

# Appendix for “Growing Like China”<sup>1</sup>

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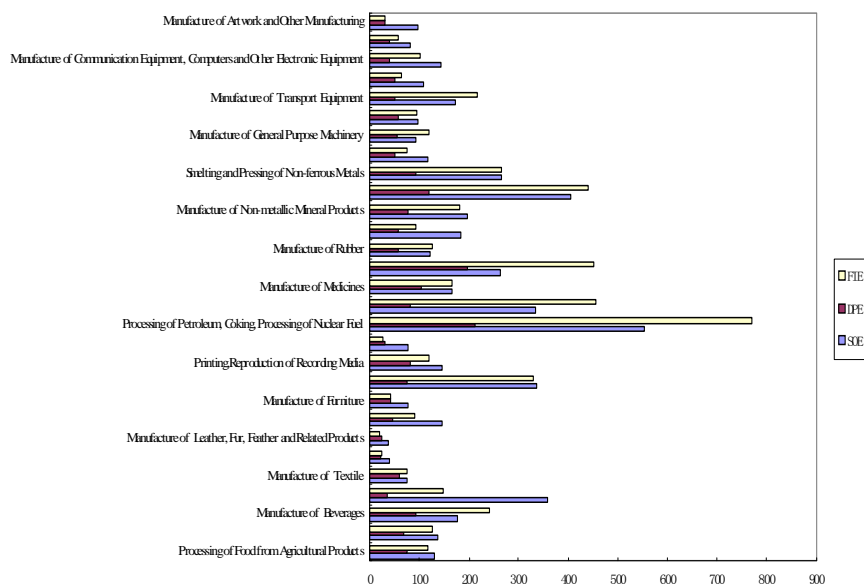
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<sup>1</sup>Equations, sections, propositions, etc. are numbered as in Song, Storesletten, and Zilibotti (“Growing Like China”, forthcoming in *American Economic Review*).

# 1 Capital-output and capital-labor ratio by ownership

Figure A1 reports capital-output and capital-labor ratios by ownership structure within three-digit manufacturing industries.



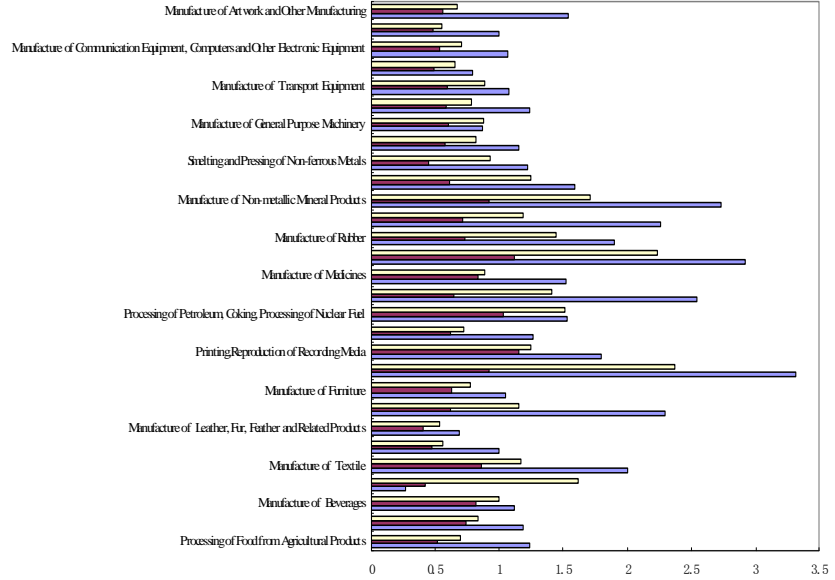


Figure A1: Capital-Labor Ratios (upper panel, thousand yuans per worker) and Capital-Output Ratios by Ownership and Sector in Manufacturing in 2006 (yellow=FIE, red=DPE, blue=SOE).

