Was The New Deal Contractionary?

Gauti B. Eggertsson

Web Appendix

# VIII. Appendix C:Proofs of Propositions (not intended for publication)

**Proof of Proposition 3:** The social planner's problem at date $t$ is

$$\min_{\pi(r^e),\hat{Y}(r^e),\hat{\omega}(r^e),i(r^e)} E_t \sum_{T=t} \beta^{T-t}\{\pi_T^2 + \lambda \tilde{Y}_T^2\}$$
$$\text{s.t. (5), (6) and (7)}$$

The minimization problem can be solved by forming the Lagrangian

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{ \frac{1}{2}\pi(r_t^e)^2 + \frac{1}{2}\lambda \tilde{Y}(r_t^e) + \psi_1(r_t^e)[\pi(r_t^e) - \kappa \tilde{Y}(r_t^e) - \kappa\varphi\hat{\omega}(r_t^e) - \beta\pi(r_{t+1}^e)] \quad (34)$$
$$\psi_2(r_t^e)[\tilde{Y}(r_t^e) - \tilde{Y}(r_{t+1}^e) + \sigma i(r_t^e) - \sigma\pi(r_t^e) - \sigma r_t^e] + \psi_3(r_t^e) i(r_t^e)\}$$

where the functions $\psi_i(r^e)$, $i = 1, 2, 3$, are Lagrangian multipliers. Under $A1$, $r_t^e$ can take only two values. Hence each of the variables can take only on one of two values, $\pi_S, \tilde{Y}_S, i_S, \omega_S$ or $\pi_L, \tilde{Y}_L, i_L, \omega_L$. I find the first-order conditions by setting the partial derivative of the Lagrangian with respect to these variables equal to zero. In A1, it is assumed that the probability of the switching from $r_L$ to $r_S$ is "remote," i.e., arbitrarily close to zero. The Lagrangian used to find the optimal value for $\pi_L, \tilde{Y}_L, i_L, \hat{\omega}_L$ (i.e., the Lagrangian conditional on being in the L state) can be simplified to yield[26]

$$L_0 = \frac{1}{1-\beta}\{\frac{1}{2}\pi_L^2 + \frac{1}{2}\lambda\tilde{Y}_L + \psi_{1L}((1-\beta)\pi_L - \kappa\tilde{Y}_L - \kappa\varphi\hat{\omega}_L) + \psi_{2L}(i_L - \pi_L - r_L) + \psi_{3L} i_L\}$$

It is easy to see that the solution to this minimization problem is

$$\pi_L = \tilde{Y}_L = \hat{\omega}_L = 0 \quad (35)$$

and that the necessary conditions for achieving this equilibrium (in terms of the policy instruments) are that

$$i_L = r_L \quad (36)$$

$$\hat{\omega}_L = 0. \quad (37)$$

[26] In the Lagrangian, we drop the terms involving the L state because these terms are weighted by a probability that is assumed to be arbitrarily small.

Taking this solution as given and substituting it into equations (5) and (6), the social planner's feasibility constraints in the states in which $r_t^n = r_S$ are

$$(1-\beta\mu)\pi_S = \kappa\tilde{Y}_S + \kappa\varphi\hat{\omega}_S$$

$$(1-\mu)\tilde{Y}_S = -\sigma i_S + \sigma\mu\pi_S + \sigma r_S^e)$$

$$i_S \geq 0$$

Consider the Lagrangian (34) given the solution (35)-(37). There is a part of this Lagrangian that is weighted by the arbitrarily small probability that the low state happens (which was ignored in our previous calculation). Conditional on being in that state and substituting for (35)-(37) the Lagrangian at a date $t$ in which the economy is in the low state can be written as:

$$\begin{aligned} L_t &= E_t \sum_{T=t}^{\infty} \beta^{T-t}\{\frac{1}{2}\pi(r_T^e)^2 + \frac{1}{2}\lambda\tilde{Y}(r_T^e) + \psi_1(r_T^e)[\pi(r_T^e) - \kappa\tilde{Y}(r_T^e) - \kappa\varphi\hat{\omega}(r_T^e) - \beta\pi(r_{T+1}^e)] \\ & +\psi_2(r_T^e)[\tilde{Y}(r_T^e) - \tilde{Y}(r_{T+1}^e) + \sigma i(r_T^e) - \sigma\pi(r_T^e) - \sigma r_T^e] + \psi_3(r_T^e)i(r_T^e)\} \\ &= \frac{1}{1-\beta\mu}\{\frac{1}{2}\pi_S^2 + \frac{1}{2}\lambda\tilde{Y}_S^2 + \psi_{1L}((1-\beta\mu)\pi_S - \kappa\tilde{Y}_S - \frac{\kappa}{\sigma^{-1}+\upsilon}\hat{\omega}_S) \\ & +\psi_{2L}((1-\mu)\tilde{Y}_S + \sigma i_S - \sigma\mu\pi_S - \sigma r_S^n) + \psi_{3L} i_S\} \end{aligned}$$

Differentiating this Lagrangian yields

$$\pi_S + (1-\beta\mu)\psi_{1S} - \sigma\mu\psi_{2S} = 0 \tag{38}$$

$$\lambda\tilde{Y}_S - \kappa\psi_{1S} + \alpha\psi_{2S} = 0 \tag{39}$$

$$-\kappa\varphi\psi_{1S} = 0 \tag{40}$$

$$\sigma\psi_{2S} + \psi_{3S} = 0 \tag{41}$$

and the complementary slackness condition

$$i_S \geq 0,\ \psi_{3S} \geq 0,\ i_S\psi_{3S} = 0 \tag{42}$$

where $\psi_{1S,}$ $\psi_{2S}$ and $\psi_{3S}$ are the Lagrangian multipliers associated with the constraints. Consider now the optimal second best solution in which the government can use both policy instruments. Observe first that $i_S = 0$. This leaves six equations with six unknowns $(\pi_S, \tilde{Y}_S, \omega_{S,}\psi_{1S,}\psi_{2S,}\psi_{3S})$ and equations (38)-(41) together with AD and AS equations) that can be solved to yield

$$\begin{aligned} \tilde{Y}_S &= \frac{\sigma\delta_c}{[1-\mu+\lambda\delta_c^2\sigma^2\frac{\mu^2}{1-\mu}]} r_S^e < 0, \quad \pi_S = -\frac{\sigma^2\lambda\frac{\mu}{1-\mu}}{[1-\mu+\lambda\delta_c^2\sigma^2\frac{\mu^2}{1-\mu}]} r_S^e > 0, \\ \hat{\omega}_S &= -\varphi^{-1}\frac{\sigma+\sigma^2\lambda\frac{\mu}{1-\mu}[1-\beta\mu]\kappa^{-1}}{[1-\mu+\lambda\delta_c^2\sigma^2\frac{\mu^2}{1-\mu}]} r_S^e > 0 \end{aligned}$$

The proposition follows directly.

