TECHNICAL APPENDIX (not intended for publication)

Understanding International Prices: Customers as Capital

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Contents

Abstract

The document includes more detailed derivations, proofs and robustness checks that are referenced but not included in the paper.

1 Data

Here, we provide extended tables supporting the empirical part of the paper.

1.1 Definition of Aggregate Prices

The real export (import) price is constructed by dividing the nominal deflator price of exports[1] (imports) by the all-items CPI,

$$p_x = \frac{EPI}{CPI} \tag{1}$$

$$p_m = \frac{IPI}{CPI}, \tag{2}$$

where EPI (IPI) denotes the nominal deflator prices of exports (imports) constructed by dividing the value of exports in current prices by the value of export is constant prices. p_x^T (p_m^T) has been constructed from the formula stated in the Data Section of the paper, using the CPI for housing and services to measure the prices of non-tradables P^N.

In the case of Table 2, *the real exchange rate* is constructed by dividing the trade-weighted foreign price level index by the corresponding domestic price level index, after prior conversion to a common numéraire (using nominal exchange rate),

$$x_i \equiv \Pi_{j=1}^{N}(e_{ij}P_j)^{\omega_{ij}}/P_i, \tag{3}$$

where x_i denotes the real exchange rate of country i, e_{ij} denotes bilateral nominal exchange rate between country i and country j (j currency units in terms of domestic currency), ω_{ij} denotes the weight of country j in total trade ($\sum_j^N \omega_j = 1$) of country i, and P_i is the price index used to measured the overall price level. In all other cases we used the trade weighted time-series from the IMF-IFS database.

[1] The nominal deflator price of exports (imports) is defined as the ratio of value of exports (imports) in current prices to the value of exports (imports) in constant prices.

The *terms of trade* is constructed as follows

$$p = \frac{IPI}{EPI} = \frac{p_m}{p_x}. \tag{4}$$

2 Extended Discussion of the Puzzles

Here, we set the quantitative goal for our theory by defining the discrepancy between the predictions of the standard international macroeconomic model[2] and international price data. We use data for both disaggregated prices and aggregate prices. Our aggregate data is based on H-P-filtered quarterly price data for the time period 1980 to 2005, and our sample includes the time series for the following countries: Belgium, Australia, Canada, France, Germany, Italy, Japan, the Netherlands, United Kingdom, United States, Sweden, and Switzerland. Our disaggregated data are based on the disaggregated producer and wholesale price data for Japan.

2.1 Export-Import Price Correlation Puzzle

One of the central predictions of the standard theory for international relative price movements is that the price of the exported goods, evaluated relative to the overall home price level, moves in the opposite direction to the similarly constructed import price. Intuitively, this implication follows from the fact that, by the law of one price, export prices are tied to the prices of domestically-produced and domestically-sold goods, and import prices are tied to corresponding prices abroad, expressed in home units. As a result, whenever the real exchange rate depreciates[3], import prices rise relative to home prices due to their direct link to the overall foreign price level, and export prices fall relative to home prices, as home prices additionally include the higher priced imports.

To show the above implication formally, we first derive it in a simple model without a distinction between tradable and non-tradable goods, and then generalize the results to a model that makes such distinction explicit.

In the standard model without non-tradable goods, the overall home price level measured by the CPI can be approximated by a trade-share-weighted geometric average of the prices of the

[2] Backus, Kehoe & Kydland (1995). See Stockman & Tesar (1995) for a version with non-tradable goods.
[3] An increase in the foreign overall price level relative to the overall home price level.

tradable home and foreign good, d and f (home-bias toward the local good d is parameterized by $1/2 < \omega < 1$)[4]. Given the formula for the CPI, the definitions of the real export price p_x and the real import price p_m of a country (deflated by CPI) are as follows:

$$p_x = \frac{P_d}{CPI} = \frac{P_d}{P_d^{\omega} P_f^{1-\omega}} = (\frac{P_d}{P_f})^{(1-\omega)}, \quad p_m = \frac{P_f}{CPI} = \frac{P_f}{P_d^{\omega} P_f^{1-\omega}} = (\frac{P_f}{P_d})^{\omega}. \tag{5}$$

The above formulas immediately imply a negative correlation of p_x and p_m in the model for all admissible values of ω.

To contrast this prediction with the data, we calculate export and import price indices from the import and export price deflators[5], and then deflate these prices by the all-items CPI index to construct p_x and p_m, respectively (different measures of overall price level yield very similar results). As shown in Table 1, we find that the correlations between real export and import prices are highly positive across all 12 OECD countries in our sample, covering the range from 0.57 (Australia) to 0.94 (Belgium and Netherlands), with a median correlation of 0.87. We should mention that these prices are also quite volatile. Their median volatility relative to the real exchange rate is 0.56 for the real export price and 0.83 for the real import price[6].

Next, we verify whether the above results are robust to an explicit distinction between tradable and non-tradable goods. For a more general constant elasticity of substitution aggregator with explicit non-tradable component, we can derive a prediction of the model that the following two objects must be negatively correlated[7]:

$$p_m^T \equiv \left[\frac{1}{v}(\frac{P_f}{P})^{\frac{1-\mu}{\mu}} - \frac{(1-v)}{v}(\frac{P_f}{P_N})^{\frac{1-\mu}{\mu}}\right]^{\frac{\mu}{1-\mu}} = (\frac{P_f}{P_d})^{\omega}, \quad \text{and} \tag{6}$$

$$p_x^T \equiv \left[\frac{1}{v}(\frac{P_d}{P})^{\frac{1-\mu}{\mu}} - \frac{(1-v)}{v}(\frac{P_d}{P_N})^{\frac{1-\mu}{\mu}}\right]^{\frac{\mu}{1-\mu}} = (\frac{P_d}{P_f})^{(1-\omega)}. \tag{7}$$

[4]The approximation is exact when the elasticity of substitution between domestic and foreign goods is one. However, unit elasticity is within the range of values commonly used in the literature, and small departures from unity do not matter quantitatively for what follows.

[5]Constructed from the time series for constant- and current-price import and export prices at the national level. Formal definitions are stated in the Appendix.

[6]The table with all reported correlations for this and the next exercise with nontradables is available in the online Appendix.

[7]The CPI in our formulation with non-tradable goods is $CPI = (v(P_d^{\omega} P_f^{1-\omega})^{\frac{\mu-1}{\mu}} + (1-v)P_N^{\frac{\mu-1}{\mu}})^{\frac{\mu}{\mu-1}}$.

To contrast the above prediction of the model with the data, we approximate the price of non-tradable goods P_N by the CPI for housing and services, and similarly as before use all-items CPI to measure P, and export (import) price deflators to measure P_d (P_f).[8] The parameters μ and v are assumed to be in the range of estimates from the literature and are least favorable to positive correlation ($v = 0.6$ and $\mu = 0.44$)[9]. Using the above construct to compute import and export prices leaves the previously reported results almost intact: as shown in Table 1, the median correlation of p_x^T and p_m^T is 0.84, covering a range from 0.51 to 0.92. The reason behind this result is a high positive correlation of P_d/P and P_d/P_N (median correlation is 0.98) and their similar volatility[10]. Because $1/v \approx 2$ and $(1-v)/v \approx 1$, not surprisingly the properties of the time series for p_x^T and p_m^T are similar to p_x and p_m. We conclude that the existence of a non-tradable goods' sector *per se* cannot account for the export-import price correlation puzzle.

2.2 Terms of Trade Relative Volatility Puzzle

The second firm prediction of the standard theory is the excess volatility of the terms of trade $p = \frac{P_f}{P_d}$ (price of imports in terms of exports) relative to the real exchange x. In this respect, the standard theory predicts that the terms of trade should be exactly equal to the PPI-based real exchange rate,[11] and thus exactly as volatile. The reason is that, by the law of one price, the price index of exported goods is equal to the home producer price index and the price index of the imported goods is equal to the foreign country producer price index measured in the home numeraire units. In contrast, in the data export and import prices are highly positively correlated and the terms of trade—defined as their ratio—carries a significantly smaller volatility than the volatility of the CPI based real exchange rates[12]. In our sample of countries, the median volatility of the terms of

[8]To generate the time series for p_m^T, p_x^T, we first detrend the time series for $P_d/P, P_d/P_N$ (same for P_f) and normalize them so that they oscillate around unity.

[9]$v = 0.6$ is taken from Corsetti, Dedola & Leduc (2008), and is close to the upper bound on the estimates of the share of non-tradables in CPI basket. $\mu = 0.44$ is taken from Stockman & Tesar (1995). Generally, higher v and lower μ helps the model by disconnecting tradables and nontradables in the CPI. Other papers use more moderate numbers, for example, Corsetti, Dedola & Leduc (2008) follow Mendoza (1991) and use the elasticity of substitution between tradable and non-tradable goods equal to 0.76.

[10]Similarly for P_f/P and P_f/P_N.

[11]The PPI-based real exchange rate is the foreign producer price index relative to the home producer price index, when both measured in common numéraire.

[12]The same conclusions also hold true if we use the PPI-based real exchange rates or the nominal exchange rates. The table with all reported correlations for this exercise is available in the online Appendix.

trade relative to the CPI-based real exchange rate is 0.54 with a range from 0.21 (Sweden) to 0.83 (Germany)[13]. For details see Tables 2.

2.3 Pricing-to-Market Puzzle

In addition to the aggregate anomalies shown above, there is pervasive direct evidence that the law of one price is systematically violated between countries regardless of the level of disaggregation.[14] Here we document this feature of the data using a sample of the disaggregated price data from the Japanese manufacturing industry.

Our dataset includes quarterly time series for producer/wholesale level price indices for 31 highly disaggregated and highly traded manufactured commodity classifications. For each commodity classification, we have information on the price of the good when exported (export price EPI) and when sold on the domestic market (domestic wholesale price DPI).[15]

To emphasize the analogy to our aggregate analysis, we construct similar objects to the aggregate real export price indices considered before, but instead computed separately for each single commodity classification. More specifically, for each commodity i, we divide its export price index (EPI) by the overall Japanese CPI and use the identity relation

$$p_x^i \equiv \frac{EPI_i}{DPI_i}\frac{DPI_i}{CPI} \tag{8}$$

to decompose the fluctuations of the real export price of each commodity into two distinct components: (i) the pricing-to-market term $\frac{EPI_i}{DPI_i}$ – capturing the deviations of the export price of commodity i from its corresponding home price – and (ii) the residual term $\frac{DPI_i}{CPI}$ – capturing the deviations of the home price of commodity i from the overall CPI.

Before we discuss any results pertaining to the above decomposition, we should first note that the commodity-level prices p_x^i exhibit similar patterns as the aggregate data: the median relative

[13]When we clean the US import price data from the influence of the highly volatile crude oil prices the volatility of the terms of trade relative to the real exchange rate falls below 1/3.

[14]Our analysis here will be a reminiscent of the incomplete pass-through/pricing-to-market literature that documents related facts using regression analysis. For example, similar analysis to ours can be found in Marston (1990).

[15]Standard PPI or WPI [wholesale price index] measures would include export prices or import prices, respectively. Price indices used here come from the producer survey data and together account for 59% of the value of Japanese exports and 18% of the value of domestic shipments (as of year 2000). Examples of commodities are: ball bearings, copying machines, silicon wafers, agricultural tractors, etc. For a complete list, see the online Appendix.

volatility of p_x^i to the real exchange rate is as high as 88%, and the median correlation of p_x^i with the real exchange rate is as high as 0.82. With our decomposition at hand, we can now look what happens behind the scene.

Variance driven by pricing-to-market To measure the contribution of the volatility of each term to the overall volatility of the export price index, we use variance decomposition:

$$\text{median}_i\left(\frac{\text{var}(\frac{EPI_i}{DPI_i})}{\text{var}(\frac{EPI_i}{DPI_i}) + \text{var}(\frac{DPI_i}{CPI})}\right), \tag{9}$$

where var$(\cdot)$ in the formula above refers to the logged and H-P-filtered quarterly time series (with smoothing parameter $\lambda = 1600$). In our analysis, we omit the covariance terms, as the two terms actually covary negatively in the data. Clearly, under the law of one price, one should expect that the first term $\frac{EPI_i}{DPI_i}$ be almost constant, and all the variation in the real export prices p_x^i come from the fluctuations of the residual term $\frac{DPI_i}{CPI}$. The data shows the opposite pattern. The pricing-to-market term $\frac{EPI_i}{DPI_i}$ carries about 93% of the total volatility, and the residual term $\frac{DPI_i}{CPI}$ carries only 7%.

Pricing-to-market related to the real exchange rate The data also leaves little ambiguity as to which term drives the high positive correlation of real export prices p_x^i with the real exchange rate (median=0.82). The median correlation of $\frac{EPI_i}{DPI_i}$ with the Japanese real exchange rate is as high as 0.84, and the median correlation of the residual term $\frac{DPI_i}{CPI}$ is actually slightly negative (-0.15).

Summarizing, we find that both aggregate and disaggregated data point to robust pricing patterns for which the standard theory fails to account. We next proceed with the presentation of our model.

2.4 Robustness of Export-Import Correlation Puzzle

Here, we show that our results do not come from price deflation using aggregate indices, tables 3-4 report analogous statistics to the paper but for nominal export and import prices. We report the results for both H-P-filtered and linearly detrended data.

Table 1: Correlation of Real Export and Real Import Prices

Country	Correlation					
	p_x, p_m	p_x, x	p_m, x	p_x^T, p_m^T	p_x^T, x	p_m^T, x
Australia	0.57	0.45	0.95	n/a	n/a	n/a
Belgium	0.94	0.72	0.74	0.91	0.64	0.65
Canada	0.71	0.50	0.92	0.72	0.48	0.86
France	0.90	0.61	0.66	0.89	0.60	0.62
Germany	0.62	0.50	0.85	0.47	0.28	0.84
Italy	0.88	0.68	0.72	0.84	0.65	0.67
Japan	0.85	0.92	0.85	0.84	0.90	0.85
Netherlands	0.94	0.76	0.80	0.92	0.75	0.78
Sweden	0.89	0.60	0.74	n/a	n/a	n/a
Switzerland	0.60	0.51	0.83	0.51	0.44	0.86
UK	0.90	0.61	0.79	n/a	n/a	n/a
US	0.75	0.46	0.69	0.68	0.46	0.69
MEDIAN	0.87	0.61	0.80	0.84	0.60	0.78

Notes: Prices indices as defined in the Section 1. Statistics based on logged & H-P-filtered quarterly time series with smoothing parameter 1600. Except for T series, which ends in year 2000, the series range from 1980:1 to 2004:2. Sources are listed at the end of the document.

Table 2: Volatility of Terms of Trade Relative to Real Exchange Rate

Country	Volatility of p relative to x (in %)		
	Price index used to construct[a] x		
	CPI all-items	WPI or PPI	None (nominal)
Australia	0.51	0.54	0.60
Belgium	0.57	0.70	0.47
Canada	0.56	0.76	0.61
France	0.80	0.74	0.73
Germany	0.83	0.81	0.80
Italy	0.75	0.79	0.77
Japan	0.52	0.54	0.55
Netherlands	0.52	0.49	0.44
Sweden	0.21	0.21	0.37
Switzerland	0.71	0.68	0.67
UK	0.30	0.32	0.37
US	0.31	0.33	0.28
MEDIAN	0.54	0.61	0.57

Notes: We have constructed trade-weighted exchange rates using weights and bilateral exchange rates for the set of 11 fixed trading partners for each country. The trading partners included in the sample are the countries listed in this table. Statistics are computed from logged and H-P-filtered quarterly time-series for the time period 1980:1-2000.01 (smoothing parameter 1600). Data sources are listed at the end of the document.
[a]Definitions as stated in Section 1.

Table 3: Correlation of Nominal and PPI-deflated Real Export Price and Real Import Price (HP filtered data).

	Correlation					
Country	EPI, IPI	EPI, e	IPI, e	p_x^{ppi}, p_m^{ppi}	p_x^{ppi}, x	p_m^{ppi}, x
Belgium	0.96	0.77	0.76	0.64	0.72	0.74
Canada	0.65	0.20	0.71	0.42	0.50	0.92
Switzerland	0.72	0.59	0.80	0.40	0.51	0.83
France	0.95	0.71	0.72	0.58	0.61	0.66
Germany	0.87	0.63	0.80	-0.16	0.50	0.85
Italy	0.89	0.62	0.72	0.67	0.68	0.72
Japan	0.88	0.88	0.76	0.77	0.92	0.85
Netherlands	0.95	0.72	0.76	0.90	0.76	0.80
US	0.82	0.13	0.44	0.47	0.46	0.69
Australia	0.53	0.35	0.91	0.56	0.45	0.95
Sweden	0.91	0.54	0.67	0.38	0.60	0.74
UK	0.87	0.34	0.61	0.83	0.61	0.79
MEDIAN	0.88	0.60	0.74	0.57	0.61	0.80

Notes: EPI denotes nominal export price index, IPI denotes nominal import price index, e denotes trade-weighted nominal exchange rate (from IMF-IFS database), x denotes an analogous real exchange rate construct. All statistics based on logged & Hodrick-Prescott filtered quarterly time series for the period 1980:1-2004:2 except for CPI of tradables series which ends in 2000 (HP filter uses smoothing $\lambda = 1600$). Sources are listed at the end of the document.

Table 4: Correlation of Nominal and PPI-deflated Real Export Price and Real Import Price (HP filtered data).

Country	Correlation					
	EPI, IPI	EPI, e	IPI, e	p_x^{ppi}, p_m^{ppi}	p_x^{ppi}, x	p_m^{ppi}, x
Belgium	0.95	0.59	0.62	0.56	0.58	0.55
Canada	0.82	0.26	0.53	0.62	0.31	0.52
Switzerland	0.47	0.23	0.75	0.54	0.36	0.56
France	0.93	0.32	0.53	0.67	0.43	0.53
Germany	0.53	0.33	0.68	-0.09	0.29	0.66
Italy	0.71	0.36	0.65	0.87	0.36	0.37
Japan	0.68	0.79	0.45	0.66	0.65	0.42
Netherlands	0.98	0.50	0.55	0.97	0.58	0.64
US	0.81	0.12	0.41	0.62	0.47	0.64
Australia	0.83	0.21	0.47	0.43	0.38	0.89
Sweden	0.90	0.22	0.45	-0.22	0.46	0.45
UK	0.97	0.21	0.30	0.78	0.60	0.70
MEDIAN	0.83	0.29	0.53	0.62	0.45	0.55

Notes: EPI denotes nominal export price index, IPI denotes nominal import price index, e denotes trade-weighted nominal exchange rate (from IMF-IFS database), x denotes analogous real exchange rate. All statistics based on logged & linearly detrended quarterly time series for the period 1980:1-2004:2 except for CPI of tradables series which ends in 2000 (HP filter uses smoothing parameter $\lambda = 1600$). Sources are listed at the end of the document.

3 Model (w/ extended proofs)

The overall structure of the model is similar to Backus, Kehoe & Kydland (1995) model (BKK hereafter). Time is discrete and assumed finite, $t = 0, 1, 2, ..., T$, but the model trivially extends to $T = \infty$. There are two ex-ante symmetric countries labeled *domestic* and *foreign*. Each country is populated by identical households. The only source of uncertainty in the economy are country-specific productivity shocks. Tradable goods are country-specific: d is produced in the domestic country, and f in the foreign country. d and f are traded internationally at the *wholesale* level between *producer* and *retailers*[16]. Retailers resell these goods in a perfectly competitive *retail* market to households.

In terms of notation, we distinguish foreign country-related variables from the domestic ones using an asterisk. The history of shocks $s_i \in S$ up to and including period t is denoted by $s^t = (s_0, s_1, ..., s_t)$, where the initial symmetric realization s_0, the time invariant probability $\mu(s^t)$, as well as the discrete and finite set $S \subset N^2$, are given. In the presentation of the model, whenever possible, we exploit symmetry and present the model from the domestic country's perspective.

3.1 Uncertainty and Production

Each country is assumed to have access to a constant returns to scale production function $zF(k, l)$ that uses country-specific capital k and labor l, and is subject to a country-specific stochastic technology z following an exogenous AR(1) process, i.e. $\log z(s^t) = \psi \log z(s^{t-1}) + \varepsilon_t$ and $\log z^*(s^t) = \psi \log z^*(s^{t-1}) + \varepsilon_t^*$, where $0 < \psi < 1$ is a common persistence parameter, and $s_t \equiv (\varepsilon_t, \varepsilon_t^*) \in S$ is an i.i.d. discrete random variable with zero mean and compact support.

Since the production function is assumed to be constant returns to scale, we summarize the production process by an economy-wide marginal cost v. Given domestic factor prices w, r and domestic shock z, the marginal cost, equal to per unit cost, is given by:

$$v\left(s^t\right) \equiv \min_{k,l} \left\{ w\left(s^t\right) l + r\left(s^t\right) k \;\; \text{subject to} \;\; z\left(s^t\right) F\left(k, l\right) = 1 \right\}. \tag{10}$$

[16]Interpretation of retailers in our model should not be confined to the retail sector only. The label is introduced to clearly distinguish the two sides of matching. By retailers we mean all other producers who participate in the overall production process of bringing goods to the consumer. Distribution of the value added across different types of producers is not critical for any of the results.

3.2 Households

The problem of the household is standard and identical to a decentralized version of the standard model under complete asset markets.