## 2 Proofs of Lemmas and of Proposition 2

**Proof.** [Proof of Lemma 1] That the capital-output ratio is higher in F firms follows immediately from the fact that  $\kappa_E < \kappa_F$  (shown in the text), since  $k_E/y_E = \kappa_E^{1-\alpha} < \kappa_F^{1-\alpha} = k_F/y_F$ . Similarly, that the capital-labor ratio is higher in F firms follows from observing that

$$\frac{k_F/n_F}{k_E/n_E} = \frac{\kappa_F/A}{\kappa_E/\chi A} = \left(\frac{\chi}{\chi}\right)^{\frac{1-\alpha}{\alpha}} > 1$$

where the inequality again follows from Assumption 1. ■

**Proof.** [Proof of Lemma 2] Due to constant-return-to-scale, aggregation holds, thus we can replace individual-firm variables (lower case) by aggregate variables (upper case). Since  $\kappa_E \equiv K_{Et}/(\chi A_t N_{Et})$  is constant and  $N_{Ft} = N_t - N_{Et}$ , then

$$N_{Et} = \frac{K_{Et}}{\chi A_t \kappa_E}, \quad N_{Ft} = N_t - \frac{K_{Et}}{\chi A_t \kappa_E}, \quad (23)$$

where  $\kappa_E$  is given by (9).

The next-period capital is given by  $k_{t+1}^E = s_t^E + l_t^E = R^l / (R^l - \eta\rho_t^E) s_t^E = R^l / (R^l - \eta\rho_t^E) \zeta^E m_t$ . Using (4), and aggregating over all entrepreneurs yields:

$$K_{Et+1} = R^l / (R^l - \eta\rho_t^E) \zeta^E \psi \kappa_E^\alpha A_t N_{Et}. \quad (24)$$

Dividing both sides of (24) by  $K_{Et}$ , and substituting  $\kappa_E$  by its equilibrium expression, we obtain (10). That  $N_{Et+1}/N_{Et} = (K_{Et+1}/K_{Et}) / (1+z)$  follows from (23).

Recall that the condition  $\nu_E > \nu$  is equivalent to

$$\frac{R^l}{R^l - \eta\rho_E} \left( 1 + \beta^{-\theta} \left( \frac{(1-\eta)\rho_E R^l}{R^l - \eta\rho_E} \right)^{1-\theta} \right)^{-1} \frac{\psi}{(1-\psi)} \frac{\rho_E}{\alpha} > (1+\nu)(1+z). \quad (25)$$

Using the fact that,

$$\frac{R^l - \eta\rho_E}{R^l \rho_E} = \frac{1}{\rho_E} - \frac{\eta}{R^l} = \frac{1}{R^l} \left( (1-\psi)^{-\frac{1}{\alpha}} \chi^{-\frac{1-\alpha}{\alpha}} - \eta \right).$$

and rearranging terms allows us to rewrite (25) as

$$\begin{aligned} \frac{\psi}{(1-\psi)} \frac{1}{\alpha(1+\nu)(1+z)} &> \frac{1}{R^l} \left( (1-\psi)^{-\frac{1}{\alpha}} \chi^{-\frac{1-\alpha}{\alpha}} - \eta \right) \\ &+ \beta^{-\theta} (1-\eta)^{1-\theta} \left( \frac{1}{R^l} \left( (1-\psi)^{-\frac{1}{\alpha}} \chi^{-\frac{1-\alpha}{\alpha}} - \eta \right) \right)^\theta. \end{aligned} \quad (26)$$

The right-hand side of equation (26) is monotonically decreasing in  $\chi$ , while the left-hand side is constant. Moreover, since the right-hand side tends to  $\infty$  (0) as  $\chi \rightarrow 0$  ( $\infty$ ), there exists a unique  $\hat{\chi}$  such that

$$\begin{aligned} \frac{\psi}{(1-\psi)} \frac{1}{\alpha(1+\nu)(1+z)} &= \frac{1}{R^l} \left( (1-\psi)^{-\frac{1}{\alpha}} \hat{\chi}^{-\frac{1-\alpha}{\alpha}} - \eta \right) \\ &+ \beta^{-\theta} (1-\eta)^{1-\theta} \left( \frac{1}{R^l} \left( (1-\psi)^{-\frac{1}{\alpha}} \hat{\chi}^{-\frac{1-\alpha}{\alpha}} - \eta \right) \right)^\theta. \end{aligned}$$

Therefore, the condition  $\nu_E > \nu$  will be satisfied when  $\chi > \hat{\chi}$ .

The following results are immediate.

1. The right-hand side of (26) is decreasing in  $\beta$ ,  $\eta$  and  $R^l$  so this inequality must hold for sufficiently large  $\beta$ ,  $\eta$  and  $R^l$ .