**Proof of Proposition 4:** Start by deriving A2, the condition for positive interest rates. Assuming $i_S > 0$, then $i_S = r_S^e + \pi^* + \phi_\pi(\pi_S - \pi^*) + \phi_y(\tilde{Y}_S - Y^*)$. Substituting this into the AD equation and solving the AD and AS equations together we obtain

$$\tilde{Y}_S - Y^* = -\frac{\sigma(\phi_\pi - \mu)\kappa\psi}{(1-\mu+\sigma\phi_y)(1-\mu\beta)+\kappa\sigma(\phi_\pi-\mu)}\omega_S + \frac{(1-\mu)(1-\beta\mu)}{(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}\hat{G}_S$$

$$\pi_S - \pi^* = \frac{(1-\mu)\kappa}{1-\beta\mu+\kappa\sigma(\phi_\pi-\mu)}\hat{G}_S + \frac{\kappa\psi(1-\mu+\sigma\phi_y)}{1-\mu\beta+\kappa\sigma(\phi_\pi-\mu)}\omega_S$$

which defines $Y^G \equiv \frac{(1-\mu)(1-\beta\mu)}{(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}$and $\pi^G \equiv \frac{(1-\mu)\kappa}{1-\beta\mu+\kappa\sigma(\phi_\pi-\mu)}$. Substituting this solution into the policy rule (20), we see that for $\omega_S \geq 0$ then A2 has to be satisfied, which proves part (ii) of the proposition. Consider now the first part of the proposition. If A2, then interest rates are zero and solving the AD and AS equation gives

$$\begin{aligned}(\tilde{Y}_S - \tilde{Y}^*) &= \frac{\kappa\psi}{(1-\mu)(1-\beta\mu)-\sigma\mu\kappa}\omega_S + \frac{(1-\beta\mu)\sigma}{(1-\mu)(1-\beta\mu)-\sigma\mu\kappa}\pi^* \\ &+\frac{(1-\beta\mu)(1-\mu)}{(1-\mu)(1-\beta\mu)-\sigma\mu\kappa}\hat{G}_S + \frac{\sigma(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\sigma\mu\kappa}r_S^e\end{aligned}$$

and

$$(\pi_S - \pi^*) = \frac{1}{1-\beta\mu}[\kappa(Y_S - Y^*) + \kappa\psi\omega_S]$$

which proves part (i).

**Proof of Proposition 6:** For solution at $t \geq \tau$, substitute $\tilde{Y}_t = \tilde{Y}^w\omega_t, \pi_t = \pi^w\omega_t$ into $\tilde{Y}_t = \tilde{Y}_{t+1} - \sigma\phi_\pi\pi_t - \sigma\phi_y\tilde{Y}_t + \sigma\pi_{t+1}$, $\pi_t = \kappa\tilde{Y}_t + \beta\pi_{t+1} + \kappa\varphi\omega_t$ using $\omega_t = \delta\omega_{t-1}$ and match coefficients to yield $\tilde{Y}^w = -\frac{\varphi}{1+\frac{(1-\beta\delta)(1+\sigma\phi_y-\delta)}{\kappa\varphi\sigma(\phi_\pi-\delta)}} < 0$, $\pi^w = \frac{(1+\sigma\phi_y-\delta)\kappa\varphi}{[(1-\beta\delta)(1+\sigma\phi_y-\delta)+\kappa\sigma(\phi_\pi-\delta)]} > 0$. Note that for $\omega_t$ to be positive at $t \geq \tau$ we need $\delta > 0$. This proves the first part of the proposition. To prove the second part of the proposition, substitute this into the AD and AS equation for $t \leq \tau$

$$\tilde{Y}_S = \mu\tilde{Y}_S + (1-\mu)Y^w\delta\omega_S + \sigma\mu\pi_S + \sigma(1-\mu)\pi^w\delta\omega_S + \sigma r_S$$

$$\pi_S = \kappa\tilde{Y}_S + \beta\mu\pi_S + \beta(1-\mu)\pi^w\delta\omega_S + \kappa\varphi\omega_S$$

solving these together to yield

$$\tilde{Y}_S = \frac{(1-\beta\mu)\sigma}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa}r_S + \{\frac{(1-\mu)(1-\beta\mu)\delta[1-\frac{1+\sigma\phi_y-\delta}{\phi_\pi-\delta}]}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa}Y^w + \frac{\sigma\mu[\beta(1-\mu)\pi^w\delta+\kappa\varphi]}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa}\}\omega_S$$

Manipulating the term in the curly bracket, using the expression for $\pi^w$, proves the result.

# IX. Appendix D: Notes on Alternative Nominal Frictions (not intended for publication)

The purpose of these notes is to ask whether the main result is sensitive to the source of nominal rigidities. In the paper we assumed that prices were rigid, while wages were perfectly flexible.

Keynes originally focused on wage rigidities as opposed to rigid prices, and some of the literature, such as the paper by Bordo et al (2000) assumes that wages are rigid, while prices are flexible. Section D.1 shows that the results of the paper are unchanged if prices are perfectly flexible but wages are set in a staggered way. Section D.2 shows that the same applies under another common specification for nominal rigidities. My conjecture is that a similar results are likely to be found in any model with nominal rigidities, because any theory of nominal rigidities that I am aware of will result in an upward sloping AS curve, i.e. there will be a positive relationship between inflation and output, and the NIRA will shift this relationship upward in a output-inflation space. This conjecture is further discussed in section D.3

### D.1 Rigid Wages

This section incorporates wage rigidities, following the work of Erceg, Henderson and Levin (2000). The exposition follows chapter 4.1 in Woodford (2003). The basic structure of the model is the same as the text but with some modification in the wage setting as outlined below. The production function of each firm is no longer given by (F.) but instead by

$$y_t(i) = L_t(i),$$

where $L_t(i)$ is a CES of the individual labor types of labor supply

$$L_t(i) \equiv \left[\int_0^1 l_t(i,j)^{(\theta_w-1)/\theta_w} dj\right]^{\theta_w/(\theta_w-1)} \tag{43}$$

where $l_t(i,j)$ is the labor of type $j$ hired by firm $i$. It follows that the aggregate demand for labor of type $j$ on the part of wage-taking firms is given by

$$L_t(j) = L_t(\frac{W_t(j)}{W_t})^{-\theta_w}$$

where $L_t$ is aggregate labor demand (because there is continuum of firms of measure 1 then $L_t = L_t(j)$), $W_t(j)$ is the wage of labor of type $j$, and $W_t$ is the aggregate wage Dixit-Stiglitz index