Each country is populated by a unit measure of identical, finitely-lived households. In each period, they choose the level of consumption c, investment in physical capital i, labor supply l, purchases of tradable goods d, f, and purchases of a set of one-period s_{t+1}-contingent bonds $b\left(s_{t+1}|s^t\right)$, to maximize the expected discounted lifetime utility $U = \sum_{t=0}^{T} \beta^t \sum_{s^t \in S^t} u(c(s^t), l(s^t))\mu(s^t)$, where u satisfies the usual assumptions and $0 < \beta < 1$. The preferences over domestic and foreign goods are modeled by the Armington aggregator $G\left(d, f\right)$ with an assumed exogenous elasticity of substitution (Armington elasticity) γ, and an assumed home-bias parameter ω,

$$G\left(d, f\right) = \left(\omega d^{\frac{\gamma-1}{\gamma}} + (1-\omega) f^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}, \ \gamma > 0, \ \frac{1}{2} < \omega < 1. \tag{11}$$

Households combine goods d and f through the above aggregator into a composite good which they use for consumption and investment, according to the aggregation constraint $c(s^t) + i(s^t) = G(d(s^t), f(s^t))$. Physical capital follows the standard law of motion, $k(s^t) = (1-\delta)k(s^{t-1}) + i(s^t)$, with $0 < \delta < 1$. Asset markets are complete, and the budget constraint of the domestic household is given by

$$\begin{aligned} & P_d\left(s^t\right) d\left(s^t\right) + P_f\left(s^t\right) f\left(s^t\right) + \sum_{s_{t+1} \in S} Q(s_{t+1}|s^t) b(s_{t+1}|s^t) \mu(s^{t+1}) \\ & = b(s^t) + w\left(s^t\right) l\left(s^t\right) + r\left(s^t\right) k\left(s^{t-1}\right) + \Pi\left(s^t\right), \text{ all } s^t. \end{aligned} \tag{12}$$

The expenditure side of the budget constraint consists of purchases of domestic and foreign goods and purchases of one-period-forward s_{t+1}-state contingent bonds. The income side consists of income from maturing bonds purchased at history s^{t-1}, labor income, rental income from physical capital, and the dividends paid out by local firms.

The foreign budget constraint is analogous, with the exception of an additional adjustment of the price of bonds given instead by $Q(s_{t+1}, s^t)^* \equiv \frac{x(s^{t+1})}{x(s^t)} Q(s_{t+1}|s^t)$. In this equation, $x(s^t)$ is the ideal real exchange rate, which translates foreign units to domestic ones, as implied by the assumptions of integrated world asset market and the composite consumption in each country being the numéraire[17].

Summarizing, given the initial values of the state variables, households choose their allocations to maximize lifetime utility U subject to the aggregation constraint, the law of motion for physical capital, the budget constraint (12), the standard no–Ponzi scheme condition, and the numéraire normalization. In further analysis, we will use two optimality conditions derived from the household problem: the demand equations linking retail prices to household valuations

$$P_d\left(s^t\right) = G_d\left(s^t\right), \quad P_f\left(s^t\right) = G_f\left(s^t\right), \tag{13}$$

and the efficient risk sharing condition[18] implied by the complete asset market structure

$$x\left(s^t\right) = x(s_0)\frac{u_c^*\left(s^t\right)}{u_c\left(s^t\right)}, \tag{14}$$

where $u_c\left(s^t\right), G_d\left(s^t\right), G_f\left(s^t\right)$ denote partial derivatives and by ex-ante symmetry $x(s_0) = 1$.

3.3 Producers

Tradable (intermediate) goods d and f are country specific and are produced by a unit measure of atomless competitive producers residing in each country. They employ local capital and labor, and use local technology, which gives rise to production cost given by (10).

The novel feature introduced in this paper is that producers need to first match with the retailers in order to sell their goods. Matching is costly and time consuming, and trade involves bargaining. In the sections that follow, we describe the details of matching and state the profit maximization problem of the producers. We provide a formal treatment of the bargaining problem in a later section, as it is not essential to define the producer problem.

[17] Specifically, we assume that he ideal-CPI in each country is normalized to one. The ideal CPI is defined by the lowest cost of acquiring a unit of composite consumption (c in the domestic country, c^* in the foreign country). Since the foreign budget constraint is expressed in foreign consumption, and so is foreign b^*, integrated asset markets imply that $Q(s_{t+1}, s^t)^* = x(s^{t+1})Q(s_{t+1}, s^t)/x(s^t)$. In the data, the real exchange rate is measured using fixed-weight CPI rather than ideal CPI indices. Quantitatively, this distinction turns out not to matter in this particular class of models.

[18] This condition says that households fully share risk internationally, and equalize MRS from consumption across the border with the relative price x of their consumption. It is known to imply counterfactual connection of real exchange rate to quantities. As we show later, our results are robust to relaxing this tight relation by considering a different asset market structure.

List of customers and market shares To match with retailers, the producers have access to an explicitly formulated *marketing technology* and accumulate a form of capital labeled *marketing capital*, m. Marketing capital is accumulated separately in each country a producer sells in, and the marketing capital a producer holds in each country relative to other producers, determines the contact probabilities with the searching retailers. For example, an exporter from the domestic country with marketing capital $m_d^*(s^t)$ in the foreign country attracts a fraction $\frac{m_d^*(s^t)}{\bar{m}_d^*(s^t)+\bar{m}_f^*(s^t)}$ of the searching retailers from the foreign country, where $\bar{m}_f^*(s^t)$ and $\bar{m}_d^*(s^t)$ denote the average levels of marketing capital an f and d good producer holds in that country. Retailers who join the customer list of this producer, $H(s^t)$, will stay on the list until the match is dissolved with exogenous probability δ_H.

Formally, given the measure $h(s^t)$ of searching retailers in a given country, who are *potential customers*, the arrival of *new customers* to the customer list of a given producer is given by $\frac{m_d(s^t)}{\bar{m}_d(s^t)+\bar{m}_f(s^t)}h\left(s^t\right)$. We assume that each match with a retailer is long-lasting and is subject to an exogenous destruction rate δ_H, and thus the evolution of the endogenous list of customers $H_d(s^t)$ is described by the following law of motion:

$$H_d(s^t) = (1-\delta_H)H_d(s^{t-1}) + \frac{m_d(s^t)}{\bar{m}_d(s^t)+\bar{m}_f(s^t)}h(s^t),\ 0 < \delta_H \leq 1 \tag{15}$$

The size of this list is critical for the producer, as it determines the amount of goods this producer can sell in a given market (country). Specifically, we assume here that in each match, one unit of the good can be traded per period—to reflect the fact that each match is somewhat specific to a particular task at hand[19]. Thus, sales of a given producer cannot exceed the size of the customer list H. For example, the sales constraint[20] of a producer of good d in the foreign country with a customer list $H_d^*(s^t)$ is given by $d^*(s^t) \leq H_d^*(s^t)$.

[19]One interpretation could be that each match trades a different good, and there is a Dixit-Stiglitz aggregator on the retail level. In such case, the implied capacity constraint would be continuous rather than a discrete zero/one. We can conjecture that the results of the paper would not differ much as long as this capacity constraint would be tight enough—looser/tighter capacity constraints would work similarly to a lower/high value of ϕ. We therefore omit such considerations from the paper.

[20]Due to always positive markups, this condition *always* binds on the simulation path.

Marketing capital Producers accumulate marketing capital m to attract searching retailers. Given last period's level of marketing capital $m_d(s^{t-1})$ and the current level of instantaneous marketing input $a_d\left(s^t\right)$, current period marketing capital $m_d(s^t)$ is given by

$$m_d(s^t) = (1-\delta_m)\, m_d(s^{t-1}) + a_d(s^t) - \frac{\phi}{2} m_d\left(s^{t-1}\right)\left(\frac{a_d\left(s^t\right)}{m_d\left(s^{t-1}\right)} - \delta_m\right)^2. \tag{16}$$

The above specification nests two key features: (i) decreasing returns from the instantaneous marketing input $a_d(s^t)$ and (ii) capital-theoretic specification of marketing. These features, parameterized by the market expansion friction parameter ϕ and depreciation rate δ_m, are intended to capture the idea that marketing-related assets like brand awareness, reputation or distribution network are capital for a firm and their buildup takes time.

Profit maximization Producers sell goods in the domestic country for the wholesale price p_d and in the foreign country for the wholesale export price $p_x \equiv xp_d^*$, measured in domestic numéraire. These prices are determined by bargaining with the domestic and foreign retailers. However, to set up the profit maximization of the producers, we can abstract from bargaining at this stage and assume that the prices are taken as given[21].

The instantaneous profit function Π of the producer is determined by the difference between the profit from sales in each market and the total cost of marketing, i.e. $\Pi = (p_d - v)d + (xp_d^* - v)d^* - vA_d(s^t)a_d - xv^*A_d^*(s^t)a_d^*,$. The state-dependent input requirements A_d and A_d^* are introduced here only for the sake of the analytical characterization of the sources of deviations from LOP in the model. Unless explicitly noted, we maintain the assumption $A_d = A_d^* = 1$.

Given the instantaneous profit function Π, the representative producer from the domestic country, who enters period t in state s^t with the customer lists $H_d(s^{t-1})$, $H_d^*(s^{t-1})$ and marketing capitals $m_d(s^{t-1})$, $m_d^*(s^{t-1})$, chooses the allocation $a_d(s^t)$, $a_d^*(s^t)$, $m_d(s^t)$, $m_d^*(s^t)$, $d(s^t)$, $d^*(s^t)$, $H_d(s^t)$, $H_d^*(s^t)$, to maximize the present discounted stream of future profits given by $\sum_{\tau=t}^{T} \int Q(s^\tau)\Pi\left(s^\tau\right)\mu\left(ds^\tau|s^t\right)$, subject to the law of motion for marketing capital, sales constraints $(d(s^t) \leq H_d(s^t), d^*(s^t) \leq$

[21] This is because the producer can perfectly anticipate the outcome of bargaining at every contingency, and cannot strategically influence it beforehand by making a different choice—as we will see later, neither the state variables nor decision variables chosen in the problem below affect the outcome of bargaining. This property follows from the 3 key assumptions of the model: (i) production, marketing and search are all constant returns to scale activities, (ii) atomistic agents, (iii) expensed search cost and marketing cost cannot be retrieved by breaking a match.

$H_d^*(s^t)$), and the laws of motion for customer lists (15). The discount factor $Q(s^t)$ is defined by the recursion on the conditional pricing kernel derived from the household's problem, $Q(s^t) = Q(s_t|s^{t-1})Q(s^{t-1})$.

3.4 Retailers

In each country there is a sector of atomless retailers who purchase goods from producers and resell them in a local competitive market to households. It is assumed that new retailers who enter into the market must incur an initial search cost χv in order to find a producer with whom they can match and trade. Each match lasts until it exogenously dissolves with a per-period probability δ_H. As long as the match lasts, the producer and the retailer have an option to trade one unit of the good per period[22]. In equilibrium, the industry dynamics is governed by a free entry and exit condition, which endogenously determines the measure h of new entrants (searching retailers). Trade between households and retailers takes place in a local competitive market at prices P_d for good d and P_f for good f. In equilibrium, these prices are given by (13), and throughout the paper we refer to them as *retail prices*.

In each period, there is a mass of retailers already matched with the producers H and a mass of new entrants h (searching retailers). A new entrant, upon paying the up-front search cost χv, meets with probability π a producer from the domestic country and with probability $1 - \pi$ the producer from the foreign country (selling in the domestic country). The entrant takes this probability as given, but in equilibrium it is determined by the marketing capital levels accumulated by the producers, according to

$$\pi(s^t) = \frac{\bar{m}_d\left(s^t\right)}{\bar{m}_d\left(s^t\right) + \bar{m}_f\left(s^t\right)}. \tag{17}$$

The measures of matched retailers H evolve in consistency with (15).

As is clear the above formulation of the matching process, search by the retailers is guided directly only by the marketing capital accumulated by the producers. Thus, in our model, prices and the anticipated surplus from trade with each type of producer, only indirectly influence the

[22]One can more generally think of each match as effectively providing a different type of intermediate good with a low elasticity of substitution. d is then an integral over all matches. The link between exchange rate and prices will be qualitatively robust to this modification—albeit not as tractable as our formulation.

basket of goods which is consumed on the aggregate level[23]. This a central feature of our model environment, and a key departure from the standard models.

Bargaining and wholesale prices We assume that each retailer bargains with the producer over the total future surplus from a given match. This surplus is split in consistency with Nash bargaining solution with continual renegotiation. Nash bargaining as a surplus splitting rule is an important assumption for our results, but not the only modeling option delivering our results. Any departures from our setup can be mapped onto a time-varying Nash bargaining power, and as long as its variation is independent from exchange rate movements or limited in size, our results will largely remain unchanged.

To set the stage for the bargaining problem, we first need to define the value functions from the match for the producer and for the retailer. We assume that they trade at history s^t at some arbitrary wholesale price p, and in the future they will trade according to an equilibrium price schedule $p(s^t)$. The value functions are

$$W_d\left(p;s^t\right)=\max\left\{0,\,p-v\left(s^t\right)\right\}+\left(1-\delta_h\right)E_tQ\left(s_{t+1}|s^t\right)W_d\left(p_d(s^{t+1});s^{t+1}\right),\tag{18}$$

$$J_d\left(p;s^t\right)=\max\left\{0,\,P_d\left(s^t\right)-p\right\}+\left(1-\delta_h\right)E_tQ\left(s_{t+1}|s^t\right)J_d\left(p_d(s^{t+1});s^{t+1}\right),\tag{19}$$

where W_d is the value of the domestic producer selling in the domestic country and J_d is the value for the domestic retailer matched with a domestic producer.

The flow part of the above Bellman equation for the producer is determined by the difference between the wholesale price of the good, p, and the production cost, v, whereas for the retailer, it is determined by the difference between the retail price (resell price) of the good P_d and the wholesale price paid to the producer p. These equations additionally imply that markups will be necessarily

[23] Under some assumptions this disconnect would not matter. Two other features of the model may potentially render it relevant: (1) long-lasting matches modeled by $\delta_H < 1$ and (2) market-share adjustment friction, $\phi > 0$. In our quantitative specification, most action will come from (2). However, it should be noted that (1) with low enough depreciation of customer base and *directed search* of retailers can also give rise to similar dynamics of prices, but in an environment like ours it is infeasible to solve (as it requires global solution methods due to corner $h = 0$). The intuition is that when matches are expected to be persistent, even if search of retailers can be directed, retailers' search intensity will depend on the present discounted value of future surpluses, and not only on the current surplus. This will generate PTM (unless shocks are permanent). The view of downplaying current prices in long-lasting partnerships is consistent with the anecdotal evidence in Egan & Mody (1992).

bounded below by zero[24] (as no trade is an option in any given period).

With the values from a match at hand, we are now ready to set up the Nash bargaining problem, which imposes the following restriction on the equilibrium schedule of the wholesale prices[25] $p(s^t)$, given bargaining power $0 < \theta < 1$,

$$p_f(s^t) \in \arg\max_p \{J_f(p; s^t)^\theta W_f\left(p; s^t\right)^{1-\theta}\}, \text{ all } s^t. \tag{20}$$

Note that the threat-points of both sides are zero by three assumptions of the model: (i) search cost and marketing cost cannot be retrieved by breaking the match, (ii) there is free entry and exit to retail sector (zero profit condition), (iii) production, marketing and search are all constant returns to scale activities.

The proposition below establishes that under continual renegotiation, the price schedule resulting from (20) allocates θ fraction of the total instantaneous (static) trade surplus given by $P_f - xv^*$ to the producer and fraction $1-\theta$ to the retailer. Intuitively, this follows from the fact that since it is impossible to split the trade surplus from tomorrow onward in any other proportion than θ and $1-\theta$, the *static* surplus today must be split the same way.

Proposition 1 *Assume that trade takes place at* s^t. *Then, the solution to the bargaining problem stated in (20) is given by*

$$\begin{aligned} p_d(s^t) &= \theta P_d(s^t) + (1-\theta)v(s^t), \\ p_x(s^t) &\equiv x(s^t)p_d^*(s^t) = \theta x(s^t)P_d^*(s^t) + (1-\theta)v(s^t). \end{aligned} \tag{21}$$

Proof. Add both sides of (18) and (19) to obtain a Bellman equation for total surplus $S \equiv W + J$. Multiply both sides of this Bellman equation by θ, and then subtract from both sides of it equation (18). By bargaining, note $W = \theta S$ at each state, and hence (21) follows.

Free entry and exit condition Free entry and exit into the retail sector governs the measures of searching retailers in each country h. It relates the expected surplus for the retailer from matching

[24] On the simulation patch markups never hit zero, but for some parameterization this may be the case (featuring low steady state markups and large shocks).

[25] Other prices are defined by analogy.

with a producer from the domestic or the foreign country to the search cost incurred to identify a match opportunity, i.e. $\pi(s^t)J_d(p_d(s^t);s^t)+(1-\pi(s^t))J_f(p_d(s^t);s^t) \leq \chi(s^t)v(s^t)$, with the condition holding with equality whenever $h>0$. The state-dependent search cost $\chi(s^t)$, assumed uniformly bounded away from zero, is introduced here only for the sake of the analytical characterization of the sources of deviations from LOP in the model. Unless explicitly noted, we maintain the assumption $\chi(s^t)=\chi>0$.

3.5 Feasibility

Equilibrium must satisfy several market clearing conditions and feasibility constraints. The aggregate resource constraint is given by

$$d(s^t)+d^*(s^t)+A_d(s^t)a_d(s^t)+A_f(s^t)a_f(s^t)+h(s^t)\chi(s^t) \leq z(s^t)F(k(s^{t-1}),l(s^t)), \text{ all } s^t. \tag{22}$$

By representativeness, all producers and retailers choose the same allocation. The aggregate levels of marketing capital are thus given by $\bar{m}_f(s^t)=m_f(s^t)+\zeta$ and $\bar{m}_d(s^t)=m_d(s^t)+\zeta$ for all s^t, where $\zeta>0$ is an arbitrarily small constant that renders $\pi(s^t)$ well-defined even for $\bar{m}=0$ (needed for formal proof of existence). Finally, the contact probability $\pi(s^t)$ is consistent with the average relative marketing capital accumulated by the producers of each type, according to (17), and the world asset market clears, i.e. $b(s^t)+x\left(s^t\right)b^*\left(s^t\right)=0$.

3.6 Equilibrium

Equilibrium of this economy is, for all histories, an allocation for the domestic country d, f, a_d, a_f, m_d, m_f, H_d, H_f, c, l, b, i, k, an analogous allocation for the foreign country, prices in the domestic country P_d, P_f, p_d, p_f, v, Q, analogous prices in the foreign country, meeting probabilities π and π^*, the aggregates $\bar{m}_d, \bar{m}_d^*$ and $\bar{m}_f, \bar{m}_f^*$, and the real exchange rate x, such the allocation satisfies the feasibility conditions, and, given prices, the allocation solves the household problem, producer problem, and satisfies the retailer zero profit condition.

Proposition 2 *Assuming strictly positive initial values of all state variables, and neoclassical assumptions on utility and production function, the equilibrium exists.*

Proof. We prove existence by considering an operator $\mathcal{T}$, which, by definition, returns the equilibrium allocation at its fixed point. This operator is essentially a planning problem that emulates the distortions implied by bargaining and search. Below, we first define this problem (operator $\mathcal{T}$), and, using the Brouwer fixed point theorem, we establish that the fixed point exists in a properly defined space. At the end, we explicitly verify that this fixed point satisfies all the requirements for equilibrium.

The operator $\mathcal{T}$ is assumed to take as given the finite dimensional positive vector:

$$\mathcal{A} \equiv \{h^x(s^t), h^{*x}(s^t), a_d^x(s^t), a_f^x(s^t), a_d^{*x}(s^t), a_f^{*x}(s^t)\}_{t=1..T, s^t \in S^t}$$

and return an analogous vector implied by the solution of the following problem (Lagrange multiplier in brackets, set $Omega$ will be defined below):

$$\mathcal{T} : \max_{...\in\Omega} \sum_{t=1..T} \beta^t \sum_{s^t} \left(u(c(s^t), l(s^t)) + u(c^*(s^t), l^*(s^t)) \right) \mu(s^t) \tag{23}$$

subject to

$$\begin{aligned} (Q) &: c(s^t) + i(s^t) = G(d(s), f(s)), \\ (xQ) &: c^*(s^t) + i^*(s^t) = G(f^*(s), d^*(s)), \end{aligned} \tag{24}$$

$$\begin{aligned} (QP_d) &: d(s^t) \leq H_d(s^t) \\ (xQP_d^*) &: d^*(s^t) \leq H_d^*(s^t) \\ (QP_f) &: f(s^t) \leq H_f(s^t) \\ (xQP_f^*) &: f^*(s^t) \leq H_f^*(s^t) \end{aligned} \tag{25}$$

$$(QS_d) : H_d(s^t) = H_d(s^{t-1})(1-\delta_H) + \theta \frac{m_d(s^t)}{\bar{m}^x(s^t)} h^x(s^t) + (1-\theta) \frac{m_d^x(s^t)}{\bar{m}^x(s^t)} h(s^t) \tag{26}$$

$$(QS_d^*) : H_d^*(s^t) = H_d^*(s^{t-1})(1-\delta_H) + \theta \frac{m_d^*(s^t)}{\bar{m}^{x*}(s^t)} h^{x*}(s^t) + (1-\theta) \frac{m_d^x(s^t)}{\bar{m}^x(s^t)} h^*(s^t)$$

$$(QS_f) : H_f(s^t) = H_f(s^{t-1})(1-\delta_H) + \theta \frac{m_f(s^t)}{\bar{m}^x(s^t)} h^x(s^t) + (1-\theta) \frac{m_f^x(s^t)}{\bar{m}^x(s^t)} h(s^t)$$

$$(QS_f^*) : H_f^*(s^t) = H_f^*(s^{t-1})(1-\delta_H) + \theta \frac{m_f^*(s^t)}{\bar{m}^{x*}(s^t)} h^{x*}(s^t) + (1-\theta) \frac{m_f^{*x}(s^t)}{\bar{m}^{*x}(s^t)} h^*(s^t)$$

where $\bar{m}^x$ is defined as before (note: ζ is an assumed arbitrarily small positive constant), i.e.

$$\bar{m}^x(s^t) \equiv m_d^x(s^t) + m_f^x(s^t) + \zeta; \tag{27}$$

and $m_i^x(s^t)$ are assumed to be generated from the given sequence of $a_i^x(s^t)$ and the equilibrium law of motion (initial value $m_i^x(0)$ given and assumed equal to $m_i(0)$ in $T; i = d, f$; $m_i^x(s^t)$ is non-negative at every date and state by assumption imposed on the domain — the domain of $\mathcal{T}$ is discussed below) :

$$m_i^x(s^t) = m_i^x(s^{t-1})(1-\delta_m) + a_i^x(s^t) - (\phi/2) m_i^x(s^{t-1}) (\frac{a_i^x(s^t)}{m_d^x(s^{t-1})} - \delta_m)^2; \tag{28}$$

$$(Q\psi_d) : m_d(s^t) = m_d(s^{t-1})(1-\delta_m) + a_d(s^t) - (\phi/2) m_d(s^{t-1}) (\frac{a_d(s^t)}{m_d(s^{t-1})} - \delta_m)^2 \tag{29}$$

$$(Q\psi_d^*) : m_d^*(s^t) = m_d^*(s^{t-1})(1-\delta_m) + a_d^*(s^t) - (\phi/2) m_d^*(s^{t-1}) (\frac{a_d^*(s^t)}{m_d^*(s^{t-1})} - \delta_m)^2$$

$$(Q\psi_f) : m_f(s^t) = m_f(s^{t-1})(1-\delta_m) + a_f(s^t) - (\phi/2) m_f(s^{t-1}) (\frac{a_f(s^t)}{m_f(s^{t-1})} - \delta_m)^2$$

$$(Q\psi_f^*) : m_f^*(s^t) = m_f^*(s^{t-1})(1-\delta_m) + a_f^*(s^t) - (\phi/2) m_f^*(s^{t-1}) (\frac{a_f^*(s^t)}{m_f^*(s^{t-1})} - \delta_m)^2$$

$$(\lambda) : k(s^t) = k(s^{t-1})(1-\delta) + i(s^t) \tag{30}$$

$$(\lambda^*) : k^*(s^t) = k^*(s^{t-1})(1-\delta) + i^*(s^t)$$

$$(vQ) : z(s^t)F)(k(s^t), l(s^t)) \geq a_f(s^t) + a_d(s^t) + \chi h(s^t) + d(s^t) + d^*(s^t) \tag{31}$$
$$(xv^*Q) : z^*(s^t)F(k^*(s^t), l^*(s^t)) \geq a_f^*(s^t) + a_d^*(s^t) + \chi h^*(s^t) + f(s^t) + f^*(s^t),$$

Finally, the sequence $\mathcal{A}$ implied by the above problem is required to be contained in a set Ω that we now define. The fixed point of operator $\mathcal{T}$ is an element $\mathcal{A}^*$ such that $\mathcal{T}(\mathcal{A}^*) = \mathcal{A}^*$.