2. The left-hand side of equation (26) is decreasing in  $\nu$  and  $z$ . Thus, the condition  $\nu_E > \nu$  is satisfied for sufficiently small  $\nu$  and  $z$ .

■

**Proof.** [Proof of Lemma 3] Using equation (23), and recalling that  $\kappa_E$  and  $\kappa_F$  are constant, we can rewrite (12) as:

$$\begin{aligned}
B_t &= \left( \zeta^W w_{t-1} N_{t-1} - K_{Ft} - \frac{\eta \rho_E}{R^l} K_{Et} \right) \\
&= \left( \zeta^W \frac{w_{t-1}}{A_t} \frac{N_{t-1}}{N_t} - \kappa_F \frac{N_{Ft}}{N_t} - \frac{\eta \rho_E}{R^l} \kappa_E \frac{\chi N_{Et}}{N_t} \right) A_t N_t \\
&= \left( \zeta^W (1 - \alpha) \kappa_F^\alpha \frac{A_{t-1}}{A_t} \frac{N_{t-1}}{N_t} - \kappa_F \left( 1 - \frac{N_{Et}}{N_t} \right) - \frac{\eta \rho_E}{R^l} \kappa_E \frac{\chi N_{Et}}{N_t} \right) A_t N_t \\
&= \left( \zeta^W \frac{(1 - \alpha) \kappa_F^{\alpha-1}}{(1 + z)(1 + \nu)} - 1 + (1 - \eta) \frac{N_{Et}}{N_t} \right) \kappa_F A_t N_t
\end{aligned}$$

which proves the Lemma. ■

**Proof.** [Proof of Lemma 4] Part (i). We start by proving that  $\rho_E^l = (1 - \psi)^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R$ . To this aim, observe that, since (assuming that the incentive constraint is binding)  $m_t = \psi P_t^l y_{Et}^l$ , then

$$\Xi_t^l(k_{Et}^l) = \max_{n_{Et}} \left\{ (1 - \psi) P_t^l (k_{Et}^l)^\alpha (A_{Et} n_{Et})^{1-\alpha} - w_t n_{Et} \right\}$$

The first order condition yields:

$$n_{Et} = \frac{k_{Et}^l}{A_{Et}^l} \left( \frac{(1 - \psi) (1 - \alpha) P_t^l A_{Et}^l}{w_t} \right)^{\frac{1}{\alpha}}$$

Then, plugging (19) and (20) into the first order condition yields

$$n_{Et} = ((1 - \psi) \chi)^{\frac{1}{\alpha}} \left( \frac{P_t^l \alpha}{R} \right)^{-\frac{1}{1-\alpha}} \frac{k_{Et}^l}{A_{Et}^l} \quad (27)$$

Finally, plugging the optimal  $n_{Et}$  into the profit function, and simplifying term, yields the value of a E firm in the labor-intensive sector:

$$\begin{aligned}
\Xi_t(k_{Et}^l) &= (1 - \psi) k_{Et}^l \left( ((1 - \psi) \chi)^{\frac{1}{\alpha}} \right)^{1-\alpha} \left( \frac{\alpha}{R} \right)^{-1} \\
&\quad - (1 - \alpha) \chi^{-1} ((1 - \psi) \chi)^{\frac{1}{\alpha}} \left( \frac{\alpha}{R} \right)^{-1} k_{Et}^l \\
&= (1 - \psi)^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R k_{Et}^l \equiv \rho_E^l k_{Et}^l, \quad (28)
\end{aligned}$$

where  $\rho_E^l$  is identical to  $\rho_E$  in the one-sector model of section II of the paper (see equation (6)). This is the rate of return for E firms when F firms are active in the labor-intensive industry.

Next, we show that, when F firms are active in both industries, the return to investment in the capital-intensive sector for E firms,  $\rho_E^k$ , is lower than  $\rho_E^l$ . When F firms are active in the capital-intensive industry, the value of a E firm in the labor-intensive sector is

$$\begin{aligned}\Xi_t^k(k_{Et}^k) &= (1 - \psi) P_t^k (A_{Et}^k)^{1-\alpha} k_{Et}^k \\ &= (1 - \psi) \chi^{1-\alpha} R k_{Et}^k \equiv \rho_E^k k_{Et}^k\end{aligned}$$

where we have used equation (21) to eliminate  $P_t^k$ . Finally, Assumption 1 ensures that  $\rho_E^l > \rho_E^k$  (since  $(1 - \psi)^{\frac{1}{1-\alpha}} \chi > 1 \Leftrightarrow (1 - \psi)^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} > (1 - \psi) \chi^{1-\alpha}$ ). Thus, E firms will not invest in the capital-intensive sector. This completes the proof of part (i) of the Lemma.