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}}$$

Once again conditions (30),the transversality condition, and the zero bound condition, are required for a rational expectation equilibrium consistent with households maximization. Note, however, that condition (1), derived by the household optimal labor supply, is not included in this list. This is because we now assume nominal frictions in the wage setting. More specifically we assume that the wage for each type of labor is set by the monopoly supplier of that type of labor, who then stands ready to supply as many hours of work as turns out the be demanded at that wage. We assume independent wage setting decision of each type $j$, made under the assumption that the choice of that wage setter has no effect upon the aggregate wage or hours. Furthermore, as in the Calvo model of staggered price setting, each wage is adjusted with a probability $1-\alpha_w$ each period, for some $0<\alpha_w<1$. In particular each wage $W_t(j)$ is chosen to maximize

$$E_t\sum_{T=t}^{\infty}(\alpha_w\beta)^{T-t}[\Lambda_T(1-\omega_{1t}(j))W_t(j)L_T(W_t(j))+\Lambda_T\omega_{1t}(j)\int_{i\in[0,1]\text{ and }i\neq j}W_t(i)L_T(i)di-v(L_T(W_t(j))\xi_T]$$

where $\Lambda_T$ is the representative households' marginal utility of nominal income in period T. Analogous to our firm price setting assumption we assume that only a fraction $(1-\omega_{1t}(j))$ of the wages of each wage setters accrue to the household that supplies that type of labor, while the remainder $\omega_{1t}(j)$ is redistributed to the other suppliers of labor. The term $\omega_{1t}(j)$ represents a wage collusion term, a positive $\omega_{1t}$ implies that the wage setters will set wages at a level that is higher than implied by the monopolistic competitive equilibrium and thus corresponds to a "labor wedge".

The solution to this maximization problem satisfies the first order condition

$$E_t\sum_{T=t}^{\infty}(\alpha_w\beta)^{T-t}\frac{u_{c,T}}{P_T}\xi_T L_T W_T^{\theta_w}(1-\omega_{1T})[W_t^*-\frac{\theta_w}{\theta_w-1}\frac{1}{1-\omega_{1T}}\frac{v_l}{u_c}P_T]=0 \tag{44}$$

where we assume that the wedge $\omega_{1t}(j)$ is set symmetrically across labor types and $W_t^*$ denotes the optimal wage of the wage setters that adjust their price at time t. The wage index satisfies the law of motion

$$W_t=[(1-\alpha_w)EW_t^{*1-\theta_w}+\alpha_w W_{t-1}^{1-\theta_w}]^{1/(1-\theta_w)} \tag{45}$$

The firm's profits are again given by

$$Z_t(i)=(1-\omega_{2t}(i))y_t(i)p_t(i)-\omega_{2t}(i)y_t^i-W_tL_t(i)$$

The only difference with respect to firm profit in the text is that the last term, because now the firm does not hire only one labor type, but the bundle given by (43) at the price $W_t$. Because we assume prices are flexible, optimal price setting then implies

$$p_t(i)=\frac{\theta}{\theta-1}\frac{1}{1-\omega_{2t}}W_t=0$$

It follows that the aggregate price level is given by

$$P_t=\frac{\theta}{\theta-1}\frac{1}{1-\omega_{2t}}W_t \tag{46}$$

An sticky price equilibrium can now be defined as a collection of stochastic processes for $\{Y_t,P_t,W_t^*,W_t,i_t,\omega_{1t},\omega_{2t}\}$ that satisfy the conditions outlined above for a given stochastic process for the exogenous shock $\{\xi_t\}$ and an initial condition $(Y_{-1},P_{-1})$. The model is linearized around the same steady state as before, with the only difference that it is assumed that $1-\bar{\omega}_2=\theta/(\theta-1)$ and $1-\bar{\omega}_1=\theta^w/(\theta^w-1)$ i.e. each wedge is eliminates the monopoly distortions in the goods and labor markets respectively. Again $\omega_t$ is defined as the ratio of the two distortions, but for simplicity I only allow for variations in $\omega_{2t}$. A linear approximation around the steady state will once again yield an unchanged AD equation and the zero bound. Instead of the AS equation, however, we now have a log-linear approximation of (44) and (45) that yields

$$AS1 \qquad \pi_t^w = \kappa_w \tilde{Y}_t + \beta E_t \pi_{t+1}^w + \kappa_w \varphi \omega_t \tag{47}$$

where $\pi_t^w \equiv \log W_t - \log W_{t-1}$ and $\kappa_w \equiv \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w}\frac{\sigma^{-1}+\nu}{1+\nu\theta_w}$. A linear approximation of 46 yields

$$AS2 \qquad \pi_t = \pi_t^w \tag{48}$$

An approximate sticky wage equilibrium is now defined as a collection of stochastic processes for the endogenous variables $\{\tilde{Y}_t, \pi_t, \pi_t^w, i_t, \hat{\omega}_t\}$ that satisfy (5),(7), (47) and (48) for a given stochastic process for the exogenous shock $\{r_t^e\}$. Observe that these equations, one AS2 is substituted into AS1, are precisely the same as before, and hence all the propositions in the paper follow unchanged if we assume wage frictions instead of pricing frictions. To summarize:

**Proposition 7** *Wage and Price friction equivalence. Suppose that wages are set in a staggered way as in Calvo (1983) but prices flexible and the wedges determined as explained in the text above. Then Proposition 1-6 follow unchanged replacing $\kappa$ with $\kappa_w$.*

**Proof.** See equation (47) and (48) ■

### D.2 New Classical Phillips Curve

Consider now an alternative pricing Euler equation, namely the one common in the earlier literature on price frictions. Suppose that the AS equation takes the form

$$AS \qquad \pi_t = \kappa_p \tilde{Y}_t + E_{t-1}\pi_t + \kappa_p \varphi \omega_t \tag{49}$$

where $\kappa_p$ is a coefficient greater than zero. This form of "expectation augmented" or "New Classical" Phillips curve is common in the early rational expectation literature, see e.g. Kydland and Prescott (1977) and Barro and Gordon (1983) classic papers. A Phillips curve of this form is derived from the same microfoundations as in the main text in Woodford (2003) under the assumption that a fraction $\iota$ set their prices one period in advance and a fraction $1-\iota$ has flexible prices. The term involving the wedge can be derived in exactly the same way as in the main text. Under these alternative microfoundations $\kappa_p \equiv (\iota/(1-\iota))\left((\nu+\sigma^{-1})/(1+\nu\theta)\right)$ and the AD equation and zero bound are unchanged.