To define the set Ω, we use the constrains of our original economy to first define the set of feasible allocations $\mathcal{F}$. It consists of the following equations:

$$c(s^t) + i(s^t) = G(d(s), f(s)), \tag{32}$$
$$c^*(s^t) + i^*(s^t) = G(f^*(s), d^*(s)),$$

$$\begin{aligned} d(s^t) &\leq H_d(s^t) \\ d^*(s^t) &\leq H_d^*(s^t) \\ f(s^t) &\leq H_f(s^t) \\ f^*(s^t) &\leq H_f^*(s^t) \end{aligned} \tag{33}$$

$$\begin{aligned} H_d(s^t) &= H_d(s^{t-1})(1-\delta_H) + \frac{m_d(s^t)}{\bar{m}(s^t)}h(s^t) \\ H_d^*(s^t) &= H_d^*(s^{t-1})(1-\delta_H) + \frac{m_d^*(s^t)}{\bar{m}^*(s^t)}h^*(s^t) \\ H_f(s^t) &= H_f(s^{t-1})(1-\delta_H) + \frac{m_f(s^t)}{\bar{m}(s^t)}h(s^t) \\ H_f^*(s^t) &= H_f^*(s^{t-1})(1-\delta_H) + \frac{m_f^*(s^t)}{\bar{m}^*(s^t)}h^*(s^t) \end{aligned} \tag{34}$$

where

$$\bar{m}(s^t) \equiv m_d(s^t) + m_f(s^t) + \zeta; \tag{35}$$

$$m_d(s^t) = m_d(s^{t-1})(1-\delta_m) + a_d(s^t) - (\phi/2)m_d(s^{t-1})(\frac{a_d(s^t)}{m_d(s^{t-1})} - \delta_m)^2 \tag{36}$$
$$m_d^*(s^t) = m_d^*(s^{t-1})(1-\delta_m) + a_d^*(s^t) - (\phi/2)m_d^*(s^{t-1})(\frac{a_d^*(s^t)}{m_d^*(s^{t-1})} - \delta_m)^2$$
$$m_f(s^t) = m_f(s^{t-1})(1-\delta_m) + a_f(s^t) - (\phi/2)m_f(s^{t-1})(\frac{a_f(s^t)}{m_f(s^{t-1})} - \delta_m)^2$$
$$m_f^*(s^t) = m_f^*(s^{t-1})(1-\delta_m) + a_f^*(s^t) - (\phi/2)m_f^*(s^{t-1})(\frac{a_f^*(s^t)}{m_f^*(s^{t-1})} - \delta_m)^2$$

$$k(s^t) = k(s^{t-1})(1-\delta_m) + i(s^t) \tag{37}$$
$$k^*(s^t) = k^*(s^{t-1})(1-\delta_m) + i^*(s^t)$$

$$z(s^t)F(k(s^t), l(s^t)) \geq a_f(s^t) + a_d(s^t) + \chi h(s^t) + d(s^t) + d^*(s^t) \tag{38}$$
$$z^*(s^t)F(k^*(s^t), l^*(s^t)) \geq a_f^*(s^t) + a_d^*(s^t) + \chi h^*(s^t) + f(s^t) + f^*(s^t),$$

non-negativity constraints imposed on all variables (it is sufficient for a to be bounded from below), upper bound on the labor supply l, law of motions for z, z^*, and the constraints on the forcing process as stated in the paper (shocks on finite and discrete support, z, z^* uniformly bounded away from zero).

The above system of equilibrium constraints (within any finite time horizon T) is non-empty, as long as positive initial values for state variables are assumed. To see this, take, for example, a sequence of controls arbitrarily close to $c = d = f = i = 0, a = 0, h = 0$, and note that due to the assumed exponential decay in the laws of motion and positive initial values of state variables, the values of all state variables are guaranteed to remain strictly positive over any finite horizon (for arbitrarily low values of controls). Moreover, since within each period, the constraints are defined by continuous functions and strict inequalities, for any feasible choice of control variables (non-negative) these constraints define a compact set of future state values, and thus a compact set of sequences of variables that can be generated from the feasible sequences of controls. Additionally, this set is also convex, as these defining functions are either linear or convex (under appropriate inequality warranting convexity of the implied set). Summarizing, the set of feasible allocations $\mathcal{F}$

is non-empty, convex, and compact.

Using the defined above set of feasible allocations $\mathcal{F}$ (of the original economy), we next define Ω as a truncation of $\mathcal{F}$ to the set of selected variables that are included in the vector $\mathcal{A}$ (given above), i.e. $\Omega \equiv \mathcal{F}|_{\mathcal{A}}$. Clearly, such set Ω is trivially non-empty, convex and also compact. To satisfy the requirements of Brouwer fixed point theorem, we next must establish that $\mathcal{T}$ is a well-defined continuous function on this space, and maps Ω onto itself.

To establish non-emptiness of the constraints in $\mathcal{T}$, note that the planner can always choose the allocations that yields $\mathcal{A}$ as the solving sequence of $\mathcal{T}$. Since for the values of $a = a^x, h = h^x$ chosen under maximization in $\mathcal{T}$, by construction, the constraints (24)-(31) collapse to the set of equilibrium constraints that have been used to define $\mathcal{F}$ (and Ω) and shown non-empty, such choice must be feasible. For reasons analogous to the ones given above (in the context of definition of $\mathcal{F}$ and Ω), given any sequence $\mathcal{A} \in \Omega$, the constraint set in $\mathcal{T}$ is thus not only non-empty, but also convex and compact. (Here, in the context of other variables, note that the constraint set underlying the max operator of $\mathcal{T}$ is required to be in Ω, but since this set is convex and compact, these properties are preserved. Non-emptiness of the intersection follows from the fact that Ω shares one point with the constraints underlying the operator $\mathcal{T}$ [26]) Now, since the objective function of the problem defining operator $\mathcal{T}$ is a continuous and concave function, by standard results in optimization, we know that the solution to $\mathcal{T}$ exists and is unique. Moreover, by Maximum Theorem, we know that this solution thus defines a continuous function w.r.t. $\mathcal{A}$, and it maps Ω onto itself by construction (recall: at the end, the constraint set underlying the definition of $\mathcal{T}$ is intersected with Ω). We thus conclude that $\mathcal{T} : \Omega \to \Omega$ satisfies the requirements of the Brouwer fixed point theorem (i.e. $\mathcal{T}$ is a continuous function defined on a non-empty, convex, compact and finite dimensional subset of R^n, Ω, that maps this set onto itself.)

To show that the fixed point returns the equilibrium allocation, note that the set of first order conditions are necessary and sufficient for the solution to $mathcalT$ (by the above properties and concavity of the objective function), and thus we can compare them to the equilibrium conditions. In the the decentralized equilibrium, the set of first order conditions is also necessary and sufficient to solve the household problem and the producer problem, and thus this system can be used as an

[26]The constraints collapse to the equilibrium ones when the controls equal the ones from the exogenous sequence $\mathcal{A}$.

equivalent formulation of the definition of equilibrium given in text.

To show equivalence, first note that the constraint set of the operator $\mathcal{T}$ collapses to the equilibrium constraint set at the fixed point. Furthermore, note that the remaining first order conditions are also identical (compare with first order conditions included in the next section). Using the Lagrange multipliers imposed as stated in brackets, with a normalization by dividing the entire Lagrangian by the first multiplier at s^t, we derive the following first order conditions for operator T (the algebra is tedious and the supporting Mathematica file may be helpful here):

- Derivative wrt c gives the intertemporal price (compare with (50), and the defining relation $Q(s^t) \equiv Q(s_t|s^{t-1})Q(s^{t-1})$)
$$Q(s^t) = u'(c),$$

- The perfect risk sharing condition (under symmetry) is obtained by combining (1) above with the derivative wrt c^* (compare with (14))
$$x(s^t) = \frac{u'(c^*)}{u'(c)},$$

- Derivative wrt d gives the household demand equations (compare with (13))
$$P_d(s^t) = G_d(d(s^t), f(s^t))$$

- Derivative wrt H_d gives the definition of total surplus from trade (compare with the result of summation of $J(s^t) + W(s^t)$ using (54) and (63) and bargaining equations (77))
$$S_d(s^t) = P_d(s^t) - v(s^t) + E_{s^t}\left[Q(s_{t+1}|s^t)S_d(s^{t+1})\right],$$

- Derivative wrt a_d gives the producer first order condition wrt a_d (compare with (62))
$$\psi_d = \frac{v(s^t)}{1 - \phi(\frac{a_d(s^t)}{m_d(s^{t-1})} - \delta_m)},$$

- Derivative wrt m_d gives the producer first order condition wrt m (compare with (64) and (77))

$$\psi_d(s^t) = E\left[\psi_d(1-\delta_m + \frac{\phi}{2}((\frac{a_d}{m_d})^2 - \delta_m^2))\right] + \frac{h^x}{\bar{m}^x}\theta S_d,$$

- Derivative wrt h gives the analog of the retailer zero profit condition (compare with (54) and (77))

$$(1-\theta)\frac{m_d}{\bar{m}^x}S_d + (1-\theta)\frac{m_f}{\bar{m}^x}S_d = \chi v,$$

  and finally,

- Euler equation and labor leisure choice equation can be similarly obtained by evaluating derivatives wrt i, k and l.

The foreign country related conditions follow by symmetry and are omitted. QED.

4 Equilibrium Conditions

4.1 Dynamic Market Expansion Friction (Benchmark)

Here, we derive equilibrium condition for $t = 1, T-1$ under the assumption that market expansion friction is given by the *dynamic* formulation as defined in the setup of the model from previous section. These conditions include all enumerated equations, except for (77), (50), (64) and (40). We use these conditions to solve the model.

Domestic households solve

$$\max \sum_{t=0}^{T} \beta^t \sum_{s^t} (u(c(s^t), l(s^t))\mu(s^t)$$

subject to

$$c(s^t) + k(s^{t+1}) = G(d, f) + (1-\delta_k)\, k(s^t), \quad \left(\mu\left(s^t\right)\lambda\left(s^t\right)\right) \tag{39}$$

$$P_d\left(s^t\right) d\left(s^t\right) + P_f\left(s^t\right) f\left(s^t\right) + \sum_{s_{t+1}} Q(s_{t+1}|s^t) b(s_{t+1}, s^t)\mu(s_{t+1}|s^t)$$

$$= b\left(s^t\right) + w\left(s^t\right) l\left(s^t\right) + r\left(s^t\right) k\left(s^t\right) + \Pi\left(s^t\right), \quad \left(\mu\left(s^t\right)\sigma\left(s^t\right)\right)$$

$$b\left(s^{t+1}\right) \geq B,\ k\left(s^0\right), b\left(s^0\right) \text{ given,}$$

where $(s_{t+1}, s^t) \equiv s^{t+1}$, $\mu(s_{t+1}|s^t) \equiv \mu(s^{t+1})/\mu(s^t)$ and similarly $Q(s_{t+1}|s^t) \equiv Q(s^{t+1})/Q(s^t)$.

First order conditions (excluding constraints) are:

(Lagrange multipliers σ, λ defined in the brackets next to the constraints.)

$$
\begin{aligned}
c &: \beta^t u_c\left(s^t\right) = \lambda\left(s^t\right), \\
n &: \beta^t u_l\left(s^t\right) = -\sigma\left(s^t\right) w\left(s^t\right), \\
d &: \sigma\left(s^t\right) P_d\left(s^t\right) = G_d\left(s^t\right) \lambda\left(s^t\right), \\
\hat{f} &: \sigma\left(s^t\right) P_f\left(s^t\right) = G_f\left(s^t\right) \lambda\left(s^t\right), \\
b &: Q(s_{t+1}|s^t) = \frac{\sigma\left(s^{t+1}\right)}{\sigma\left(s^t\right)}, \\
k_{+1} &: \lambda\left(s^t\right) = E_{s^t}\left[\left(1-\delta_k\right)\lambda\left(s^{t+1}\right) + r\left(s^{t+1}\right)\sigma\left(s^{t+1}\right)\right].
\end{aligned}
$$

Normalize prices using the numéraire assumption, to derive

$$
\begin{aligned}
P_d\left(s^t\right) &= G_d\left(s^t\right), \\
P_f\left(s^t\right) &= G_f\left(s^t\right),
\end{aligned} \tag{40}
$$

and

$$\lambda\left(s^t\right) = \sigma\left(s^t\right).$$

Simplify, to obtain

$$\frac{u_l\left(s^t\right)}{u_c\left(s^t\right)} = -w\left(s^t\right), \tag{41}$$

$$P_d\left(s^t\right) = G_d\left(s^t\right), \tag{42}$$

$$P_f\left(s^t\right) = G_f\left(s^t\right), \tag{43}$$

$$Q(s_{t+1}|s^t) = \frac{\sigma\left(s^{t+1}\right)}{\sigma\left(s^t\right)} = \beta\frac{u_{1c}\left(s^{t+1}\right)}{u_{1c}\left(s^t\right)},$$

$$u_{1c}\left(s^t\right) = \beta E_{s^t} u_{1c}\left(s^{t+1}\right)\left[\left(1-\delta_k\right) + r\left(s^{t+1}\right)\right]. \tag{44}$$

Foreign households solve

$$\max \sum_{t=0}^{T} \beta^t \sum_{s^t} (u(c^*(s^t), l^*(s^t))\mu(s^t)$$

subject to

$$c(s^t) + k(s^{t+1}) = G\left(d^*, f^*\right) + (1-\delta_k)\, k(s^t), \quad \left(\mu\left(s^t\right)\lambda^*\left(s^t\right)\right) \tag{45}$$
$$P_d^*\left(s^t\right) d^*\left(s^t\right) + P_f^*\left(s^t\right) f^*\left(s^t\right) + \sum_{s_{t+1}} Q^*(s_{t+1}|s^t) b^*(s_{t+1}, s^t)\mu(s_{t+1}|s^t)$$
$$= b^*\left(s^t\right) + w^*\left(s^t\right) l^*\left(s^t\right) + r^*\left(s^t\right) k^*\left(s^t\right), \quad \left(\mu\left(s^t\right)\sigma^*\left(s^t\right)\right)$$
$$b^*\left(s^{t+1}\right) \geq B, k\left(s^1\right), b\left(s^1\right) \text{ given}$$

which gives

$$\frac{u_l\left(s^t\right)}{u_c\left(s^t\right)} = -w^*\left(s^t\right), \tag{46}$$
$$P_d^*\left(s^t\right) = G_d^*\left(s^t\right), \tag{47}$$
$$P_f^*\left(s^t\right) = G_f^*\left(s^t\right), \tag{48}$$
$$Q^*(s_{t+1}|s^t) = \frac{\sigma^*\left(s^{t+1}\right)}{\sigma^*\left(s^t\right)} = \beta\frac{u_{2c}\left(s^{t+1}\right)}{u_{2c}\left(s^t\right)},$$
$$u_{2c}\left(s^t\right) = \beta E_{s^t} u_{2c}\left(s^{t+1}\right)\left[(1-\delta_k) + r^*\left(s^{t+1}\right)\right]. \tag{49}$$

Non-arbitrage condition imposed on asset prices, under ex-ante symmetry between countries, implies

$$Q^*(s_{t+1}|s^t) = \beta\frac{u_c^*\left(s^{t+1}\right)}{u_c^*\left(s^t\right)}, \tag{50}$$
$$Q(s_{t+1}|s^t) = \beta\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)},$$
$$Q^*(s_{t+1}|s^t) = \frac{x\left(s^{t+1}\right)}{x\left(s^t\right)} Q\left(s_{t+1}|s^t\right),$$
$$\frac{u_c^*\left(s^{t+1}\right)}{u_c^*\left(s^t\right)} = \frac{x\left(s^{t+1}\right)}{x\left(s^t\right)}\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)},$$
$$x\left(s^t\right) = x(s^0)\frac{u_c^*\left(s^t\right)}{u_c\left(s^t\right)}, \; x(s^1) = 1. \tag{51}$$

Retailers zero profit condition implies

$$J_d\left(s^t\right)\pi\left(s^t\right)+\left(1-\pi\left(s^t\right)\right)J_f\left(s^t\right)=v\left(s^t\right)\chi(s^t), \tag{52}$$

$$\left(1-\pi^*\left(s^t\right)\right)J_d^*\left(s^t\right)+\pi^*\left(s^t\right)J_f^*\left(s^t\right)=v^*\left(s^t\right)\chi^*(s^t), \tag{53}$$

where

$$J_d\left(s^t\right)=\left(P_d\left(s^t\right)-p_d\left(s^t\right)\right)+\left(1-\delta_H\right)E_{s^t}\left[Q\left(s_{t+1}|s^t\right)J_d\left(s^{t+1}\right)\right], \tag{54}$$

$$J_f\left(s^t\right)=\left(P_f\left(s^t\right)-p_f\left(s^t\right)\right)+\left(1-\delta_H\right)E_{s^t}\left[Q\left(s_{t+1}|s^t\right)J_f\left(s^{t+1}\right)\right], \tag{55}$$

$$J_d^*\left(s^t\right)=\left(P_d^*\left(s^t\right)-p_d^*\left(s^t\right)\right)+\left(1-\delta_H\right)E_{s^t}\left[\frac{x\left(s^{t+1}\right)}{x\left(s^t\right)}Q\left(s_{t+1}|s^t\right)J_d^*\left(s^{t+1}\right)\right], \tag{56}$$

$$J_f^*\left(s^t\right)=\left(P_f^*\left(s^t\right)-p_f^*\left(s^t\right)\right)+\left(1-\delta_H\right)E_{s^t}\left[\frac{x\left(s^{t+1}\right)}{x\left(s^t\right)}Q\left(s_{t+1}|s^t\right)J_f^*\left(s^{t+1}\right)\right]. \tag{57}$$

Domestic country producers solve

(where, we define recursively: $Q(s^t)\equiv Q(s_t|s^{t-1})Q(s^{t-1})$)

$$\max\sum_{t=0}^{T}\sum_{s^t}Q\left(s^t\right)\left[\left(p_d\left(s^t\right)-v_d\left(s^t\right)\right)d\left(s^t\right)+\left(x\left(s^t\right)p_d^*\left(s^t\right)-v\left(s^t\right)\right)d^*\left(s^t\right)+\right.$$

$$\left.-(A(s^t)v\left(s^t\right)a_d\left(s^t\right)+A^*(s^t)x\left(s^t\right)v^*\left(s^t\right)a_d^*\left(s^t\right))\right]\mu(s^t)$$

subject to (note: ϕ is a function here, its derivatives are defined at the end of the section)

$$d(s^t)\leq H_d(s^t),\qquad \left(Q\left(s^t\right)\psi_d(s^t)\mu(s^t)\right)$$

$$d^*(s^t)\leq H_d^*(s^t),\qquad \left(Q\left(s^t\right)\psi_d^*(s^t)\mu(s^t)\right)$$

and

$$H_d(s^t) = \frac{m_d(s^t)}{\bar{m}(s^t)} h(s^t) + (1-\delta_H) H_d(s^{t-1}), \quad \left(Q\left(s^t\right) W_d\left(s^t\right) \mu(s^t)\right) \tag{58}$$

$$H_d^*(s^t) = \frac{m_d^*(s^t)}{\bar{m}^*(s^t)} h^*(s^t) + (1-\delta_H) H_d^*(s^{t-1}), \quad \left(Q\left(s^t\right) W_d^*\left(s^t\right) \mu(s^t)\right) \tag{59}$$

$$m_d\left(s^t\right) = (1-\delta_m) m_d(s^{t-1}) + a_d(s^t) - \frac{\phi}{2} m_d(s^{t-1}) \left(\frac{a_d(s^t)}{m_d(s^{t-1})} - \delta_m\right)^2, \quad \left(Q\left(s^t\right) \mu\left(s^t\right)\right) \tag{60}$$

$$m_d^*(s^t) = (1-\delta_m) m_d^*(s^{t-1}) + a_d^*(s^t) - \frac{\phi}{2} m_d^*(s^{t-1}) \left(\frac{a_d^*(s^t)}{m_d^*(s^{t-1})} - \delta_m\right)^2. \quad \left(Q\left(s^t\right) \mu\left(s^t\right)\right) \tag{61}$$

where

$$v(s^t) = \arg\min_{k,l} \left\{rk + wl \;\mid\; zF\left(k,l\right) = 1\right\}.$$