Part (ii). We prove the argument by constructing a contradiction. Suppose that, when  $K_{Et}^l > 0$  and  $K_{Et}^k > 0$ ,  $K_{Ft}^l > 0$ . Then, (19) and (20) hold true, and  $\rho_E^l = (1 - \psi)^{\frac{1}{\alpha}} \chi^{\frac{1-\alpha}{\alpha}} R$  as shown in the first part of the proof, see (28). Moreover,  $\rho_E^l = \rho_E^k = (1 - \psi) \chi^{1-\alpha} R$ , since otherwise E firms would not invest in both industries. Solving for  $P_t^k$  yields

$$P_t^k = (1 - \psi)^{\frac{1-\alpha}{\alpha}} \chi^{\frac{1}{\alpha}} \frac{R}{(A_{Ft}^k)^{1-\alpha}} > \frac{R}{(A_{Ft}^k)^{1-\alpha}}$$

where the inequality follows from Assumption 1, and  $P_t^k = R / (A_{Ft}^k)^{1-\alpha}$  is the condition that guarantees that F firms make zero profits in the capital-intensive industries. Thus, the inequality establishes that F firms would be making positive profits in the capital-intensive sector, which is impossible in a competitive equilibrium. Thus,  $K_{Ft}^l = 0$  when E firms are active in both sectors. This concludes the proof of part (ii) of the Lemma. ■

**Proof.** [Proof of Proposition 2] The problem of the monopolist is:

$$\max_{K^k} (P^k - R) K^k,$$

subject to (16), and the equilibrium conditions, (17), (18) and (20). Replacing  $K^k$  with  $Y^k = (\varphi P^l / P^k)^\sigma Y^l$  (by equation 17), we can rewrite the problem as

$$\max_{P^k} \left( (P^k)^{1-\sigma} - R (P^k)^{-\sigma} \right) (P^l)^\sigma Y^l.$$

Here,  $Y^l$  is given by

$$Y^l = \left(\frac{P^l \alpha}{R}\right)^{-1} \psi (\chi (1 - \psi))^{\frac{1-\alpha}{\alpha}} K_E^l + \left(\frac{P^l \alpha}{R}\right)^{\frac{\alpha}{1-\alpha}} A_F N. \quad (29)$$

as proven in Section 5 below. The first-order condition yields:

$$0 = \left( (1 - \sigma) + \sigma R (P^k)^{-1} \right) + \left( 1 - R (P^k)^{-1} \right) \left( \sigma \frac{dP^l}{dP^k} \frac{P^k}{P^l} + \frac{dY^l}{dP^k} \frac{P^k}{Y^l} \right).$$

Now we compute the elasticities  $\frac{dP^l}{dP^k} \frac{P^k}{P^l}$  and  $\frac{dY^l}{dP^k} \frac{P^k}{Y^l}$ . Differentiating (18) w.r.t.  $P^k$  yields:

$$\frac{dP^l}{dP^k} \frac{P^k}{P^l} = \frac{-\varphi^\sigma (P^k)^{1-\sigma}}{1 - \varphi^\sigma (P^k)^{1-\sigma}}.$$

Differentiating (29) w.r.t.  $P^k$  yields:

$$\frac{dY^l}{dP^k} \frac{P^k}{Y^l} = - \left( 1 - \frac{1}{1 - \alpha} \frac{Y_F^l}{Y^l} \right) \frac{dP^l}{dP^k} \frac{P^k}{P^l}.$$