Under A1 the AS equation 49 can be written as

$$(1-\mu)\pi_S = \kappa_p \tilde{Y}_S + \kappa_p \varphi \hat{\omega}_S \tag{50}$$

Observe that this equation is identical to the $AS$ equation 18 when $\beta = 1$. To summarize:

**Proposition 8** *Expectation Augmented Phillips curve equivalence. Suppose that the AS equation is replaced with an expectation augmented Phillips curve from the microfoundations explained above. Then Proposition 1-6 follow unchanged for replacing $\kappa$ with $\kappa_p$ and setting $\beta = 1$ in our previous expression.*

**Proof.** See equation 50 ■

Observe that we do not need $\beta = 1$ in the microfoundation that underlie the expectation augmented Phillips curve. We only set $\beta = 1$ in the expressions in the text to make the Calvo model equivalent to the model with the expectation augmented Phillips curve (where $\beta$ can take any value).

### D.3 General comment on other nominal frictions

The two subsections above illustrate two examples of alternative nominal frictions in which case the results are identical to those in the text. The main result of the paper, however, is more general than these examples and likely to hold in most models that incorporate nominal frictions. Figure **??** is helpful to clarify this. In the model, alternative specifications for nominal frictions only change the AS equation. All that is needed for the result, is that the AS curve is upward sloping in $(\tilde{Y}_S, \pi_S)$ space (i.e. higher quasi-growth rate of output demanded by consumer is associated with higher rate of price increase) and that this relationship is shifted to the left with the policy wedge. I am not aware of any theory of nominal frictions that does not result in a firm Euler equation in which case prices are positively related to output. Moreover, any theory of monopolistic competition will result in price increases if the government facilitates cartelization , i.e., the AS curve will shift to the left. Hence my conjecture is that the key result will hold for any reasonable description of nominal frictions, even if the exact expressions in the main text may change a bit.

## Appendix E: Notes on Endogenous Capital (not intended for publication)

Appendix D was concerned with variations in the model that change or replace the firm Euler equation (AS equation). We now turn to alternative specification for the AD equation, which determines spending decisions. Perhaps the most obvious source of spending variations abstracted from in the paper is investment spending, but all production is consumed in the model in the text. The purpose of these notes is to consider whether investment spending changes the results in a fundamental way. This has been suggested by some authors such as Christiano (2004) in a related context. We find that endogenous capital accumulation has very little effect as long as we consider intertemporal disturbances that affect the consumption and investment Euler equation in the same way (an assumption that is consistent with the criteria for the shock in the paper, i.e., that the shock reduces the efficient rate of interest). The basic finding is in line with recent results in the literature, such as Woodford (2005), that argues that the fixed capital stock model provides a reasonable approximation to a model with endogenous capital stock. For simplicity this section only considers the most simple variation of the model in the text by assuming no habits, as in Woodford (2005).

### E.1 Model

The household maximization problem is the same as in the paper and the same set of equations apply. For the firms I assume a firm specific convex cost of investment as in Christiano (2004) and Woodford (2005). To increase the capital stock to $K_{t+1}(i)$ in the next period from $K_t(i)$ the firm needs to buy

$$I_t(i) = I(\frac{K_{t+1}(i)}{K_t(i)}, \xi_t)K_t(i) \tag{51}$$

of the consumption good. The function $I$ satisfies $I(1, \bar{\xi}) = \lambda$, $I'(1, \bar{\xi}) = 1$, $I''(1, \bar{\xi}) = \phi^{II} \geq 0$, $I^{\xi}(1, \bar{\xi}) = 0$, $I^{I\xi}(1, \bar{\xi}) \neq 0$. The variable $\lambda$ corresponds to the depreciation rate of capital. At time $t$ the capital stock is predetermined. I allow for the vector of fundamental shocks to appear in the cost of adjustment function. This is important to generate the same kind of shocks as considered in the paper (namely variations in the efficient rate of interest) and is the key difference relative to Christiano (2004). The shock in the cost of adjustment, in addition to the wedges, is the only difference relative to Woodford (2005). Accordingly the description of the model below is brief [readers can refer to Woodford (2005) for details].

Here $I_t(i)$ represents to purchases of firm $i$ of the composite good, defined over all the Dixit-Stiglitz good varieties, so that we can write

$$y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}.$$

Output is produced with the Cobb Douglas function

$$y_t(i) = AK_t(i)^{1/\phi_h - 1}l_t(i)^{1/\phi_h}$$

Firm $i$ in industry $j$ maximize present discounted value of profits and where the period profit is now given by

$$Z_t(i) = [1 - \omega_{2t}(j)]p_t(i)y_t(i) + \omega_{2t}(j)p_t^j Y_t(p_t^j/P_t)^{-\theta} - W_t(j)l_t(i) - P_t I_t(i)$$

which is identical to the period profit (2) apart from the presence of the variable firm specific investment represented by the last term. Let us denote $I_t^N(i) \equiv \frac{K_{t+1}(i)}{K_t(i)}$ as the net increase in the capital stock in each period. Endogenous capital accumulation gives rise to the following first order condition.

$$-I'(I_t^N(i), \xi_t) + E_t Q_{t+1}\Pi_{t+1}[\rho_{t+1}(i) + I'(I_{t+1}^N(i), \xi_{t+1})I_{t+1}^N - I(I_{t+1}^N(i), \xi_{t+1})] \tag{52}$$

where

$$\rho_t(i) \equiv \frac{\alpha}{1-\alpha}\frac{l_t(i)}{K_t(i)}W_t(j) \tag{53}$$

There is an analogous Euler equation to (31) in the text for the price setting that is complicated by the fact that we need to keep track of the capital stock of each firm (see Woodford [2005] for details).