First order conditions, with Lagrange multipliers imposed as indicated next to the constraints, give:

$$a_d : A(s^t) v\left(s^t\right) = \psi_d\left(s^t\right) \left(1 - \phi\left(\frac{a_d(s^t)}{m_d(s^{t-1})} - \delta_m\right)\right), \tag{62}$$

$$a_d^* : A^*(s^t) x\left(s^t\right) v^*\left(s^t\right) = \psi_d^*\left(s^t\right) \left(1 - \phi\left(\frac{a_d^*(s^t)}{m_d^*(s^{t-1})} - \delta_m\right)\right),$$

$$H_d : p_d\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h) E_{s^t}\left[Q\left(s^{t+1}|s^t\right) W_d\left(s^{t+1}\right)\right] = W_d\left(s^t\right), \tag{63}$$

$$H_d^* : x\left(s^t\right) p_d^*\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h) E_{s^t}[Q\left(s^{t+1}|s^t\right) W_d^*\left(s^{t+1}\right)] = W_d^*\left(s^t\right),$$

$$m_d : -\psi_d\left(s^t\right) + E_{s^t}\left[Q\left(s_{t+1}|s^t\right)\left(1-\delta_m - \frac{\phi}{2}\left[\delta_m^2 - \left(\frac{a_d(s^{t+1})}{m_d(s^t)}\right)^2\right]\right)\right] + \frac{h\left(s^t\right)}{\bar{m}\left(s^t\right)} W_d\left(s^t\right) = 0, \tag{64}$$

$$m_d^* : -\psi_d^*\left(s^t\right) + E_{s^t}\left[Q\left(s_{t+1}|s^t\right)\left(1-\delta_m - \frac{\phi}{2}\left[\delta_m^2 - \left(\frac{a_d^*(s^{t+1})}{m_d^*(s^t)}\right)^2\right]\right)\right] + \frac{h^*\left(s^t\right)}{\bar{m}^*\left(s^t\right)} W_d^*\left(s^t\right) = 0.$$

After simplifications, we obtain

$$H_d : W_d\left(s^t\right) = p_d\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h)\, E_{s^t}\left[Q\left(s_{t+1}|s^t\right) W_d\left(s^{t+1}\right)\right], \tag{65}$$

$$H_d^* : W_d^*\left(s^t\right) = x\left(s^t\right) p_d^*\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h)\, E_{s^t}\left[Q\left(s_{t+1}|s^t\right) W_d^*\left(s^{t+1}\right)\right], \tag{66}$$

$$\begin{aligned} m_d : W_d\left(s^t\right) = {} & \frac{\bar{m}\left(s^t\right)}{h\left(s^t\right)} \frac{A(s^t) v\left(s^t\right)}{1-\phi\left(\frac{a_d(s^t)}{m_d(s^{t-1})}-\delta_m\right)} + \\ & - \frac{\bar{m}\left(s^t\right)}{h\left(s^t\right)} E_{s^t}\left[Q\left(s_{t+1}, s^t\right) A(s^{t+1}) v\left(s^{t+1}\right) \frac{1-\delta_m-\frac{\phi}{2}\left[\delta_m^2-\left(\frac{a_d(s^{t+1})}{m_d(s^t)}\right)^2\right]}{1-\phi\left(\frac{a_d(s^t)}{m_d(s^{t-1})}-\delta_m\right)}\right], \end{aligned} \tag{67}$$

$$\begin{aligned} m_d^* : W_d^*\left(s^t\right) = {} & \frac{\bar{m}^*\left(s^t\right)}{h^*\left(s^t\right)} \frac{A^*(s^t) x\left(s^t\right) v^*\left(s^t\right)}{1-\phi\left(\frac{a_d^*(s^t)}{m_d^*(s^{t-1})}-\delta_m\right)} + \\ & - \frac{\bar{m}^*\left(s^t\right)}{h^*\left(s^t\right)} E_{s^t}\left[Q\left(s_{t+1}, s^t\right) x\left(s^{t+1}\right) A^*(s^{t+1}) v^*\left(s^{t+1}\right) \frac{1-\delta_m-\frac{\phi}{2}\left[\delta_m^2-\left(\frac{a_d^*(s^{t+1})}{m_d^*(s^t)}\right)^2\right]}{1-\phi\left(\frac{a_d^*(s^t)}{m_d^*(s^{t-1})}-\delta_m\right)}\right]. \end{aligned} \tag{68}$$

Foreign producers solve

$$\begin{aligned} \max \sum_{t=0}^{T} \sum_{s^t} Q^*\left(s^t\right) [& \left(\frac{p_f\left(s^t\right)}{x\left(s^t\right)} - v^*\left(s^t\right)\right) f\left(s^t\right) + \left(p_f^*\left(s^t\right) - v_f\left(s^t\right)\right) f^*\left(s^t\right) + \\ & - (A(s^t) \frac{v\left(s^t\right)}{x(s^t)} a^f\left(s^t\right) - A^*(s^t) v^*\left(s^t\right) a_f^*\left(s^t\right))]\mu(s^t) \end{aligned}$$

subject to

$$f(s^t) \leq H_f(s^t), \qquad \left(Q^*\left(s^t\right) \psi_f(s^t) \mu(s^t)\right)$$

$$f^*(s^t) \leq H_f^*(s^t), \qquad \left(Q^*\left(s^t\right) \psi_f^*(s^t) \mu(s^t)\right)$$

and

$$H_f\left(s^t\right) = \frac{m_f\left(s^t\right)}{\bar{m}\left(s^t\right)} h\left(s^t\right) + (1-\delta) H_f\left(s^{t-1}\right), \quad \left(Q^*\left(s^t\right) W_f\left(s^t\right) \mu(s^t)\right) \tag{69}$$

$$H_f^*\left(s^t\right) = \frac{m_f^*\left(s^t\right)}{\bar{m}^*\left(s^t\right)} h^*\left(s^t\right) + (1-\delta) H_f^*\left(s^{t-1}\right), \quad \left(Q^*\left(s^t\right) W_f^*\left(s^t\right) \mu(s^t)\right) \tag{70}$$

$$m_f(s^t) = m_f(s^{t-1})(1-\delta_m) + a_f(s^t) - \frac{\phi}{2} m_f(s^{t-1}) \left(\frac{a_f(s^t)}{m_f(s^{t-1})} - \delta_m\right)^2, \quad \left(Q^*\left(s^t\right) \mu\left(s^t\right)\right) \tag{71}$$

$$m_f^*(s^t) = (1-\delta_m) m_f^*(s^{t-1}) + a_f^*(s^t) - \frac{\phi}{2} m_f^*(s^{t-1}) \left(\frac{a_f^*(s^t)}{m_f^*(s^{t-1})} - \delta_m\right)^2, \quad \left(Q^*\left(s^t\right) \mu^*\left(s^t\right)\right) \tag{72}$$

where

$$v(s^t) = \arg\min_{k,l} \left\{rk + wl \;\mid\; zF(k,l) = 1\right\},$$

and $Q(s^t)$ is given by recursive law: $Q(s^t) = Q(s^{t-1})Q(s_t|s^{t-1})$, with $Q(s_t|s^{t-1})$ being the pricing kernel derived from the household problem.

After simplifications, the first order conditions give (Lagrange multipliers imposed as indicated above):

$$H_f : W_f\left(s^t\right) = \frac{p_f\left(s^t\right)}{x\left(s^t\right)} - v^*\left(s^t\right) + (1-\delta_H) E_{s^t}\left[Q^*\left(s_{t+1}|s^t\right) W_f\left(s^{t+1}\right)\right], \tag{73}$$

$$H_f^* : W_f^*\left(s^t\right) = p_f^*\left(s^t\right) - v_f\left(s^t\right) + (1-\delta_H) E_{s^t}\left[Q^*\left(s_{t+1}|s^t\right) W_f^*\left(s^{t+1}\right)\right], \tag{74}$$

$$\begin{aligned} m_f : W_f\left(s^t\right) = {} & \frac{\bar{m}\left(s^t\right)}{h\left(s^t\right)} \frac{A(s^t) v\left(s^t\right)/x\left(s^t\right)}{\left(1-\phi\left(\frac{a_f(s^t)}{m_f(s^{t-1})} - \delta_m\right)\right)} + \\ & - \frac{\bar{m}\left(s^t\right)}{h\left(s^t\right)} E_{s^t}\left[\frac{x\left(s^{t+1}\right)}{x\left(s^t\right)} Q\left(s_{t+1}|s^t\right) A(s^t) \frac{v\left(s^{t+1}\right)}{x\left(s^{t+1}\right)} \frac{1-\delta_m - \frac{\phi}{2}\left[\delta_m^2 - \left(\frac{a_f(s^{t+1})}{m_f(s^t)}\right)^2\right]}{1-\phi\left(\frac{a_f(s^t)}{m_f(s^{t-1})} - \delta_m\right)}\right], \end{aligned} \tag{75}$$

$$\begin{aligned} m_f^* : W_f^*\left(s^t\right) = {} & \frac{\bar{m}^*\left(s^t\right)}{h^*\left(s^t\right)} \frac{A^*(s^t) v^*\left(s^t\right)}{\left(1-\phi\left(\frac{a_f^*(s^t)}{m_f^*(s^{t-1})} - \delta_m\right)\right)} + \\ & - \frac{\bar{m}^*\left(s^t\right)}{h^*\left(s^t\right)} E_{s^t}\left[\frac{x\left(s^{t+1}\right)}{x\left(s^t\right)} Q\left(s_{t+1}|s^t\right) A^*(s^{t+1}) v^*\left(s^{t+1}\right) \frac{1-\delta_m - \frac{\phi}{2}\left[\delta_m^2 - \left(\frac{a_f^*(s^{t+1})}{m_f^*(s^t)}\right)^2\right]}{1-\phi\left(\frac{a_f^*(s^t)}{m_f^*(s^{t-1})} - \delta_m\right)}\right]. \end{aligned} \tag{76}$$

Bargaining gives

$$W_d(s^t) = \theta(W_d(s^t) + J_d(s^t)) \equiv \theta S_d(s^t), \tag{77}$$

$$W_d^*(s^t) = \theta(W_d^*(s^t) + x(s^t)J_d^*(s^t)) \equiv \theta S_d^*(s^t), \tag{78}$$

$$W_f(s^t)/x(s^t) = \theta(W_f(s^t)/x(s^t) + J_f(s^t)) \equiv \theta S_f(s^t), \tag{79}$$

$$x(s^t)W_f^*(s^t) = \theta x(s^t)(W_f^*(s^t) + J_f^*(s^t)) \equiv \theta S_f^*(s^t), \tag{80}$$

where we define total surplus S to be expressed in the domestic country units.

Cost minimization for Cobb-Douglas production function $zk^{\alpha}l^{1-\alpha}$ implies

$$v(s^t) = \frac{w(s^t)^{1-\alpha}r(s^t)^{\alpha}}{z(s^t)}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}, \tag{81}$$

$$v(s^t) = \frac{w^*(s^t)^{1-\alpha}r^*(s^t)^{\alpha}}{z^*(s^t)}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}, \tag{82}$$

and

$$\frac{k(s^t)}{l(s^t)} = \frac{w(s^t)}{r(s^t)}\frac{\alpha}{1-\alpha}, \tag{83}$$

$$\frac{k^*(s^t)}{l^*(s^t)} = \frac{w^*(s^t)}{r^*(s^t)}\frac{\alpha}{1-\alpha}. \tag{84}$$

Resource feasibility requires

$$d(s^t) + d^*(s^t) + A(s^t)(a_d(s^t) + a_f(s^t)) + \chi(s^t)h(s^t) = z(s^t)F\left(k, l\right)(s^t), \tag{85}$$

$$f(s^t) + f^*(s^t) + A^*(s^t)(a_d^*(s^t) + a_f^*(s^t)) + \chi^*(s^t)h^*(s^t) = z^*(s^t)F\left(k^*, l^*\right)(s^t), \tag{86}$$

$$f(s^t) = H_f(s^t), \tag{87}$$

$$f^*(s^t) = H_f^*(s^t), \tag{88}$$

$$d(s^t) = H_d(s^t), \tag{89}$$

$$d^*(s^t) = H_d^*(s^t), \tag{90}$$

$$\bar{m}_f^*(s^t) = m_f^*(s^t), \tag{91}$$

$$\bar{m}_d^*(s^t) = m_d^*(s^t), \tag{92}$$

$$\bar{m}_f(s^t) = m_f(s^t), \tag{93}$$

$$\bar{m}_d(s^t) = m_d(s^t), \tag{94}$$

$$\pi(s^t) = \frac{\bar{m}_d(s^t)}{\bar{m}(s^t)}, \tag{95}$$

$$\pi^*(s^t) = \frac{\bar{m}_f^*(s^t)}{\bar{m}^*(s^t)}, \tag{96}$$

$$\bar{m}(s^t) = \bar{m}_d(s^t) + \bar{m}_f(s^t), \tag{97}$$

$$\bar{m}^*(s^t) = \bar{m}_d^*(s^t) + \bar{m}_f^*(s^t), \tag{98}$$

$$b(s^t) + x(s^t)b(s^t) = 0.$$

Exogenous process

$$\log z(s^{t+1}) = \psi \log z(s^t) + \varepsilon(s^t), \tag{99}$$

$$\log z^*(s^{t+1}) = \psi \log z^*(s^t) + \varepsilon^*(s^t). \tag{100}$$

For the calculation of the deterministic steady state around which we linearize the model, refer to section "Calibration".

4.2 Static Market Expansion Friction (Setup Used in Section 3)

Here, we derive equilibrium condition for $t = 1, T-1$ under the assumption that market expansion friction is given by the *static* formulation as defined in Section 3 of the paper:

$$m_d(s^t) = a_d(s^t) - \frac{\phi}{2}(\frac{a_d(s^t)}{a_d^{ss}} - 1)^2,$$

where a_d^{ss} is a parameter assumed equal to the deterministic steady state value of $a_d(s^t)$ (other goods/countries by analogy). This specification of the model is used in the analytic section, and in the quantitative section we parameterize it to verify that it yields similar results for prices. Equilibrium conditions include all enumerated equations.

Domestic households solve

$$\max \sum_{t=0}^{T} \beta^t \sum_{s^t} (u(c(s^t), l(s^t))\mu(s^t)$$

subject to

$$c(s^t) + k(s^{t+1}) = G(d, f) + (1 - \delta_k)\, k(s^t), \quad \left(\mu\left(s^t\right)\lambda\left(s^t\right)\right) \tag{101}$$

$$P_d\left(s^t\right) d\left(s^t\right) + P_f\left(s^t\right) f\left(s^t\right) + \sum_{s_{t+1}\in S} Q(s_{t+1}|s^t) b(s_{t+1}, s^t)\mu(s_{t+1}|s^t)$$
$$= b\left(s^t\right) + w\left(s^t\right) l\left(s^t\right) + r\left(s^t\right) k\left(s^t\right) + \Pi\left(s^t\right), \quad \left(\mu\left(s^t\right)\sigma\left(s^t\right)\right)$$

$$b\left(s^{t+1}\right) \geq B,\ k\left(s^0\right), b\left(s^0\right) \text{ given} \tag{102}$$

where $\mu(s_{t+1}|s^t) \equiv \mu(s^{t+1})/\mu(s^t)$ and similarly $Q(s_{t+1}|s^t) \equiv Q(s^{t+1})/Q(s^t)$.

First order conditions (excluding constraints) are:

(Lagrange multipliers σ, λ defined in the brackets next to the constraints.)

$$c : \beta^t u_c\left(s^t\right) = \lambda\left(s^t\right),$$
$$n : \beta^t u_l\left(s^t\right) = -\sigma\left(s^t\right) w\left(s^t\right),$$
$$d : \sigma\left(s^t\right) P_d\left(s^t\right) = G_d\left(s^t\right)\lambda\left(s^t\right),$$
$$\hat{f} : \sigma\left(s^t\right) P_f\left(s^t\right) = G_f\left(s^t\right)\lambda\left(s^t\right),$$
$$b : Q(s_{t+1}|s^t) = \frac{\sigma\left(s^{t+1}\right)}{\sigma\left(s^t\right)},$$
$$k_{+1} : \lambda\left(s^t\right) = E_{s^t}\left[(1-\delta_k)\lambda\left(s^{t+1}\right) + r\left(s^{t+1}\right)\sigma\left(s^{t+1}\right)\right].$$

Normalize prices using the numéraire assumption, to derive

$$P_d\left(s^t\right) = G_d\left(s^t\right),$$
$$P_f\left(s^t\right) = G_f\left(s^t\right),$$

and

$$\lambda\left(s^t\right) = \sigma\left(s^t\right).$$

Simplify, to obtain

$$\frac{u_l\left(s^t\right)}{u_c\left(s^t\right)} = -w\left(s^t\right), \tag{103}$$
$$P_d\left(s^t\right) = G_d\left(s^t\right), \tag{104}$$
$$P_f\left(s^t\right) = G_f\left(s^t\right), \tag{105}$$
$$Q(s_{t+1}|s^t) = \frac{\sigma\left(s^{t+1}\right)}{\sigma\left(s^t\right)} = \beta\frac{u_{1c}\left(s^{t+1}\right)}{u_{1c}\left(s^t\right)}, \tag{106}$$
$$u_{1c}\left(s^t\right) = \beta E_{s^t} u_{1c}\left(s^{t+1}\right)\left[(1-\delta_k) + r\left(s^{t+1}\right)\right]. \tag{107}$$

Foreign households solve

$$\max \sum_{t=0}^{T} \beta^t \sum_{s^t} (u(c^*(s^t), l^*(s^t))\mu(s^t)$$

subject to

$$c(s^t) + k(s^{t+1}) = G\left(d^*, f^*\right) + \left(1-\delta_k\right)k(s^t), \quad \left(\mu\left(s^t\right)\lambda^*\left(s^t\right)\right) \tag{108}$$
$$P_d^*\left(s^t\right)d^*\left(s^t\right) + P_f^*\left(s^t\right)f^*\left(s^t\right) + \sum_{s_{t+1}\in S} Q^*(s_{t+1}|s^t)b^*(s_{t+1}, s^t)\mu(s_{t+1}|s^t)$$
$$= b^*\left(s^t\right) + w^*\left(s^t\right)l^*\left(s^t\right) + r^*\left(s^t\right)k^*\left(s^t\right), \quad \left(\pi\left(s^t\right)\sigma^*\left(s^t\right)\right)$$
$$b^*\left(s^{t+1}\right) \geq B, k\left(s^1\right), b\left(s^1\right) \text{ given} \tag{109}$$

which gives

$$\frac{u_l\left(s^t\right)}{u_c\left(s^t\right)} = -w^*\left(s^t\right), \tag{110}$$

$$P_d^*\left(s^t\right) = G_d^*\left(s^t\right), \tag{111}$$

$$P_f^*\left(s^t\right) = G_f^*\left(s^t\right), \tag{112}$$

$$Q^*(s_{t+1}|s^t) = \beta\frac{u_{2c}\left(s^{t+1}\right)}{u_{2c}\left(s^t\right)},$$

$$u_{2c}\left(s^t\right) = \beta E_{s^t} u_{2c}\left(s^{t+1}\right)\left[(1-\delta_k) + r^*\left(s^{t+1}\right)\right]. \tag{113}$$

Non-arbitrage condition imposed on asset prices, under ex-ante symmetry between countries, implies

$$\begin{aligned}
Q^*(s_{t+1}|s^t) &= \beta\frac{u_c^*\left(s^{t+1}\right)}{u_c^*\left(s^t\right)}, \\
Q(s_{t+1}|s^t) &= \beta\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)}, \\
Q^*(s_{t+1}|s^t) &= \frac{x\left(s^{t+1}\right)}{x\left(s^t\right)}Q(s_{t+1}|s^t), \\
\frac{u_c^*\left(s^{t+1}\right)}{u_c^*\left(s^t\right)} &= \frac{x\left(s^{t+1}\right)}{x\left(s^t\right)}\frac{u_c\left(s^{t+1}\right)}{u_c\left(s^t\right)}, \\
x\left(s^t\right) &= x(s^0)\frac{u_c^*\left(s^t\right)}{u_c\left(s^t\right)},\ x(s^1) = 1.
\end{aligned} \tag{114}$$

Retailers zero profit condition implies

$$J_d\left(s^t\right)\pi\left(s^t\right) + \left(1-\pi\left(s^t\right)\right)J_f\left(s^t\right) = v\left(s^t\right)\chi(s^t), \tag{115}$$

$$\left(1-\pi^*\left(s^t\right)\right)J_d^*\left(s^t\right) + \pi^*\left(s^t\right)J_f^*\left(s^t\right) = v^*\left(s^t\right)\chi^*(s^t), \tag{116}$$

where

$$J_d\left(s^t\right) = \left(P_d\left(s^t\right) - p_d\left(s^t\right)\right) + \left(1-\delta_H\right) E_{s^t}\left[Q\left(s_{t+1}|s^t\right) J_d\left(s^{t+1}\right)\right], \tag{117}$$

$$J_f\left(s^t\right) = \left(P_f\left(s^t\right) - p_f\left(s^t\right)\right) + \left(1-\delta_H\right) E_{s^t}\left[Q\left(s_{t+1}|s^t\right) J_f\left(s^{t+1}\right)\right], \tag{118}$$

$$J_d^*\left(s^t\right) = \left(P_d^*\left(s^t\right) - p_d^*\left(s^t\right)\right) + \left(1-\delta_H\right) E_{s^t}\left[\frac{x\left(s^{t+1}\right)}{x\left(s^t\right)} Q\left(s_{t+1}|s^t\right) J_d^*\left(s^{t+1}\right)\right], \tag{119}$$

$$J_f^*\left(s^t\right) = \left(P_f^*\left(s^t\right) - p_f^*\left(s^t\right)\right) + \left(1-\delta_H\right) E_{s^t}\left[\frac{x\left(s^{t+1}\right)}{x\left(s^t\right)} Q\left(s_{t+1}|s^t\right) J_f^*\left(s^{t+1}\right)\right]. \tag{120}$$

Domestic country producers solve

$$\max \sum_{t=0}^{T} \sum_{s^t} Q\left(s^t\right) \left[\left(p_d\left(s^t\right) - v_d\left(s^t\right)\right) d\left(s^t\right) + \left(x\left(s^t\right) p_d^*\left(s^t\right) - v\left(s^t\right)\right) d^*\left(s^t\right) + \right.$$
$$\left. -(A(s^t) v_d\left(s^t\right) a_d\left(s^t\right) + A^*(s^t) x\left(s^t\right) v^*\left(s^t\right) a_d^*\left(s^t\right))\right] \mu(s^t)$$

subject to (note: ϕ is a function here, its derivatives are defined at the end of the section)

$$d(s^t) \leq H_d(s^t), \quad \left(Q\left(s^t\right) \psi_d\left(s^t\right) \mu(s^t)\right)$$
$$d^*(s^t) \leq H_d^*(s^t), \quad \left(Q\left(s^t\right) \psi_d\left(s^t\right) \mu(s^t)\right)$$

and

$$H_d(s^t) = \frac{m_d(s^t)}{\bar{m}(s^t)} h(s^t) + \left(1-\delta_H\right) H_d(s^{t-1}), \quad \left(Q\left(s^t\right) W_d\left(s^t\right) \mu(s^t)\right) \tag{121}$$

$$H_d^*(s^t) = \frac{m_d^*(s^t)}{\bar{m}^*(s^t)} h^*(s^t) + \left(1-\delta_H\right) H_d^*(s^{t-1}), \quad \left(Q\left(s^t\right) W_d^*\left(s^t\right) \mu(s^t)\right) \tag{122}$$

$$m_d\left(s^t\right) = a_d(s^t) - \frac{\phi}{2}\left(\frac{a_d(s^t)}{a_d^{ss}} - 1\right)^2, \quad \left(Q\left(s^t\right) \mu\left(s^t\right)\right) \tag{123}$$

$$m_d^*(s^t) = a_d^*(s^t) - \frac{\phi}{2}\left(\frac{a_d^*(s^t)}{a_d^{*ss}} - 1\right)^2. \quad \left(Q\left(s^t\right) \mu\left(s^t\right)\right) \tag{124}$$

where

$$v(s^t) = \arg\min_{k,l} \left\{rk + wl \;\mid\; zF\left(k,l\right) = 1\right\},$$

and $Q(s^t)$ is given by recursive law: $Q(s^t) \equiv Q(s^{t-1})Q(s_t|s^{t-1})$, with $Q(s_t|s^{t-1})$ being the pricing kernel derived from the household problem.