Therefore, the first-order condition can be rewritten as

$$\begin{aligned} & \left( (1 - \sigma) + \sigma R (P^k)^{-1} \right) \\ & = \left( 1 - R (P^k)^{-1} \right) \left( \sigma - \left( 1 - \frac{1}{1 - \alpha} \frac{Y_F^l}{Y^l} \right) \right) \frac{\varphi^\sigma (P^k)^{1-\sigma}}{1 - \varphi^\sigma (P^k)^{1-\sigma}}, \end{aligned}$$

which is expression (22) in the text. ■

### 3 Post-Transition Equilibrium (Section II.E)

In this section, we provide the details of the analysis in Section II.E of the paper. Under log utility, the equilibrium wage, rate of return on capital, output and foreign balance are given by:

$$\begin{aligned} w_t &= A_{Et} (1 - \alpha) (1 - \psi) (\kappa_{Et})^\alpha \\ \rho_t &= \rho_{E,t} = \alpha (1 - \psi) (\kappa_{Et})^{\alpha-1} \\ Y_t &= A_{Et} N_t (\kappa_{Et})^\alpha \\ \frac{B_t}{A_t N_t} &= \frac{\beta}{1 + \beta} w_t \frac{N_t}{A_t N_t} = \frac{\beta}{1 + \beta} \chi (1 - \alpha) (1 - \psi) (\kappa_{Et})^\alpha \end{aligned}$$

If

$$\alpha(1-\eta)(1-\psi) > \frac{\beta}{1+\beta} \frac{\psi R}{(1+z)(1+\nu)}, \quad (30)$$

then capital in E firms evolves according to (14) and eventually converges to a steady state, where

$$\kappa_E^* = \left( \frac{\beta}{1+\beta} \frac{\psi}{(1+z)(1+\nu)} + \frac{\eta\alpha(1-\psi)}{R} \right)^{\frac{1}{1-\alpha}}.$$

Here we let  $R^l = R$  in the steady state. The steady state rate of return to capital is thus equal to

$$\rho_E^* = \frac{\alpha(1-\psi)}{\frac{\beta}{1+\beta} \frac{\psi}{(1+z)(1+\nu)} + \frac{\eta\alpha(1-\psi)}{R}}.$$

Condition (30) ensures that  $\rho_E^* > R$ ; i.e., entrepreneurs never invest in bonds. Otherwise, entrepreneurs will eventually place part of their savings in bank deposits.



## 4 Analysis of Footnote 33 in Section III.C

In this section, we provide a complete formal argument of the discussion in footnote 33. Assume, for simplicity, log preferences and  $\eta = 0$ . Let  $\chi_i$  denote firm  $i$ 's productivity and  $K_i$  be the corresponding capital stock. Then, the rate of return to capital for firm  $i$  is

$$\rho_{iE} = (1 - \psi)^{\frac{1}{\alpha}} \chi_i^{\frac{1-\alpha}{\alpha}} R^l = \omega^\rho \cdot \chi_i^{\frac{1}{\alpha}},$$

where  $\omega^\rho$  is a unimportant constant. The law of motion of capital for firm  $i$  can be written as

$$\frac{K_{iEt+1}}{K_{iEt}} = \omega^K \chi_i^{\frac{1}{\alpha}}, \quad (31)$$

where  $\omega^K$  is also a unimportant constant. Denote  $\rho_{Et} = \sum \rho_{iE} K_{iEt} / K_{Et}$  the average rate of return of E firms. We now show that  $\rho_{Et}$  grows over time, since the growth rate of  $K_{it}$  is increasing in  $\chi_i$  as shown by (31). Specifically, using (31), the next-period average rate of return of E firms is equal to:

$$\rho_{Et+1} = \frac{\sum \omega^\rho \chi_i^{\frac{2}{\alpha}} K_{iEt}}{\sum \chi_i^{\frac{1}{\alpha}} K_{iEt}}.$$

Standard algebra establishes that

$$\frac{\sum \omega^\rho \chi_i^{\frac{2}{\alpha}} K_{iEt}}{\sum \chi_i^{\frac{1}{\alpha}} K_{iEt}} = \sum \frac{\omega^\rho \chi_i^{\frac{2}{\alpha}} K_{iEt}}{\sum \chi_i^{\frac{1}{\alpha}} K_{iEt}} \chi_i^{\frac{1}{\alpha}} > \sum \frac{\omega^\rho K_{iEt}}{\sum K_{iEt}} \chi_i^{\frac{1}{\alpha}} = \frac{\sum \omega^\rho \chi_i^{\frac{1}{\alpha}} K_{iEt}}{\sum K_{iEt}},$$

implying that  $\rho_{Et+1} > \rho_{Et}$ . Thus, the average rate of return of E firms increases over time in this case.

## 5 Equilibrium in Section IV.A

In this section, we provide a formal characterization of the equilibrium in the two-sector economy of Section IV.A. The equilibrium entails four stages, described in the text. For notational convenience, we let  $A_{Jt}^k = A_{Jt}^l = A_J$ .