## E.2 Approximate Equilibrium

Let us now linearize the model around the efficient steady state with zero inflation. The firm Euler equation is:

$$\pi_t = \zeta\hat{s}_t + \beta E_t\pi_{t+1} + \zeta\hat{\omega}_{2t} \tag{54}$$

where $\zeta \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha\phi}$ and $\pi_t \equiv \log\Pi_t$. The coefficient $\phi$ is defined in equation 3.23 in Woodford (2005) (note that the arguments of this equation involve the solutions of several polynomials in that paper). The variable $\hat{s}_t$ is a log-linerization of real marginal costs (in deviation from steady state) given by[27]

$$\hat{s}_t = (1+\nu)\hat{L}_t + \tilde{\sigma}^{-1}\delta_c^{-1}\hat{C}_t - \hat{Y}_t \tag{55}$$

where $\tilde{\sigma} \equiv -\frac{u_c}{u_{cc}C}$, $\delta_c \equiv \frac{C}{Y}, \nu \equiv \frac{v_{;;}L}{v_l}, \hat{C}_t \equiv \log\frac{C_t}{Y}$, $\hat{L}_t \equiv \log\frac{L_t}{L}$. Equation (52) is

$$\hat{I}_t^N = \beta E_t\hat{I}_{t+1}^N - \frac{1}{\phi^{II}}(i_t - E_t\pi_{t+1} - r_t^I) + \frac{1}{\phi^{II}}\beta\bar{\rho}E_t\hat{\rho}_{t+1} \tag{56}$$

where $r_t^I \equiv \log\beta^{-1} - I^{I\xi}\xi_t + \beta I^{I\xi}E_t\xi_{t+1}$ and $\hat{\rho}_t = \log\frac{\rho_t}{\bar{\rho}}$, $\hat{I}_t^N = \log I_t^N$.

Observe that this IS equation takes the same form as the consumption Euler equation and this is the reason for why the extension yields similar results once it is assumed that $r_t^I$ – the shock to the investment Euler equation – parallels the shock to the consumption Euler equation (more on this below). Linearizing the definition of $\rho_t$ yields

$$\hat{\rho}_t = (1+\nu)\hat{L}_t + \tilde{\sigma}^{-1}\delta_c^{-1}\hat{C}_t - \hat{K}_t \tag{57}$$

where $\hat{K}_t \equiv \log\frac{K_t}{K}$. Linearizing the definition of $I_t^N$ yields

$$\hat{I}_t^N \equiv \hat{K}_{t+1} - \hat{K}_t \tag{58}$$

Linearizing (51) yields

$$\hat{I}_t = \delta_K\hat{I}_t^N + \lambda\delta_K\hat{K}_t \tag{59}$$

$\delta_K \equiv \frac{K}{Y}$. Linearizing the resource constraint $Y_t = C_t + I_t$ yields

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t = \hat{C}_t + \delta_K\hat{I}_t^N + \lambda\delta_K\hat{K}_t \tag{60}$$

Linearizing the consumption Euler equation yields

$$\hat{C}_t = E_t\hat{C}_{t+1} - \tilde{\sigma}\delta_c(i_t - E_t\pi_{t+1} - r_t^c) \tag{61}$$

where $r_t^e \equiv \log\beta^{-1} + \frac{\bar{u}_{c\xi}}{\bar{u}_c}\xi_t - \frac{\bar{u}_{c\xi}}{\bar{u}_c}E_t\xi_{t+1}$. The production function is

$$\hat{Y}_t = (1-\phi_h^{-1})\hat{K}_t + \phi_h^{-1}\hat{L}_t \tag{62}$$

[27] The real marginal cost for firm i in industry j is

$$s_t(i) = \frac{W(j)}{\phi_h^{-1}K_t(i)^{1-\phi_h^{-1}}l_t(i)^{\phi_h^{-1}-1}}$$

where I have assumed no productivity shocks. An approximate equilibrium is a collection of stochastic processes for $\{\hat{Y}_t, \hat{L}_t, \hat{C}_t, \hat{K}_{t+1}, \hat{I}_t^N, \hat{I}_t, \hat{\rho}_t, i_t, \hat{s}_t, \hat{\omega}_t\}$ for a given a stochastic process for the exogenous shocks $\{r_t^I, r_t^C\}$. Observe that to close the model we need two equations to determine policy (rules that governs $\omega_t$ and $i_t$) and need to specify the exogenous processes $r_t^I$ and $r_t^C$.Observe that if $\phi^{II} \to \infty$ then this model collapses to the one in the text. The question is whether the main result is overturned for intermediate values of $\phi^{II}$.

### E.3 The efficient rate of interest

Observe that in the current model we have two spending Euler equations – (56) and (61) – the first relating investment to current and expected future short-term real interest rates and the second consumption to current and expected future real short term interest rates. Our definition of the shocks in the paper was that it was they correspond to intertemporal disturbances that only changes the efficient rate of interest, leaving the efficient level of output and consumption constant (this criteria for the shock is made more explicit in somewhat more detail in Eggertsson [2008] who argues that a shock of this kind is natural candidate for the Great Depression). It is easy to see that in the model with endogenous capital, the disturbance that satisfies this criteria is one in which $r_t^c = r_t^I = r_t^e$. This kind of disturbance leads to a decline in the efficient rate of interest, leaving the efficient level of output, capital, investment and consumption constant.

This shock has different properties than the one studied in Christiano (2004). In his model the shock he considers is only a shock to the discount factor in the household utility. This does not satisfy the criteria in the paper, because a shock that only affects the consumption Euler equation will then lead to an increase in investment that offsets this shock, having a much smaller effect on the efficient interest rate. The fact that the shock only appears in the consumption Euler equation also has the implication that it perturbs the efficient allocation for investment, capital, output and consumption. This kind of shock is less appealing for my purposes because it would imply that the Great Depression was associated with an investment boom. Instead investment collapsed together with output and consumption during the Great Depression, consistent with the assumption that the investment Euler equation was subject to an identical shock. More generally if one thinks of the intertemporal disturbance as a reduced form representation of financial frictions it makes sense to assume that it affected the cost of lending by both consumers and firms in the same way.

### E.4 Calibration

To calibrate the model we re-estimate the fixed capital stock model abstracting from habits. This yields the following estimate for the structural parameters $\tilde{\sigma} = 0.9956, \nu = 0.8795$, $\alpha = 0.7846$, $\theta = 10.07$. I do not estimate the variable capital stock model, but instead parameterize it using these estimate and assume the following values for the other parameters, $\phi_h^{-1} = 0.75$ and $\lambda = 0.05$ (which is the depreciation rate). In the steady state $\delta_K = 2.72$ and $\delta_C = 0.7832$. To calibrate the cost function $\phi^{II}$ I follow Christiano and Davis (2006) by assuming that $\phi^{II} = \lambda^{-1}$ (see further discussion in that paper). As $\phi^{II}$ increases the variable capital stock model collapses to the fixed capital stock model. Using these parameter values the implied value of $\phi$ is 8.08.