First order conditions, with Lagrange multipliers imposed as indicated above (next to constraints), give:

$$a_d : A(s^t)v\left(s^t\right) = \mu\left(s^t\right)\left(1 - \phi\left(\frac{a_d\left(s^t\right)}{a_d^{ss}} - 1\right)\right),$$
$$a_d^* : A^*(s^t)x\left(s^t\right)v^*\left(s^t\right) = \mu^*\left(s^t\right)\left(1 - \phi\left(\frac{a_d^*\left(s^t\right)}{a_d^{*ss}} - 1\right)\right),$$

$$H_d : p_d\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h)\, E_{s^t}[Q\left(s_{t+1}|s^t\right)W_d\left(s^{t+1}\right)] = W_d\left(s^t\right),$$
$$H_d^* : x\left(s^t\right)p_d^*\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h)\, E_{s^t}[Q\left(s_{t+1}|s^t\right)W_d^*\left(s^{t+1}\right)] = W_d^*\left(s^t\right),$$

$$m_d : -\psi_d\left(s^t\right) + \frac{h\left(s^t\right)}{\bar{m}\left(s^t\right)}W_d\left(s^t\right) = 0,$$
$$m_d^* : -\psi^*\left(s^t\right) + \frac{h^*\left(s^t\right)}{\bar{m}^*\left(s^t\right)}W_d^*\left(s^t\right) = 0.$$

After simplifications, we obtain

$$H_d : W_d\left(s^t\right) = p_d\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h)\, E_{s^t}[Q\left(s_{t+1}|s^t\right)W_d\left(s^{t+1}\right)], \tag{125}$$
$$H_d^* : W_d^*\left(s^t\right) = x\left(s^t\right)p_d^*\left(s^t\right) - v_d\left(s^t\right) + (1-\delta_h)\, E_{s^t}[Q\left(s_{t+1}|s^t\right)W_d^*\left(s^{t+1}\right)], \tag{126}$$
$$m_d : W_d\left(s^t\right) = \frac{\bar{m}\left(s^t\right)}{h\left(s^t\right)}\frac{A(s^t)v\left(s^t\right)}{1-\phi\left(\frac{a_d(s^t)}{a_d^{ss}} - 1\right)}, \tag{127}$$
$$m_d^* : W_d^*\left(s^t\right) = \frac{\bar{m}^*\left(s^t\right)}{h^*\left(s^t\right)}\frac{A^*(s^t)x\left(s^t\right)v^*\left(s^t\right)}{1-\phi\left(\frac{a_d^*(s^t)}{a_d^{*ss}} - 1\right)}. \tag{128}$$

Foreign producers solve

$$\max \sum_{t=0}^{T} \sum_{s^t} Q^*\left(s^t\right) [\left(\frac{p_f\left(s^t\right)}{x\left(s^t\right)} - v^*\left(s^t\right)\right) f\left(s^t\right) + \left(p_f^*\left(s^t\right) - v_f\left(s^t\right)\right) f^*\left(s^t\right) +$$
$$- \left(A(s^t)\frac{v\left(s^t\right)}{x(s^t)} a^f\left(s^t\right) - A^*(s^t) v^*\left(s^t\right) a_f^*\left(s^t\right)\right)]\mu(s^t)$$

subject to

$$f(s^t) \leq H_f(s^t), \quad \left(Q^*\left(s^t\right)\psi_f\left(s^t\right)\mu\left(s^t\right)\right)$$
$$f^*(s^t) \leq H_f^*(s^t), \quad \left(Q^*\left(s^t\right)\psi_f\left(s^t\right)\mu\left(s^t\right)\right)$$

and

$$H_f\left(s^t\right) = \frac{m_f\left(s^t\right)}{\bar{m}\left(s^t\right)} h\left(s^t\right) + (1-\delta) H_f\left(s^{t-1}\right), \quad \left(Q^*\left(s^t\right) W_f\left(s^t\right)\mu\left(s^t\right)\right) \tag{129}$$
$$H_f^*\left(s^t\right) = \frac{m_f^*\left(s^t\right)}{\bar{m}^*\left(s^t\right)} h^*\left(s^t\right) + (1-\delta) H_f^*\left(s^{t-1}\right), \quad \left(Q^*\left(s^t\right) W_f^*\left(s^t\right)\mu\left(s^t\right)\right) \tag{130}$$
$$m_f(s^t) = a_f(s^t) - \frac{\phi}{2}\left(\frac{a_f(s^t)}{a_f^{ss}} - 1\right)^2, \quad \left(Q^*\left(s^t\right)\mu\left(s^t\right)\right) \tag{131}$$
$$m_f^*(s^t) = a_f^*(s^t) - \frac{\phi}{2}\left(\frac{a_f^*(s^t)}{a_f^{*ss}} - 1\right)^2, \quad \left(Q^*\left(s^t\right)\mu^*\left(s^t\right)\right) \tag{132}$$

where

$$v(s^t) = \arg\min_{k,l}\left\{rk + wl \ \mid \ zF(k,l) = 1\right\}.$$

After simplifications, the first order conditions give:

$$H_f : W_f\left(s^t\right) = \frac{p_f\left(s^t\right)}{x\left(s^t\right)} - v^*\left(s^t\right) + (1-\delta_H) E_{s^t}\left[Q^*\left(s_{t+1}|s^t\right) W_f\left(s^{t+1}\right)\right], \tag{133}$$
$$H_f^* : W_f^*\left(s^t\right) = p_f^*\left(s^t\right) - v_f\left(s^t\right) + (1-\delta_H) E_{s^t}\left[Q^*\left(s_{t+1}|s^t\right) W_f^*\left(s^{t+1}\right)\right], \tag{134}$$
$$m_f : W_f\left(s^t\right) = \frac{\bar{m}\left(s^t\right)}{h\left(s^t\right)} \frac{A(s^t) v\left(s^t\right)/x\left(s^t\right)}{1 - \frac{\phi}{2}\left(\frac{a_f(s^t)}{a_f^{ss}} - 1\right)^2}, \tag{135}$$
$$m_f^* : W_f^*\left(s^t\right) = \frac{\bar{m}^*\left(s^t\right)}{h^*\left(s^t\right)} \frac{A^*(s^t) v^*\left(s^t\right)}{1 - \frac{\phi}{2}\left(\frac{a_f^*(s^t)}{a_f^{*ss}} - 1\right)^2}. \tag{136}$$

Bargaining implies

$$W_d(s^t) = \theta(W_d(s^t) + J_d(s^t)) \equiv \theta S_d(s^t), \tag{137}$$

$$W_d^*(s^t) = \theta(W_d^*(s^t) + x(s^t)J_d^*(s^t)) \equiv \theta S_d^*(s^t), \tag{138}$$

$$W_f(s^t)/x(s^t) = \theta(W_f(s^t)/x(s^t) + J_f(s^t)) \equiv \theta S_f(s^t), \tag{139}$$

$$x(s^t)W_f^*(s^t) = \theta x(s^t)(W_f^*(s^t) + J_f^*(s^t)) \equiv \theta S_f^*(s^t), \tag{140}$$

where we define S to be expressed in the domestic country units.

Cost minimization for Cobb-Douglas production function $zk^\alpha l^{1-\alpha}$ implies

$$v(s^t) = \frac{w(s^t)^{1-\alpha} r(s^t)^\alpha}{z(s^t)} \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{1}{1-\alpha}\right)^{1-\alpha}, \tag{141}$$

$$v(s^t) = \frac{w^*(s^t)^{1-\alpha} r^*(s^t)^\alpha}{z^*(s^t)} \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{1}{1-\alpha}\right)^{1-\alpha}, \tag{142}$$

and

$$\frac{k(s^t)}{l(s^t)} = \frac{w(s^t)}{r(s^t)} \frac{\alpha}{1-\alpha}, \tag{143}$$

$$\frac{k^*(s^t)}{l^*(s^t)} = \frac{w^*(s^t)}{r^*(s^t)} \frac{\alpha}{1-\alpha}. \tag{144}$$

Resource feasibility requires

$$d(s^t) + d^*(s^t) + A(s^t)(a_d(s^t) + a_f(s^t)) + \chi(s^t)h(s^t) = z(s^t)F\left(k, l\right)(s^t), \tag{145}$$

$$f(s^t) + f^*(s^t) + A^*(s^t)(a_d^*(s^t) + a_f^*(s^t)) + \chi^*(s^t)h^*(s^t) = z^*(s^t)F\left(k^*, l^*\right)(s^t), \tag{146}$$

$$f(s^t) = H_f(s^t), \tag{147}$$

$$f^*(s^t) = H_f^*(s^t), \tag{148}$$

$$d(s^t) = H_d(s^t), \tag{149}$$

$$d^*(s^t) = H_d^*(s^t), \tag{150}$$

$$\bar{m}_f^*(s^t) = m_f^*(s^t), \tag{151}$$

$$\bar{m}_d^*(s^t) = m_d^*(s^t), \tag{152}$$

$$\bar{m}_f(s^t) = m_f(s^t), \tag{153}$$

$$\bar{m}_d(s^t) = m_d(s^t), \tag{154}$$

$$\pi(s^t) = \frac{\bar{m}_d(s^t)}{\bar{m}(s^t)}, \tag{155}$$

$$\pi^*(s^t) = \frac{\bar{m}_f^*(s^t)}{\bar{m}^*(s^t)}, \tag{156}$$

$$\bar{m}(s^t) = \bar{m}_d(s^t) + \bar{m}_f(s^t), \tag{157}$$

$$\bar{m}^*(s^t) = \bar{m}_d^*(s^t) + \bar{m}_f^*(s^t), \tag{158}$$

$$b(s^t) + x(s^t)b(s^t) = 0.$$

Exogenous process

$$\log z(s^{t+1}) = \psi \log z(s^t) + \varepsilon(s^t), \tag{159}$$

$$\log z^*(s^{t+1}) = \psi \log z^*(s^t) + \varepsilon^*(s^t). \tag{160}$$

Equilibrium conditions are comprised of all numbered equations, excluding (77), (50), (64) and (40) which are already included in the numbered equations (or not needed). For the calculation of the deterministic steady state, refer to the Calibration Section at the end.

5 Analytical Results (w/ extended proofs)

(All algebraic calculations are in a supporting *Mathematica* notebook: Calculations.nb.)

In order to gain intuition on the pricing predictions of our theory, in this section we explore analytically the determinants of the international price differential $xp_d^* - p_d$ in our setup. Since this key measure of pricing to market is hardwired in our setup to a similar differential on the retail level by bargaining that implies

$$xp_d^* - p_d = \theta(xP_d^* - P_d), \tag{161}$$

in what follows next, we focus on the analysis of $xP_d^* - P_d$, and refer to the fluctuations of this object as deviations from the law of one price broadly defined.

We establish two sets of results as far as the dynamics of $xP_d^* - P_d$ is concerned. First, using a setup with market expansion friction ($\phi > 0$) and by introducing an auxiliary notion of law of one price for marketing and search cost, we provide a set of necessary conditions for deviations from LOP (i.e. $xP_d^* \neq P_d$) in the benchmark model. Second, we characterize the force which plays quantitatively a dominant role (80% of deviations) in the benchmark model —the market expansion friction. In doing so, we shut down the residual source of deviations from LOP identified in the section that follows next.

5.1 Sources of Deviations from the Law of One Price

As our first step, we derive the set of conditions under which LOP holds in the model. To this end, we introduce an auxiliary notion of the law of one price for marketing and search cost, as defined below. This notion assures that the price of marketing investment and the cost of search is identical across countries. Our main result is summarized in Proposition 3 below. It essentially states that any deviations from LOP in the model comes either from (i) international differences in marketing/search cost, or (ii) the market expansion friction ($\phi > 0$). In what follows next, we turn to the analysis of the sole effect of the second force, which turns out to be the dominant one. Specifically, in the quantitative section, we establish that as much as 80% of the overall deviations from LOP come from this source.

Definition 1 *The **law of one price for marketing and search costs** holds iff*

$$\chi(s^t)v(s^t) = \chi^*(s^t)x(s^t)v^*(s^t) \text{ and} \tag{162}$$
$$A_d(s^t)v(s^t) = A_f(s^t)v(s^t) = A_d^*(s^t)x(s^t)v^*(s^t) = A_f^*(s^t)x(s^t)v^*(s^t).$$

Proposition 3 *(Proposition 3 in the paper) Suppose (162) holds and $\phi = 0$. Then, law of one price (LOP) holds in the benchmark model.*

Proof. (Identical to the one in the paper) By bargaining under continual renegotiation, the surplus from a match that goes to the producer is given by θS_d, and the surplus that goes to retailer is $(1-\theta)S_d$, where $S_i \equiv W_i + J_i$, $i = d, f$. Furthermore, given (162) and $\phi = 0$, we know that both the domestic producer and the foreign importer face the same marginal cost of matching in the domestic country, which is given by $\frac{\bar{m}}{h}(Av - (1-\delta_m)E\left[Q_{+1}A_{+1}v_{+1}\right])$. Given equality of the cost, we conclude that the surplus from a match must be equal and so: $S_d = S_f$ and $S_d^* = S_f^*$ (all expressed to domestic numéraire[27]). By (162) and domestic and foreign retailer zero profit condition,

$$S_d(s^t) = \frac{\chi(s^t)v(s^t)}{1-\theta}\ ,\ S_d^*(s^t) = \frac{\chi^*(s^t)x(s^t)v^*(s^t)}{1-\theta},\ \text{all } s^t, \tag{163}$$

we additionally conclude that surpluses across the border are equal. Now, using the Bellman equations defining total surpluses from a match,

$$x(s^t)P_d^*(s^t) - P_d(s^t) = [S_d^*(s^t) - S_d(s^t)] - (1-\delta_H)E[Q(s^{t+1}, s^t)(S_d^*(s^{t+1}) - S_d(s^{t+1}))], \tag{164}$$

we obtain $x(s^t)P_d^*(s^t) = P_d(s^t)$, all s^t.

In what follows next, we turn to the analysis of the sole effect of (ii), which turns out to be the dominant force in the benchmark model. Specifically, in the quantitative section, we establish that as much as 80% of the overall deviations from LOP come from this source.

[27]To save on notation, we abuse our usual convention and define valuations of foreign producers and retailers in domestic country numéraire, i.e. $S_d^* \equiv W_d + xJ_d^*$, $S_f^* \equiv xW_f + J_f$ and $S_f^* \equiv xW_f^* + xJ_f^*$.

5.2 Effects of Sluggish Market Shares

To expose the mechanism through which market expansion friction $\phi > 0$ leads to deviations from LOP, here we consider a simplified analytic version of our setup by making the following changes, that we later show are innocuous for the results on prices (see comparison of benchmark model to *Static Friction* in tables with results): (A1) we dispense with physical capital accumulation and labor-leisure choice from the household problem (labor the only input); (A2) we assume (162) holds[28] to isolate the sole effect of ϕ, as implied by Proposition 3; (A3) we simplify the capital theoretic formulation of marketing capital by replacing (16) with a static one[29]

$$m_i = a_i - \frac{\phi a_i^s (a_i/a_i^s - 1)^2}{2}, i = d, f, \tag{165}$$

where a_i^s is assumed to be the deterministic steady state value of a_i.

The two central results of this section are stated in Propositions 4 and 5 below. Proposition 4 establishes that, in this setup, whenever the real exchange rate changes, it must necessarily imply deviations from the LOP. In other words, firms price to market in which they sell. Proposition 5 completes this result by only linking real exchange rate movements in our model to productivity shocks. Given the uncertainty about the channel generating real exchange rate movements in international economics, we view Proposition 4 as central. It shows that pricing to market is a universal feature of novel aspects of our model and occurs independently from the exact driving forces that move the real exchange rate in the model.

The aforementioned results essentially follow from a technical lemma stated below (Lemma 1). To a first order approximation, this lemma shows that any differential between retailer's valuations between the two markets are linked to the market share dynamics. The important implication of the lemma, used repeatedly in the proofs of Proposition 4 and 5, is its immediate corollary (Corollary 1). It shows that in the presence of market expansion friction, the law of one price is only consistent with market shares being constant across all dates and states. To see why this allows

[28]For example, consider: $A_d(s^t) = \frac{xv^*+v}{2}$ and $\chi(s^t) = \chi\frac{xv^*+v}{2v}$, and combine it with (162).

[29]In terms of analytics, this simpler formulation of market expansion friction emulates the crucial properties of benchmark specification, but assumes away an analytically intractable dynamic link between current marketing expenditures and the future cost of market share expansion. We choose capital theoretic formulation in benchmark model as it allows us to more naturally relate market expansion friction to the elasticity puzzle. Note that in this case model implied long-run and short-run elasticities will no longer differ.

us to establish the results listed in Proposition 4 and 5, note that if market shares across all dates and states are constant and the real exchange rate moves, this immediately contradicts Corollary 1. This is because, by linearization of 13, we know that market shares are constant and so are all retail prices in local unit (i.e. P_d and P_d^* would be constant).

