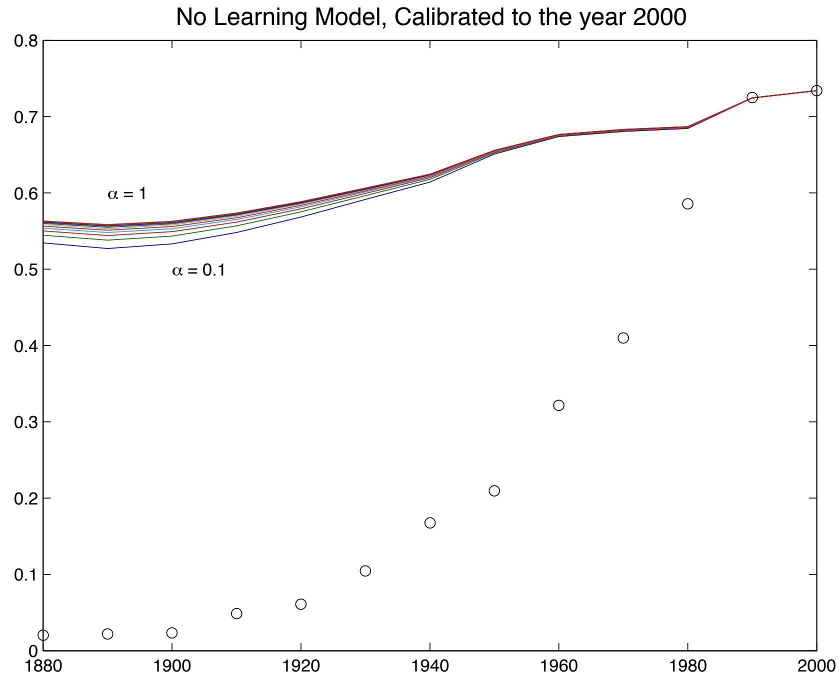

Figure 12: Parameters: $\gamma = 0.503$, $\beta = 0.321$, and $\sigma_L = 2.293$. α is the fraction of husband's earnings that enters a wife's utility via consumption.

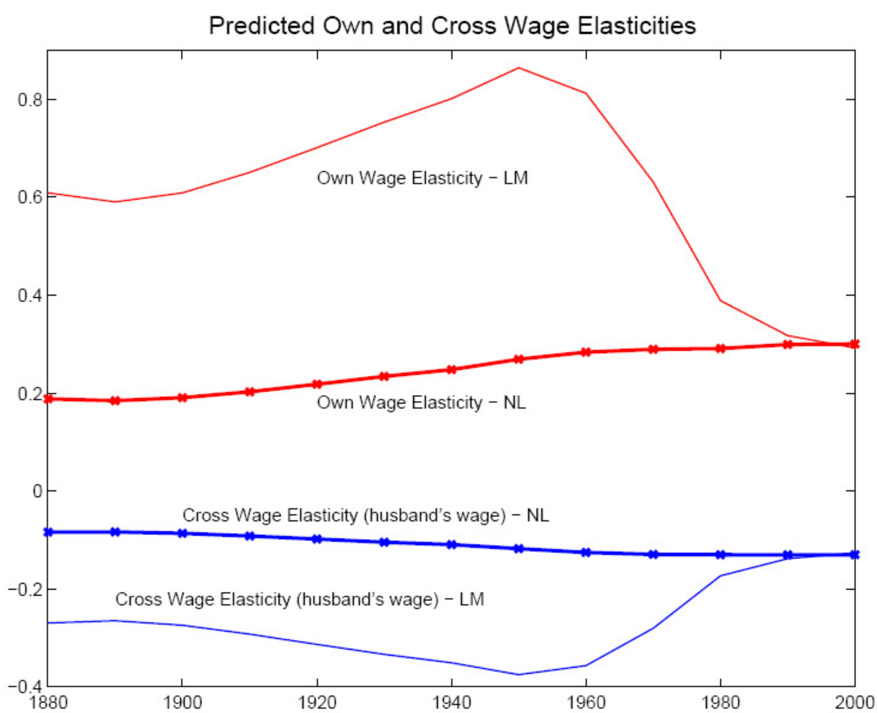

Figure 13: Parameter values from calibrated model. See the online Appendix for a description of how the elasticities were calculated. LM = Learning Model, NL = No Learning Model.