**Proposition 3** *Stage 1 is defined as*

$$\frac{K_{Et}}{A_EN} < ((1 - \psi)\chi)^{-\frac{1}{\alpha}} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}}, \quad (32)$$

where

$$P_t^l = \left( 1 - \varphi^\sigma (P_t^k)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad (33)$$

$$P_t^k = \frac{R}{A_F^{1-\alpha}}. \quad (34)$$

In the first stage, both of the  $E$  and  $F$  firms are active in the labor-intensive industry while only the  $F$  firms produce capital-intensive goods. Specifically, prices of labor- and capital-intensive goods are determined by (33) and (34). Labor, capital and output in the labor- and capital-intensive industries are such that

$$N_{Ft}^l = N - N_{Et}^l, \quad N_{Et}^l = ((1 - \psi)\chi)^{\frac{1}{\alpha}} \left( \frac{P_t^l \alpha}{R} \right)^{-\frac{1}{1-\alpha}} \frac{K_{Et}^l}{A_E}, \quad (35)$$

$$K_{Ft}^l = \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} A_F N_{Ft}^l, \quad K_{Et}^l = K_{Et}, \quad (36)$$

$$Y_t^l = \left( \frac{P_t^l \alpha}{R} \right)^{-1} \psi (\chi(1 - \psi))^{\frac{1-\alpha}{\alpha}} K_{Et}^l + \left( \frac{P_t^l \alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} A_F N, \quad (37)$$

$$Y_t^k = (\varphi P_t^l / P_t^k)^\sigma Y_t^l, \quad K_{Ft}^k = \frac{Y_t^k}{A_F^{1-\alpha}}, \quad K_{Et}^k = 0, \quad (38)$$

respectively. Moreover, capital of  $E$  firms evolves according to

$$\frac{K_{Et+1}}{A_EN} = \frac{\beta\psi}{1 + \beta} P_t^l \left( \frac{K_{Et}}{A_EN} \right)^\alpha, \quad (39)$$

and the aggregate output is equal to

$$Y_t = (P_t^l)^\sigma Y_t^l, \quad (40)$$

**Proof.** When  $K_{Ft}^l > 0$  and  $K_{Ft}^k > 0$ , it is straightforward from Lemma 4 that  $K_{Et}^k = 0$ . (33) follows immediately from (18), whereas (34) follows from the zero-profit condition (20) for F firms in the capital-intensive industry. The first part of (36) comes from (20). The first part of (38) follows from (17). Using the condition that final-good firms make zero profits, together with, (17) and (18) leads to

$$\begin{aligned} Y_t &= P_t^l Y_t^l + P_t^k Y_t^k \\ &= \left( 1 + \varphi^\sigma \left( \frac{P_t^k}{P_t^l} \right)^{1-\sigma} \right) P_t^l Y_t^l = (P_t^l)^\sigma Y_t^l, \end{aligned}$$

which establishes (40). (35) follows immediately from (27). To derive (37), observe that

$$\begin{aligned} Y_t^l &= (\kappa_F^l)^\alpha \left( \frac{\psi}{1-\psi} \frac{N_{Et}}{N} + 1 \right) A_F N \\ &= \left( \frac{P_t^l \alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} \left( \psi \chi^{\frac{1}{\alpha}} (1-\psi)^{\frac{1-\alpha}{\alpha}} \left( \frac{P_t^l \alpha}{R} \right)^{-\frac{1}{1-\alpha}} \frac{K_{Et}^l}{A_E} + N \right) A_F \\ &= \frac{R}{P_t^l \alpha} \left( \psi \chi^{\frac{1}{\alpha}} (1-\psi)^{\frac{1-\alpha}{\alpha}} \frac{K_{Et}^l}{A_E} + N \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} \right) A_F \\ &= \left( \frac{P_t^l \alpha}{R} \right)^{-1} \psi (\chi (1-\psi))^{\frac{1-\alpha}{\alpha}} K_{Et}^l + \left( \frac{P_t^l \alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} A_F N. \end{aligned}$$

Finally, (32) ensures that  $K_{Ft}^l > 0$ , according to (27). The rest is immediate. ■