Lemma 1 *(Lemma 1 in the paper) Under A1-A3, producer optimization and retailer zero profit condition imply that*

$$\hat{\mathcal{P}}_d^*(s^t) - \hat{\mathcal{P}}_d(s^t) = \phi \times \frac{\chi + (1-\theta)}{\chi(1-(1-\delta_H)\beta)} \times \left[\widehat{1-\pi^*}(s^t) - \widehat{\pi}(s^t)\right], \tag{166}$$

where $\widehat{}$ *denotes log deviation from the deterministic steady state, and* $\mathcal{P}_d, \mathcal{P}_d^*$ *are given by*

$$\mathcal{P}_d(s^t) = P_d(s^t) + (1-\delta_H)E\left[Q(s_{t+1}|s^t)\mathcal{P}_d(s^{t+1})\right], \tag{167}$$

$$\mathcal{P}_d^*(s^t) = x(s^t)P_d^*(s^t) + (1-\delta_H)E\left[Q(s_{t+1}|s^t)\mathcal{P}_d^*(s^{t+1})\right], \tag{168}$$

Proof. (Extended) The producer's first order conditions in this case are:

$$\theta S_d(s^t) = \frac{\bar{m}(s^t)}{h(s^t)} \frac{A_d(s^t)v(s^t)}{1-\phi(\frac{a_d(s^t)}{a_d^s}-1)},$$

$$\theta S_f(s^t) = \frac{\bar{m}(s^t)}{h(s^t)} \frac{A_f(s^t)v(s^t)}{1-\phi(\frac{a_f(s^t)}{a_f^s}-1)},$$

$$\theta S_d^*(s^t) = \frac{\bar{m}^*(s^t)}{h^*(s^t)} \frac{A_d^*(s^t)x(s^t)v(s^t)}{1-\phi(\frac{a_d^*(s^t)}{a_d^s}-1)},$$

$$\theta S_f^*(s^t) = \frac{\bar{m}^*(s^t)}{h^*(s^t)} \frac{A_d^*(s^t)x(s^t)v^*(s^t)}{1-\phi(\frac{a_f^*(s^t)}{a_d^s}-1)},$$

where $\cdot^s$(or $\cdot^{ss}$) denotes the steady state value of the underlying variable and $S_d(s^t)$ is the present discounted value of the total surplus from trade in a match:

$$
\begin{aligned}
S_d(s^t) &= P_d(s^t) - v(s^t) + (1-\delta_H)E_{s^t}\left[Q(s_{t+1}|s^t)S_d(s_{t+1},s^t)\right], \qquad (169)\\
S_d^*(s^t) &= x(s^t)P_d^*(s^t) - v(s^t) + (1-\delta_H)E_{s^t}\left[Q(s_{t+1}|s^t)S_d^*(s_{t+1},s^t)\right],\\
S_f^*(s^t) &= x(s^t)P_f^*(s^t) - x(s^t)v^*(s^t) + (1-\delta_H)E_{s^t}\left[Q(s_{t+1}|s^t)S_f^*(s_{t+1},s^t)\right],\\
S_f(s^t) &= P_f(s^t) - x(s^t)v^*(s^t) + (1-\delta_H)E_{s^t}\left[Q(s_{t+1}|s^t)S_f(s_{t+1},s^t)\right].
\end{aligned}
$$

Log-linearizing the above conditions (see online *Mathematica* file), we obtain

$$
\begin{aligned}
\hat{S}_d &= \frac{1-\theta}{\theta\chi}\frac{\bar{m}^s}{h^s}(\widehat{\frac{\bar{m}}{h}} + \phi\hat{a}_d + \widehat{A_d v}), \qquad (170)\\
\hat{S}_f &= \frac{1-\theta}{\theta\chi}\frac{\bar{m}^s}{h^s}(\widehat{\frac{\bar{m}}{h}} + \phi\hat{a}_f + \widehat{A_f v}),\\
\hat{S}_d^* &= \frac{1-\theta}{\theta\chi}\frac{\bar{m}^s}{h^s}(\widehat{\frac{\bar{m}^*}{h^*}} + \phi\hat{a}_d^* + \widehat{A_d^* x v^*}),\\
\hat{S}_f^* &= \frac{1-\theta}{\theta\chi}\frac{\bar{m}^s}{h^s}(\widehat{\frac{\bar{m}^*}{h^*}} + \phi\hat{a}_f^* + \widehat{A_f^* x v^*}).
\end{aligned}
$$

To simplify the above expressions, we derive $\frac{\bar{m}^s}{h^s}$ from the fact that in the deterministic steady state $S_d = S_f \equiv S^s$, and by the retailer zero profit conditions applied to the deterministic steady state, we have:

$$S^s = \frac{\chi}{1-\theta}v^s.$$

From the producer first order condition evaluated at the steady state (see above),

$$\theta S^s = \frac{\bar{m}^s}{h^s}v^s,$$

we derive

$$\frac{\bar{m}^s}{h^s} = \frac{\chi\theta}{1-\theta}.$$

Under the assumption of LOP for marketing/search cost, implying $\widehat{A_i v} = \widehat{A_i^* x v^*}$, $i = d, f$, from

(170), we derive:

$$\hat{S}_d - \hat{S}_f = \phi(\hat{a}_d - \hat{a}_f), \tag{171}$$
$$\hat{S}^*_d - \hat{S}^*_f = \phi(\hat{a}^*_d - \hat{a}^*_f),$$

Next, we link the RHS of (171) to market shares using the following: (1) linearized law of motion for marketing capital (16)

$$\hat{m}_d = \hat{a}_d, \tag{172}$$
$$\hat{m}_f = \hat{a}_f,$$
$$\hat{m}^*_d = \hat{a}^*_d,$$
$$\hat{m}^*_f = \hat{a}^*_f.$$

(2) log-linearized definition of $\pi = \frac{m_d}{m_f + m_f}$

$$\hat{\pi} = (1 - \pi^s)(\hat{m}_d - \hat{m}_f), \tag{173}$$
$$\widehat{1 - \pi^*} = \pi^s(\hat{m}^*_d - \hat{m}^*_f),$$

and (3) log-linearized retailer zero profit condition

$$\hat{S}_d - \hat{S}_f = \frac{\hat{S}_d}{1 - \pi^s} - \frac{\widehat{\chi v}}{1 - \pi^s},$$
$$\hat{S}^*_d - \hat{S}^*_f = \frac{\hat{S}^*_d}{\pi^s} - \frac{\widehat{\chi^* x v^*}}{\pi^s}.$$

Using these relations, and $\widehat{\chi v} = \widehat{\chi^* x v^*}$, from (171), we derive:

$$\hat{S}_d = \phi\hat{\pi},$$
$$\hat{S}^*_d = \phi\widehat{1 - \pi^*},$$

and

$$\hat{S}^*_d - \hat{S}_d = \phi\left[\widehat{1 - \pi^*} - \hat{\pi}\right]. \tag{174}$$

Next, we link the RHS of (171) to prices. Splitting $\mathcal{S}$,

$$S_d(s^t) = \mathcal{P}_d(s^t) - \mathcal{V}_d(s^t),$$

where

$$\mathcal{P}_d(s^t) \equiv P_d(s^t) + (1-\delta_H)E_{s^t}[Q(s_{t+1}|s^t)\mathcal{P}_d(s_{t+1}, s^t)],$$
$$\mathcal{V}_d(s^t) \equiv v(s^t) + (1-\delta_H)E_{s^t}[Q(s_{t+1}|s^t)\mathcal{V}_d(s_{t+1}, s^t)].$$

we derive

$$\hat{S}_d S^s = \hat{\mathcal{P}}_d \mathcal{P}_d^s - \hat{\mathcal{V}}_d \mathcal{V}_d^s,$$
$$\hat{S}_d^* S^s = \hat{\mathcal{P}}_d^* \mathcal{P}_d^s - \hat{\mathcal{V}}_d \mathcal{V}_d^s,$$

and thus

$$\hat{S}_d^* - \hat{S}_d = \left(\hat{\mathcal{P}}_d^* - \hat{\mathcal{P}}_d\right) \frac{\mathcal{P}_d^s}{S^s}. \tag{175}$$

Since in the steady state[30]:

$$S^s = \frac{P^s}{1-(1-\delta_H)\beta} = \frac{\chi}{(1-\theta)} v^s, \mathcal{P}_d^s = \frac{P^s}{1-(1-\delta_H)\beta}, \frac{P^s}{v^s} = \frac{\chi}{1-\theta} + 1,$$

we derive

$$\frac{\mathcal{P}_d^s}{S^s} = \frac{\frac{P^s}{1-(1-\delta_H)\beta}}{\frac{\chi}{(1-\theta)} v^s} = \frac{\chi + (1-\theta)}{\chi(1-(1-\delta_H)\beta)}.$$

and combining (174) with (175), we obtain (i):

$$\hat{\mathcal{P}}_d^* - \hat{\mathcal{P}}_d = \phi \times \frac{\chi + (1-\theta)}{\chi(1-(1-\delta_H)\beta)} \times \left[\widehat{1-\pi^*} - \widehat{\pi}\right].$$

■

Corollary 1 *(Corollary 1 in the paper) In the presence of home-bias in trade in the steady state (i.e. $\pi^s > 1/2$), we additionally have $\widehat{xP_d^*}(s^t) = \widehat{P_d}(s^t)$ iff $\hat{\pi}(s^t) = \widehat{\pi}^*(s^t) = 0$.*

[30]By symmetry of steady state, $P_d^s = P_f^s = P$.

Proof. (Identical as the one in the paper) ($\Rightarrow$) Suppose, by contradiction, that $\widehat{LOP}$ holds in all markets as indicated in (ii), but market shares do adjust in response to shocks. Since LOP implies $\hat{\mathcal{P}}_d^* - \hat{\mathcal{P}}_d = 0$, by (166), w.l.o.g. we can assume $\widehat{1-\pi^*} = \widehat{\pi} = \Delta \neq 0$. Since an analogous condition holds for the foreign country as well, in that case implying $\widehat{1-\pi} = \widehat{\pi}^*$, by home-bias ($\pi^s > 1/2$), we obtain $|\widehat{1-\pi}| > \Delta$, $|\widehat{\pi}^*| > \Delta$, and $|\widehat{1-\pi^*}| > \Delta$. The last inequality is a contradiction. ($\Leftarrow$) By (166), it is enough to show $\hat{\mathcal{P}}_d^* \equiv \hat{\mathcal{P}}_d$ implies $\widehat{LOP}$, which follows from the following evaluation based on (167)[31]

$$\underbrace{\mathcal{P}_d^*(s^t) - \mathcal{P}_d(s^t)}_{\text{all 1st order terms=0}} \equiv xP_d^*(s^t) - P_d(s^t) + (1-\delta_H)E_{s^t}\{\underbrace{Q(s_{t+1}|s^t)[\mathcal{P}_d^*(s^{t+1}) - \mathcal{P}_d(s^{t+1})]}_{\text{all 1st order terms = 0}}\}. \tag{176}$$

■

Proposition 4 *(Proposition 4 in the paper) Under A1-A3, under home-bias in trade, the real exchange rate fluctuations imply deviations from the law of one price.*

Proof. (Identical to the one in the paper) Suppose LOP holds. By Corollary 1: $1-\widehat{\pi^*(s^t)} = \widehat{\pi}(s^t) = 0$. By linearization of 13, we observe that $\hat{P}_d^* = \hat{P}_d = 0$ whenever $\hat{\pi} = \hat{\pi}^* = 0$. But, since $\hat{x} \neq 0$ and $1-\widehat{\pi^*(s^t)} = \widehat{\pi}(s^t) = 0$, we conclude: $x\hat{P}_d^* = \hat{P}_d = 0$. This is a contradiction. ■

Proposition 5 *(Proposition 5 in the paper) Under A1-A3, and in the presence of home-bias in trade, the equilibrium response to a relative productivity shock $z \neq z^*$, to a first order approximation, result in real exchange rate fluctuations, both under perfect risk sharing (benchmark case) and under financial autarky.*

Proof. (Extended) By contradiction, assume that LOP holds in equilibrium ($xP_d^* \equiv P_d$). From Lemma 1 (ii), it must be that $\hat{\pi} = \hat{\pi}^* = 0$ and thus $\hat{S}_d - \hat{S}_f = 0$, which also implies $\widehat{xv^*} = \hat{v} = 0$ by evaluation similar to (176), but applied to $S_d - S_f$. As summarized by Lemma 2 stated below, under such conditions, $z \neq z^*$ necessarily implies $h \neq h^*$. The complete proof of this lemma is included below, and follows by subtracting the log-linearized foreign feasibility (22) (first multiplied by $\frac{xv^*}{v}$ to use (162)) from each side of the of log-linearized domestic feasibility (22), and the fact that $\hat{a}_i = \frac{\hat{h}-\widehat{Av}}{1+\phi}, \hat{a}_i^* = \frac{\hat{h}-\widehat{Av}}{1+\phi}, i = d, f$, as derived from: (i) the first part of the proof of Lemma 1, implying

[31] Note: By symmetry, $\mathcal{P}_d^{*s} = \mathcal{P}_d^s$.

$\hat{S}_i = \hat{S}_i^* = 0$, (ii) log-linearized producer FOCs, given by $\theta S_i = (\frac{m_d+m_f}{h})Av/(1-\phi(a_i/a_i^s - 1))$, and (iii) log-linearized equation (165), which together with $\hat{\pi} = \hat{\pi}^* = 0$, gives $\hat{m}_d = \hat{a}_d = \hat{m}_f = \hat{a}_f$.

Lemma 2 *Suppose in equilibrium $\hat{\pi} = \hat{\pi}^* = 0$. Then, $z > (<) z^*$ implies $h > (<) h^*$, as implied by the equation:*

$$\left(\hat{h} - \hat{h}^*\right) \times (\frac{a_d^s + a_f^s}{1+\phi} + h^s(2\pi^s + \chi - 1)) = \hat{z} - \hat{z}^*.$$

(Proof of the lemma) From the proof of Lemma 1, equations (172) and (173), we note that $\hat{\pi} = \hat{\pi}^* = 0$ implies $\hat{m}_d = \hat{a}_d = \hat{m}_f = \hat{a}_f$ (same for the foreign country) and, since $\hat{S}_i = 0$, $i = d, f$, using an equation (170) from proof of Lemma 1, we obtain

$$(*) : \widehat{\frac{\bar{m}}{h}} + \phi \hat{a}_i + \widehat{Av} = 0, i = d, f.$$

Log-linearizing the definition of market tightness $(\widehat{\frac{\bar{m}}{h}})$, and using the fact that by definition $\bar{m} = m_f + m_d$, we have

$$\widehat{\frac{m}{h}} \frac{\bar{m}^s}{h^s} = \frac{m_d^s}{h^s}\hat{m}_d + \frac{m_d^s}{h^s}\hat{m}_f - \frac{\bar{m}^s}{h^s}\hat{h},$$

which, given the above, implies

$$\widehat{\frac{m}{h}} \frac{\bar{m}^s}{h^s} = \frac{\bar{m}^s}{h^s}(\hat{a}_d - \hat{h}) = \frac{\bar{m}^s}{h^s}(\hat{a}_f - \hat{h}),$$

and simplifies to

$$(**) : \widehat{\frac{m}{h}} = (\hat{a}_i - \hat{h}), i = d, f.$$

Next, plugging in (**) to (*), we derive

$$(\hat{a}_i - \hat{h}) + \phi \hat{a}_i + \widehat{Av} = 0,$$

from which, we calculate

$$\hat{a}_i = \frac{\hat{h} - \widehat{Av}}{1+\phi}, i = d, f. \tag{177}$$

The analogous expression for the foreign country is given by

$$\hat{a}_i^* = \frac{\hat{h}^* - \widehat{Axv^*}}{1+\phi} = \frac{\hat{h}^* - \widehat{Av}}{1+\phi}, i = d, f, \tag{178}$$

where the last equality follows from the assumption of LOP for marketing and search costs: $\widehat{Av} = \widehat{Axv^*}$.

Using the equations for $\hat{a}_i, \hat{a}_i^*$, we next log-linearize domestic feasibility, given by (we deal with inequality later)

$$A(s^t)(a_f(s^t) + a_d(s^t)) + \chi h(s^t) + d(s^t) + d^*(s^t) = z.$$

Under the assumption $\hat{\pi} = \hat{\pi}^* = 0$, we obtain

$$\hat{z} = (\hat{A} + \frac{\hat{h} - \widehat{Av}}{1+\phi})(a_f^s + a_d^s) + h^s \left[(\pi^s + \chi)\hat{h} + \chi\hat{\chi} + (1-\pi^s)\hat{h}^*\right],$$

for the domestic country. As far as the foreign feasibility condition is concerned,

$$A^*(s^t)(a_f^*(s^t) + a_d^*(s^t)) + \chi^*(s^t)h^*(s^t) + f^*(s^t) + f(s^t) = z^*(s^t),$$

we first multiply both sides of it by $\frac{x(s^t)v^*(s^t)}{v(s^t)}$, and next use the assumption of LOP for marketing/search cost to convert the foreign country specific input requirements into the domestic country ones. Specifically, we log-linearize the following equivalent expression:

$$A(s^t)(a_f^*(s^t) + a_d^*(s^t)) + \chi^*(s^t)h^*(s^t) + (f^*(s^t) + f(s^t))\frac{x(s^t)v^*(s^t)}{v(s^t)} = z^*(s^t)\frac{x(s^t)v^*(s^t)}{v(s^t)},$$

and using the property that $\widehat{\frac{xv^*}{v}} = 0$ (discussed above), we obtain

$$\hat{z}^* = (\hat{A} + \frac{\hat{h}^* - \widehat{Axv^*}}{1+\phi})(a_f^s + a_d^s) + h^s \left[(\pi^s + \chi)\hat{h}^* + \chi^*\hat{\chi}^* + (1-\pi^s)\hat{h}\right].$$

The statement of the lemma follows now by subtracting the linearized foreign feasibility equation above from the domestic one, and the implication of LOP for search and marketing cost ($Av \equiv Axv^*$,

$\chi v \equiv \chi^* x v^*$):

$$\left(\hat{h} - \hat{h}^*\right) \times \left(\frac{a_d^s + a_f^s}{1+\phi} + h^s(2\pi^s + \chi - 1)\right) = \hat{z} - \hat{z}^*.$$

Finally, we note that our assumption that feasibility holds with equality is without loss of generality, as otherwise it would not solve the planning problem used in the proof of existence of equilibrium (operator T). Clearly, the planner, given increasing utility, should adjust market shares to utilize the wasted this way resources[32]. Under financial autarky, since we do not prove existence, we assume that an analogous equality holds, as required by our approach of linear approximation.

(Proof of the proposition (Proposition 5) continued) (as in the paper) To see why Lemma 2 leads to a contradiction, observe that under efficient risk sharing, by log-linearization of (11) for the steady state assumptions, $H_d^s = \pi^s h^s/\delta_H$, $H_f^s = (1-\pi^s)h^s/\delta_H$, implies $\hat{c} = \hat{h}$, thus $\hat{c} \neq \hat{c}^*$ and $\hat{x} \neq 0$ by (14). Since the aggregator G is homogenous of degree 1, by linearization of (13), we have $\hat{P}_d^* = \hat{P}_d = 0$ whenever $\hat{\pi} = \hat{\pi}^* = 0$, thus implying $\widehat{xP_d^*} \neq \widehat{P_d}$ – a contradiction. To contradict the hypothesis but under financial autarky, observe that $\hat{h} \neq \hat{h}^*$ violates the requirement that current account is zero at all states and dates, namely $xp_d^*(1-\pi^*)h^* - p_f(1-\pi)h + Av(a_d^* - a_f) = 0$. To see this implication, log-linearize this condition after plugging in for p_d^*, p_f from bargaining equation, and note that when ($\hat{P}_d = \hat{P}_f = \hat{x} = \hat{\pi} = \hat{\pi}^* = \hat{v}^* = \hat{v} = 0$) and (177), (178) hold, the current account condition requires that

$$\left(\hat{h} - \hat{h}^*\right) \times \left(\frac{v^s a^s}{1-\phi} - h^s(1-\pi^s)(v^s(1-\theta) - \theta)\right) = 0,$$

which is a contradiction since $\hat{h} \neq \hat{h}^*$. ■

(NOTE: The section does not end here in the paper).

[32] Given initial symmetry (equal marginal utility across countries for $z = z^*$), the planner could achieve a strictly higher value of the program by adjusting market shares or setting instead $h > h^*$, to preserve equality of both feasibility conditions. Since at the initial point shadow value each type of good across countries is equalized (by symmetry), by utilizing the wasted resources this way, due to their positive shadow value (increasing utility), the value of the program must increase (to a first order approximation).

6 Calibration

To calibrate all of the parameters of the model, we use the following result (for both static and dynamic formulations of the market expansion friction).

Definition 2 *By **static calibration targets**, we mean:*

- Wholesale markups $\mathcal{U} \equiv \frac{p}{v}$
- Share of labor in time endowment l
- Growth adjusted real interest rate r
- Physical capital depreciation rate (directly pins down δ)
- Share of payments to labor in GDP $\mathcal{L}$
- Import share of goods to GDP $\mathcal{I} \equiv \frac{\text{Imports}}{\text{GDP}}$
- Marketing expenditure share $\mathcal{M} \equiv \frac{\text{Marketing Expenditures}}{\text{GDP}}$
- Average duration of matches (directly determines δ_H)

Proposition 6 *Given values of θ and γ, there is at most one corresponding symmetric deterministic steady state associated with the above set of static calibration targets and the values of these parameters.*

Proof. (We assume that in the steady state $A = A^* = 1$.) We take θ, and γ as given and note that by symmetry $p_d = p_f = p, P_d = P_f = P = 1$ (by numeraire normalization). We calculate β from Euler's equation: $\beta = (r + 1 - \delta)^{-1}$. Using the bargaining equations and the definition of $\mathcal{U}$, we obtain

$$\frac{P}{v} = \frac{\mathcal{U} - (1-\theta)}{\theta}.$$

From numeraire normalization $P = 1$, we derive:

$$v = \frac{\theta}{\mathcal{U} - (1-\theta)} \tag{179}$$

From the fact that $H = h/\delta_H$, we note that

$$\pi = \frac{f}{d+f},$$

and by symmetry ($P_d = P_f$), we note that since

$$\mathcal{I} = \frac{\text{Imports}}{\text{GDP}} = \frac{pf}{P(d+f)}.$$

we have

$$\pi = 1 - \mathcal{I} \times \frac{\mathcal{U} - (1-\theta)}{\theta\mathcal{U}}. \tag{180}$$

By household demand equations $G_d = G_f$, we know

$$f = (\frac{\omega}{1-\omega})^{-\gamma} d.$$

Using the above equation, and the previously calculated value of π, we recover the value of the unknown parameter ω (as a function of γ) from

$$1 - \pi \equiv \frac{f}{d+f} = \frac{(\frac{\omega}{1-\omega})^{-\gamma} d}{(\frac{\omega}{1-\omega})^{-\gamma} d + d} = \frac{(\frac{\omega}{1-\omega})^{-\gamma}}{(\frac{\omega}{1-\omega})^{-\gamma} + 1}. \tag{181}$$

Next, we note that from the value function of the retailer,

$$J = \pi(P-p) + (1-\pi)(P-p) + (1-\delta_H)\beta J = (P-p) + (1-\delta_H)\beta J,$$
$$J = \frac{P-p}{1-(1-\delta_H)\beta} = \frac{(p-v)(1-\theta)/\theta}{1-(1-\delta_H)\beta}.$$

Thus, from retailer zero profit condition,

$$J = \chi v$$

we obtain

$$\chi = \frac{1-\theta}{\theta(1-(1-\delta_H)\beta)}\mathcal{U}. \tag{182}$$

From the interest rate target and cost minimization, we derive

$$k = l(\frac{\alpha v}{r})^{\frac{1}{1-\alpha}}, \tag{183}$$

and calculate output from

$$F(k,l) = k^{\alpha} l^{1-\alpha} \tag{184}$$

From the definition of $\mathcal{M}$, we observe

$$\delta_m(m_d + m_f) = \frac{P_d}{v}\mathcal{M}(d+f).$$

and from feasibility

$$d + f + \delta_m(m_d + m_f) + \chi h = F(k,l),$$

we calculate $d + f$

$$(*) : d + f = \frac{k^{\alpha} l^{1-\alpha}}{(1 + \chi\delta_H + \chi\frac{P_d}{v}\mathcal{M})}. \tag{185}$$

(Average duration of matches pins down δ_H, and P_d/v has been calculated above, l is pinned down by calibration targets). Using definition of $\mathcal{L}$, we calculate α from

$$\mathcal{L} = \frac{\text{payments to labor}}{\text{total income}} = \frac{wl}{P(d+f)} = \frac{pF_l(k,l)l}{P(d+f)} = \frac{p}{P}(1 + \chi\delta_H + \frac{P}{v}\mathcal{M})(1-\alpha).$$

The values of the following variables can now be calculated as follows:

$$\begin{aligned} c &= (\omega d^{\frac{\gamma-1}{\gamma}} + (1-\omega) f^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}} - \delta k, \\ w &= \frac{rk}{n}\frac{1-\alpha}{\alpha}, \\ Q &= \beta, \\ \bar{a} &= \mathcal{M}\frac{P_d}{v}(d+f). \end{aligned} \tag{186}$$

Finally, we use the producer first order condition to pin down marketing capital related variables. The first order condition in the steady state is given by

$$p - v = v + \lambda(1 - (1 - \delta_H)\beta),$$

where

$$\lambda = \frac{\bar{m}}{h}(v - \beta(1 - \delta_m)),$$

and since we know $\bar{m} = \bar{a}/\delta_m$, $h = (d + f)\delta_H$, and $\mathcal{U} - 1 = \frac{p-v}{v}$, by dividing the above by v, we retrieve the unknown value of the parameter δ_m as follows:

$$\delta_m = \frac{\bar{a}(1-\beta)(1-(1-\delta_H)\beta)}{\delta_H(\mathcal{U}-1)(d+f) + \bar{a}(1-\delta_H)\beta^2 - \bar{a}\beta} = \frac{\mathcal{M}\frac{P}{v}(d+f)(1-\beta)(1-(1-\delta_H)\beta)}{(d+f)\left[\delta_H(\mathcal{U}-1) - \mathcal{M}\frac{P}{v}\beta(1-(1-\delta_H)\beta)\right]}. \tag{187}$$

Then, we calculate

$$\begin{aligned} m_d &= \pi\bar{m}, \\ m_f &= (1-\pi)\bar{m}. \end{aligned} \tag{188}$$

and

$$\begin{aligned} a_d &= \delta_m m_d, \\ a_f &= \delta_m m_f, \end{aligned} \tag{189}$$

Finally, the value of η is obtained from the labor leisure choice:

$$\begin{aligned} \frac{1-\eta}{\eta}\frac{c}{1-l} &= w \\ \eta &= \frac{1}{\frac{1-l}{c}w + 1}. \end{aligned} \tag{190}$$

Clearly, mapping from targets to parameters and quantities is unambiguous (all equilibrium objects are pinned down uniquely), which finishes the proof. ■

The above two propositions tell us there is a unique candidate steady state associated with the parameters implied by the targets. The steady state is well-defined iff all the parameters derived in the process, prices and quantities lie within their domains. This can be assured by inspection, and thus the proposition is useful to solve the model in practice using standard methods.