I consider here the consequence of a policy regime of the following kind (which is identical to the baseline policy in the main text for the fixed capital stock model).

$$\pi_t = 0 \text{ for } t \geq \tau$$

$$i_t = 0 \text{ for } t \leq \tau$$

The shock takes the form

$$r_t^e = r_t^I = r_t^C = 1/\beta - 1 \text{ for } t \geq \tau$$

$$r_t^e = r_t^I = r_t^C = r_S^e < 0 \text{ for } t \geq \tau$$

The value of the shock in the case of the fixed capital stock model is estimated as $r_S^e$ is $-1.92\%$ (in annual percentage terms) and the value of $\mu$ is 0.89. As already noted, I do not estimate the model with variable capital but instead choose the shock informally by selecting $r_S^e$ and $\mu$ such that inflation and output corresponds to the one in the model with fixed capital stock in 1933, assuming that the shock occurred in 1929.[28] This results in that $r_S^e$ is $-0.9\%$ and $\mu = 0.86$ in the model with variable capital stock. Instead of deriving the optimal second best $\omega_t$ in the variable capital model, I choose it such that the inflation outcome under the New Deal is identical to the inflation outcome in the fixed capital stock model in 1933.

Figures 7-9 shows the results. Figure 7 shows the fixed capital stock solution with dashed line and the variable capital solution with solid line. The figure shows the outcome under the assumption the shock hits in 1929 and stays there until 1939. It considers both the outcome under the baseline policy, and in the case the New Deal is implemented in 1929 (this solution corresponds to the smaller contraction in both output and inflation in Figure 7 ). In either case, the difference between the two models is small. The second figure shows how the decline in output is distributed between a decline in gross investment and consumption. Figure 3 shows the long run evolution of the model if it stays in the $r_S^e$ state for a very long time.

## A. D.5 General comments

The key difference between the result here and the one in Christiano (2004) is the way in which the shock is introduced. Once it satisfies the criteria in the paper for the efficient rate of interest, the results are similar across the two models. The shock that satisfies the criterion of the paper is slightly smaller in the variable capital model, although I do not know if this will hold for all parameterization (my conjecture is that this depends on the calibrated value of $\phi^{II}$ among other things). There appears to be little reason to believe that the extension to a MPE with external habits would yield substantially different results from those already in the paper, although this remains to be confirmed. Those results would be considerable more complicated to derive, however, because the model would have many more state variables so I would not be able to

[28] This is essentially the same criteria as used in the estimation of the fixed capital stock model, although in that case we do not only choose the shocks to satisfy this criterion but all the other parameters as well.

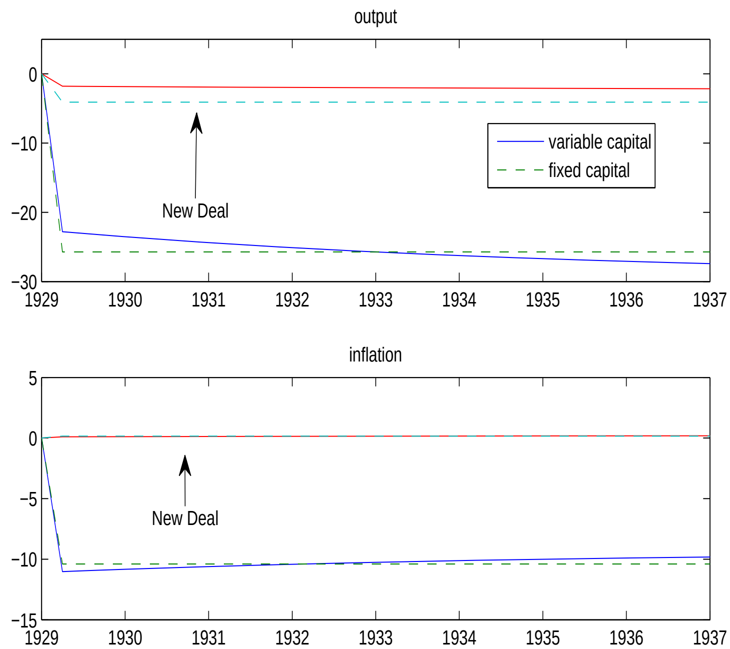


Figure 7: Comparing the solution assuming a fixed vs endogenous capital stock, conditional on a negative shock.

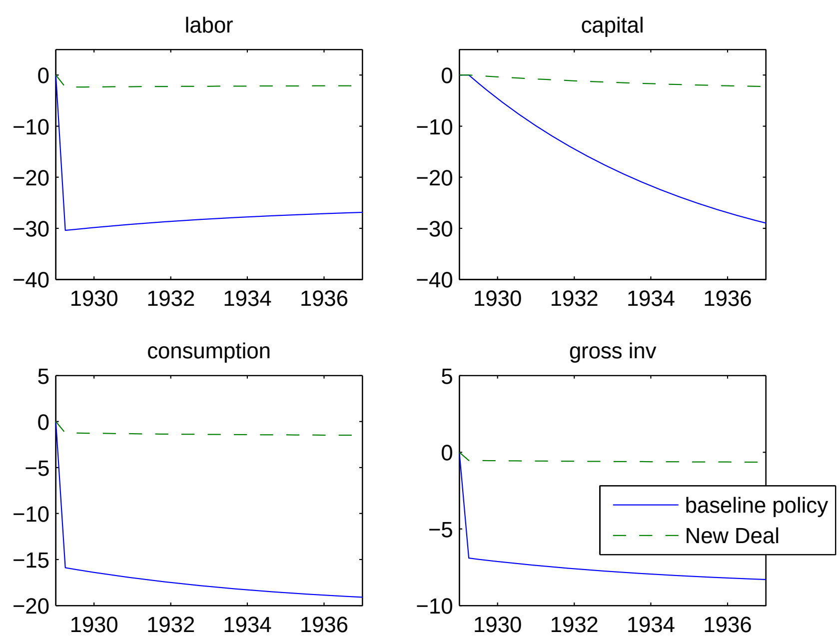


Figure 8: The solution assuming endogenous capital stock, conditional on a negative shock.