**Proposition 4** *Stage 2 is defined as*

$$((1-\psi)\chi)^{-\frac{1}{\alpha}} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} \leq \frac{K_{Et}}{A_E N} < \frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}}. \quad (41)$$

*In the second stage, F firms disappear in the labor-intensive industry. Specifically, prices of labor- and capital-intensive goods are determined by (33) and (34).  $N_{Ft}^l = 0$ ,  $N_{Et}^l = N$ , capital and output in the labor-intensive industries are such that*

$$K_{Ft}^l = 0, \quad K_{Et}^l = K_{Et}, \quad Y_t^l = (K_{Et}^l)^\alpha (A_E N)^{1-\alpha},$$

capital and output in the capital-intensive industry is identical to (38) in Stage 1. Moreover, capital in  $E$  firms also evolves according to (39) as in Stage 1.

**Proof.** The first inequality of (41) implies that  $K_{Ft}^l = 0$ . Now the wage rate is determined by the marginal product of labor in  $E$  firms.

$$w_t = P_t^l (1 - \alpha) (1 - \psi) A_E \left( \frac{K_{Et}^l}{A_E N} \right)^\alpha.$$

It is then easy to show that

$$\rho_{Et}^l = P_t^l \alpha (1 - \psi) \left( \frac{K_{Et}^l}{A_E N} \right)^{\alpha-1}.$$

Suppose that  $E$  firms are active in the capital-intensive industry. We have

$$\rho_{Et}^k = (1 - \psi) \chi^{1-\alpha} R.$$

However, the second inequality of (41) implies that  $\rho_{Et}^l > \rho_{Et}^k$ . Therefore,  $K_{Et}^k = 0$  in the second stage. Finally, (41) is non-empty by Assumption 1.

■

**Corollary 1** *If*

$$\chi^{1-\alpha} < \frac{\alpha(1+\beta)}{\beta\psi R}, \quad (42)$$

*then there are only two stages in the economy ( $E$  firms never produce capital-intensive goods).*

**Proof.** Define  $\tilde{P}^l \equiv \left( 1 - \varphi^\sigma \left( \frac{R}{(A_F)^{1-\alpha}} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$  as the constant price of labor-intensive goods in Stage 2. The law of motion (39) implies a upperbound of capital stock during the second stage of transition:

$$\frac{K_{Et}}{A_E N} \leq \left( \frac{\beta\psi\tilde{P}^l}{1+\beta} \right)^{\frac{1}{1-\alpha}}.$$

This gives the lowerbound of the rate of return:

$$\rho_{Et}^l > \tilde{P}^l \alpha (1 - \psi) \left( \frac{\beta\psi\tilde{P}^l}{1+\beta} \right)^{-1} = \frac{\alpha(1-\psi)(1+\beta)}{\beta\psi}.$$

Recall that  $\rho_{Et}^k = (1 - \psi) \chi^{1-\alpha} R$ . Therefore,  $\rho_{Et}^l > \rho_{Et}^k$  always holds under the assumption of (42). ■

**Proposition 5** *Stage 3 is defined as*

$$\frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} \leq \frac{K_{Et}}{A_E N} < \frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} + \frac{1}{A_E^{1-\alpha}} \left( \varphi \frac{P_t^l}{P_t^k} \right)^\sigma \left( \frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} \right)^\alpha. \quad (43)$$

In the third stage, the E firms start to produce capital-intensive goods. Specifically, prices of labor- and capital-intensive goods are determined by (33) and (34).  $N_{Ft}^l = 0$ ,  $N_{Et}^l = N$ , capital and output in the labor- and capital-intensive industries are such that

$$K_{Ft}^l = 0, K_{Et}^l = \frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} A_E N, Y_t^l = (K_{Et}^l)^\alpha (A_E N)^{1-\alpha}, \quad (44)$$

$$Y_t^k = \left( \varphi P_t^l / P_t^k \right)^\sigma Y_t^l, K_{Ft}^k = \frac{Y_t^k - A_E^{1-\alpha} K_{Et}^k}{A_F^{1-\alpha}}, K_{Et}^k = K_{Et} - K_{Et}^l, \quad (45)$$

respectively. Moreover, the total capital of E firms evolves according to the law of motion

$$\frac{K_{Et+1}}{A_E N} = \frac{\beta \psi}{1 + \beta} \left( P_t \left( \frac{K_{Et}^l}{A_E N} \right)^\alpha + P_t^k A_E^{1-\alpha} \left( \frac{K_{Et}}{A_E N} - \frac{K_{Et}^l}{A_E N} \right) \right). \quad (46)$$