Using the above proposition and the calibration targeted listed above, to pin down all parameters, we need three additional targets that will determine the values of θ, γ, and ϕ (the parameter on which the steady state does not depend on but the dynamics does). To this end, we use the parsimonious Nash value of the bargaining parameter, $\theta = 1/2$ (or in benchmark model 2 we also use price statistics to fit the model), and utilize the data on the empirical estimates of the short-run and the long-run trade elasticity to pin down ϕ and γ, respectively. The long-run price elasticity is interpreted as the long-run impact of a permanent change in the tariff rate T on the steady state product mix ratio $\frac{f}{d}$ or expenditure ratio $\frac{p_f f}{p_d d}$. Below, we show that such steady state response in our model is identical to the frictionless Armington model, which is behind the measurement of this elasticity in the trade literature.

Proposition 7 *When tariffs are symmetrically reduced between countries, their long-run impact on the model is identical to frictionless Armington model, i.e. it is given by*

$$\Delta \log \frac{f}{d} = \gamma \Delta T.$$

Proof. For the purpose of this exercise, we must first modify our setup to include a T percent tariff rate charged by the government on the value of imported goods by the retailer (T percent out of the dock value). The tariff is assumed to be symmetric across countries, and the revenue from the tariff is assumed to be lump-sum rebated to the (local) households. Without loss of generality, we assume that the retailer is required to pay the tariff.

Since the problem of the producer remains unchanged, given symmetry of the deterministic steady state, the producer's first order conditions imply[33]

$$p_d = p_d^*. \tag{191}$$

[33] In the steady state the producer must be indifferent where to sell, at home or abroad.

However, since retailer pays the tariff, the bargaining problem must be modified to incorporate this fact:

$$\max_p W(p)^\theta J(p)^{1-\theta}, \tag{192}$$

where

$$W(p) = p - v(s^t) + (1-\delta) W,$$
$$J(p) = P - (1+T) p + (1-\delta) J,$$

and W and J are steady state values of the surplus which goes to producer and retailer, respectively. Following the same steps as in the proof of Proposition 1, we can now obtain analogous expressions for prices to the ones that are derived in the paper. Namely, now we have

$$p = \frac{\theta P}{(1+T)} + (1-\theta) v. \tag{193}$$

Applying the above generic solution to prices p_d, p_d^*, we obtain

$$p_d = \theta P_d + (1-\theta) v, \tag{194}$$
$$p_d^* = \theta \frac{P_d^*}{1+T} + (1-\theta)\frac{v}{x}.$$

Since in the symmetric steady state $p_d = p_d^*$, we have

$$P_d = \frac{P_d^*}{1+T},$$

and by symmetry

$$P_d = \frac{P_f}{1+T}, \tag{195}$$

implying

$$p_d = p_f. \tag{196}$$

Plugging in for consumer prices from equation (13), and using (195) and (196) above, we have

$$\frac{p_f f}{p_d d} = \frac{f}{d} = (1+T)^{-\gamma}(\frac{\omega}{1-\omega})^{-\gamma}, \tag{197}$$

Rewriting the above relation in logarithms[34], we derive

$$\Delta \log \frac{f}{d} = -\gamma \Delta T. \tag{198}$$

■

To choose the value of the elasticity parameter, we use the empirical estimates due to Head & Ries (2001). These authors look at trade liberalization between Canada and US, and use 3-digit industry data for the Canadian and the US expenditure share of import to domestic shipments for the time period 1990 − 1995, from which they construct industry level import expenditure ratios, $p_f f/p_d d$. They combine this information with a detailed schedule of tariff rates by industry. Their estimates of the parameter γ range from 7.9 to 11.4 (depending on whether they run the above regression with additional industry dummies or pool the industry data together). Note that their approach effectively combines the cross-sectional evidence on tariff rate dispersion across sectors and sectoral dispersion in import shares with the time series aspect of tariff changes following NAFTA. In addition, since in their regression equation Head et al. control for year dummies, estimates of these dummies additionally confirm the sluggishness of the underlying adjustment process. Another common source is Eaton & Kortum (2002), who estimate a similar equation using cross-sectional evidence on price dispersion across countries measured from the United Nations retail price database and bilateral import shares. Their estimate of $\gamma = 7.3$ $(= \theta - 1)$ is consistent with the findings of Head et al. Finally, in the Supplement we analyze the transitional dynamics of the response to a tariff change — which in our model is different from the static trade models — and, qualitatively, is consistent with the signs of year dummies estimated by Head & Ries (2001).

To pin down the value of ϕ, we use our measure of the short-run elasticity, which is described in the paper. To this end, we target the value based on the median volatility ratio calculated for several OECD countries listed in Table 5.

[34] $\ln(1+x) \approx x$ (for x small)

Table 5: Volatility Ratio in a Cross-Section of Countries

	Detrending method	
Country	HP-1600	Linear[a]
Australia	0.94	0.93
Belgium	0.57	0.50
Canada	1.27	0.64
France	0.54	0.73
Germany	0.90	1.16
Italy	0.69	0.46
Japan	0.60	0.43
Netherlands	0.44	0.72
Switzerland	0.71	1.16
Sweden	0.95	0.95
UK	0.65	0.61
US[b]	1.23	1.02
MEDIAN	0.71	0.73

Notes: Based on quarterly time-series, $1980:1-2000:1$. Data sources are listed at the end of the document.
[a]Linear trend subtracted from logged time series. HP filter uses smoothing parameter 1600.
[b]For the entire postwar period ($1959:3-2004:2$) this ratio in U.S. is 0.88.

7 National Accounting in the Model

In the data, we measure output by GDP in constant prices, consumption by the sum of private consumption expenditures and government final consumption expenditures in constant prices, investment by gross-fixed capital formation in constant prices, and employment by aggregate of civil employment in the case of the rest of the world, and by the total aggregate hours worked in the case of the US. We map this measurement methodology onto our model as follows. The GDP in constant prices is defined by

$$P_{d,0}d_t + P_{f,0}f_t + (x_0 p^*_{d,0} d^*_t - p_{f,0} f_t), \tag{199}$$

consumption and investment in constant prices are defined by[35]

$$(P_{d,0}d_t + P_{f,0}f_t)\frac{c_t}{G(d_t, f_t)}, \tag{200}$$

$$(P_{d,0}d_t + P_{f,0}f_t)\frac{i_t}{G(d_t, f_t)}, \tag{201}$$

and employment index is defined by n_t.

Investment in marketing does not show up explicitly in the expenditure side measurement of the GDP[36] in consistency with the methodology of national income accounting. In the national accounting system expenses on R&D, marketing, advertising are all treated as expenses on intermediate goods − see United Nations, System of National Accounts, 1993, Par. 1.49, 6.149, 6.163, 6.165, or refer to McGrattan & Prescott (2005).

[35] Consumption and investment in period zero prices are not equal to c and i. The reason is that by the Euler's Law and equilibrium price relations, we have $G(d_t, f_t) = P_{d,t}d_t + P_{f,t}f_t$, which fails for period zero prices used instead of $P_{d,t}$ and $P_{f,t}$, i.e. $P_{d,0}d_t + P_{f,0}f_t \neq P_{d,t}d_t + P_{f,t}f_t$. Quantitatively the difference is negligible.

[36] There is however a small ambiguity on whether to include marketing in the trade balance from services. The real GDP would then have to include an additional term $a_f x_0 v_0^*$. Quantitatively the impact of this change on the statistical properties of the real GDP is negligible—and so we ignored it.

8 Estimation of Productivity Shock Process

To construct the TFP residuals z from the data we follow a similar procedure to Heathcote & Perri (2002). To construct the time-series for physical capital from the time-series for gross-fixed capital formation, and the perpetual inventory method with exogenously assumed depreciation rate of $\delta = 0.025$. To measure labor input, in the case of the US we use total hours worked, and due to lack of such data, we use civil employment for the other countries. Given our quarterly dataset from 1980.1 to 2004.3 for the aggregate of main 15 European countries, Japan, Canada, Switzerland, and Australia, we construct the time-series of z from the following equation

$$\log(z) = \log(y) - 0.36\log(k) - 0.64\log(n), \tag{202}$$

where y denotes GDP in constant prices, and the coefficient 0.64 denotes the share of labor income in GDP—in consistency with the parameterization of our model, and the values estimated in the literature for the developed countries.

From the linearly detrended time-series of $log(z)$ and $log(z^*)$, we estimate the parameters of the shock process with an imposed symmetry restriction. Consequently, we obtain the value for ψ equal to 0.91, and calibration targets for international correlation of z's of 0.3, and standard deviation of 0.79%.

9 Data Sources

9.1 Disaggregated Data from Japan

The dataset has been compiled by Bank of Japan from monthly survey of producer/wholesale prices: Yen based price indices for exports (f.o.b.) and domestic prices (wholesale or corporate level prices that include only domestically-produced and domestically-used goods). The quarterly series are constructed from two separate monthly compilations by Bank of Japan: 1995-2002 and 2000-2005 by taking averages and connecting the series so that the averages during the overlapping year coincide. The analysis includes 31 heavily traded manufacturing commodity categories over the time period 1995-2005. Final series have been seasonally adjusted (using Demetra 2.0, tramo-seats method), and H-P-filtered with a smoothing parameter 1600.

Real price indices have been calculated using a more refined level of disaggregation—whenever possible each subordinate category of the basic category has been divided by the corresponding domestic price index for this category, and then aggregated back using the same weights to construct the real prices reported for basic categories only.

The basic categories we include in our analysis are: (1) Engines, (2) Pumps, (3) Agricultural tractors, (4) Bearings, (5) Printing machines, (6) Copying machines, (7) Metal valves, (8) Computers, (9) Computer external memory, (10) Computer input-output devices, (11) Wire communications equipment (fax machines and telephones), (12) Radio communications equipment, (13) Color televisions, (14) Video recording and/or reproducing apparatus, (15) Home audio equipment, (16) Car audio equipment, (17) Household electrical equipment, (18) Passive components (electronic components), (19) Connecting components (electronic components), (20) Electron tubes, (21) Semiconductors, (22) Integrated circuits, (23) Electrical measuring instruments,(24) Silicon wafers, (25) Small passenger cars,(26) Other passenger cars, (27) Bicycles, (28) Photographic cameras, (29) Spectacle frames & opthalmic lenses, (30) Medical equipment & systems, (31) Watches & clocks [37].

[37] For further details about the source data, refer to the publication of Bank of Japan: "Explanation of the 1995 Base Wholesale Price Index (Revised Version)", Research and Statistics Department, Bank of Japan, May 2001.

9.2 Aggregate Data

OECD Main Economic Indicators, SourceOECD.org, International Financial Statistics by IMF (2005), OECD Main Indicators Printed Edition and SourceOECD.org (housing-services and all-items CPI series). Countries included as rest-of-the-world are: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden and the United Kingdom, Switzerland, Canada, Australia, Japan. Data for the U.S. hours worked come from the Current Population Survey by the Bureau of Labor Statistics, and has been compiled by Prescott, Ueberfeldt & Cociuba (2008). We thank Simona Cociuba and Ellen McGrattan for this dataset.

10 Supplement:

10.1 Additional Impulse Responses

Here we include additional set of impulse responses comparing more comprehensively the dynamics of quantities and prices between the benchmark model and the standard model (Figures 1-5). One can see that on the quantity side, the two models are very similar, yet, prices look very different. (By benchmark model, we mean here the model referred to in the paper as *fitted* θ case.)

10.2 Dynamic Response to Trade Liberalizations

Below, we include a figure with the dynamic response of the model to an unexpected symmetric permanent reduction in tariffs between countries. The full adjustment takes about 80 quarters, with half of the adjustment taking around 20 quarters. (By benchmark model, we mean the model referred to in the paper as fitted θ case.)

10.3 Market Expansion Friction in Light of Plant Level Evidence

There is substantial evidence of frictions of building demand in the data. The three most notable examples, discussed below, are Foster, Haltiwanger & Syverson (2009), Eaton, Eslava, Kugler & Tybout (2007) and Ruhl & Willis (2008).

Foster, Haltiwanger & Syverson (2009) study plant-specific demand in the US Census of Manufactures. In particular, they study the dependence of plant-specific idiosyncratic demand on plant age (which is not the same as firm age). They find that new plants have significantly lower idiosyncratic demand levels than incumbent plants, independent of whether these plants are part of older firms, or young firms. More specifically, Foster et al. find that entering plants of old firms have demand that is only 63% of old plants (15+ years) of old firms, young plants (5-9 years) have only 65%, while medium aged plants (10-14 years) in this group still lag old plants by 24%. For single unit plants, entrants sell 27% less than old plants and young plants (5-9 years) still lag behind by 16%. Foster et al. suggest that a demand-side explanation is likely for the slow growth of demand - a conclusion supported by evidence in this and their previous work (Foster, Haltiwanger & Syverson (AER 2008)) that entrants and incumbents have the same supply-side fundamentals. Foster et al.

(2009) summarize:

'*This similarity in supply-side fundamentals suggests that idiosyncratic demand factors might explain the well documented plant size differences. Our earlier work documents some evidence of this. There is a clear dichotomy between the age profiles of plants' physical productivity and demand-side fundamentals. While young plants' technical efficiency levels are similar to established plants' levels, they have much lower idiosyncratic demand measures. Moreover, these demand gaps close very slowly over time. Supply side fundamentals show no such slow convergence.*'

and in another part of the paper, they write:

'It therefore appears that new plants in small firms (by our crude size measure) face significantly lower idiosyncratic demand levels than do their new competitors in multi-plant firms. Nevertheless, both types of plants see the inertial convergence patterns observed in the broader sample, suggesting demand dynamics are at work in both cases.'

Foster et al. (2009) estimate a dynamic Euler equation that captures the contribution of past sales to future demand, using data on producers of the ready-mixed concrete. While we think their analysis is very promising in terms of providing a concrete estimate, their approach does not readily map onto our model. Moreover, its current narrow focus on a single product additionally limits the applicability of their estimates to our macro framework.

The descriptive statistics cited above suggest a very persistent difference between newcomers and incumbents (demand is at 65% of that of similar incumbents as long as 9 years after entry for new plants of old firms; 84% 9 years after entry for single unit plants compared to other single unit plants), which stands for evidence of a very strong friction of building demand. Our trade liberalization exercise suggests that our parameterized friction, as it is, implies that around 85% of the distance to the new steady state is covered in 10 years (see Figure **??**). This is broadly consistent with Foster et al. descriptive evidence, even implying slightly less sluggishness in our model—hence, if anything, we are erring on the side of choosing a friction that is too low.

10.4 Additional Sensitivity Analysis

Tables 6 and 7 report results from variants of our benchmark economy (fitted θ case) with lower and higher elasticity ($\gamma = 1.5$ and $\gamma = 16$, called *Low Elasticity* and *High Elasticity* in the table; we recalibrate to hit the same targets as in benchmark), as well as a variant of the economy in which we introduce technology spillovers (called *Productivity Spillover*; we do not recalibrate the productivity process, just introduce technology spillovers in the magnitude consistent with BKK, 0.088).

The elasticity parameter does not matter for anything except for the long-run elasticity implied by the model as long as it is high enough and finite. For the benchmark specification of our model (with complete markets and only productivity shocks), we can go with elasticity slightly lower than the reported 1.5 but as some point, lower and lower adjustment cost will be required to match the short run elasticity of 0.7. Hence, as we approach 0.7, we will have to shut down the adjustment friction, which will eventually imply counterfactual predictions for prices, just like in BKK.

The spillover parameter has a very similar effect on quantities of our model as it has on the standard model—and has no qualitatively important effect on prices. On the quantity side, the statistics deteriorate. However, it is a fairly well established fact that the spillover parameters in the data are typically not significantly different from zero, and in fact, most researchers use no spillovers (see for example the estimates by Heathcote & Perri (2004)). Our estimates confirm this finding, and we have not restricted the spillover parameters but first estimated that they are not significantly different from zero, and then re-estimated the process under such assumption.

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Table 6: International Prices: Comovement and Relative Volatility[a]

Statistic	Benchmark[b]	Switched Marketing	Productivity Spillover	High Elasticity[e] γ	Low Elasticity[f] γ
A. Correlation					
p_x, p_m	0.98	0.98	0.98	0.98	0.92
p_x, x	0.98	0.98	0.98	0.98	0.96
p_m, x	0.99	0.99	0.99	0.99	0.99
p, x	0.95	0.94	0.95	0.95	0.84
B. Volatility relative to[d] x					
p_x	0.37	0.37	0.38	0.37	0.37
p_m	0.64	0.64	0.63	0.64	0.65
p (no fuels[c])	0.28	0.28	0.27	0.29	0.34
p_x/p_d [e]	0.44	0.44	0.44	0.44	0.45
p_d	0.10	0.10	0.11	0.10	0.09
C. Standard deviation of x					
	0.43	0.41	0.37	0.42	0.47
D. Correlation of c/c^* *with* x					
	0.92	0.91	0.89	0.92	0.93
E. Price elasticity of trade					
Long-run	7.90	7.90	7.90	16.00	1.50
Short-run	0.70	0.70	0.64	0.70	0.70

[a]Statistics based on logged and Hodrick-Prescott filtered time series with smoothing parameter $\lambda = 1600$.
[b]US data for the period 1980:1-2004:1.
[c]Calculated using the actual national accounting formulas; see technical appendix for further details.
[d]Ratio of corresponding standard deviation to the standard deviation of GDP.
[e]Parameters as in benchmark except for $\gamma = 16$ and $\psi = 18.0$.
[f]Parameters as in benchmark except for $\gamma = 1.5$, $\theta = 0.39$ and $\psi = 4.7$.

Table 7: Quantities - Comovement and Relative Volatility[a]

Statistic	Benchmark[b]	Switched Marketing	Productivity Spillover	High Elasticity[e] γ	Low Elasticity[f] γ
A. Correlation					
domestic with foreign					
Measured TFP[c]	0.30	0.30	0.48	0.30	0.30
GDP	0.36	0.35	0.48	0.36	0.37
Consumption	0.26	0.29	0.49	0.27	0.21
Employment	0.32	0.30	0.29	0.31	0.34
Investment	0.05	0.05	0.18	0.05	0.08
GDP with					
Consumption	0.93	0.93	0.94	0.93	0.93
Employment	0.79	0.80	0.80	0.79	0.78
Investment	0.83	0.83	0.84	0.83	0.84
Net exports	-0.54	-0.55	-0.49	-0.54	-0.54
Terms of trade with					
Net exports	-0.88	-0.89	-0.88	-0.91	-0.44
B. Volatility					
relative to GDP[d]					
Consumption	0.32	0.31	0.33	0.31	0.33
Investment	3.60	3.60	3.47	3.60	3.56
Employment	0.71	0.71	0.68	0.72	0.72
Net exports	0.20	0.19	0.17	0.20	0.19

[a]Statistics based on logged and Hodrick-Prescott filtered time series with smoothing parameter $\lambda = 1600$.
[b]US data for the period 1980:1-2004:1.
[c]Calculated using the actual national accounting formulas; see technical appendix for further details.
[d]Ratio of corresponding standard deviation to the standard deviation of GDP.
[e]Parameters as in benchmark except for $\gamma = 16$ and $\psi = 18.0$.
[f]Parameters as in benchmark except for $\gamma = 1.5$, $\theta = 0.39$ and $\psi = 4.7$.