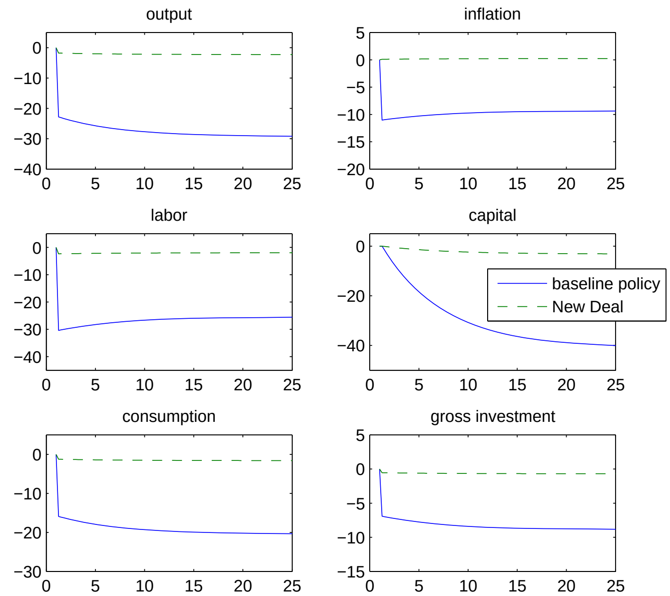


Figure 9: The long-run assuming endogenous capital and a negative shock.

produce any closed form solution (although a numerical characterization is possible). Not only would the $K_t$ be a state, but also $L_{t-1}$ and $Y_{t-1}$. The key simplification in the model with fixed capital stock was the specification of the habit which meant that the state $Y_{t-1}$ dropped out and the model could be written in terms of quasi growth rate of output. This appears no longer possible in the variable capital model because the production function is not linear in labor. As a consequence one would need to replace all the propositions and derivation in the paper numerical simulations with relatively small returns in terms of quantitative fit.

More generally it is unlikely that alternative specification of the spending side of the economy will change the main result, that inflationary policies increase output when there is excessive deflation. The key for the result is that the AD equation is upward sloping expected inflation for a given nominal interest rate. In words, this just means that demand depend on the real interest rate. Even if we introduce additional sources of spending, such as investment, these spending components will also respond positively to a reduction in real interest rates, thus preserving the property of the model that generates the main result in the paper.

## E.5 References

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## Appendix F: Notes on Alternative Microfoundations for Government Policy (not intended for publication)

The main text considered the optimal forward looking policy as microfoundations for government behaviors. These notes consider two other common characterizations of the government, the optimal policy under commitment (or Ramsey policy) and the Markov Perfect Equilibrium ([MPE], i.e., when the government cannot make any credible commitments about future policy). As we will see the MPE is almost identical to the optimal forward looking policy in the example we are considering. The Ramsey policy is a bit different from the optimal forward looking policy because the government can now commit to lower future nominal interest rates once the deflationary shocks have subsided. Yet, the Ramey policy preserves the main result of the paper, i.e., it is optimal to implement the NIRA policy for the duration of the deflationary shocks.

The appeal of the Ramsey solution is that it is the best possible outcome the planner can achieve. The main weakness for my purposes is that it requires a very sophisticated commitment that is subject to a serious dynamic inconsistency problem, especially in the example I consider. This casts doubt on how realistic it is as a description of policy making in the 1930's. The MPE, in contrast, is dynamically consistent by construct, and may thus capture actual policy making a little bit better. Its main weakness, however, is that it is not a well defined social planner's problem because each government is playing a game with future governments. The optimal MPE government strategy is therefore not a proper second best policy, as defined in the text, because showing that the government at time $t$ chooses to use a particular policy instrument (e.g. $\omega_t$) is no guarantee that this is optimal. Indeed in certain class of games it is optimal to restrict the government strategies to exclude certain policy instruments or conform to some fixed "rules" (see e.g. Kydland and Prescott (1977)).

The optimal policy from a forward looking perspective studied in the main text strikes a good middle ground between Ramsey equilibrium and the MPE. It is a well defined planner's problem and thus appropriate to illustrate the point about the policy as "optimal second best". Yet it is very close to the MPE in the example I consider and thus not subject to the same dynamic inconsistency problem as the Ramsey equilibrium (as further discussed below). Furthermore it requires a relatively simple policy commitment by the government, which makes it a more plausible description of actual policy during the Great Depression, and it accords relatively well with narrative accounts of the policy.

### F.1 Markov Perfect Equilibrium

Optimal policy under discretion is standard equilibrium concept in macroeconomics and is for example illustrated in Kydland and Prescott (1977). It is also sometimes referred to as Markov

Perfect Equilibrium (MPE).[29] The idea is that the government cannot make any commitments about future policy but instead reoptimizes every period, taking *future government actions* and the physical state as given. Observe that we have rewritten the model in terms of quasi growth rates of output and the growth rate of prices (inflation) so that the government's objective and the system of equations that determine equilibrium are completely forward looking. They only depend on the exogenous state $(r_t^e, \tilde{Y}_t^e)$. It follows that the expectations $E_t\pi_{t+1}$and $E_t\tilde{Y}_{t+1}$ are taken by the government as exogenous since they refer to expectations of variables that will be determined by future governments (I denote them by $\bar{\pi}(r_t^e)$ and $\bar{Y}(r_t^e)$ below). To solve the government's period maximization problem one can then write the Lagrangian

$$L_t = -E_t \begin{bmatrix} \frac{1}{2}\{\pi_t^2 + \lambda_y \tilde{Y}_t^2\} \\ +\phi_{1t}\{\pi_t - \kappa\tilde{Y}_t + \kappa\tilde{Y}_t^e - \frac{\kappa}{\sigma^{-1}+\upsilon}\hat{\omega}_t - \beta\bar{\pi}(r_t^e)\} \\ +\phi_{2t}\{\tilde{Y}_t - \bar{Y}(r_t^e) + \sigma(i_t - \bar{\pi}(r_t^e) - r_t^e)\} + \phi_{3t}i_t \end{bmatrix} \tag{63}$$

and obtain four first order conditions that are necessary for optimum and one complementary slackness condition

$$\pi_t + \phi_{1t} = 0 \tag{64}$$

$$\lambda_y(\tilde{Y}_t - \tilde{Y}_t^e) - \kappa\phi_{1t} + \phi_{2t} = 0 \tag{65}$$

$$-\kappa\varphi\phi_{2t} = 0 \tag{66}$$

$$\sigma\phi_{2t} + \beta^{-1}\phi_{3t} = 0 \tag{67}$$

$$\phi_{3t} \geq 0, \;\; \phi_{3t}i_t = 0 \tag{68}$$

Consider first the equilibrium in which the government does not use $\hat{\omega}_t$ to stabilize prices and output (i.e. $\hat{\omega}_t = 0$) in which case the equilibrium solves the first order conditions above apart from (66). In this case the solution is the same as the optimal forward looking policy subject to $\hat{\omega}_t = 0$ and thus also equivalent to the benchmark policy in Proposition 1.