**Proof.** Lemma 4 implies that  $K_{Ft}^l = 0$ .<sup>2</sup>  $K_{Et}^k > 0$  implies equalized rates of return across two industries.

$$\rho_{Et}^k = \rho_{Et}^l \Rightarrow (1 - \psi) \chi^{1-\alpha} R = P_t^l (1 - \psi) \alpha \left( \frac{K_{Et}^l}{A_E N} \right)^{\alpha-1} \Rightarrow$$

$$K_{Et}^l = \frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} A_E N.$$

Given total capital of E firms  $K_{Et}$ ,  $K_{Et} - K_{Et}^l$  will be allocated to the capital-intensive industry. Entrepreneurs' total income is equal to  $\psi \left( P_t^l (K_{Et}^l)^\alpha (A_E N)^{1-\alpha} + P_t^k A_E^{1-\alpha} K_{Et}^k \right)$ , which gives the law of motion of capital (46). Finally, we need  $Y_t^k > A_E^{1-\alpha} K_{Et}^k$  to ensure  $K_{Ft}^k > 0$ . This is given by the second inequality of (43). ■

<sup>2</sup>Alternatively,  $K_{Ft}^l = 0$  can be ensured by the first inequality of (43).

**Proposition 6** *Stage 4 is defined as*

$$\frac{K_{Et}}{A_EN} \geq \frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} + \frac{1}{A_E^{1-\alpha}} \left( \varphi \frac{P_t^l}{P_t^k} \right)^\sigma \left( \frac{1}{\chi} \left( \frac{P_t^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} \right)^\alpha.$$

*In the fourth stage, economic transition is complete in the sense that F firms vanish even in the capital-intensive industry. Specifically, prices of labor- and capital-intensive goods are determined by (33) and (47).*

$$P_t^k = \frac{R}{(1-\psi) A_E^{1-\alpha}}. \quad (47)$$

$N_{Ft}^l = 0$ ,  $N_{Et}^l = N$ , *capital and output in the labor- and capital-intensive industries are identical to (44) and (45), except that  $K_{Ft}^k = 0$ . The law of motion of capital in E firms also follows (46) in the third stage.*

The proof is immediate and is omitted.

Finally, we revisit the foreign balance. The balance sheets of the banks must take into account the investments of F firms in both industries:

$$K_{Ft+1}^k + K_{Ft+1}^l + B_t = \frac{\beta}{1+\beta} w_t N_t.$$

**Proposition 7** *In the first stage, the country's asset position in the international bond market increases if*

$$\frac{\alpha(1-\psi)}{\psi} > \varphi^\sigma (P^l/P^k)^{\sigma-1}, \quad (48)$$

where  $P^l$  and  $P^k$  follow (33) and (34), respectively.

**Proof.** Using (19) and (20), standard algebra shows that:

$$\begin{aligned} \frac{B_{t+1}}{A_F N} &= \frac{\beta}{1+\beta} w_t - \frac{K_{Ft+1}^l}{A_F N} - \frac{K_{Ft+1}^k}{A_F N} \\ &= \frac{\beta}{1+\beta} P^l (1-\alpha) A_F \left( \frac{P^l \alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} \\ &\quad - \left( \frac{P^l \alpha}{R} \right)^{\frac{1}{1-\alpha}} \left( 1 - \frac{((1-\psi)\chi)^{\frac{1}{\alpha}} \left( \frac{P^l \alpha}{R} \right)^{-\frac{1}{1-\alpha}} K_{Et+1}^l}{A_E N} \right) \\ &\quad - \frac{(\varphi P^l/P^k)^\sigma}{A_F^{1-\alpha}} \left( \left( \frac{P^l \alpha}{R} \right)^{-1} \psi (\chi(1-\psi))^{\frac{1-\alpha}{\alpha}} \frac{K_{Et+1}^l}{A_F N} + \left( \frac{P^l \alpha}{R} \right)^{\frac{\alpha}{1-\alpha}} \right). \end{aligned}$$

Since  $K_{Et+1}^l$  is increasing,  $B_{t+1}$  is an increasing sequence if (48) holds. ■

The main results of Proposition 1 therefore carry over to this extended model economy.