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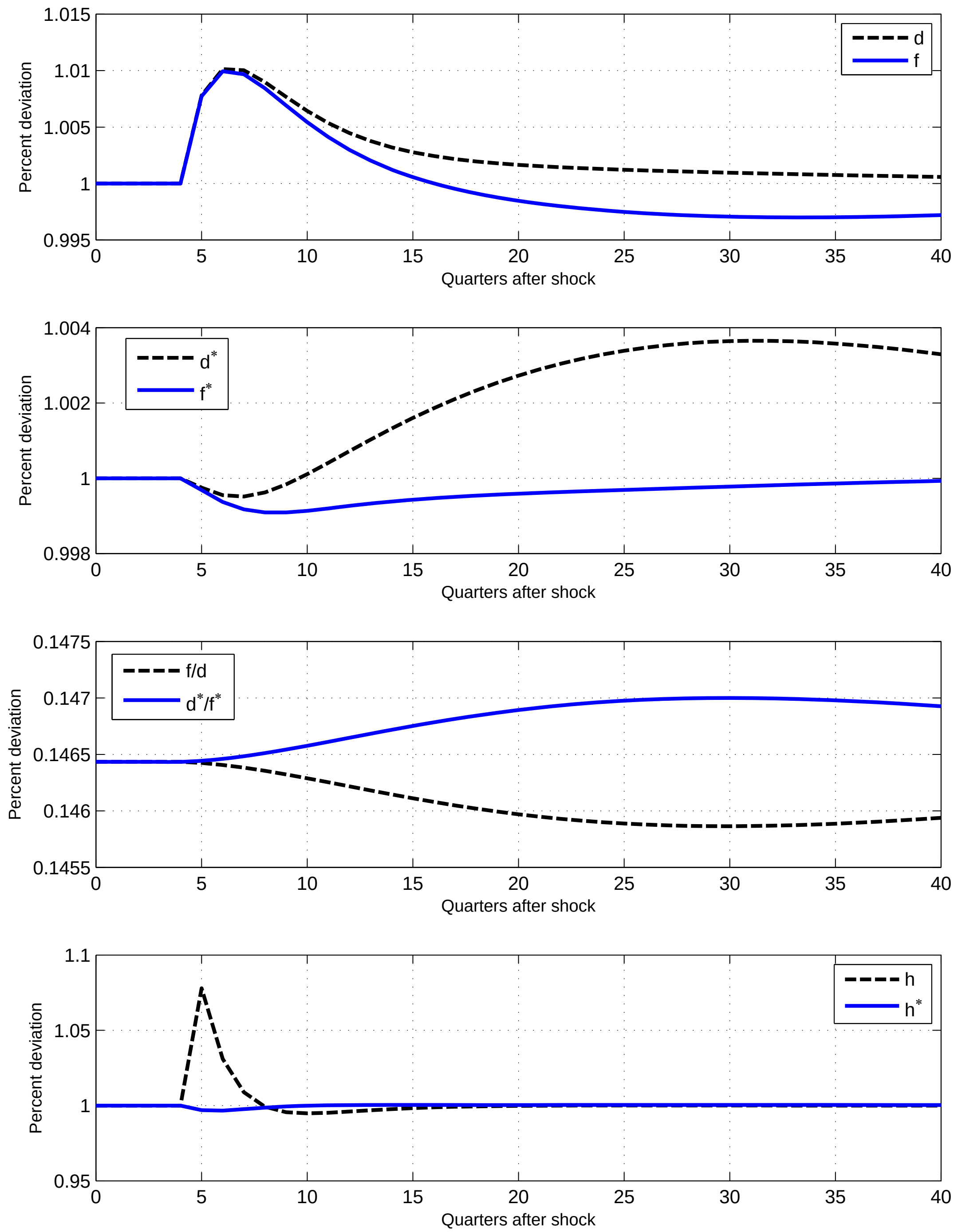

Figure 1: Response of the Benchmark Economy to a 1% Positive Productivity Shock.

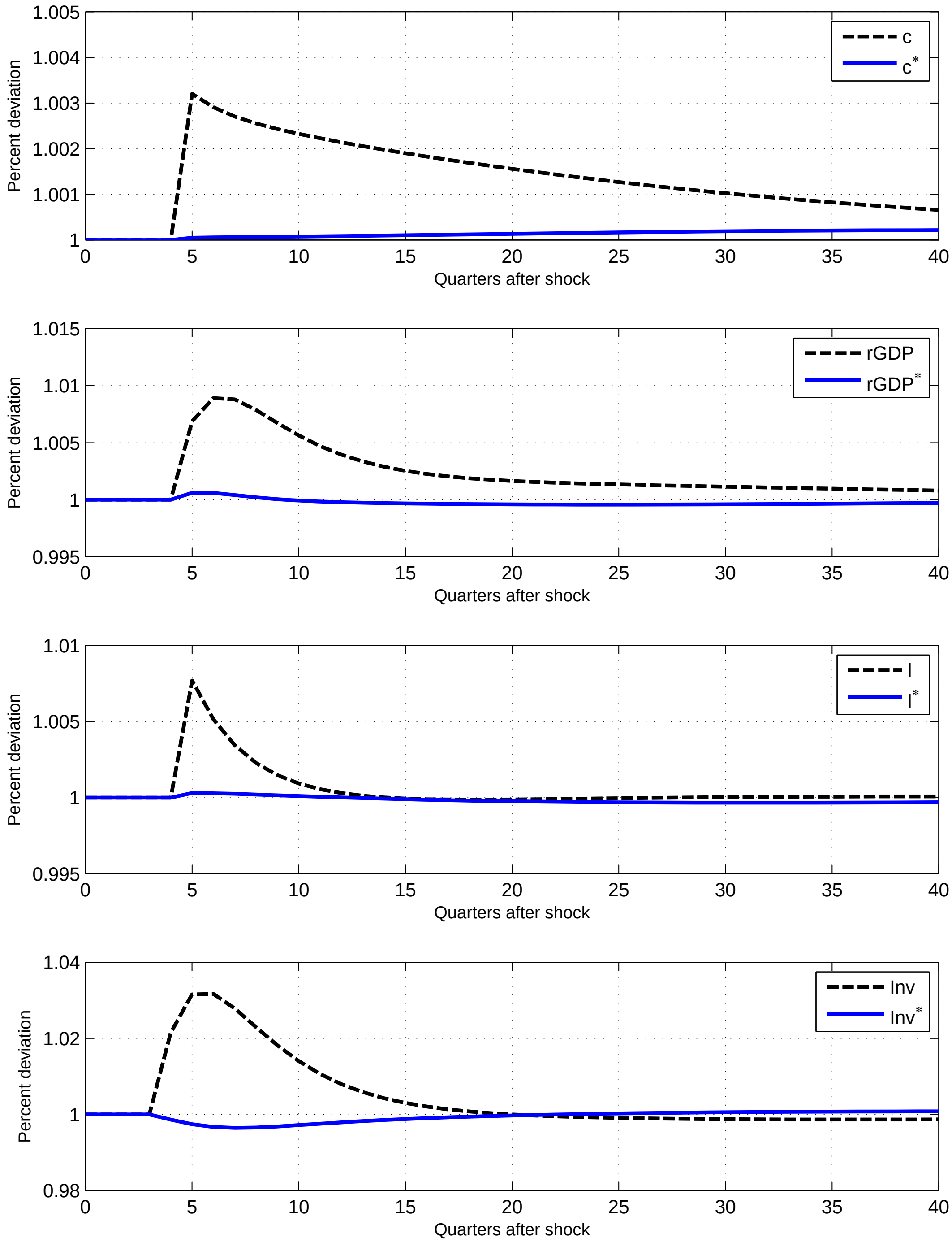

Figure 2: Response of the Benchmark Economy to a 1% Positive Productivity Shock.

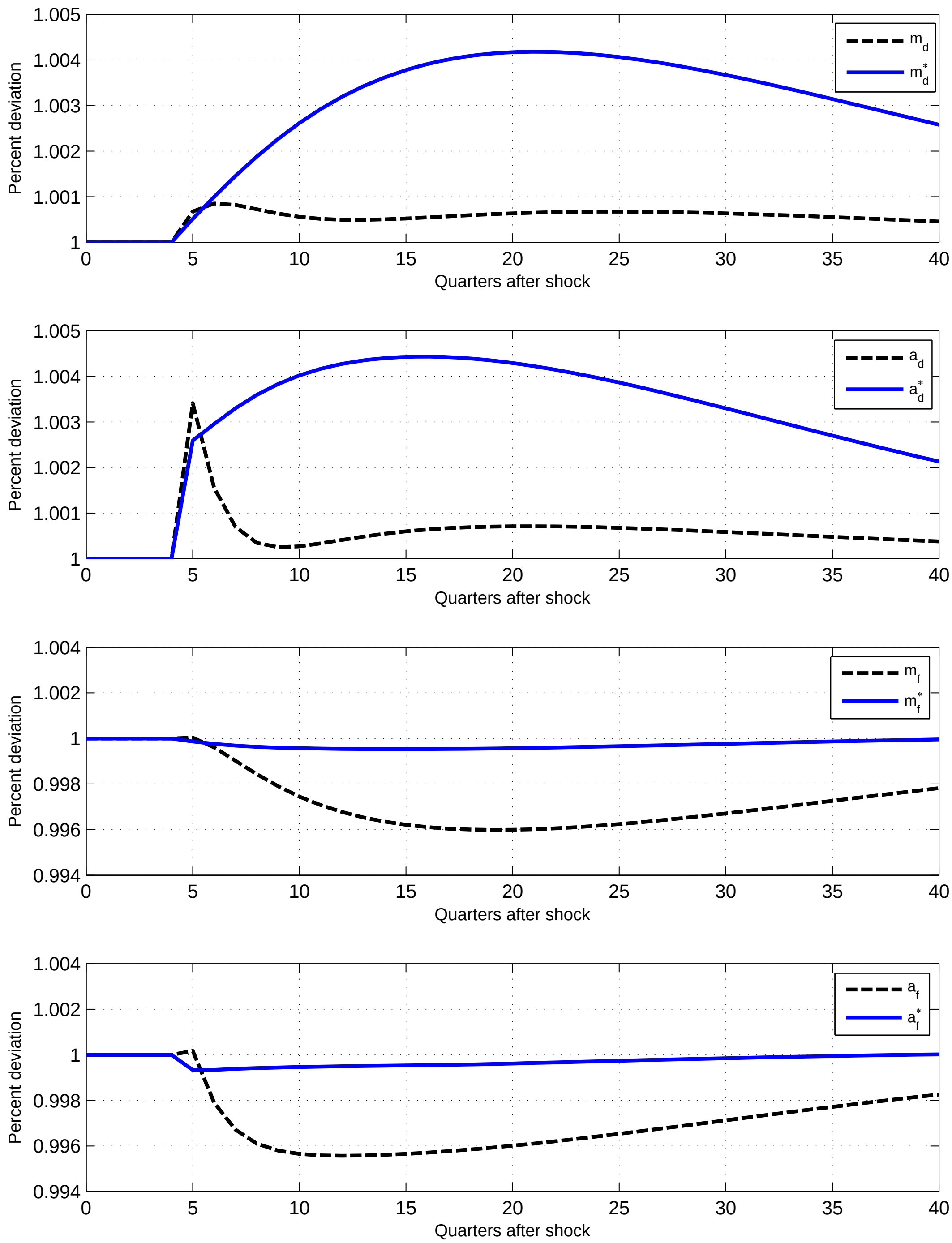

Figure 3: Response of the Benchmark Economy to a 1% Positive Productivity Shock.

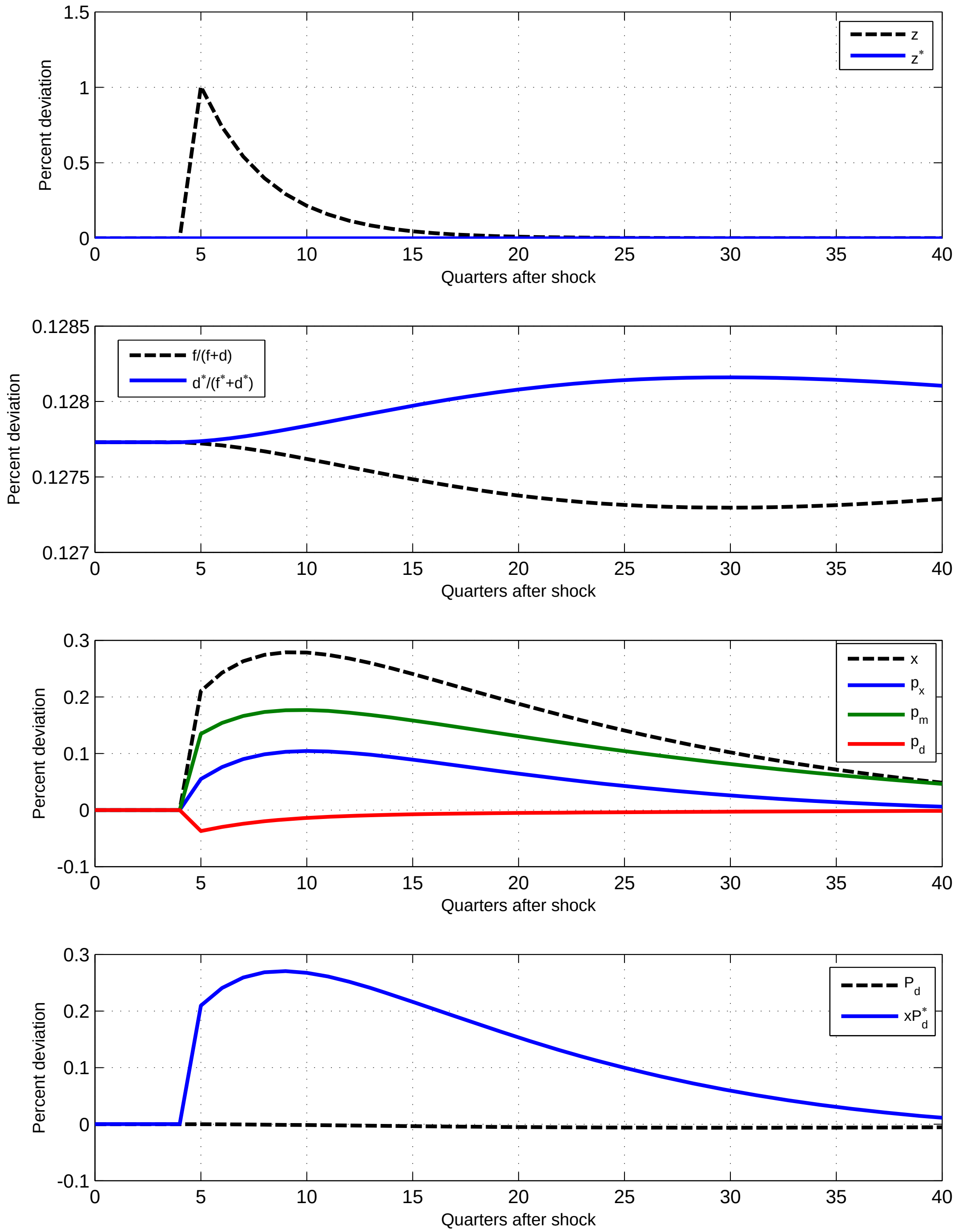

Figure 4: Response of the Benchmark Economy to a 1% Positive Productivity Shock.

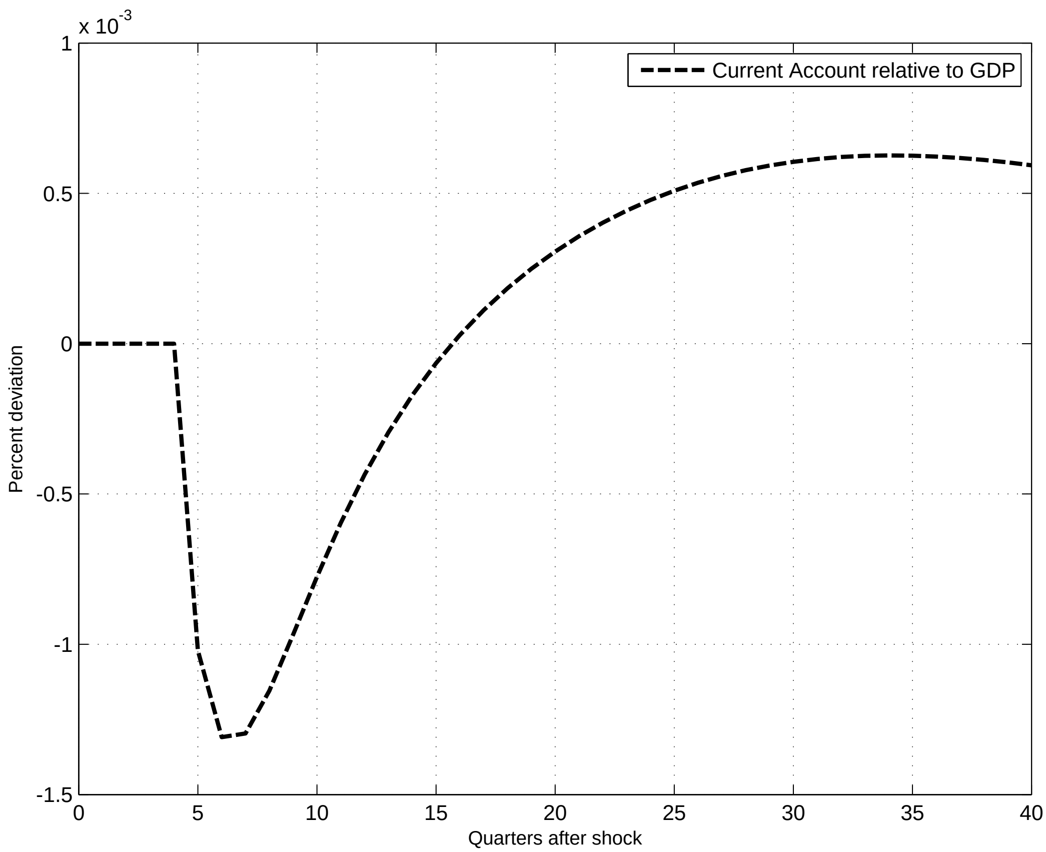

Figure 5: Response of the Benchmark Economy to a 1% Positive Productivity Shock.

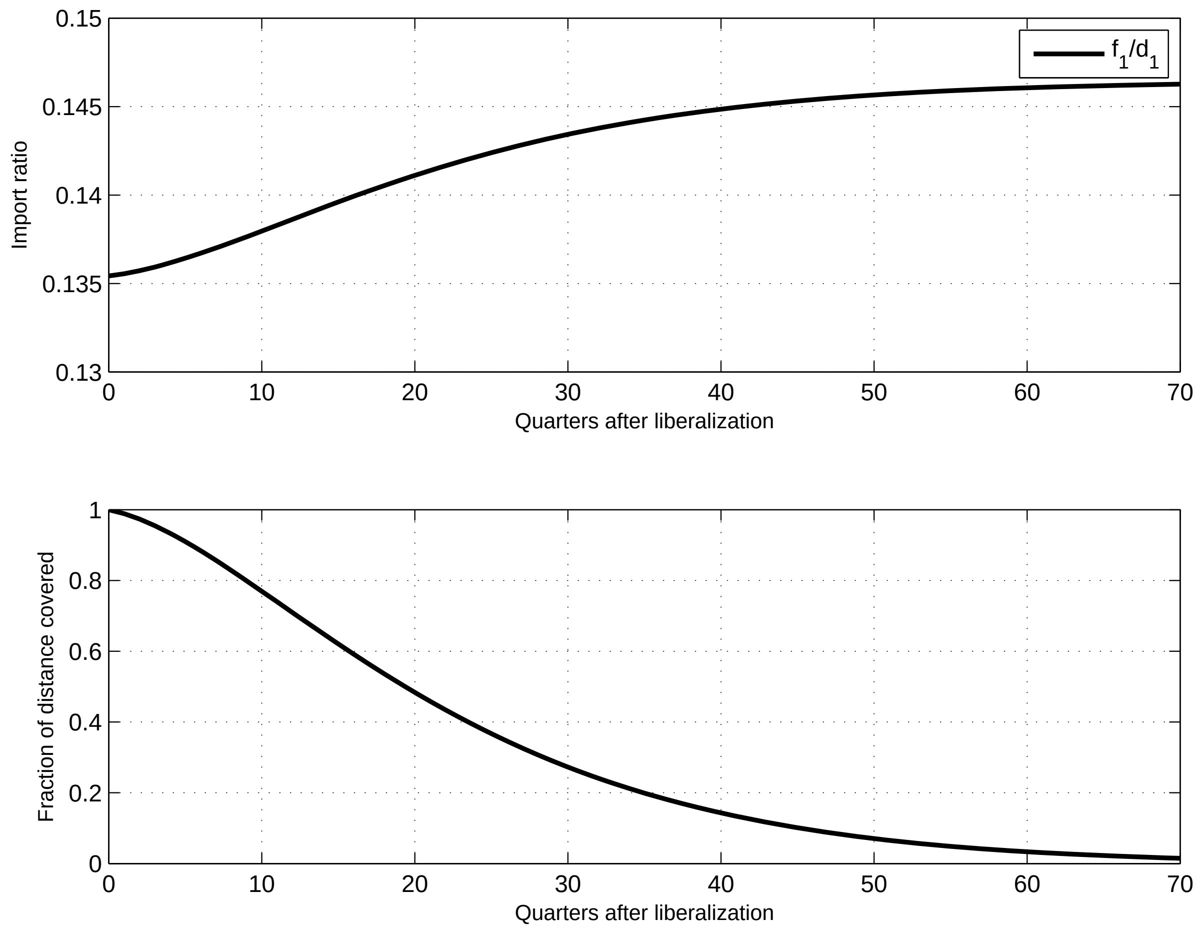

Figure 6: Theoretical Economy After an Unexpected Removal of a 1% Symmetric Tariff.