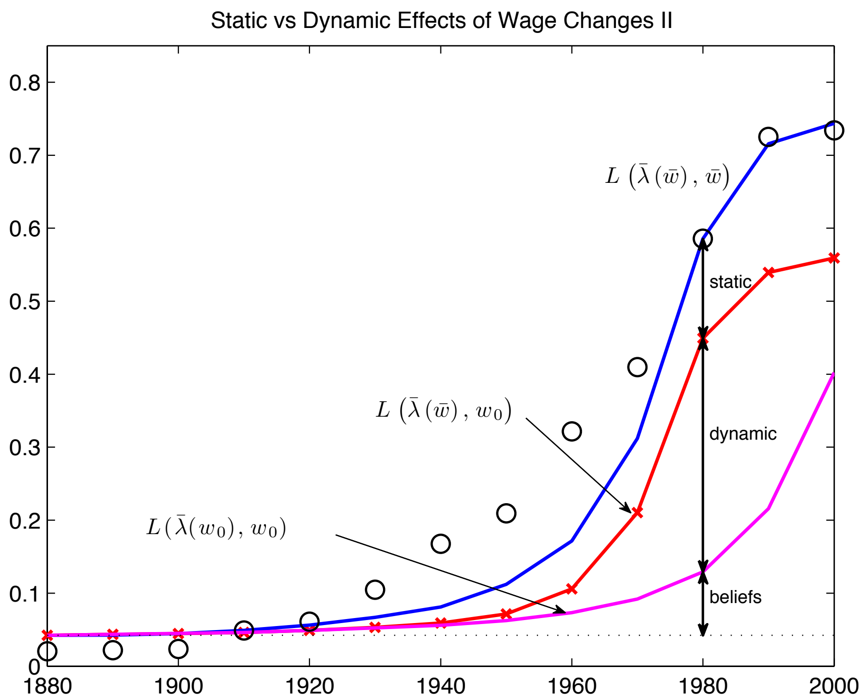

Figure 14: Alternative decomposition of LFP.

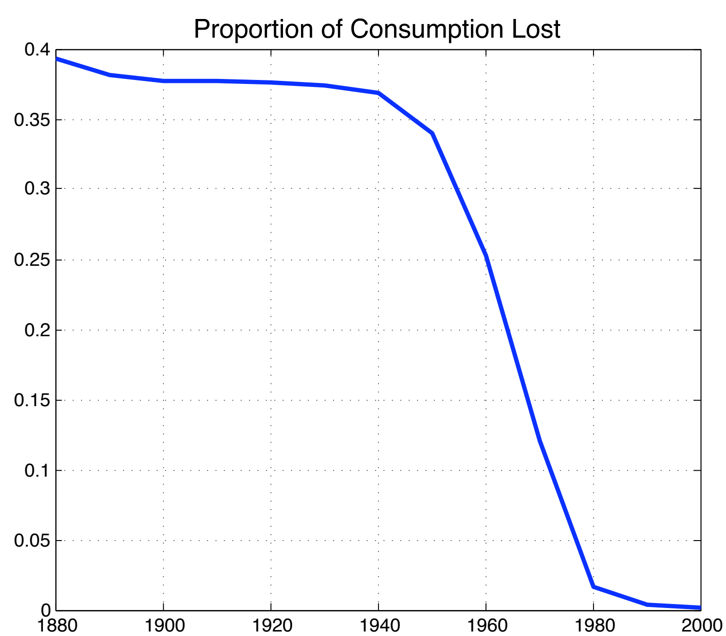

Figure 15: Percentage of welfare loss (in consumption terms) due to imperfect information about disutility from work. See text for details.

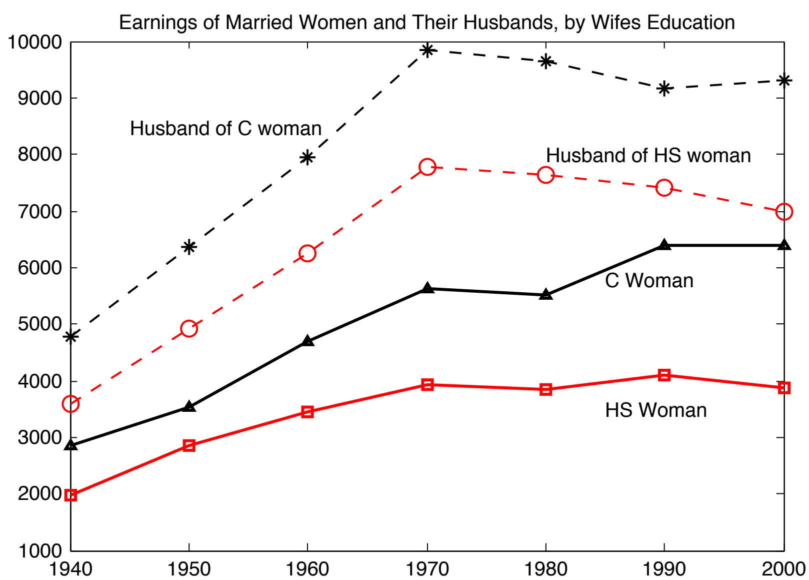

Figure 16: Median real earnings of married women by education type, college(C) and high school (HS), and median real earnings of each education type's respective husband. See text for definitions.