Next consider the optimal policy when the government can use $\hat{\omega}_t$. In this case the solution that solves (64)-(68) and the IS and AS equations is:

$$\tilde{Y}_t = \frac{\sigma}{1-\mu} r_S^e \text{ if } t < \tau \text{ and } \tilde{Y}_t = 0 \text{ if } t \geq \tau \tag{69}$$

$$\pi_t = 0 \; \forall t \tag{70}$$

$$\tilde{Y}_t^n = \frac{\sigma}{1-\mu} r_S^e \text{ if } t < \tau \text{ and } \tilde{Y}_t^n = 0 \text{ if } t \geq \tau \tag{71}$$

$$\hat{\omega}_t = -\frac{\sigma}{1-\mu}\varphi^{-1} r_S^e > 0 \text{ if } t < \tau \;\; \hat{\omega}_t = 0 \text{ if } t \geq \tau \tag{72}$$

The analytical solution above confirms the key insight of the paper, that the government will increase $\hat{\omega}_t$ to increase inflation and output when the efficient real interest rate is negative. There is however some qualitative difference between the MPE and the OFP. Under the optimal forward

[29] Although it is common in the literature that uses the term MPE to assume that the government moves before the private sector. Here, instead, the government and the private sector move simultaneously.

looking policy the social planner increases the wedge beyond the MPE to generate inflation in the low state. The reason for this is that under OFP the policy maker uses the wedge to generate expected inflation to lower the real rate of interest. In the MPE, however, this commitment is not credible and the wedge is set so that inflation is zero. The quantitative significance of the difference between MPE and OFP, however, is trivial using the parameterization of the paper.

## F.2 Ramsey Equilibrium

I now turn to the Ramsey equilibrium. In this case the government can commit to any future policy. The policy problem can then be characterized by forming the Lagrangian:

$$L_t = E_t \left[ \begin{array}{c} \frac{1}{2}\{\pi_t^2 + \lambda \hat{Y}_t^2\} + \phi_{1t}(\pi_t - \kappa \tilde{Y}_t - \frac{\kappa}{\sigma^{-1}+\upsilon}\hat{\omega}_t - \beta\pi_{t+1}) \\ +\phi_{2t}(\tilde{Y}_t - \tilde{Y}_{t+1} + \sigma i_t - \sigma\pi_{t+1} - \sigma\hat{r}_t^e) + \phi_{3t} i_t \end{array} \right] \tag{73}$$

which leads to the first order conditions:

$$\pi_t + \phi_{1t} - \phi_{1t-1} - \sigma\beta^{-1}\phi_{2t-1} = 0$$

$$\lambda\hat{Y}_t - \kappa\phi_{1t} + \phi_{2t} - \beta^{-1}\phi_{2t-1} = 0$$

$$\sigma\phi_{2t} + \phi_{3t} = 0$$

$$\phi_{1t} = 0$$

$$\phi_{3t} i_t = 0 \; i_t \geq 0 \text{ and } \phi_{3t} \geq 0$$

Figure 10 shows the solution of the endogenous variables, using the solution method suggested in Eggertsson and Woodford (2004) [the study optimal labor taxes under commitment which is identical to the case we're studying] and compares to the optimal forward looking policy studied in the main text. The calibration here is from their paper, and there is no habit persistence in the model. Again the solution implies an increase in the wedge in the periods in which the zero bound is binding. The wedge is about 5 percent initially. In the Ramsey solution, however, there is a commitment to reduce the wedge temporarily once the deflationary shocks have reverted back to steady state. There is a similar commitment on the monetary policy side. The government commits to zero interest rates for a considerable time after the shock has reverted back to steady state.

The optimal commitment thus also deviates from the first best in the periods $t \geq \tau$ both by keeping the interest rate at zero beyond what would be required to keep inflation at zero at that time and by keeping the wedge below its efficient level. This additional second best leverage – which the government is capable of using because it can fully commit to future policy – lessens the need to increase the wedge in period $t < \tau$. This is the main difference between the Ramsey equilibrium and the MPE and OFP. The central conclusion of the paper, however, is confirmed, the government increases the wedge $\omega_t$ to reduce deflation during the period of the deflationary shocks.

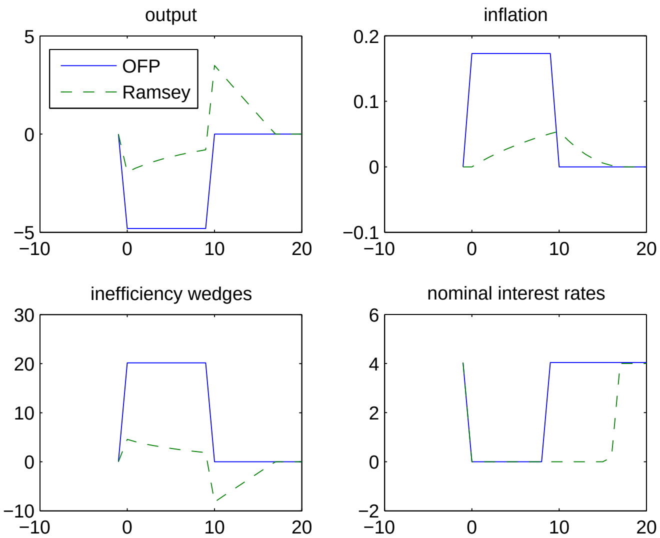


Figure 10: The qualitative features of the optimal forward looking and Ramsey policy are the same. The key difference is that the Ramsey policy achieves a better outcome by manipulating expectations about policy at the time at which the deflationary shocks have subsided.

The key weakness of this policy, as a descriptive tool, is illustrated by comparing it to the MPE. The optimal commitment is subject to a serious dynamic inconsistency problem. To see this consider the Ramsey solution in periods $t \geq \tau$ when shocks have subsided. The government can then obtain higher utility by reneging on its previous promise and achieve zero inflation and output equal to the efficient level. This incentive to renege is severe in our example, because the deflationary shocks are rare and are assumed not to reoccur. Thus the government has strong incentive to go back on its announcements. This incentive is not, however, present to the same extent under optimal forward looking policy. Under the optimal forward looking policy the commitment in periods $t \geq \tau$ is identical to the MPE.